Fisher's zeros for $SU(3)$ with N_f flavors and RG flows

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Content of the talk

- Global Aspects of RG flows for $SU(3)$ with N_f flavors
- Fisher's zeros and finite size scaling
- Two lattice matching and renormalization group flows
- Fisher's zeros for $SU(3)$ with N_f flavors
- Conclusions

Attempt to describe globally the RG flows of $SU(3)$ with N_f flavors

Generic descriptions of the the global properties of the RG flows:

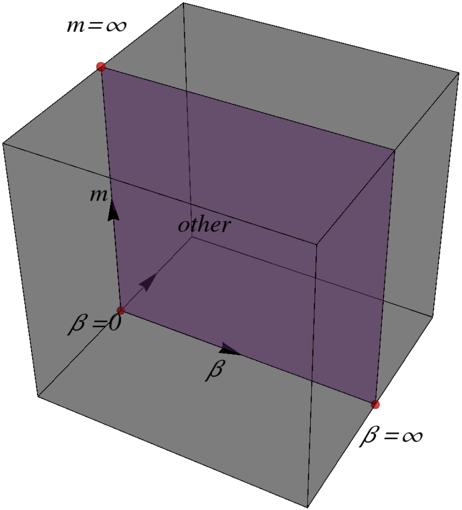
- β (inverse bare coupling) direction
- bare mass direction
- "other" directions (induced by blocking or coarse graining the bare theory)

The bare theory is a point in the β-mass plane

To start with, this could describe the situation of $SU(3)$ with N_f flavors of unimproved staggered fermions at zero temperature.

Deformations will be discussed later

The bare plane



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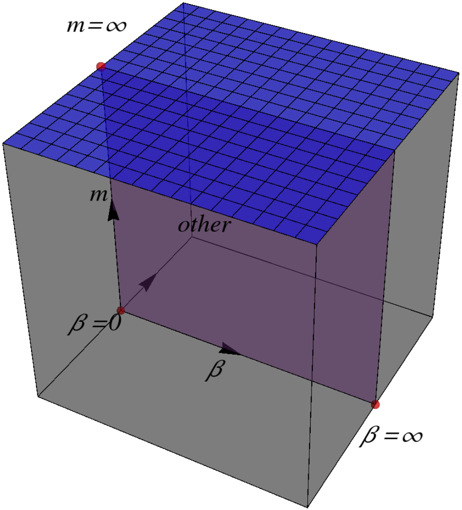
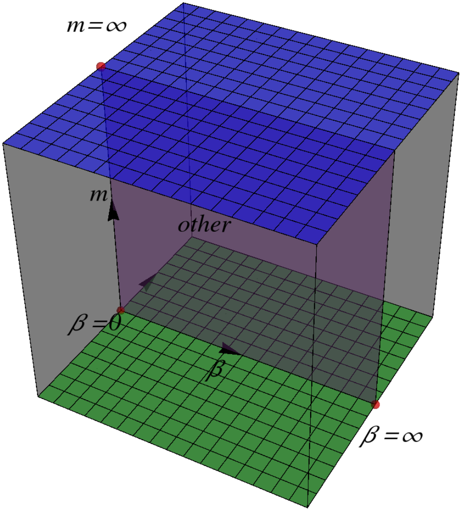
$SU(3)$ with N_f flavors: pure gauge plane



