Step Scaling Study of SU(3) 12 Flavor Theory with Larger Lattice

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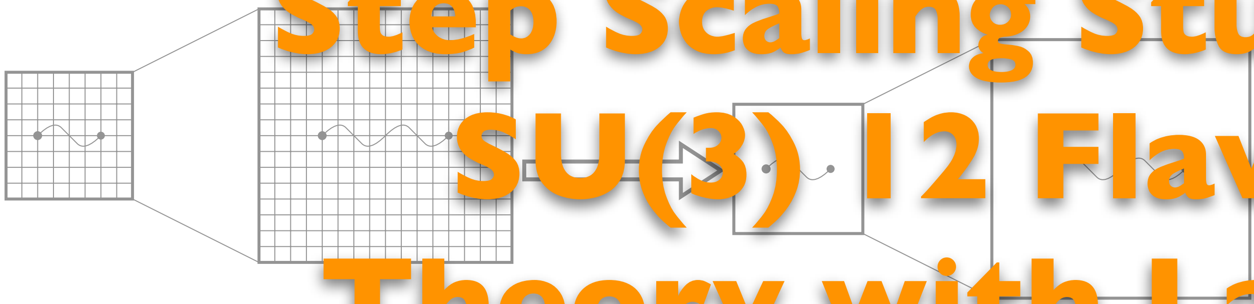
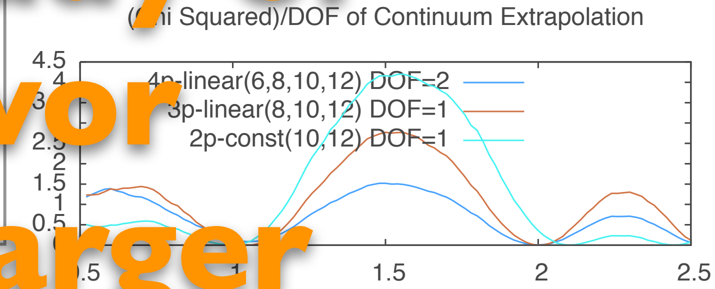
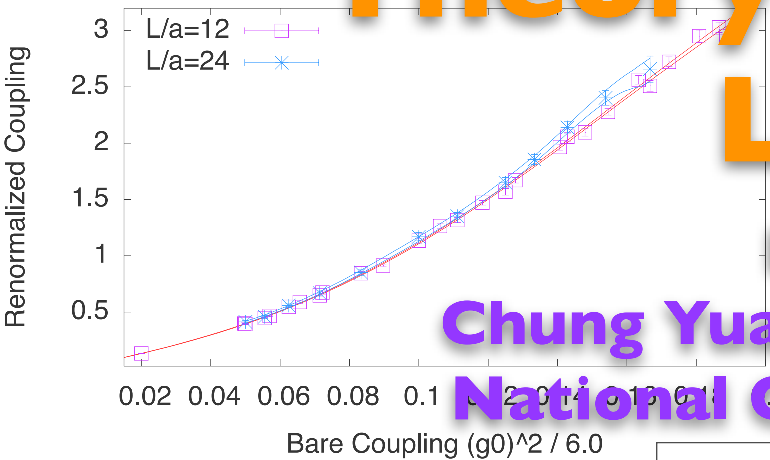
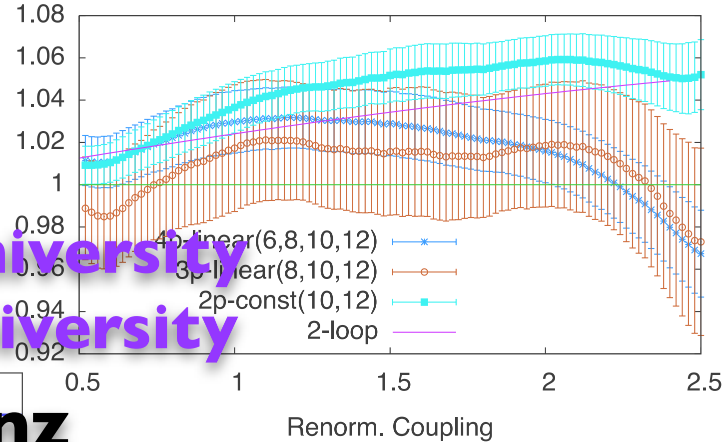
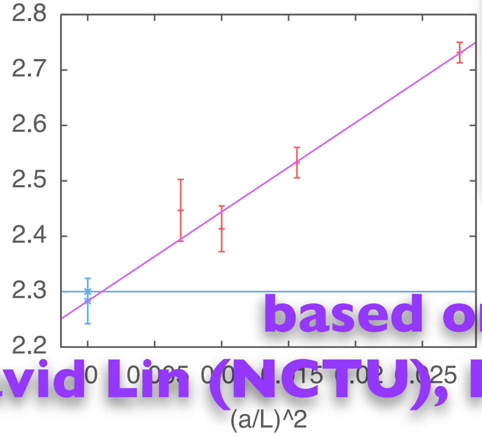
Lattice 2013, Mainz

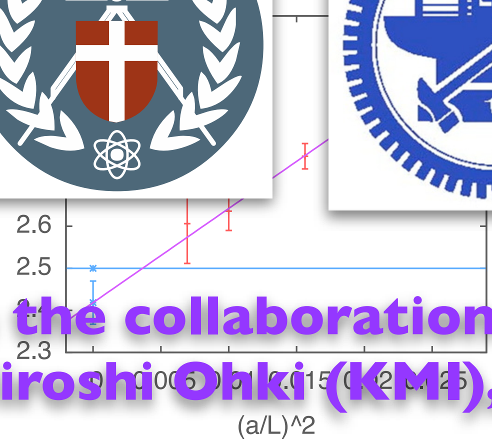


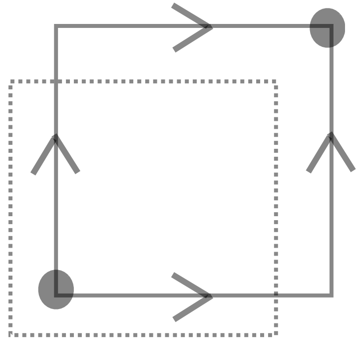












based on the collaboration work with

C.-J. David Lin (NCTU), Hiroshi Ohki (KMI), Eigo Shintani(RIKEN-BNL)

