Finite size scaling and the effect of the gauge coupling in 12 flavor systems

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University of Colorado

in collaboration with
Anqi Cheng, Greg Petropoulos and David Schaich

Boulder nHYP-BSM project

We developed several methods that are effective in investigating conformal and near-conformal systems:

David Schaich (Monday, 15:20) :

N_f=8 USBSM project, uses many of our methods

Greg Petropoulos (Tuesday, 15:40) :

Improved lattice renormalization group techniques

Anqi Cheng (Wednesday, 11:40) :

Scale dependence of the anomalous mass dimension from Dirac eigenmodes

This talk: Finite size scaling in the presence of near-marginal gauge coupling

Finite size scaling

Consider a FP with one relevant operator

$m \approx 0$ with scaling dimension $y_m > 0$

and irrelevant operators

g_i with scaling dimensions $y_i < 0$.

Renormalization group arguments in volume L^3 predict

$$M_H L = f(L m^{1/y_m}, g_i m^{-y_i/y_m}) \quad \text{as} \quad m \approx 0$$

as $m \to 0, \quad L \to \infty: \quad g_i m^{-y_i/y_0} \to 0$

$$M_H L = f(x), \quad x = L m^{1/y_m}$$

– Every physical mass has its own scaling function

– The exponent y_m is unique

Finite size scaling with a **near-marginal operator**

Consider a FP with one relevant operator

$m \approx 0$ with scaling dimension $y_m > 0$

and irrelevant operators

g_i with scaling dimensions $y_i < 0$

g_0 (near) marginal, $y_0 \lesssim 0$

Renormalization group arguments in volume L^3 predict

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as $m \to 0, \quad L \to \infty: \quad g_i m^{-y_i/y_0} \to 0$

$$g_0 \to g_0 m^{\omega}, \quad \omega = -y_0 / y_m \gtrsim 0$$

$$M_H L = f(x, g_0 m^{\omega}), \quad x = Lm^{1/y_m}$$

The scaling function depends on two variables now!

The exponent y_0

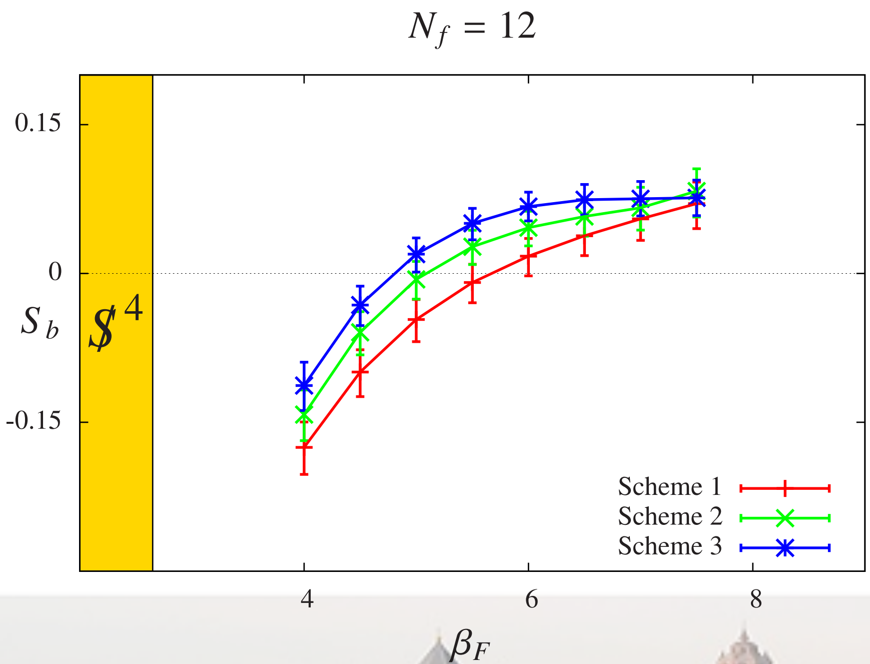
Is y_0 ever small?

Perturbatively:

- $N_f=16$: $y_0 = -0.002$ (2 loop)
- $N_f=12$: $y_0 = -0.36$ — -0.28 (2 loop /4-loop MS)

Schroedinger funct. studies suggest small y_0 in several models

MCRG for $N_f=12$ predicts $y_0 \approx -0.12(4)$



Slope of the bare step scaling function predicts y_0

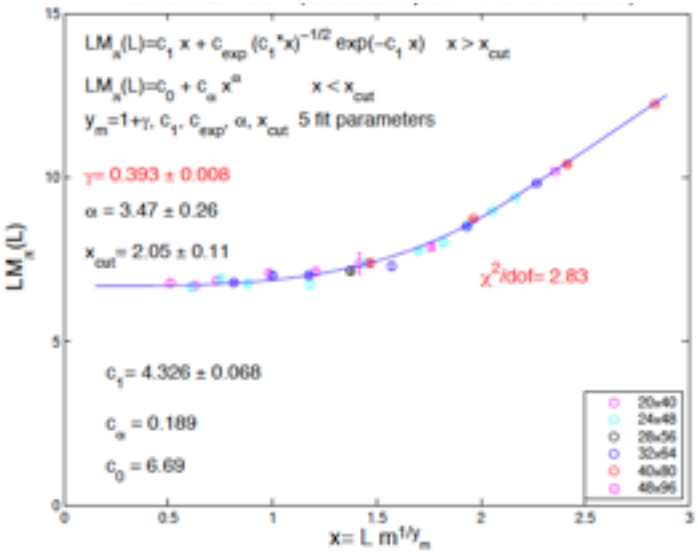
G. Petropoulos talk, 15:40 today

Finite size scaling with leading operator only

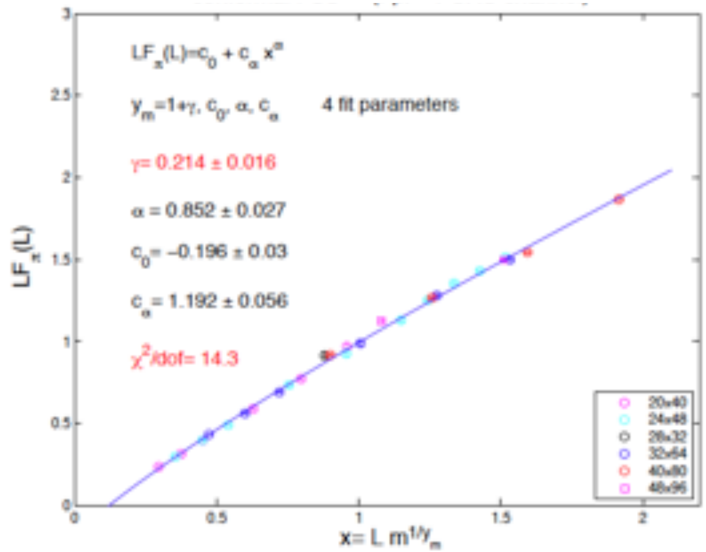
$N_f=12$: LatticeHiggsCollaboration, Lat-KMI, and other groups investigated FSS

General outcome : good (or reasonable) curve collapse but inconsistent exponents

LHC results at $\beta=2.2$ for M_π and f_π (Lat'12 PoS - R. Wong)