$SU(3)$ with N_f flavors: the massless plane



$SU(3)$ with N_f flavors: types of flows

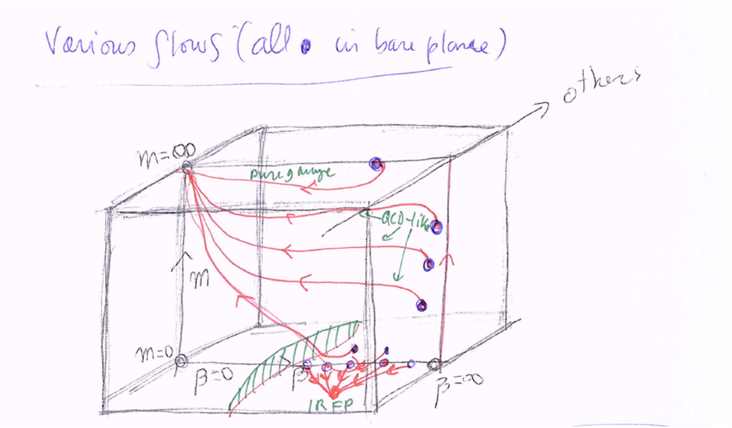


Figure: Schematic flows

$SU(3)$ with N_f flavors: bulk transitions

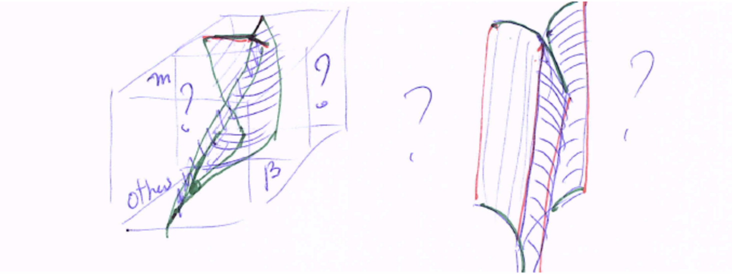


Figure: Surface of bulk transitions?

$SU(3)$ with N_f flavors

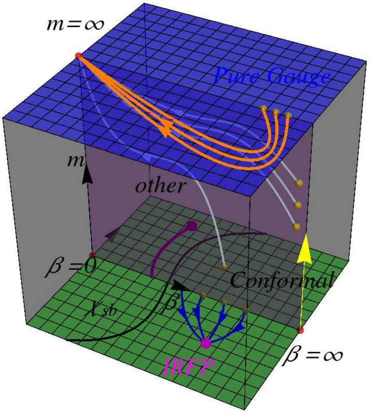


Figure: Schematic flows (a better picture).

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$SU(3)$ with N_f flavors: IR fixed point

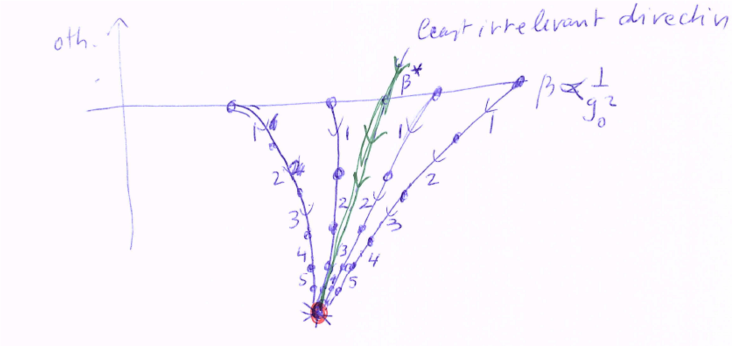

Figure: β^* from " the least irrelevant direction".

β_{bulk} and β^* as a function of N_f

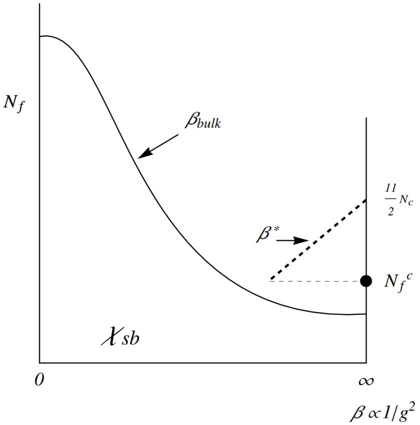

Figure: β_{bulk} and β^* (see de Forcrand et al. arXiv:1211.3374, Tomboulis arXiv:1211.4842). Could the solid line and the dotted line merge at N_f^c?

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$SU(3)$ with N_f flavors: Non-integer N_f?

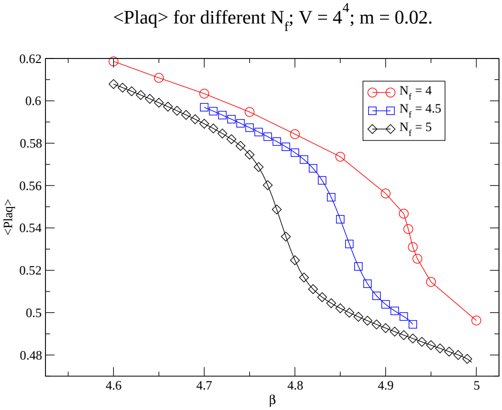

Figure: Average Plaq for integer and non-integer N_f ; V = 4^4; m = 0.02.