In this talk, I show the progress in the study of running coupling constant of SU(3) theory with 12 flavors with Polyakov loop scheme including larger lattice $L=24^4$

Continuation of JHEP08(2012)096

"Lattice study of infrared behaviour

in SU(3) gauge theory with twelve massless flavours"

Outline

Motivation

Step Scaling Function

Polyakov Loop Scheme

Settings

Results (Coup.Const, SSF)

Summary Comments

Motivation

Large Flavor Theory - Candidate of BSM

Running of the coupling constant
is important quantity to study

JHEP08(2012)096

SSF Study

IRFP $g^2 \sim 2$

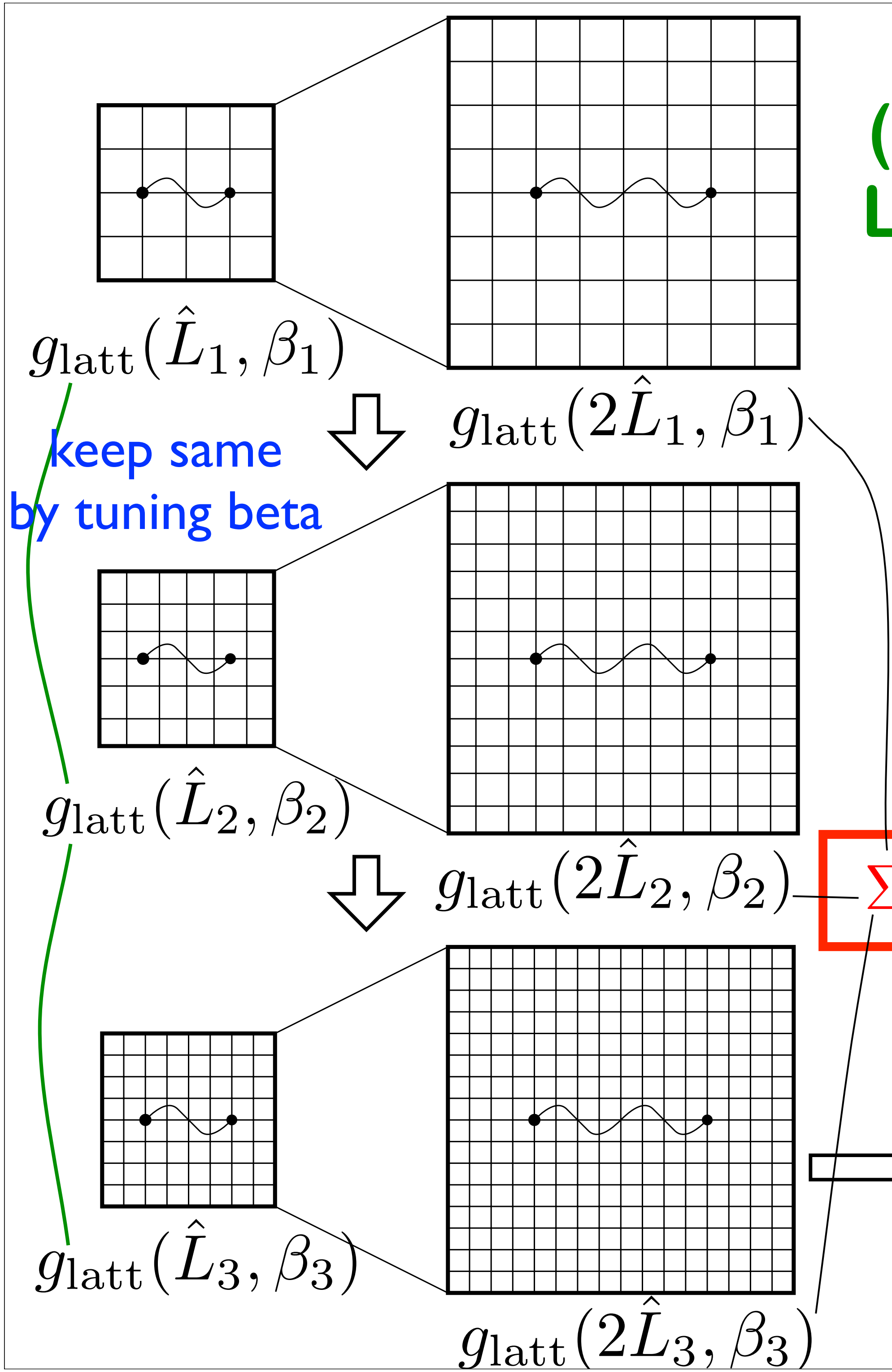
Large Systematic Error
coming from taking continuum limit

=> Further study with Larger Lattice L=24

Step Scaling Function (Integrated Beta Function) Lattice => Continuum Limit

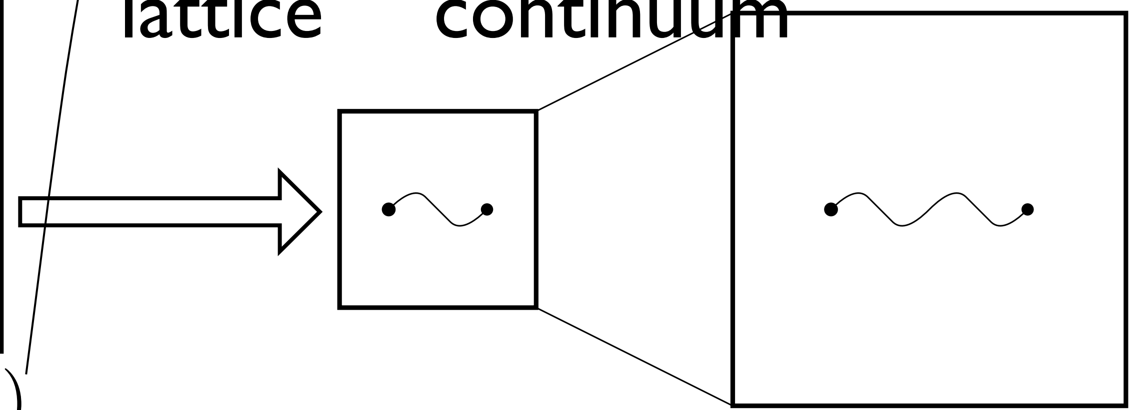
Keep Physical Box Size
= Keep observable ‘ $g(L)$ ‘
in the smaller box same
by tuning Beta

$$u = g^2_{\rm latt}(\hat{L}_1, \beta_1) = g^2_{\rm latt}(\hat{L}_2, \beta_2)$$
$$= g^2_{\rm latt}(\hat{L}_3, \beta_3)$$



$$\Sigma(u; 2; \hat{L}_i) = \sigma(u; 2) + \text{latt. artif.}$$

lattice continuum



Twisted Polyakov Loop Scheme

G.M.deDivitiis et al., ,Nucl.Phys.B**422**(1994)