M_π : $y_m=1.393(8)$

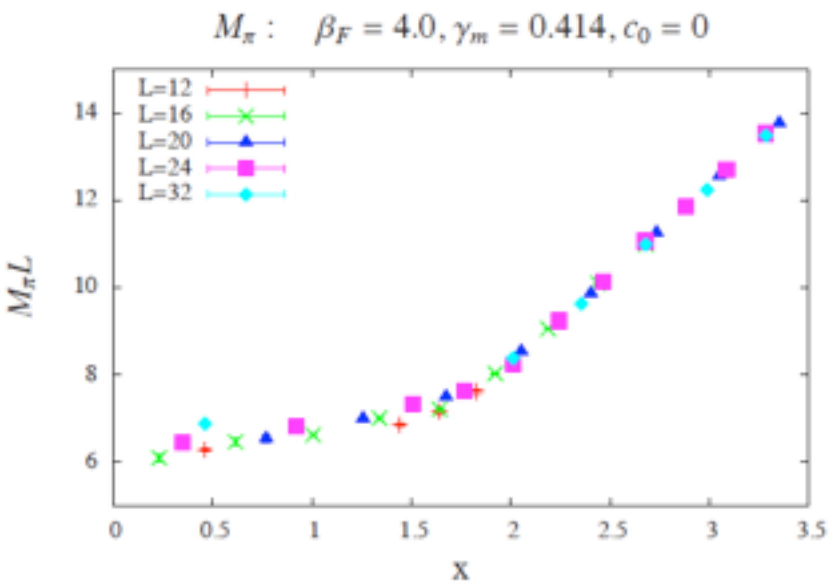


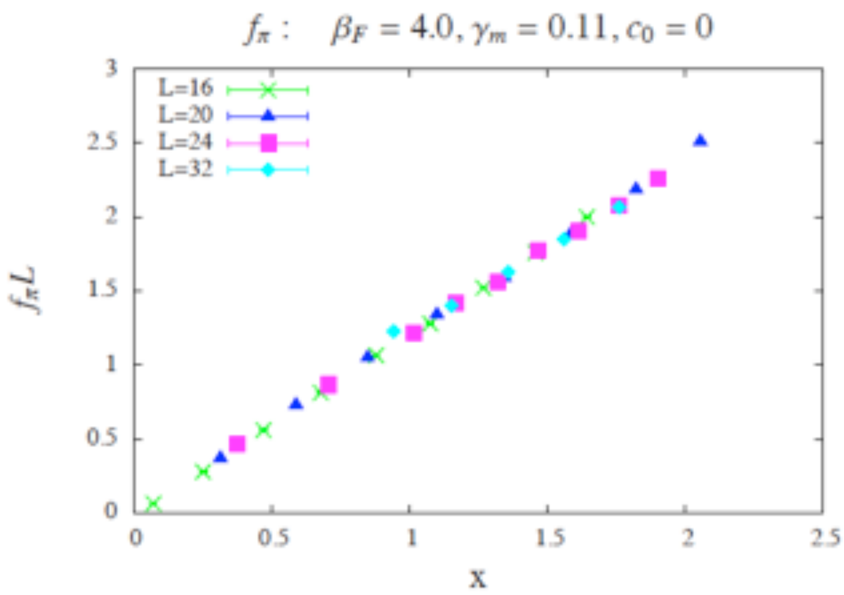
f_π : $y_m=1.214(16)$

Finite size scaling with nHYP action, N_f=12

β= 4.0 (meson spectrum matches LHC β=2.2 closely)

- good curve collapse for larger $x = Lm^{1/y_m}$
- inconsistent exponents





M_π : y_m=1.408(10)

f_π : y_m=1.11(5)

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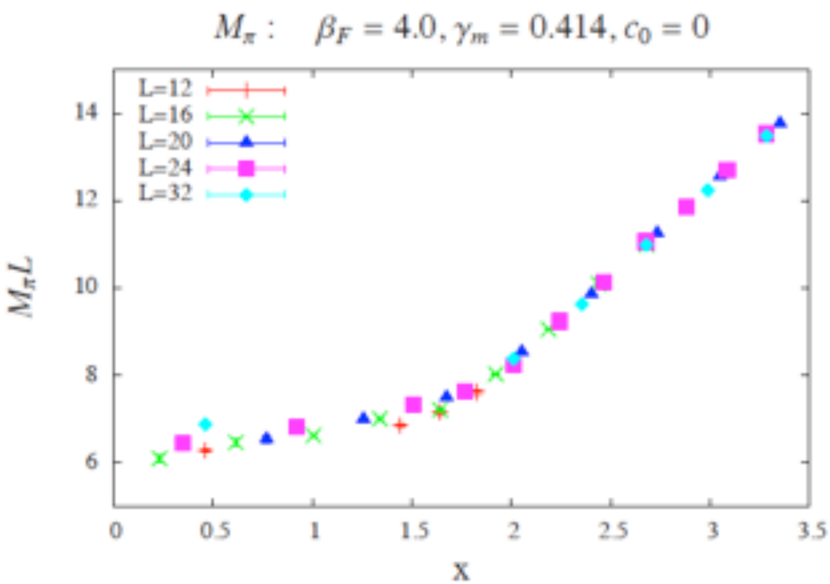
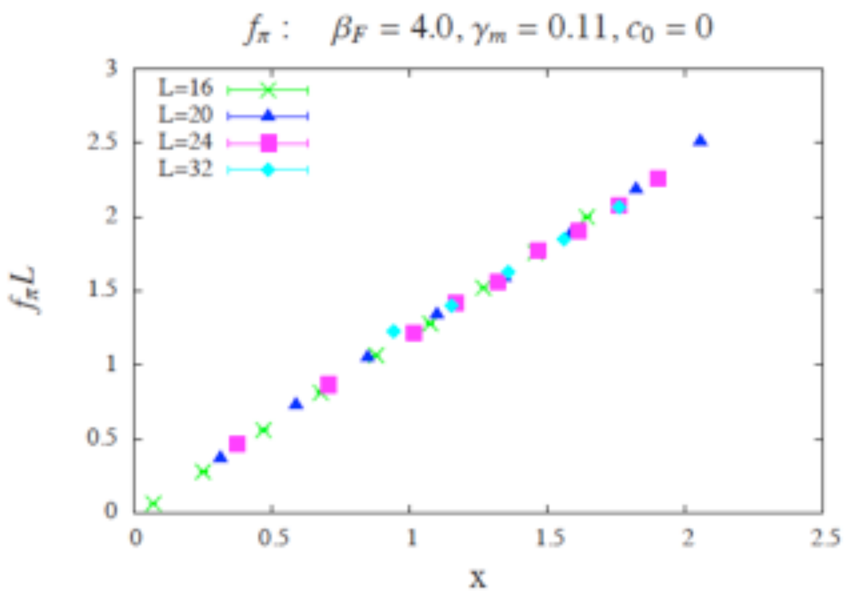
- No good curve collapse at small x
  -- cannot be fixed by changing the exponent