Finite T

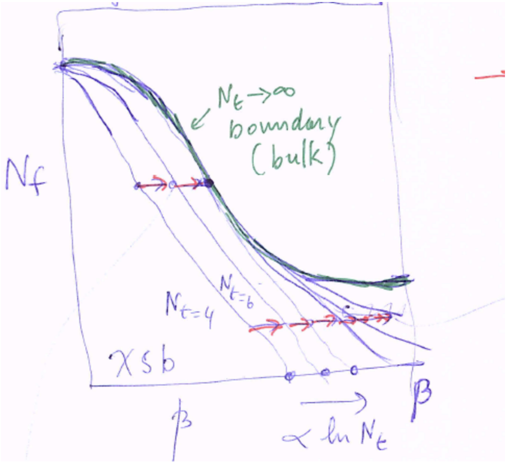

Figure: What is the lowest value of N_f for which the finite T transition turns into a bulk transition?

$SU(3)$ with $N_f = 12$, Finite T transition turning into a bulk transition

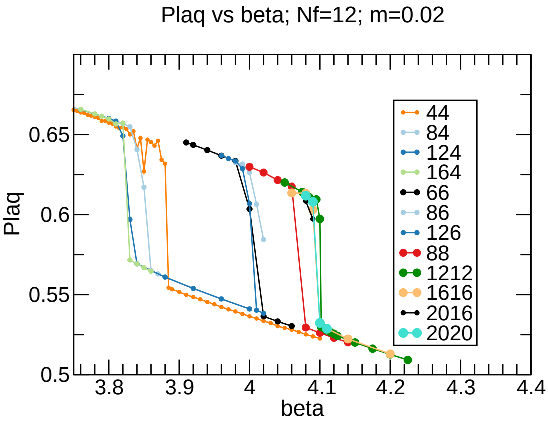

Figure: Plaquette discontinuities for various lattices. 8 4 means a $8^3 \times 4$ lattice.

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$SU(3)$ with $N_f = 12$: Finite T transition turning into a bulk transition

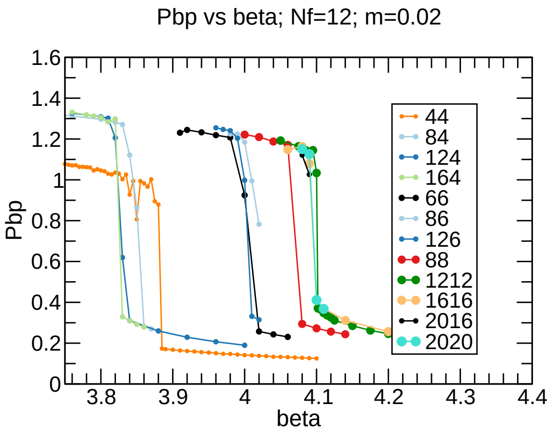

Figure: $<\bar{\psi}\psi>$ discontinuities for various lattices. 8 4 means a $8^3 \times 4$ lattice

$N_f = 12$ Finite T transition turning into a bulk transition

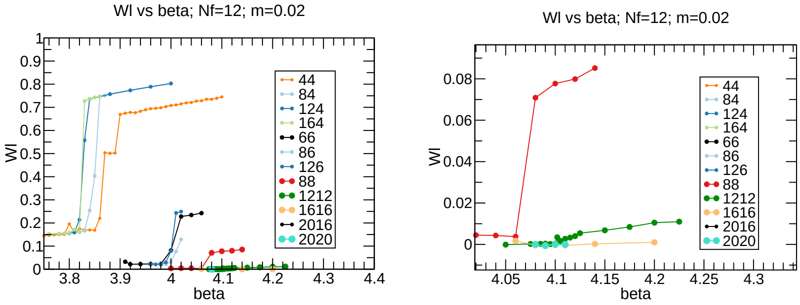

Figure: Polyakov's loop (time-periodic Wilson's line) for various lattices. 8 4 means $8^3 \times 4$ lattice.

- What is the lowest value of N_f for which the finite T transition turns into a bulk transition?
- How does the end point in (β, m) for 12 flavors (Jin and Mawhinney arXiv:1203.5855) and 8 flavors (Christ and Mawhinney, Phys.Rev. D46 (1992)) disappear? (Could it shrink to zero mass?)

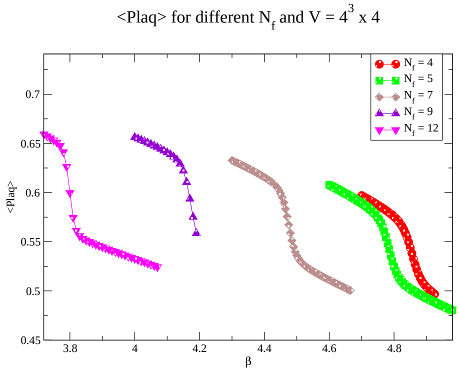

Figure: Average Plaquette for different N_f (preliminary).