Twisted Boundary Condition

$$U_\mu(x+\hat{\nu}L) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger \text{ for } \nu = 1,2$$

with $\Omega_1\Omega_2 = e^{i2\pi/3}\Omega_2\Omega_1,\ \Omega_\mu\Omega_\mu^\dagger = 1,$ single valuedness

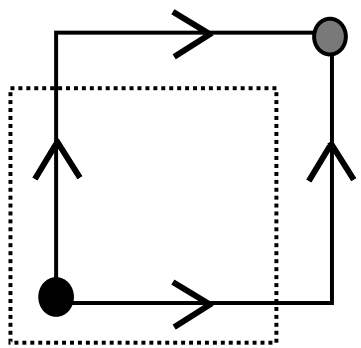
$$(\Omega_\mu)^3 = 1,\ \mathrm{Tr}\left[\Omega_\mu\right] = 0$$ SU(3)

e.g.,

$$\Omega_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad \Omega_2 = \begin{pmatrix} e^{-i2\pi/3} & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Fermion - "smell" degrees of freedom

$$\psi_\alpha^a(x+\hat{\nu}L) = e^{i\pi/3}\Omega_\nu^{ab}\psi_\beta^b(\Omega_\nu)^\dagger_{\beta\alpha}$$



Twisted Polyakov Loop Scheme

Polyakov Loop in Twisted Direction

$$P_x(y,z,t) = \mathrm{Tr}\left(\left[\prod_j U_x\left(x=j,y,z,t\right)\right]\Omega_x \underline{e^{i2\pi y/3L}}\right)$$

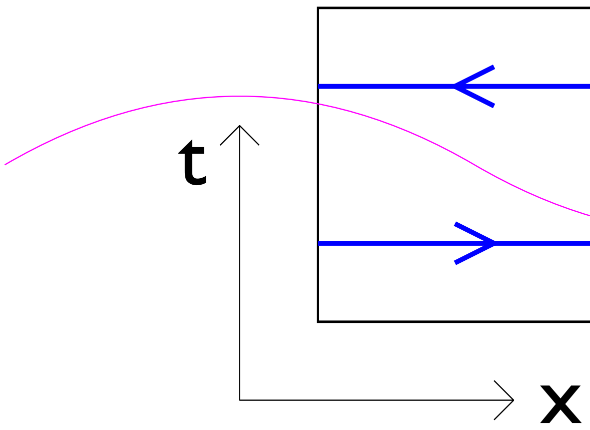
transl. inv.

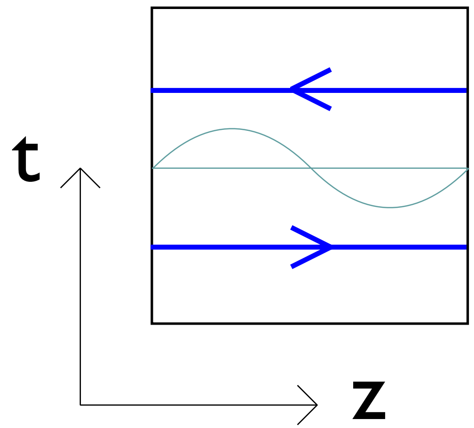
Polykov Loop Untwisted Direction

$$P_z(x,y,t) = \mathrm{Tr}\left[\prod_j U_z\left(x,y,z=j,t\right)\right]$$

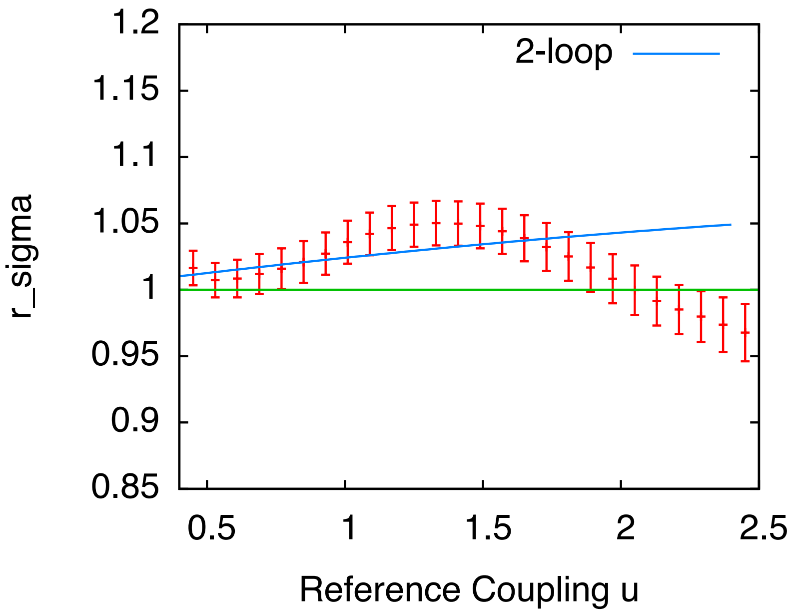
Definition of Coupling

$$R_P(L) \stackrel{\mathrm{def}}{=} \frac{\langle P_x(t=0)^\dagger P_x(t=L/2)\rangle_{L^4}}{\langle P_z(t=0)^\dagger P_z(t=L/2)\rangle_{L^4}} = k\, g^2(L)$$