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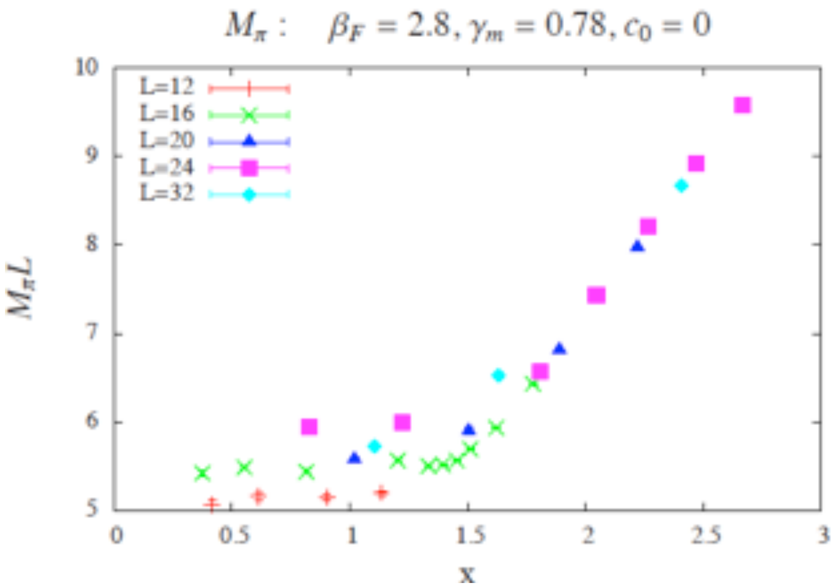
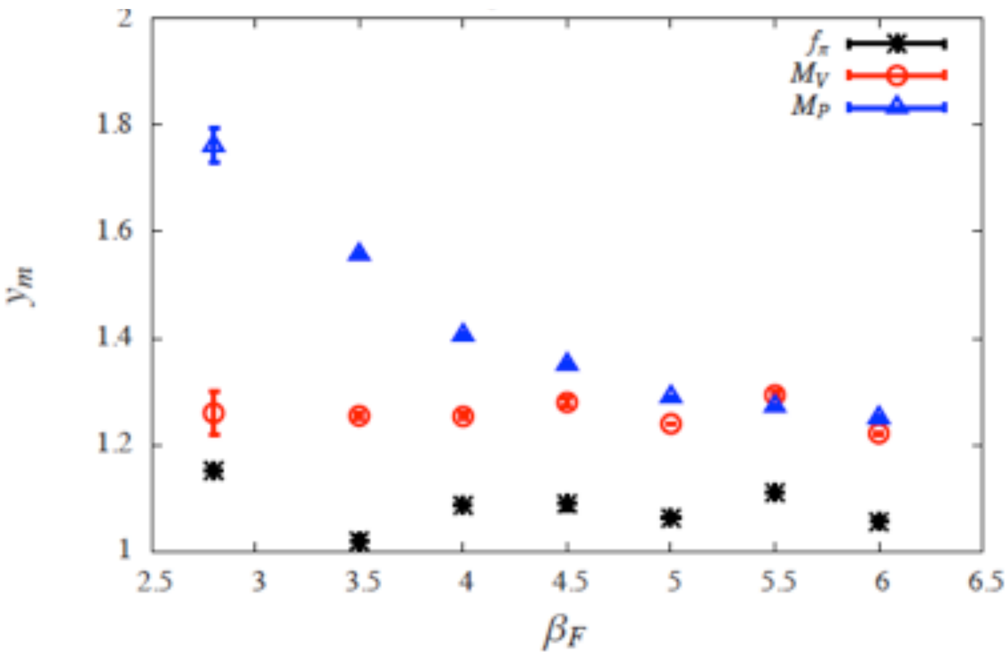


Gets worse at strong coupling! (β=2.8)

M_π : y_m=1.78(4) (β=2.8)

Scaling exponents

Use data only at large(r) x



β=2.8 — 6.0
Volumes: 12^3, 16^3, 20^3, 24^3, 32^3
$N_T = 2\ N_S$
masses: 0.005 — 0.12
such that x= 0.2 - 5

25 - 35 data points at each β
(planned, not all complete)

M_π, and M_V settle at a common value at β≈6.0
(f_π is still off)

Scaling exponents

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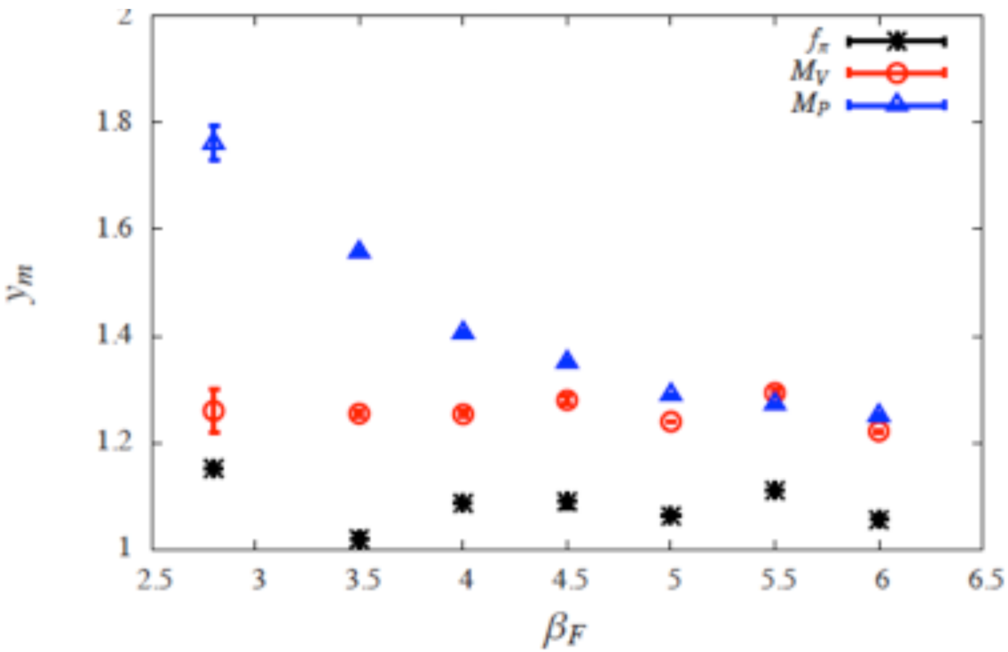

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Possible explanations:

1) N_f=12 is not conformal
2) N_f=12 is conformal but finite size scaling is strongly affected by an irrelevant operator

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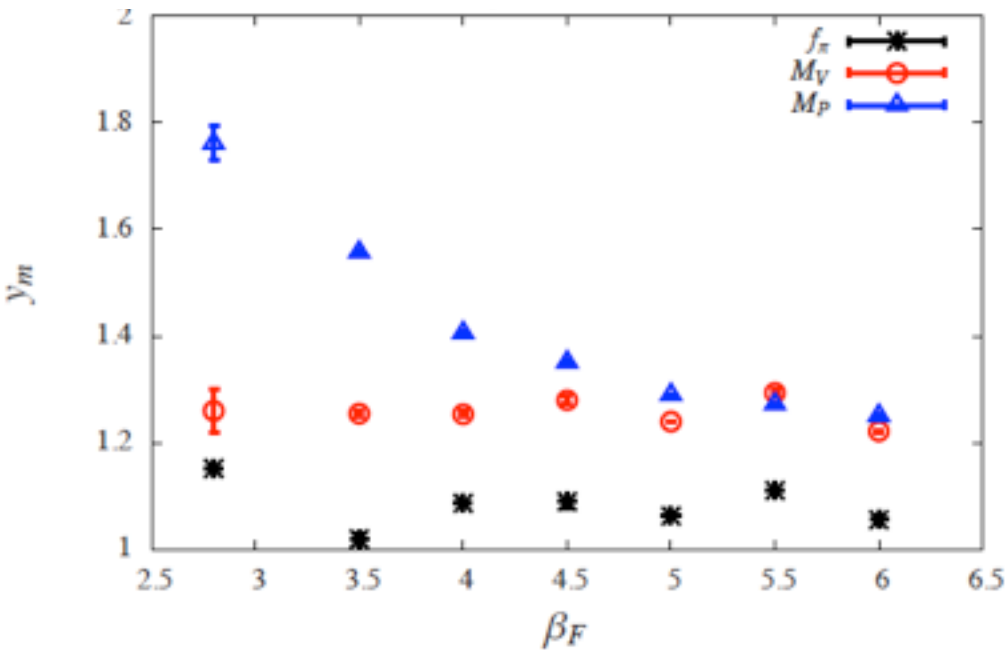

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Corrections to finite size scaling

Physical masses scale as

$$M_H = L^{-1} f(x, g_0 m^{\omega}), \quad \omega = -y_0 / y_m$$

$f(x, g_0 m^{\omega})$ is analytic both in x and g_0.