Fisher's zeros and Finite Size Scaling

Decomposition of the partition function (Niemeijer and van Leeuwen)

$$
\begin{aligned}
Z &= Z_{sing.} \mathrm{e}^{G_{bounded}} \\
Z_{sing.} &= \mathrm{e}^{-L^D f_{sing.}}
\end{aligned}
$$

RG transformation: the lattice spacing *a* increases by a scale factor *b*

$$
\begin{aligned}
a &\rightarrow ba \\
L &\rightarrow L/b \\
f_{sing.} &\rightarrow b^D f_{sing.} \\
Z_{sing.} &\rightarrow Z_{sing.}
\end{aligned}
$$

Important Conclusion (Itzykson et al. 83)

The zeros of the partition functions are RG invariant.

Fisher's zeros: zeros of the partition function in the complex β plane.

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Fisher's zeros and Finite Size Scaling

We consider discrete RG transformations.

- Lattice size in lattice spacing unit: $L \longrightarrow \frac{L}{b}$
- Scaling variables: $u_i \longrightarrow \lambda_i u_i = b^{y_i} u_i$
  - Relevant variables: $\lambda_i = b^{1/\nu_i} > 1$.
  - Irrelevant variables: $\lambda_j = b^{-\omega_j} < 1$.
- Singular part of the partition function: $Z_s = Q(\{u_i L^{1/\nu_i}\}, \{u_j L^{-\omega_j}\})$

If we only keep one relevant variable $u \simeq \beta - \beta_c$, Fisher's zeros have the following relation

$$uL^{1/\nu} = A + BL^{-\omega} + \mathcal{O}(L^{-2\omega}),$$

and the lowest Fisher's zeros

$$Re(\beta_1(L)) - \beta_c = Re(A)L^{-1/\nu} + Re(B)L^{-1/\nu-\omega},$$

$$Im(\beta_1(L)) = Im(A)L^{-1/\nu} + Im(B)L^{-1/\nu-\omega}.$$

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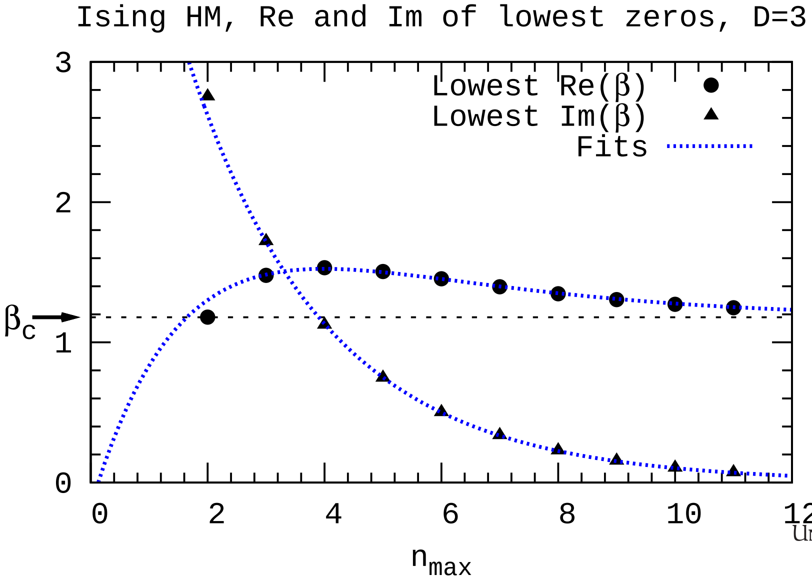
Fisher's zeros and Finite Size Scaling



Fisher's zeros and Finite Size Scaling

- Calculate both the real and imaginary part of the partition function.
- The "logarithmic residue" method

$$\frac{1}{2\pi i}\oint_C \beta^n \frac{Z'(\beta)}{Z(\beta)} = \sum_{i=1}^{k} \beta_i^n,$$

  where β_i are all the zeros in the integration region C.
- Single point reweighting $\rightarrow$ multiple point reweighting.
- Approximate the density of state via Chebyshev polynomial.
- Different methods allow us to crosscheck the final results.