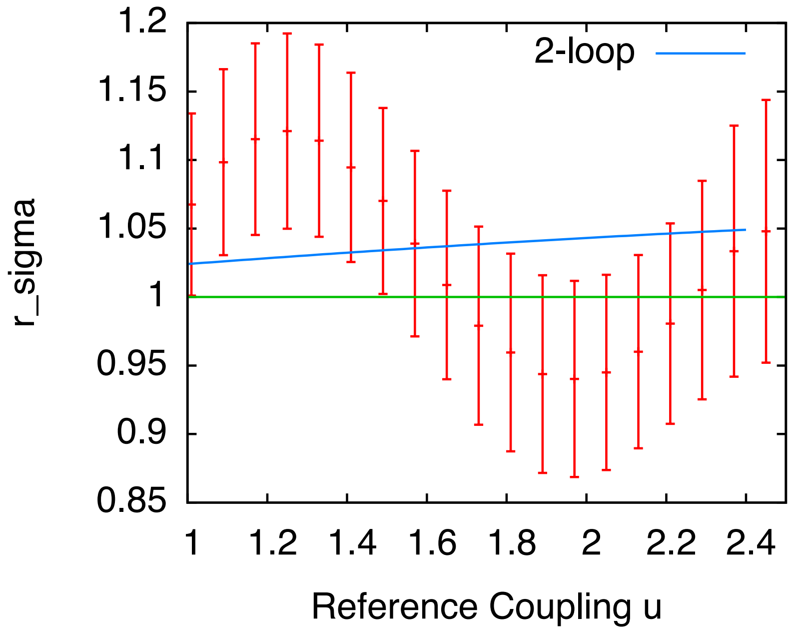




Results From L<=20 Lattice JHEP08(2012)096



Continuum limit
Linear Extrapolation
L=6,8,10



Continuum limit
Quadratic Extrapolation
L=6,(7),8,10
(Volume interpolation used)

Simulation Setting

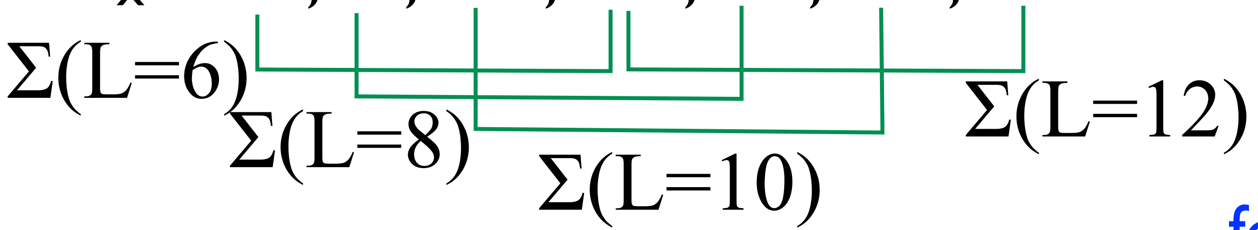
Staggered Fermion (Unimproved),
m_f=0.0, N_f =12 (4 tastes x 3 smells)

Plaquette Gauge Action, Beta = 4 ~100

Beta interpolation non-decreasing polynomial

Hyper Cubic Box, $N_x = N_y = N_z = N_t$

N_x = 6 , 8 , 10 , 12 ,16 , 20 , 24



four Σ(L)'s

Size of Step Scaling s = 2

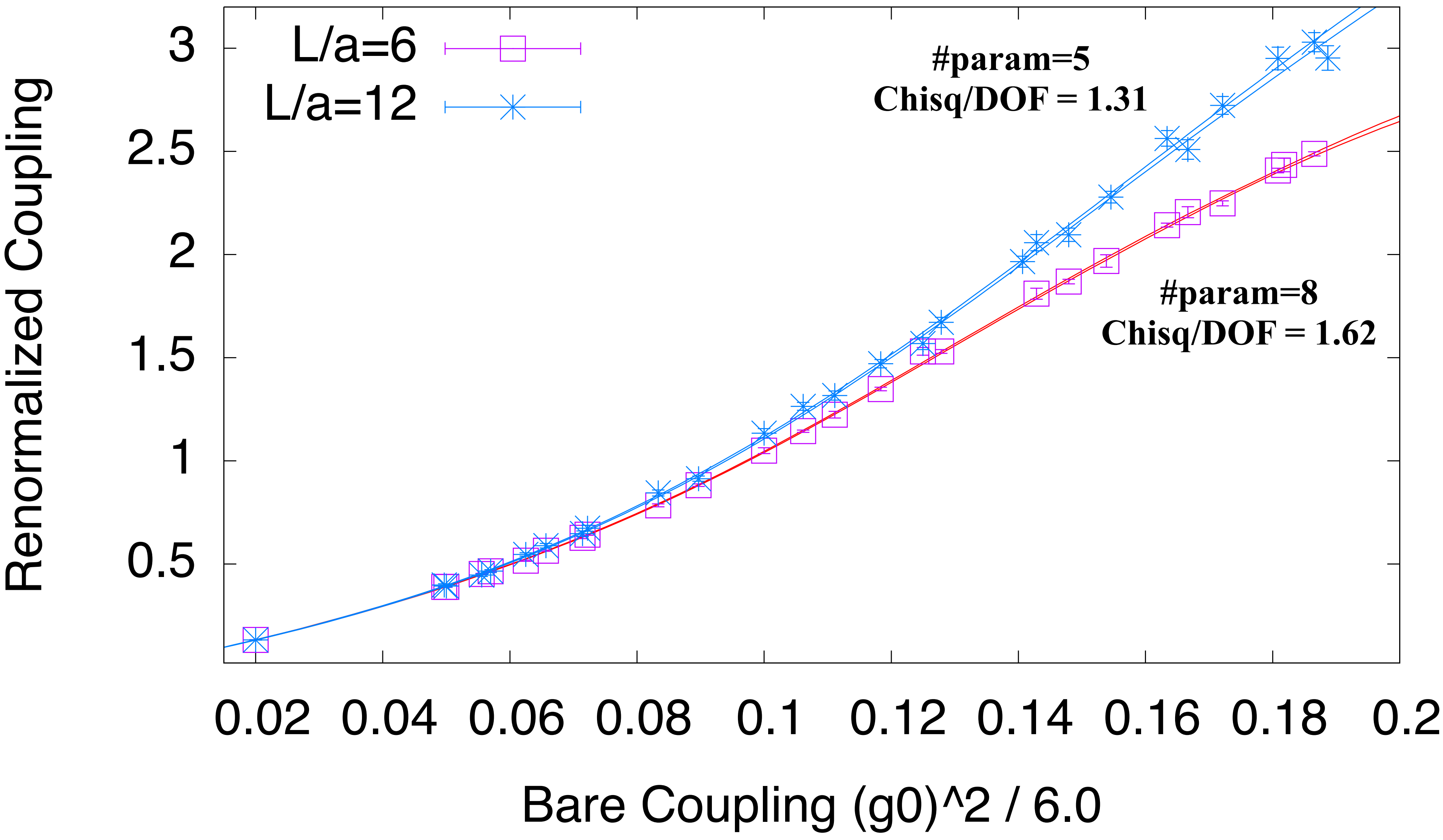
To accumulate L=24 data,
O(100) GPU are devoted for one year
NCHC(Taiwan)

