Vortex liquid in superconducting vacuum of QCD induced by strong magnetic field

V. V. Braguta, P. V. Buividovich, M. N. Chernodub, A. Yu. Kotov,
M. Müller-Preussker and M. I. Polikarpov

31^{st} International Symposium on Lattice Field Theory

Numerical part based on: arXiv:1104.3767, arXiv:1301.6590 + ongoing work

July 30, 2013

Conventional superconductivity

1. Zero electrical resistance
2. An enemy of magnetic field
   - Meissner effect (expulsion of a weak magnetic field from a superconductor)
   - Stronger magnetic field penetrates into superconductor in terms of Abrikosov vortices(type-II superconductor)
   - Strong enough magnetic field kills superconductivity

The claim:

In a background of the strong magnetic field the vacuum of QCD becomes a superconductor(due to the condensation of charged ρ-mesons).

M.Chernodub, arXiv:1008.1055, arXiv:1101.0117

Key players: ρ-mesons and vacuum

ρ-mesons:

- electrically charged($\rho^{\pm} : q = \pm e$) and neutral($\rho^{(0)} : q = 0$) particles
- vector particles($s = 1$)
- Quark content: $\rho^{+} : u\bar{d}$, $\rho^{-} : d\bar{u}$, $\rho^{(0)} : \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$
- mass: $m = 775.5$MeV
- lifetime: $c\tau = 1.35$fm

Vacuum:

- QCD+QED, zero temperature and density

Superconductivity of QCD in strong magnetic field

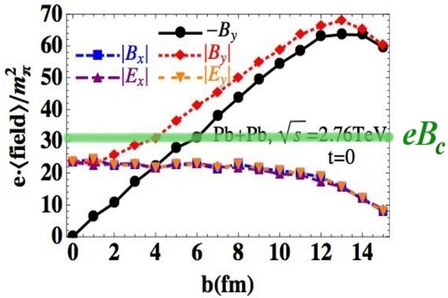
1. Spontaneously emerges above the critical value of magnetic field:

$$B_c \approx 10^{16} \text{ Tesla}$$
$$eB_c \approx m_\rho^2 \approx 0.6 \text{ GeV}^2$$

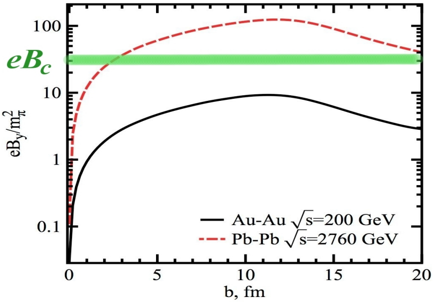
2. No Meissner effect(but vortices are formed)
3. Zero resistance along the the magnetic field
4. Insulator in other (perpendicular) directions

Strong magnetic fields

Strong magnetic fields ($eB_c = m_\rho^2 \approx 31 m_\pi^2 \approx 0.6 GeV^2$):



W. T. Deng and X. G. Huang, Phys. Rev. C85 (2012) 044907, arXiv:1201.5108



A. Bzdak and V. Skokov, Phys.Lett. B710 (2012) 171, arXiv:1111.1949

Approaches to the problem

1. General ideas;
2. Effective bosonic model (M. Chernodub, arXiv:1008.1055);
3. Effective Nambu-Jona-Lasinio fermionic model (M. Chernodub, arXiv:1101.0117);
4. Gauge/gravity duality (N. Callebaut, D. Dudal, H. Verschelde, arXiv:1105.2217; M. Ammon, J. Erdmenger, P. Kerner, M. Strydom , arXiv:1106.4551; ...)
5. Numerical calculations (ITEP Lattice Group, M. Müller-Preussker)
6. Ongoing discussions
   Y. Hidaka and A. Yamamoto, arXiv:1209.0007
   M. Chernodub, arXiv:1209.3587
   Chuan Li, Qing Wang, arXiv:1301.7009

Naive picture.

Energy of a ρ-meson in the magnetic field:

$$E^2 = m_\rho^2 + eB(2n + 1 - 2S_z) + p_z^2$$

$$\left.\begin{array}{ll} \text{Lowest Landau level:} & n = 0 \\ \text{Not moving along magnetic field:} & p_z = 0 \\ \text{Spin along magnetic field:} & S_z = 1 \end{array}\right\} \quad E^2 = m_\rho^2 - eB$$

If $eB > eB_c = m_{\rho^2} \approx 0.6 GeV^2 \Rightarrow E^2 < 0 \Rightarrow$ Condensation