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Two lattice matching and renormalization group flows

- Two lattice matching is a way to measure the running of the bare couplings and to construct the approximate RG flows. (A. Hasenfratz PRD 80 034505)
- One of the two lattice matching observables:

$$R(\beta, \mathcal{V}/a^D) \equiv \frac{\left\langle (\sum_{x \in B_1} \vec{\phi}_x)(\sum_{y \in B_2} \vec{\phi}_y) \right\rangle_\beta}{\left\langle (\sum_{x \in B_1} \vec{\phi}_x)(\sum_{y \in B_1} \vec{\phi}_y)) \right\rangle_\beta}$$

- The matching condition can be chosen as

$$R(\beta, L^D) = R(\beta', (L/b)^D)$$

$$\beta = \beta_0 \Longrightarrow \beta_1'$$

$$\beta = \beta_1' \Longrightarrow \beta_1''$$

$$\vdots$$

Two lattice matching and renormalization group flows

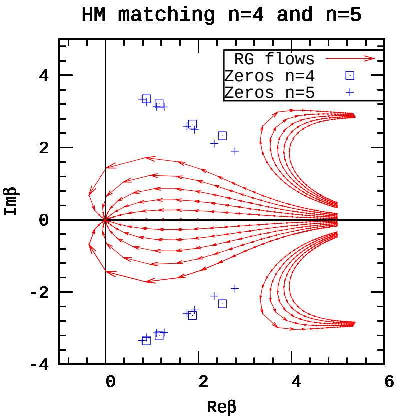

Figure: Fisher's zeros seperate complex RG flows (arrows) (PRL 104 25160).

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Fisher's zeros of the 2D $O(2)$ model: TRG and MC

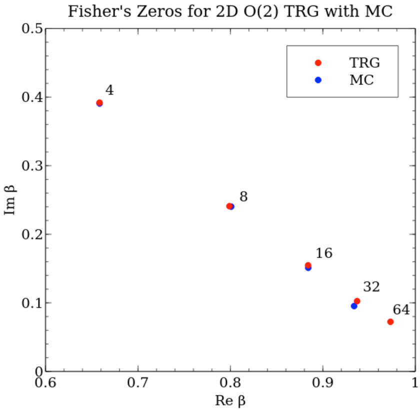
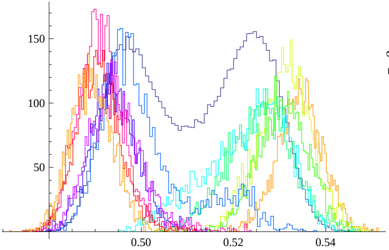


Figure: Fisher's zeros of XY model with $L = 4, 8, 16, 32, 64$ for 30 states compared to MC (Alan Denbleyker and Haiyuan Zou).

Fisher's zeros and Finite Size Scaling

- Zeros of $SU(3)$ with 3 flavors with reweighting method.



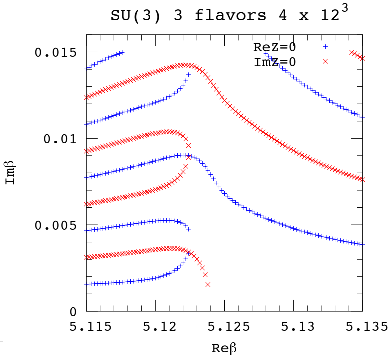


Figure: Fisher's zeros obtained by reweighting for 3 light quarks on a 4×12^3 lattice. Left: plaquette distributions for several values of β. The high-statistics (very expensive!) double peak distribution at β =5.124 is superimposed. L^{-3} scaling for the Imaginary part of lowest zero for $4 \times L^3$ lattices.

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Fisher zeros for $SU(3)$ with N_f= 4 and 12

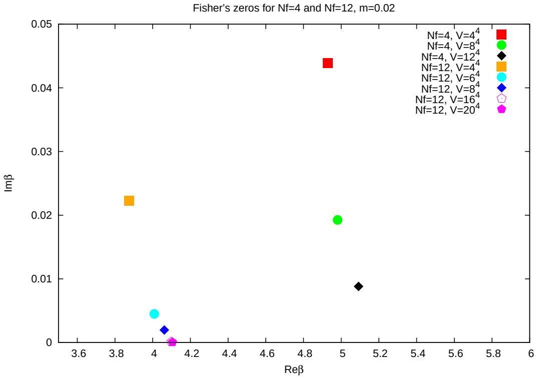

Figure: Zeros for $N_f = 4$ and $N_f = 12$ for L^4 lattices ($L = 16,\ 20$ preliminary). 12 flavors seems to pinch the real axis. Higher volumes in progress.