If the $g_0 m^{\omega}$ corrections are small, expand

$$L M_H = F(x)\left(1 + g_0 m^{\omega} G(x)\right)$$

- F(0), G(0) are finite constants
- as $L \to \infty: \quad M_H \propto m^{1/y_m} \to F(x) \propto x, \quad G(x) = \mathrm{const}$

Approximate G(x) = c (should be checked) $\to \dfrac{L M_H}{1 + c\, g_0 m^{\omega}} = F(x)$

Need minimization in y_m, ω, and $c g_0$

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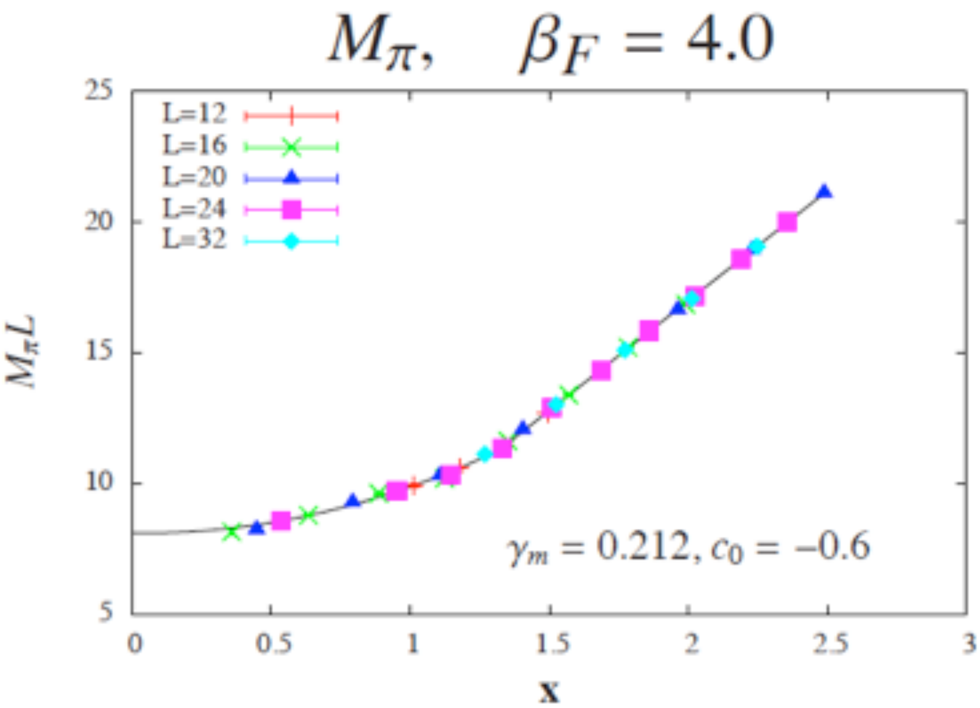
Scaling test **with** corrections

Curve collapse: 2 parameter, y_m and c_0, y_0=-0.3 fixed



Fit:
quadratic polynomial at $x < x_0$,
linear at $x > x_0$,
separation point x_0 free
(here $x_0 = 1.36$)

- Consistent curve collapse both at small and large $x = Lm^{1/y_m}$

  y_m=1.212 , c_0 = -0.6 ; $\chi^2/\mathrm{dof} = 4.5$
- Cut small x<1.2 points : y_m=1.234 , c_0 = -0.6; χ^2/dof =2.9
- Cut large x>1.3 points : y_m=1.184 , c_0 = -0.7; χ^2/dof = 0.7

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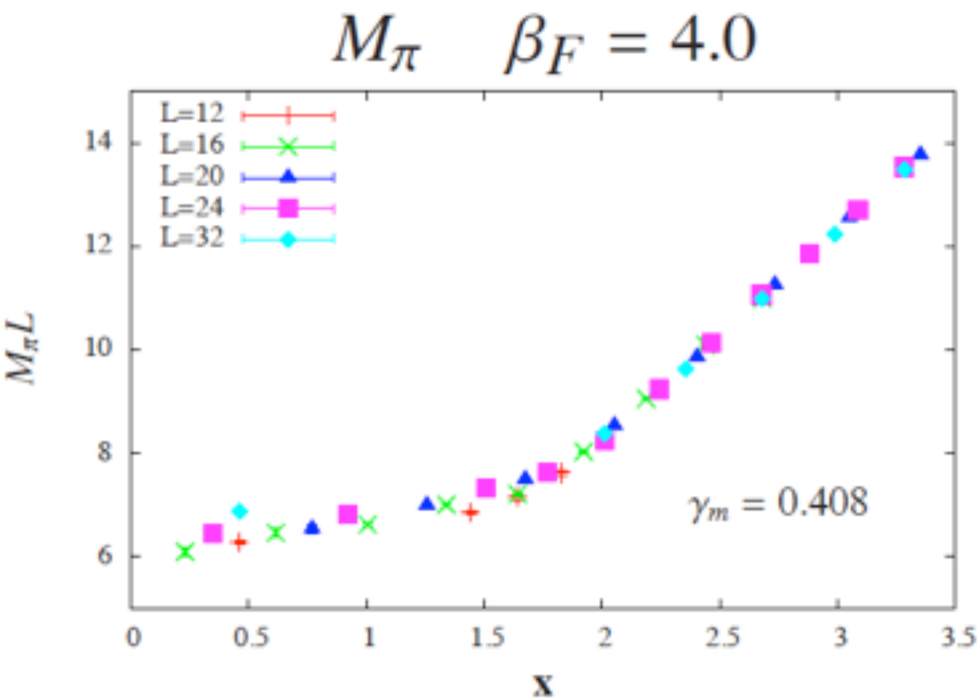