L=24 Data

Beta	g^2	stat. error	#traj.	stat. error x sqrt(#traj)
06.00	2.xx	0.116	1044850	118.573
06.50	2.xx	0.065	541040	47.811
07.00	2.xx	0.050	361180	30.0491
07.50	1.xx	0.045	341540	26.2986
08.00	1.xx	0.046	226090	21.8725
09.00	1.xx	0.033	234150	15.9684
10.00	1.xx	0.037	141910	13.9382
12.00	0.xx	0.024	138340	8.92658
14.00	0.xx	0.018	141790	6.7779
16.00	0.xx	0.015	114780	5.08188
18.00	0.xx	0.013	111870	4.34811
20.00	0.xx	0.011	85310	3.21287

Renormalized Coupling Σ(s=2,L=6)

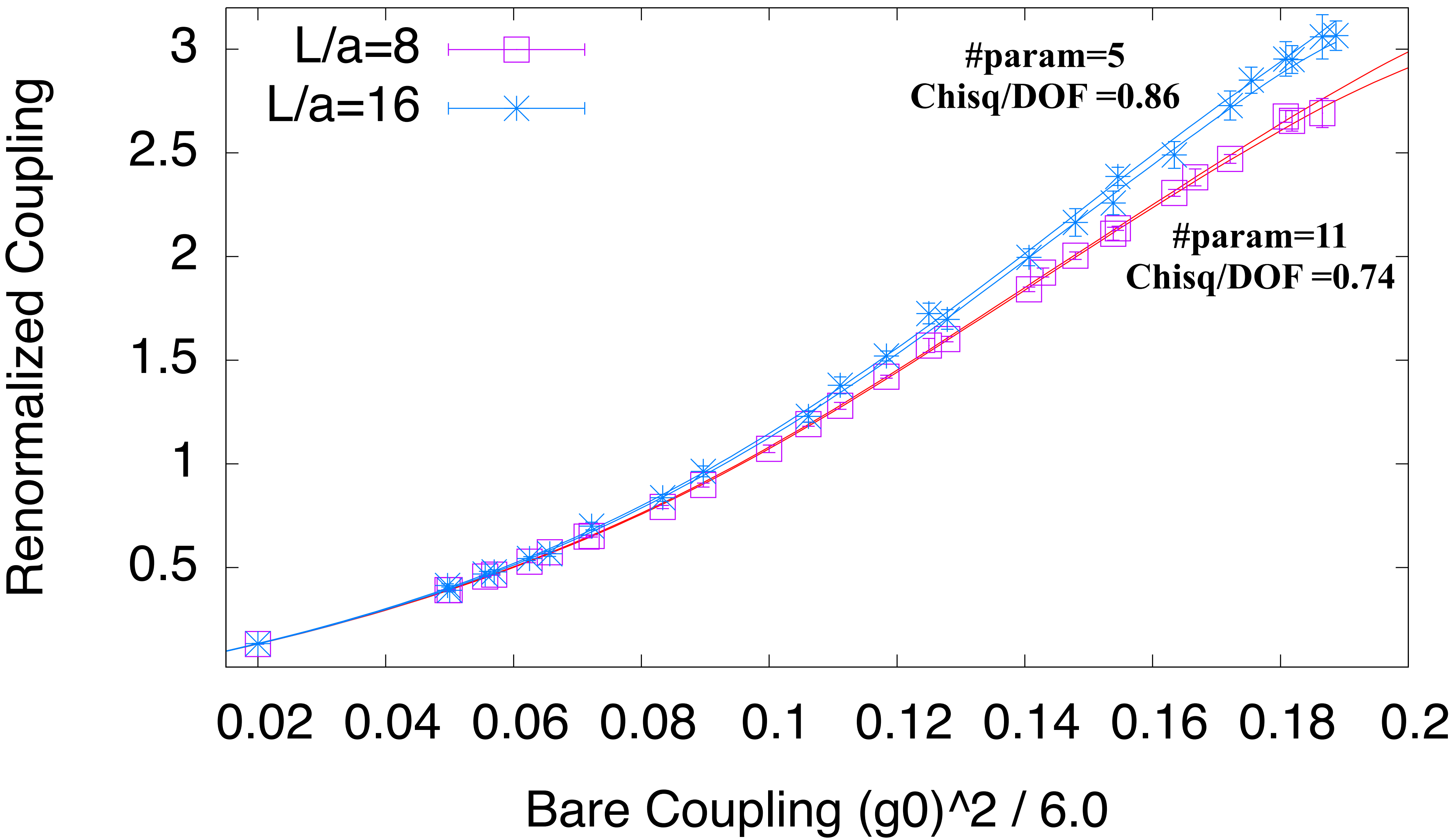
$$u = f(u_0) = \int du_0 \sum_{i=0} \{c_i {u_0}^i\}^2 \quad u = g^2, u_0 = {g_0}^2$$



Renormalized Coupling Σ(s=2,L=8)