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Fisher zeros scaling for $SU(3)$ with $N_f = 12$

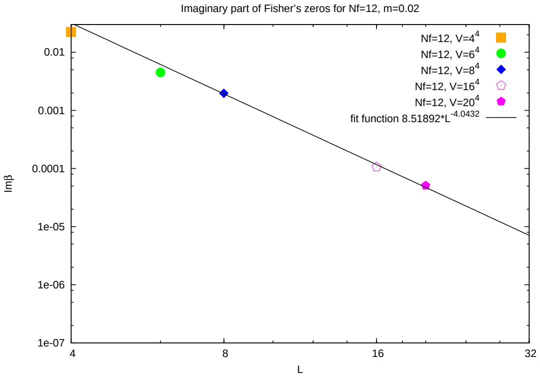

Figure: Lowest zeros scale like L^{-4} (bulk). ($L = 16,\ 20$ preliminary) .

Questions

- Will the Fisher's zero pinch the real axis like L^{-2} (ν=1/2, mean field for a free scalar) instead of L^{-4} near the end point (for $m = m_c$)?
- Is it possible to find a hint of the IR fixed point from the behavior of the zeros over a broader β interval as a function of m?
- Are the two questions related?

Conclusion

- The RG flows can be understood from the Fisher's zeros point of view.
- There is a clear first order phase transition for $N_f = 12$ from the scaling of the zeros. (L^{-4})
- We are looking for the smallest N_f for which the finite temperature transition turns into a bulk transition. It may be possible to make connection between β_{bulk} and β^*.
- Understanding the mass dependence of the transition and calculating the mass anomalous dimension γ_m from the zeros is in progress.
- Adding improvement terms to the fermion action may change the phase structure ($S4b$ phase, Zech Gelzer, in progress).
- New methods to perform the RG blocking is possible. (YM's talk on Tensor RG.)

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Backup Slides

Finite T transition turning into a bulk transition?

The two loop β function for arbitrary N_c and N_f reads

$$\beta(g) = -a\frac{\partial g}{\partial a} = -\beta_0 g^3 - \beta_1 g^5 + \cdots$$

where

$$\beta_0 = \frac{1}{16\pi^2}(\frac{11}{3}N_c - \frac{2}{3}N_f)$$

and

$$\beta_1 = \frac{1}{(16\pi^2)^2}(\frac{34N_c^2}{3} - \frac{10N_cN_f}{3} - \frac{(-1+N_c^2)N_f}{N_c})$$

The solution of the above differential equation is

$$a = \frac{1}{\Lambda_L}\exp(-\frac{1}{2\beta_0 g^2})(\beta_0 g^2)^{-\frac{\beta_1}{2\beta_0^2}}$$

Since the physical temperature T is give by $aT = N_t^{-1}$ and $\beta = 2N_c/g^2$, we can get

$$aT\Lambda_L/T = \exp(-\frac{\beta}{4N_c\beta_0})(\frac{\beta}{2N_c\beta_0})^{\frac{\beta_1}{2\beta_0^2}}$$

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Finite T transition turning into a bulk transition?

$$\ln(N_t) = \ln(\frac{\Lambda_L}{T}) + \frac{\beta}{4N_c\beta_0} - \frac{\beta_1}{2\beta_0^2}\ln(\frac{\beta}{2N_c\beta_0})$$

For $N_f = 12$ and $N_c = 3$, we will have

$$\ln(N_t) = \ln(\frac{\Lambda_L}{T}) + \frac{4\pi^2\beta}{9} + \frac{25}{9}\ln(\frac{8\pi^2\beta}{9})$$

at 2-loop level.

Assuming the physical critical temperature T_c and Λ_L is fixed, then we will have

$$\ln(\frac{N_{t2}}{N_{t1}}) = \frac{4\pi^2(\beta_{c2} - \beta_{c1})}{9} + \frac{25}{9}\ln(\frac{\beta_{c2}}{\beta_{c1}}).$$

The $\delta\beta_c = \beta_{c2} - \beta_{c1}$ is almost proportional to $\ln(\frac{N_{t2}}{N_{t1}})$ since $\frac{\beta_{c2}}{\beta_{c1}} \approx 1$ for large adjacent N_t.

$$\ln(\frac{16}{12}) = \frac{4\pi^2(\beta_{c2} - \beta_{c1})}{9} + \frac{25}{9}\ln(\frac{\beta_{c2}}{\beta_{c1}})$$

For $N_{t2} = 16$ and $N_{t1} = 12$, $\delta\beta_c = \beta_{c2} - \beta_{c1} \approx 0.058$.

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