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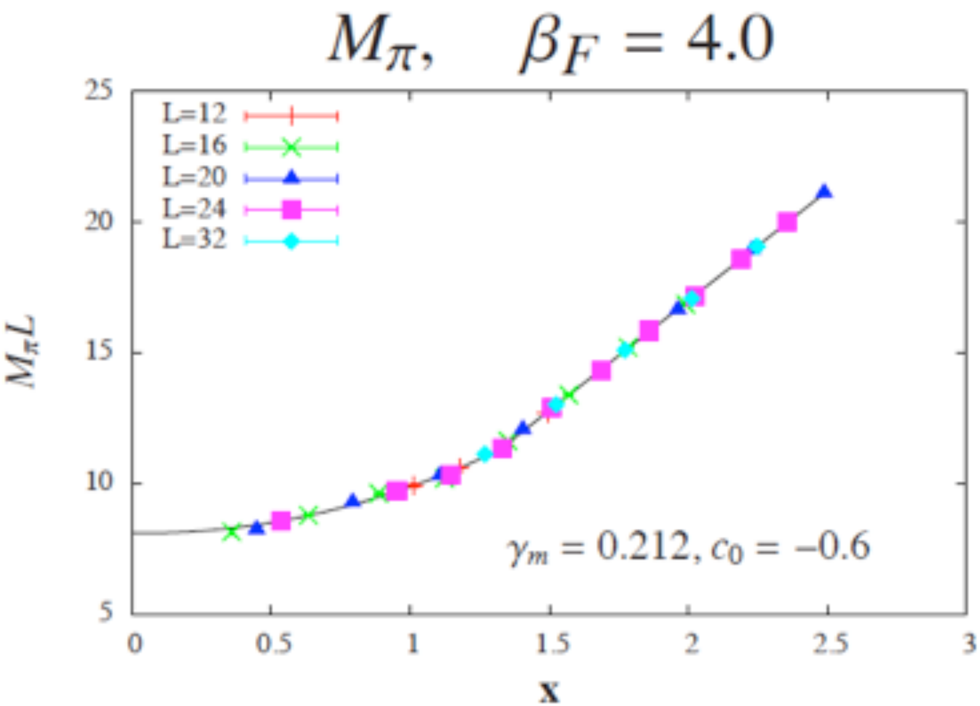

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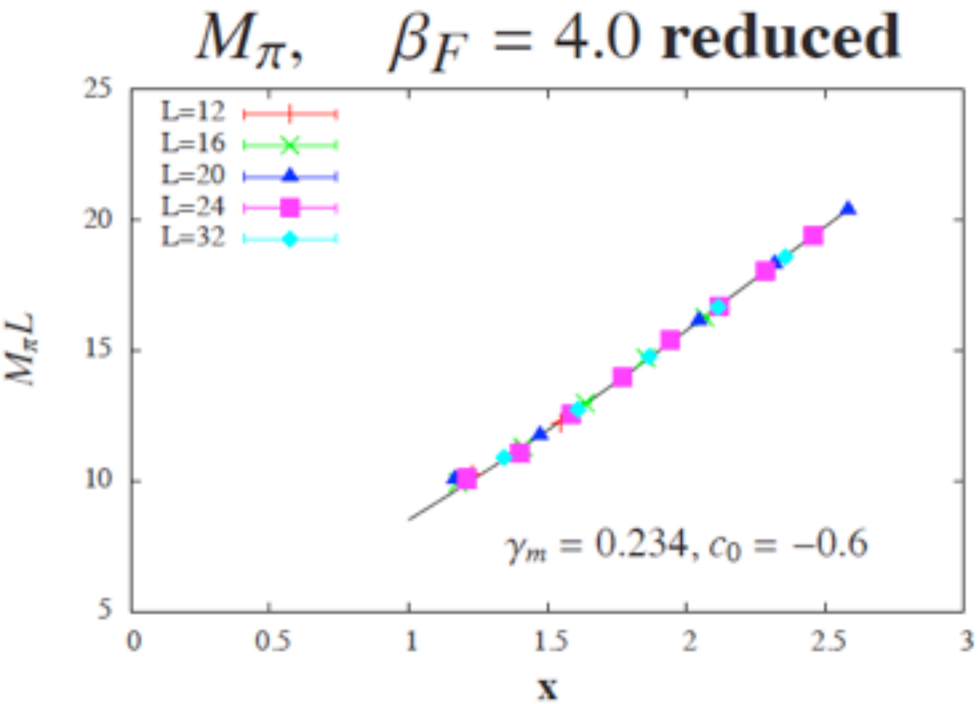

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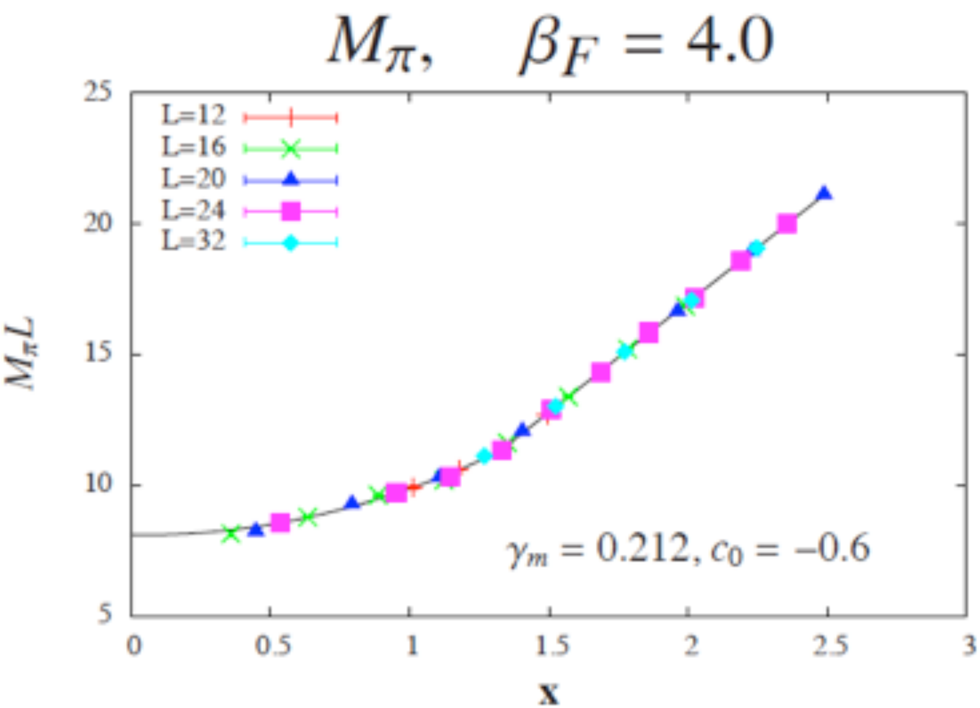

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y_m	1.212(20)	1.184(25)	1.24(2)
c_0	-0.6	-0.3	1.6
χ^2/dof	4.5	5.1	8.5
y_m c_0=0	1.406(4)	1.254(5)	1.084(5)

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- y_m=1.21 consistent for all three observables

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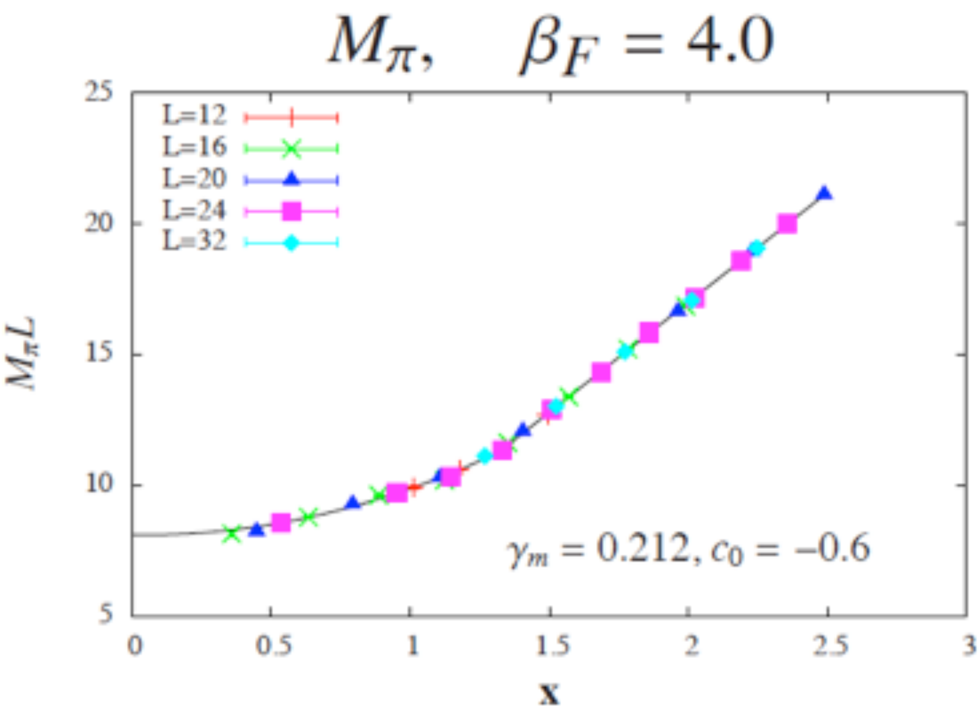