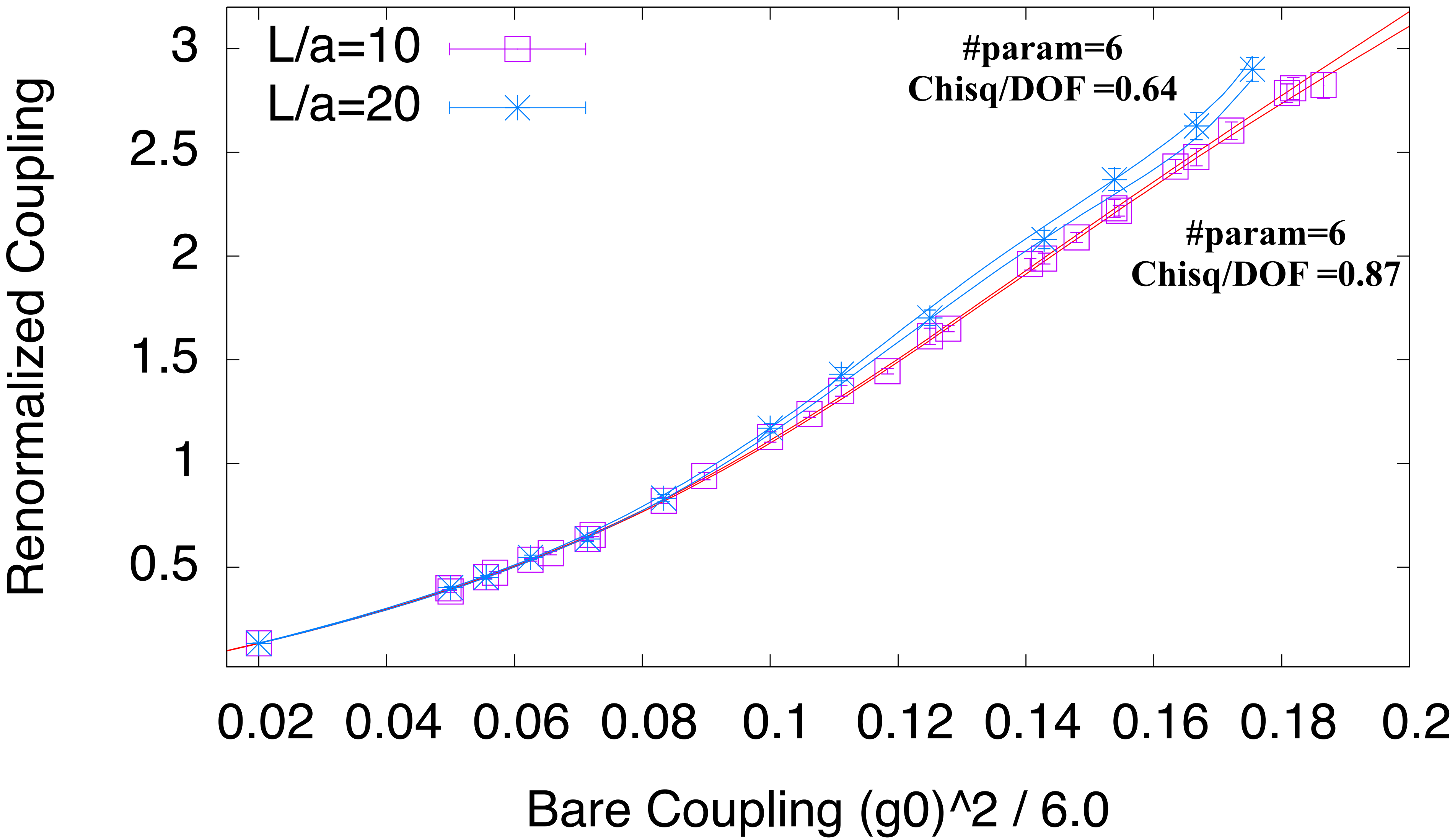
$$u = f(u_0) = \int du_0 \sum_{i=0} \{c_i {u_0}^i\}^2$$

$$u = g^2, u_0 = {g_0}^2$$



Renormalized Coupling Σ(s=2,L=10)

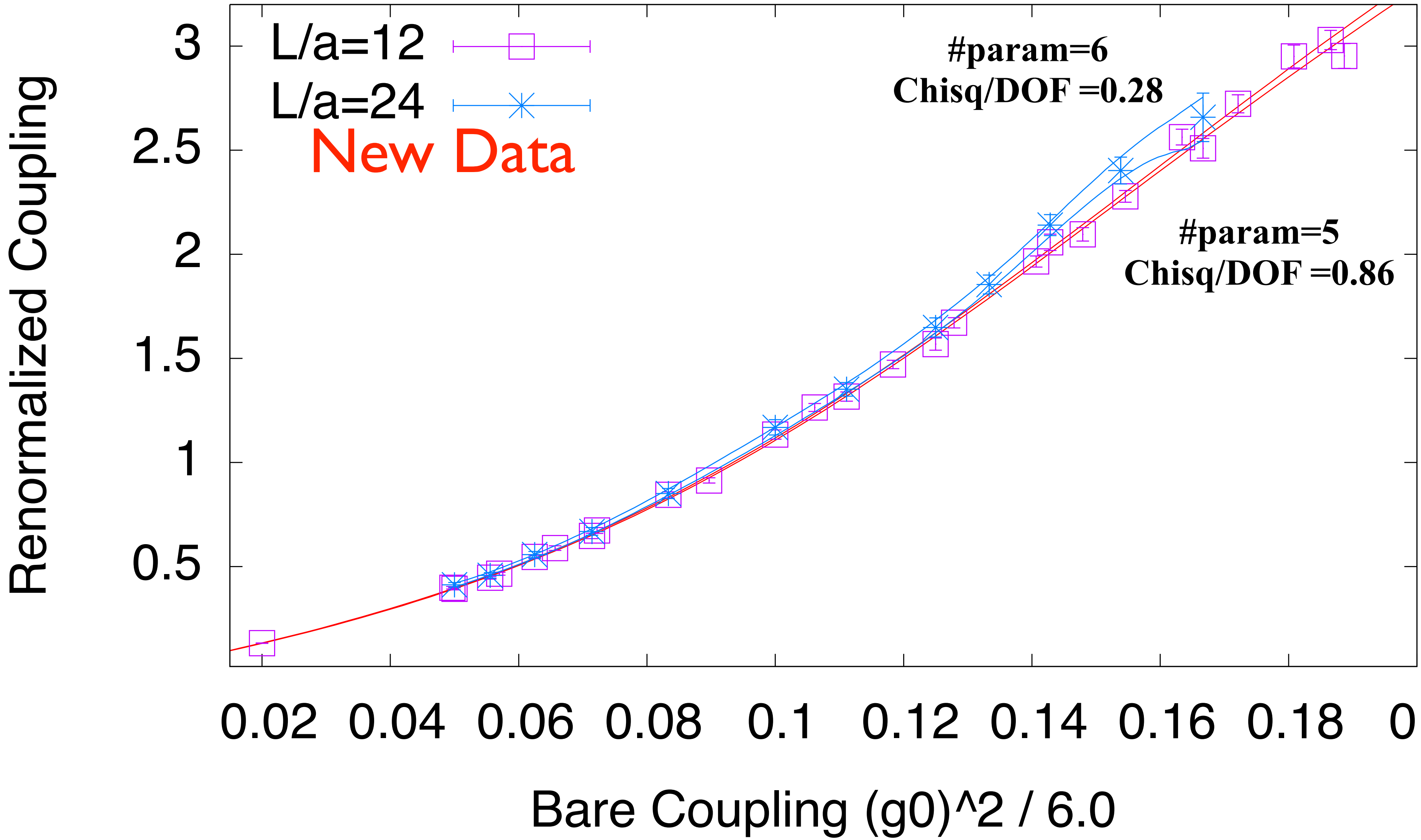
$$u = f(u_0) = \int du_0 \sum_{i=0} \{c_i {u_0}^i\}^2 \quad u = g^2, u_0 = {g_0}^2$$