Effective bosonic model

Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\rho^{\mu\nu}\rho^{\dagger}_{\mu\nu} + m_\rho^2\rho^\mu\rho^\dagger_\mu$$
$$-\frac{1}{4}\rho^{(0)}_{\mu\nu}\rho^{(0)\mu\nu} + \frac{m_\rho^2}{2}\rho^{(0)}_\mu\rho^{(0)\mu} + \frac{e}{2g_s}F^{\mu\nu}\rho^{(0)}_{\mu\nu}$$

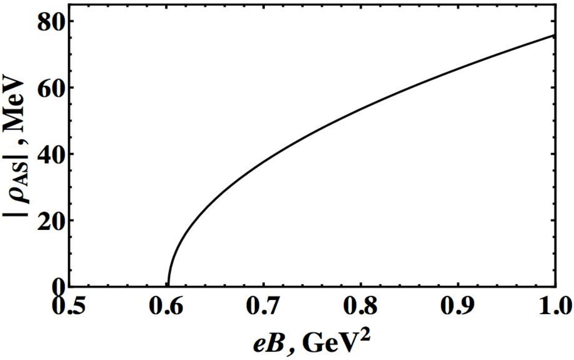
Here

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$
$$\rho^{(0)}_{\mu\nu} = \partial_\mu\rho^{(0)}_\nu - \partial_\nu\rho^{(0)}_\mu - ig_s(\rho^\dagger_\mu\rho_\nu - \rho_\mu\rho^\dagger_\nu)$$
$$\rho_{\mu\nu} = D_\mu\rho_\nu - D_\nu\rho_\mu$$
$$D_\mu = \partial_\mu + ig_s\rho^{(0)}_\mu - ieA_\mu$$

Absolute value of condensate

Condensate(absolute value):

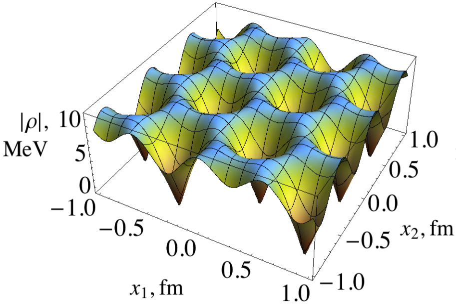
$$|\rho|_0 = \begin{cases} \sqrt{\frac{e(B_{ext}-B_c)}{2g_s^2}}, & \text{if } B_{ext} > B_c. \\ 0, & \text{if } B_{ext} < B_c \end{cases}$$



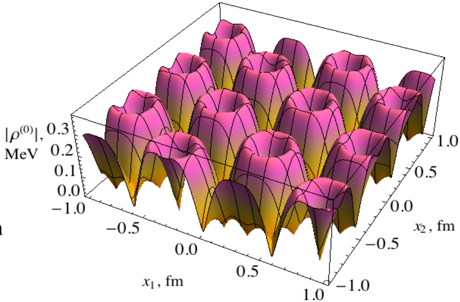
Structure of condesate

Effective bosonic model

$$B = 1.01 B_c$$



Charged ρ-mesons



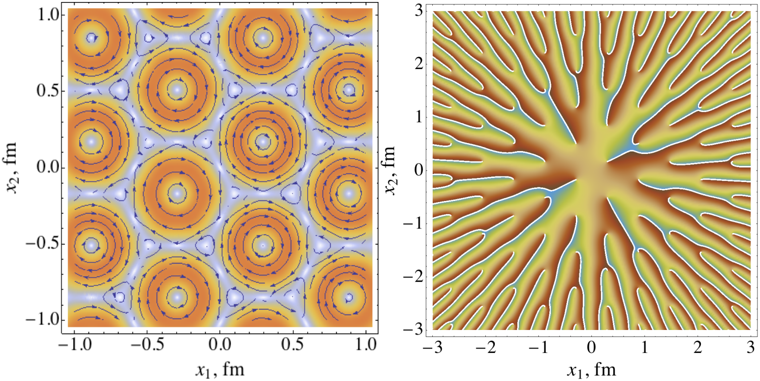
Neutral ρ-mesons

M. Chernodub, J. Doorsselaere, H. Verschelde, arXiv:1111.4401.

Similar results in holographic approach: Y.-Y. Bu, J. Erdmenger, J. P. Shock, M. Strydom, arXiv:1210.6669

Structure of condensate

Superconducting current and the phase of the condensate:



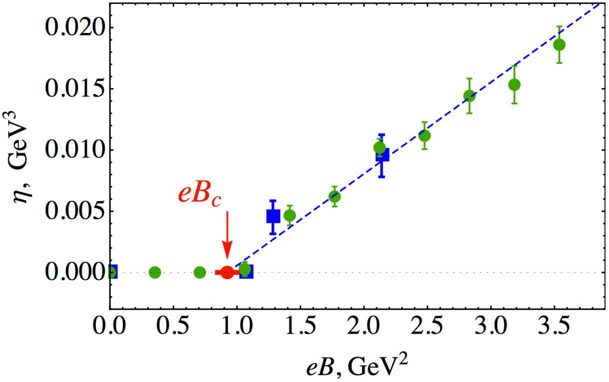
M. Chernodub, J. Doorsselaere, H. Verschelde, arXiv:1111.4401.