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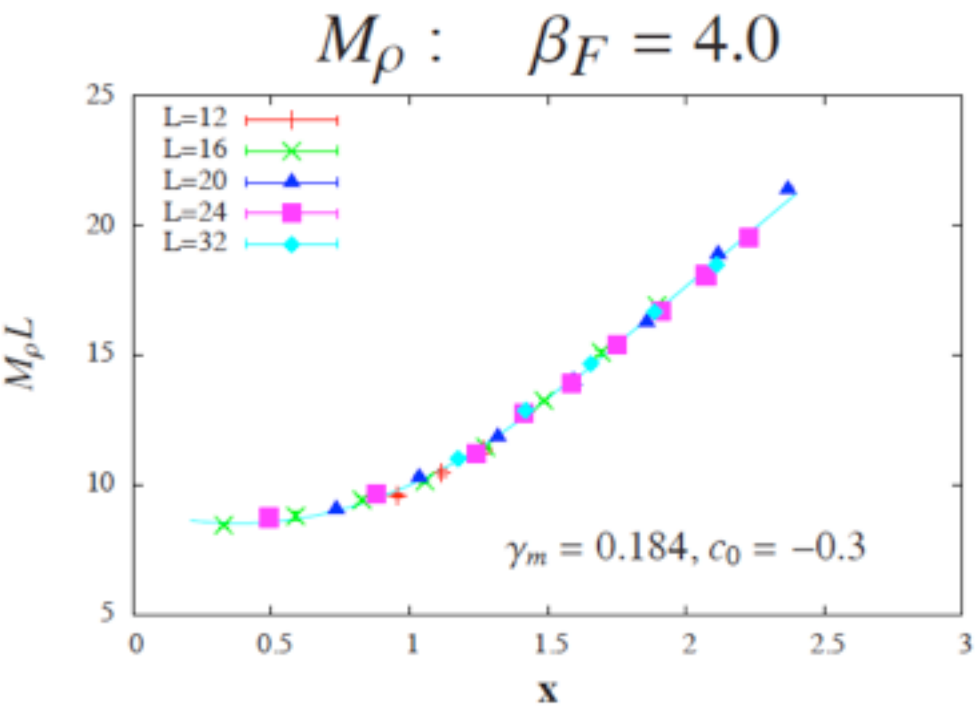

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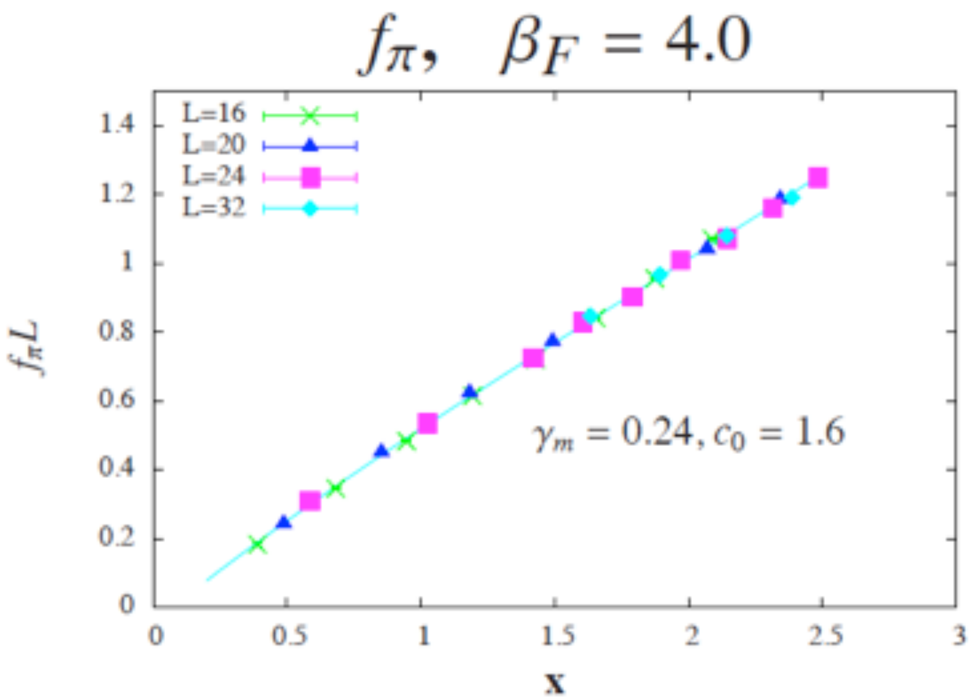