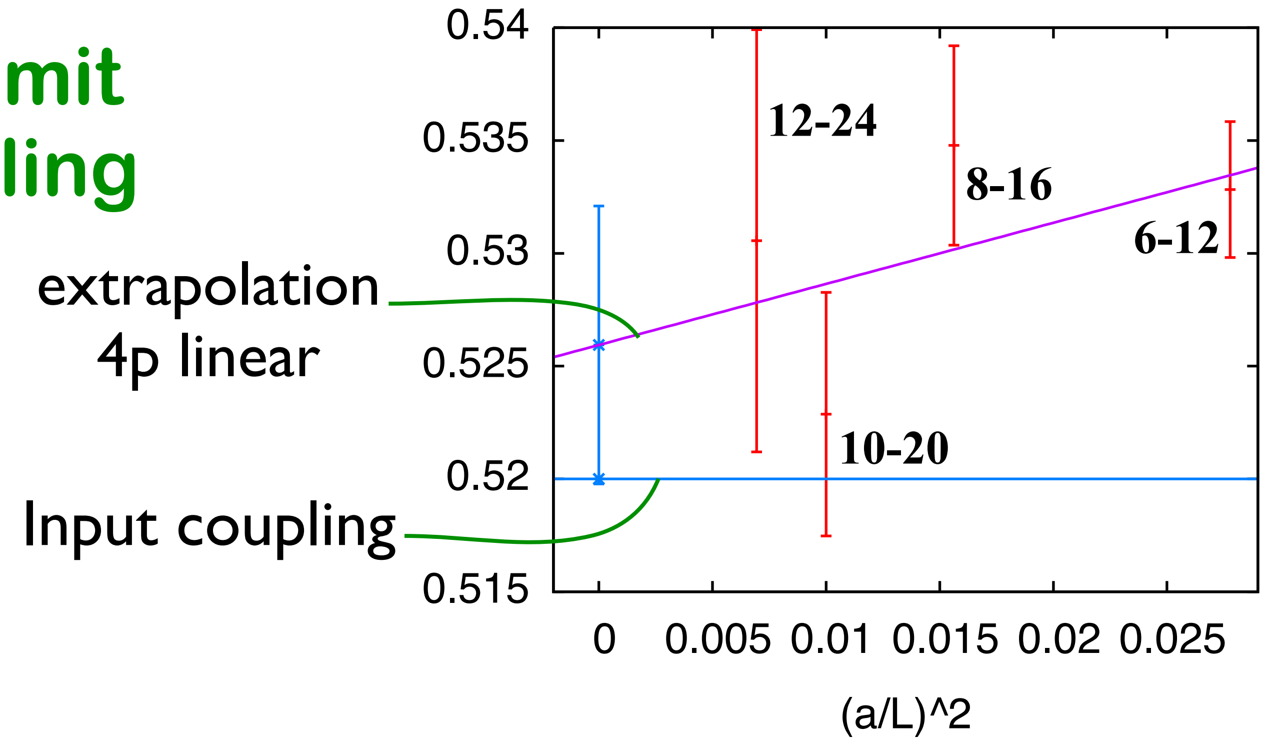
Renormalized Coupling Σ(s=2,L=12)

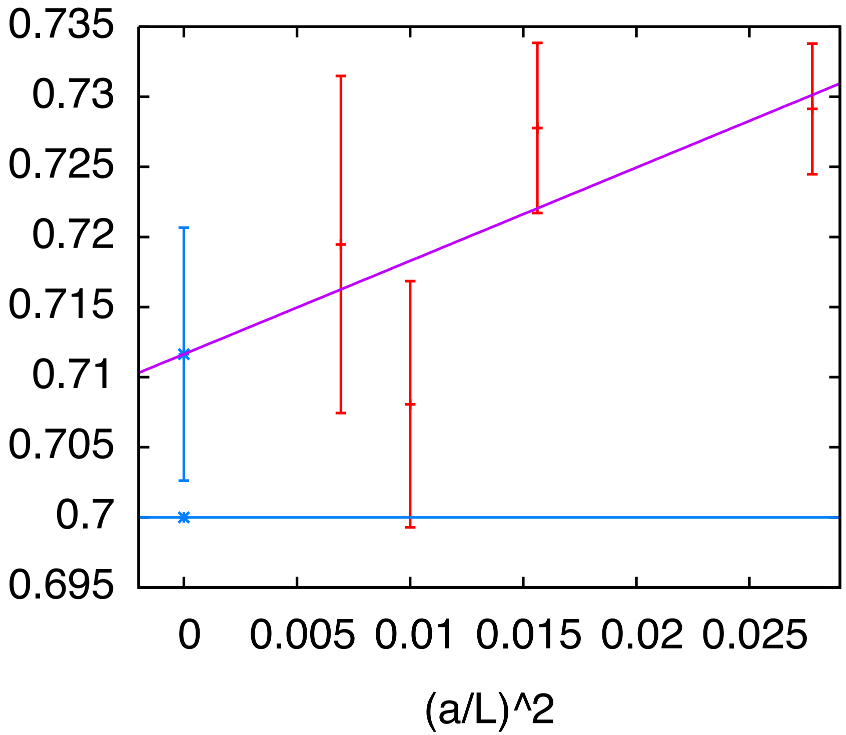
$$u = f(u_0) = \int du_0 \sum_{i=0} \left\{ c_i {u_0}^i \right\}^2$$