Numerical calculations in quenched QCD

- Two quark flavours: u, d
- Rho-meson operator: $\rho_\mu = \bar{u}\gamma_\mu d$
- Spin ± 1 along magnetic field: $\rho_\pm = \frac{1}{2}(\rho_1 \pm i\rho_2)$
- Equal time correlator: $G_\pm(z) = \langle \rho_\pm^\dagger(0)\rho_\pm(z)\rangle$
- Condensate: $\lim_{|z|\to\infty} G_+(z) = |\langle\rho\rangle|^2$
- Spin -1: $\lim_{|z|\to\infty} G_-(z) = 0$

Superconducting condensate

Quenched QCD with overlap fermions



Critical field:

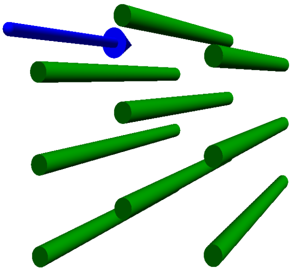
$$eB_c = 0.924(77)\, GeV^2$$
$$m_\rho^2 \sim 1.1\, GeV^2$$

Superconducting vortices.

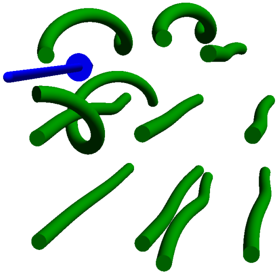
Observables:

- Rho-meson field: $\rho_\mu = \bar{u}\gamma_\mu d$ - cannot be computed in our approach!
- Correlator: $\rho_+(x) \to \phi(x) = \langle \rho_+^\dagger(0)\rho_+(x)\rangle_f$
- Energy density: $E(x) = \frac{|D_\mu \phi(x)|^2}{|\phi(x)|^2}$, $D_\mu = \partial_\mu - ieA_\mu$
- Electric current: $j_\mu(x) = \frac{\phi^* D_\mu \phi - \phi (D_\mu \phi)^*}{2i|\phi(x)|^2}$
- Vortex density: $v(x) = \text{sing} \arg \phi(x) = \frac{\epsilon^{ab}}{2\pi} \frac{\partial}{\partial x_a} \frac{\partial}{\partial x_b} \arg \phi(x)$

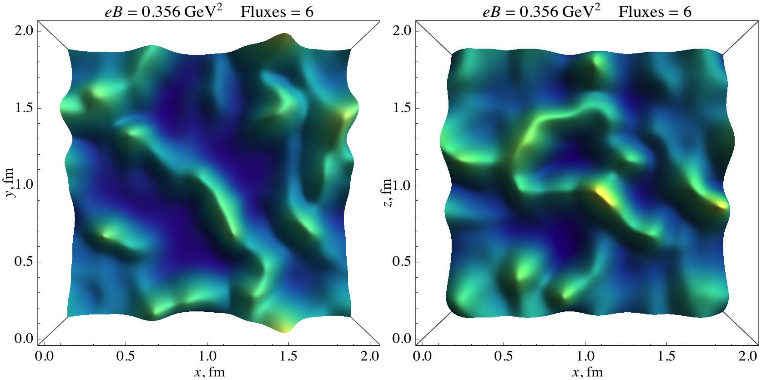
Expectations



Effective model



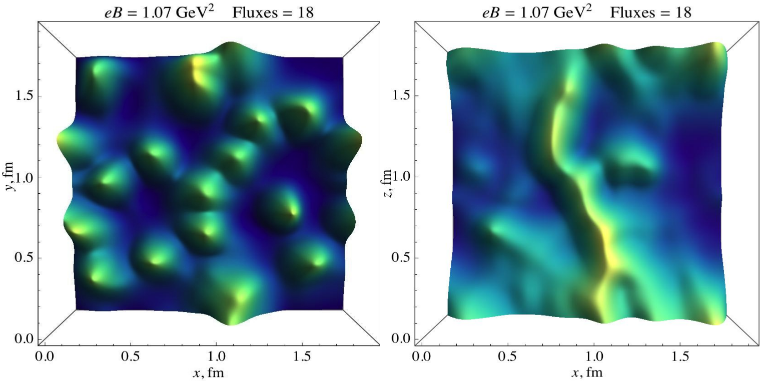
Quantum fluctuations
move and disturb vortices