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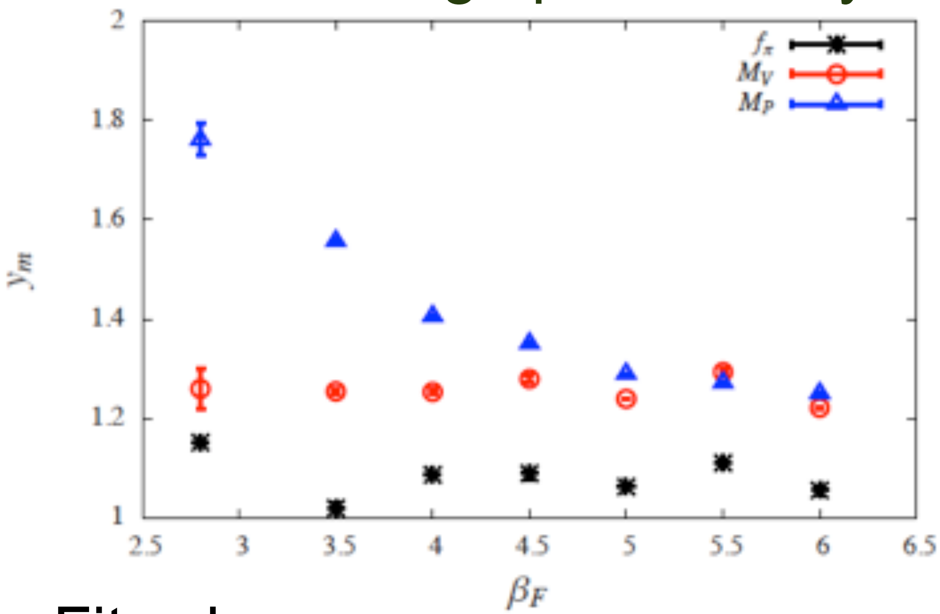
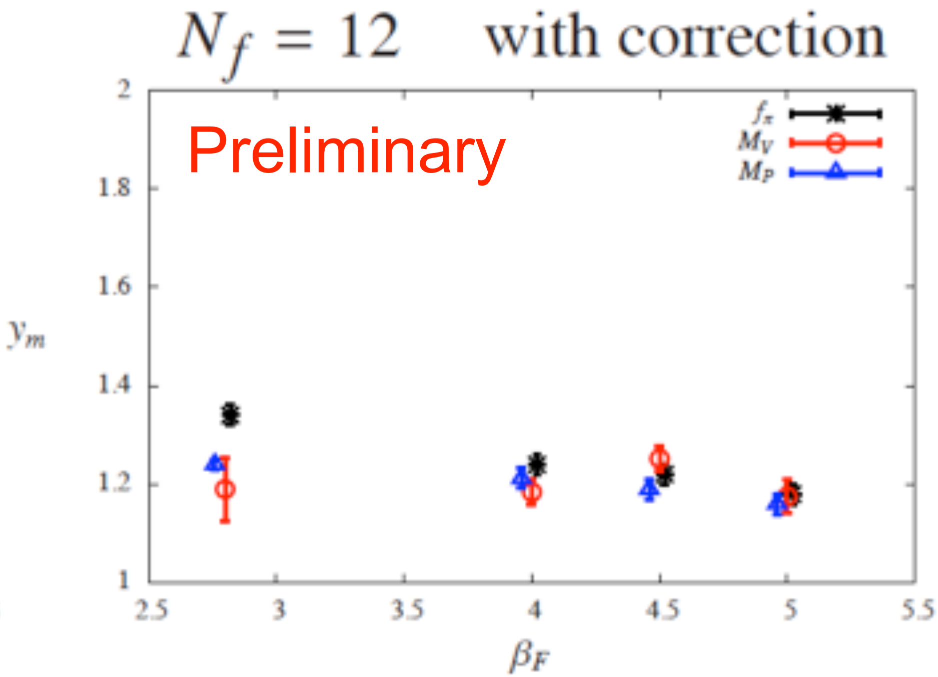
Scaling exponent **with** corrections

Include all data $M_\pi L$, $M_V L$, $f_\pi L$ points

Leading operator only





Fits show

- good curve collapse
- consistent scaling exponent γ_m=0.20(2)
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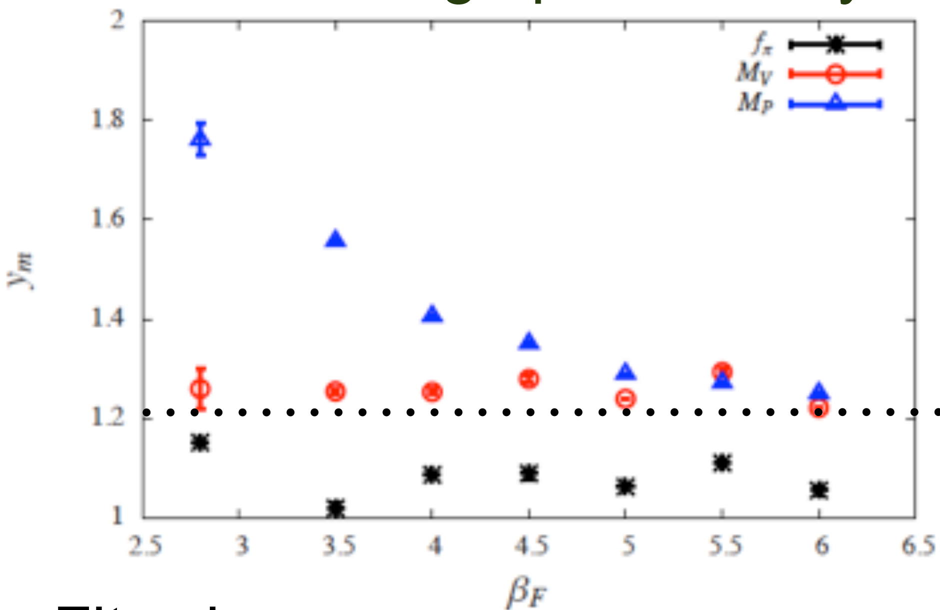
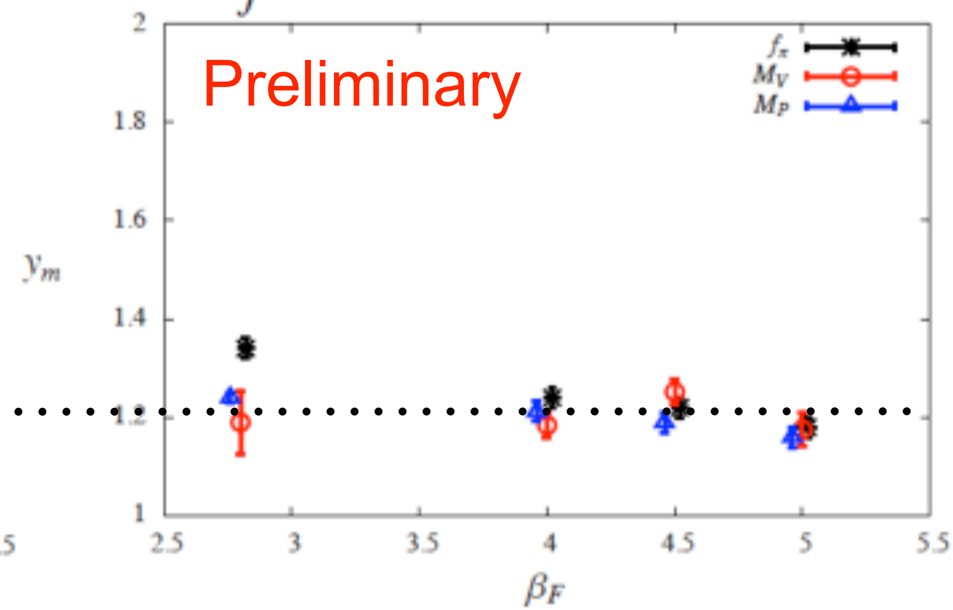


$N_f = 12$ with correction



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Conclusion

- Systems near the conformal boundary can have near marginal gauge coupling
  → this can strongly influence scaling

- Finite size scaling for N_f=12 is inconsistent;

- Accounting for the near marginal gauge coupling predicts consistent scaling exponents for all (investigated) hadrons at all gauge couplings
  - The scaling exponent for N_f=12 is small $\gamma_m \approx 0.20\,(2)$

- Similar dynamics are expected for all systems just above the conformal boundary

Backup slides

Numerical test

N_f=12 flavors nHYP smeared staggered fermions

- gauge coupling: cover a wide range

  β= 2.8, 4.0, 5.0, 6.0, (3.5, 4.5, 5.5 in progress)

  (Note: β= 2.8 is near S4b - strongest poss.
  β= 4.0 is very close to LHC β=2.2
  β= 5.5 is the IRFP based on MCRG
  and eigenmodes)