$\perp B$ $\qquad\qquad\qquad$ $\| B$

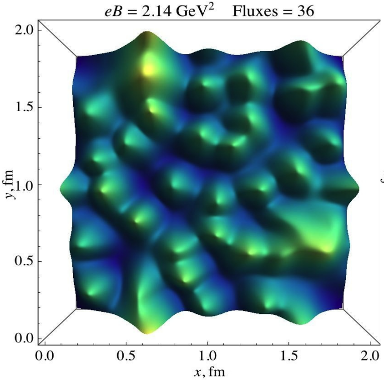
arXiv:1301.6590

Superconducting vortices. Energy density. $eB = 1.07 GeV^2$

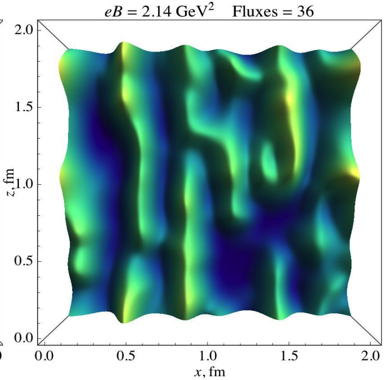


$\perp B$ $\qquad$ $\parallel B$

Superconducting vortices. Energy density. $eB = 2.14 GeV^2$

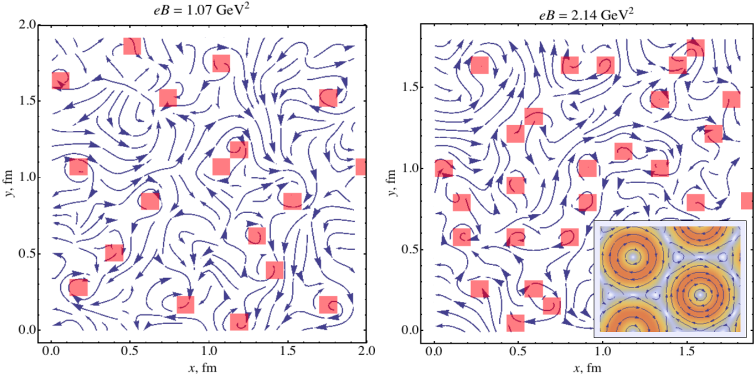


$\perp B$

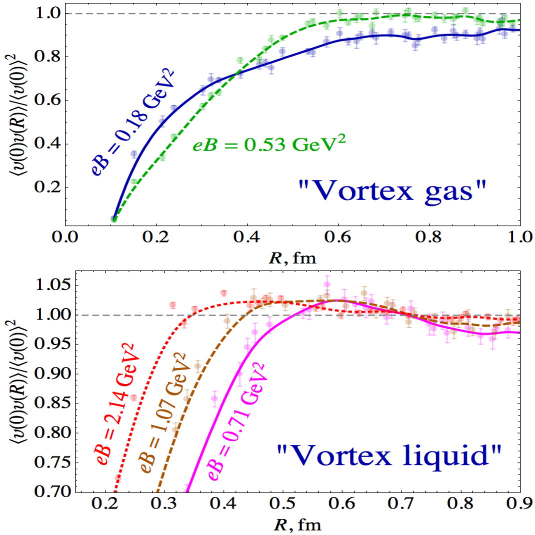


$\| B$

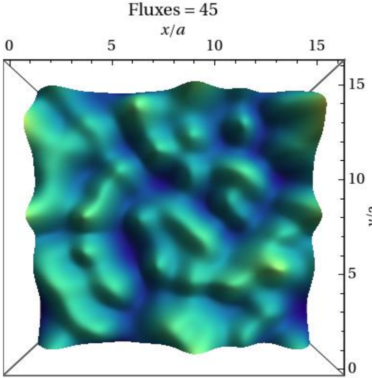
Electric current around vortices



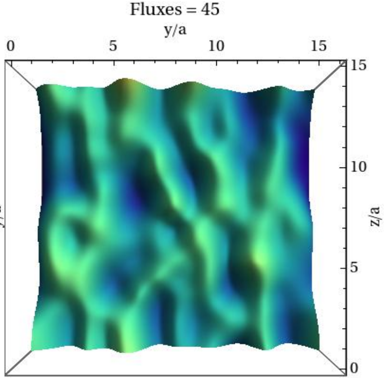
Vortex density-density correlators



First simulations with dynamical fermions



$\perp B$



$\parallel B$

$eB \sim 1.5 GeV^2$

Conclusions

- In a sufficiently strong magnetic field ρ-meson condensate is formed simultaneously.
- New type of topological defects, "ρ-vortices", emerge.
- Liquid of ρ-vortices is observed in quenched lattice calculations(cf. theory: trigonal lattice)
- Plan: simulations with dynamic fermions.