- volumes : 12^3x24, 16^3x32, 20^3x40, 24^3x48, 32^3x64

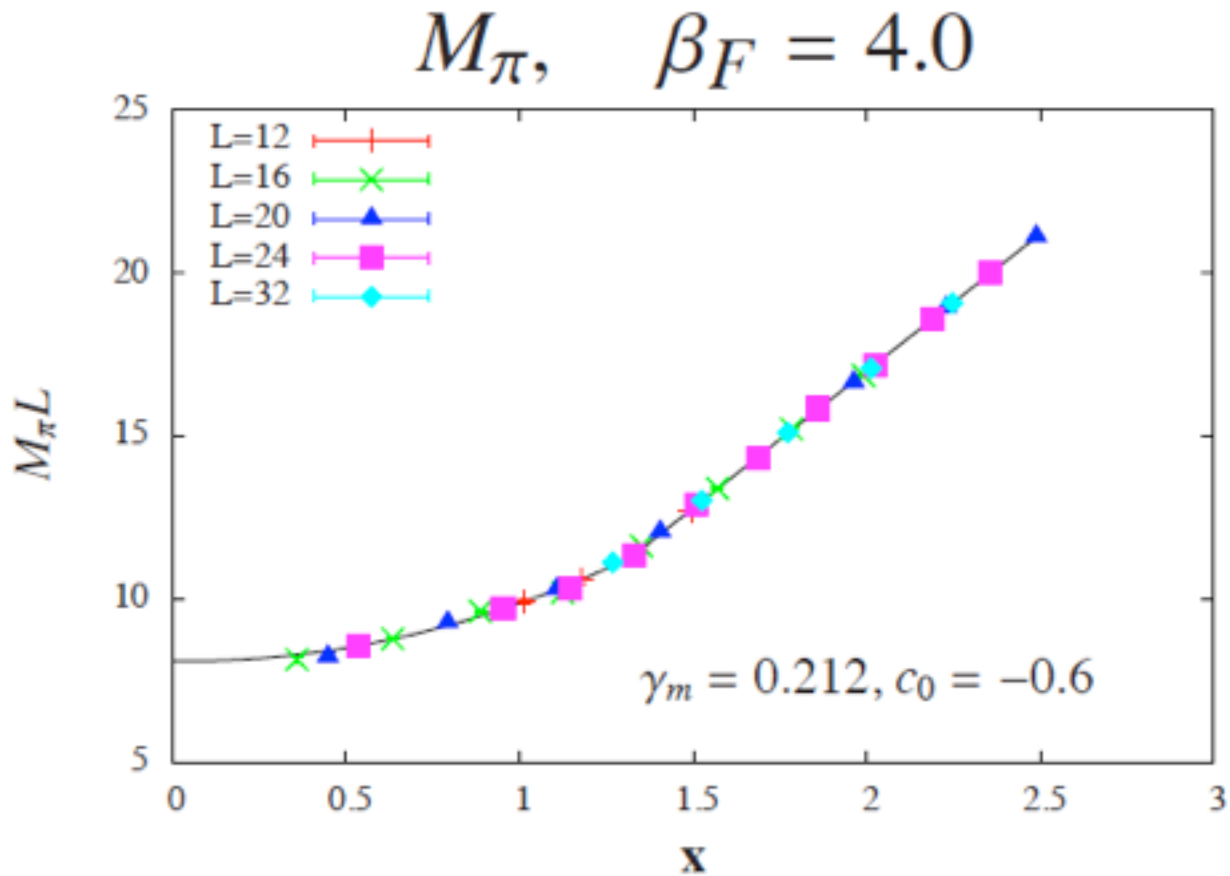
- fermion mass : m=0.01 -- 0.15 ($x = m^{1/y} L$ = 1- 6)

25-35 data points at each β

- operators: pseudoscalar, vector, f_π

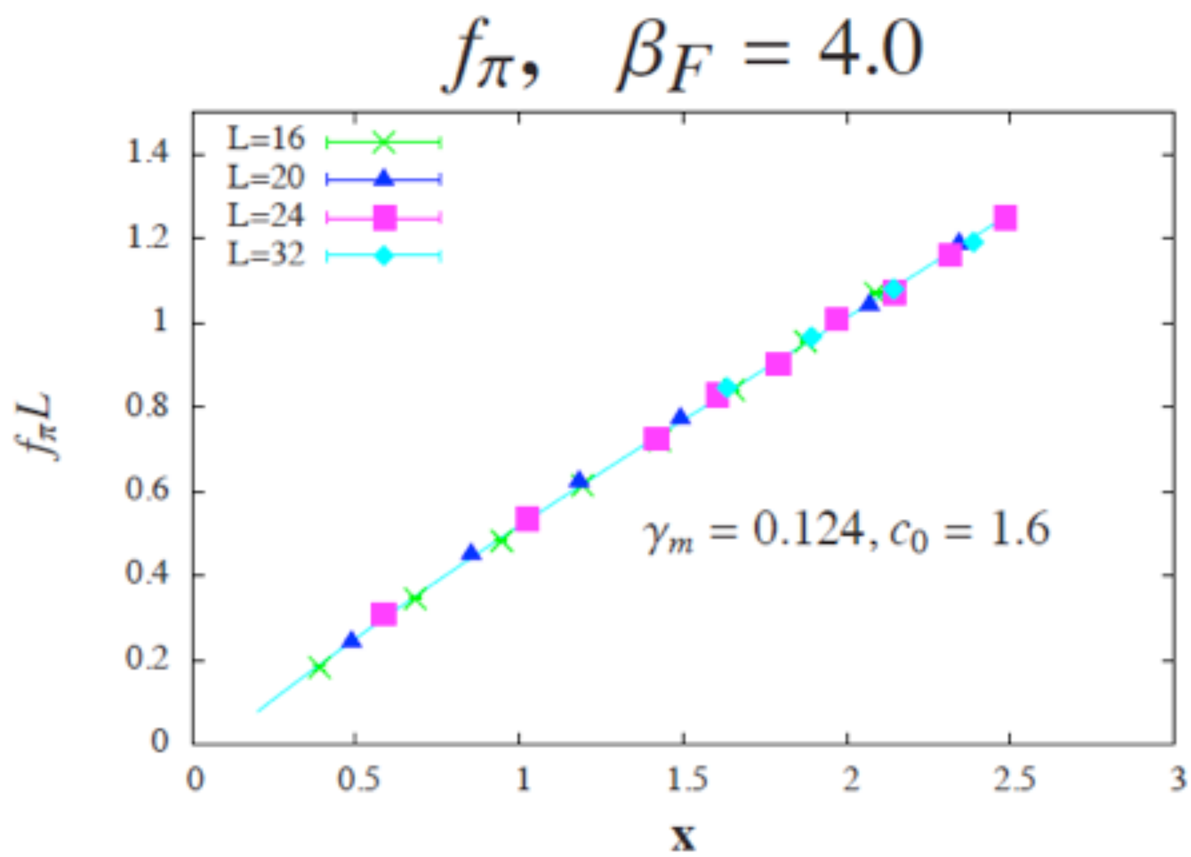
Fitting forms

M_π and M_ρ : fit quadratic at small x, linear at large.



Fitting forms

f_π : 4th order polynomial fit



Comparing different actions

LHC : 2 stout smeared fermions, Symanzik gauge

KMI : HISQ fermions without Naik, Symanzik gauge

Boulder: nHYP fermions, fundamental+adjoint plaquette gauge

Table: γ_m from fits with leading exponent only

	$6/g^2$	γ_m (M_π)	γ_m (M_ρ)	γ_m (f_π)
Boulder	1.4	0.76	0.26	0.15
Boulder	2.0	0.41	0.25	0.11
LHC	2.2	0.39	0.30	0.21
Boulder	2.5	0.29	0.24	0.06
KMI	3.7	0.43	0.46	0.52
KMI	4.0	0.41	0.46	0.58

Lattice artifacts are not universal!

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