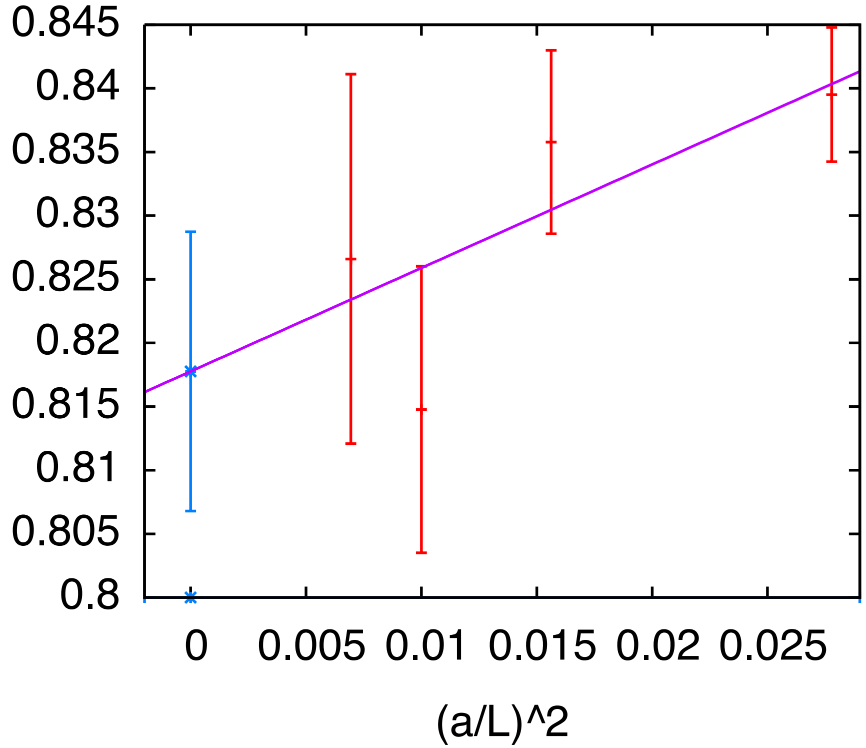
$$u = g^2, u_0 = {g_0}^2$$



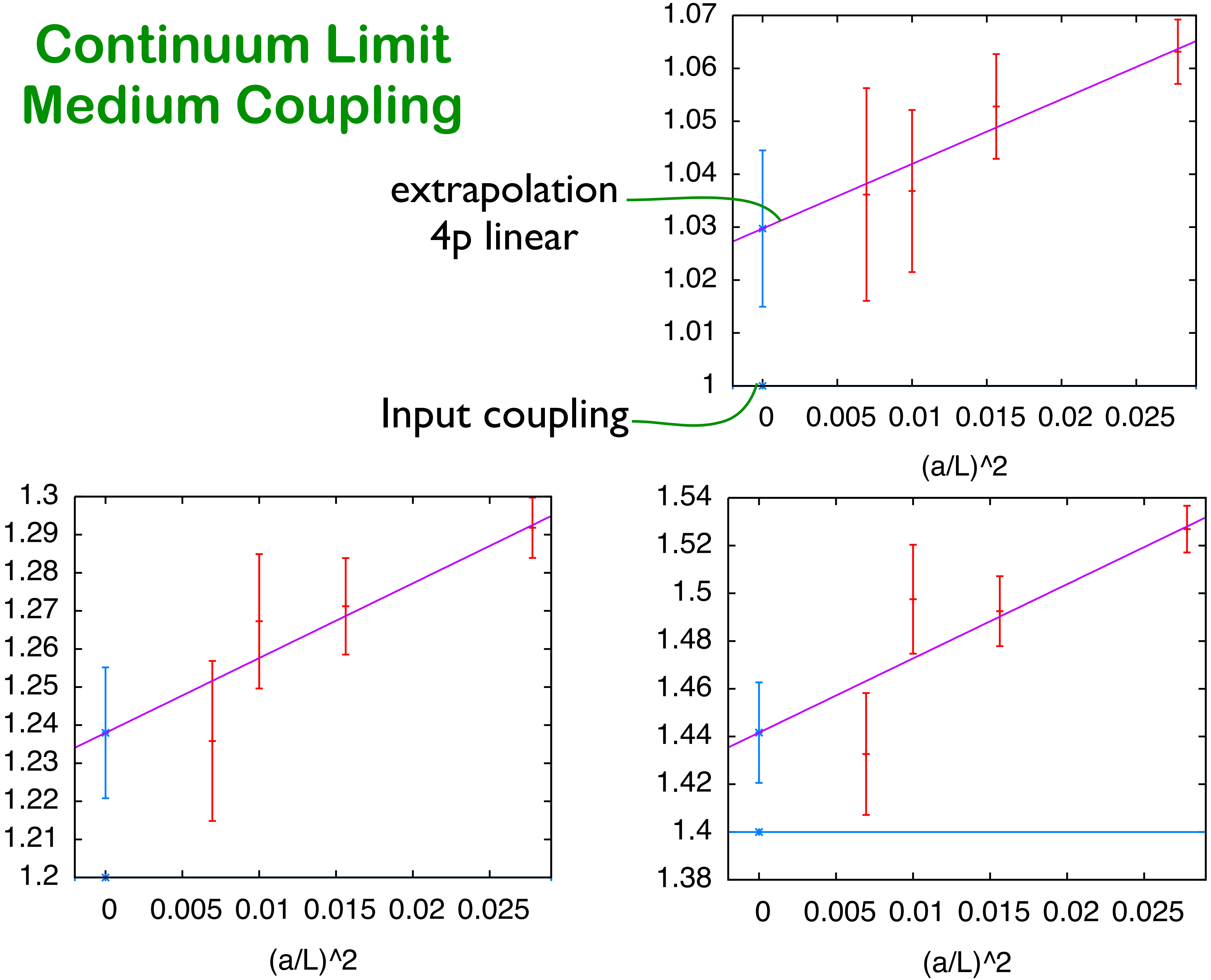
Continuum Limit
Smaller Coupling



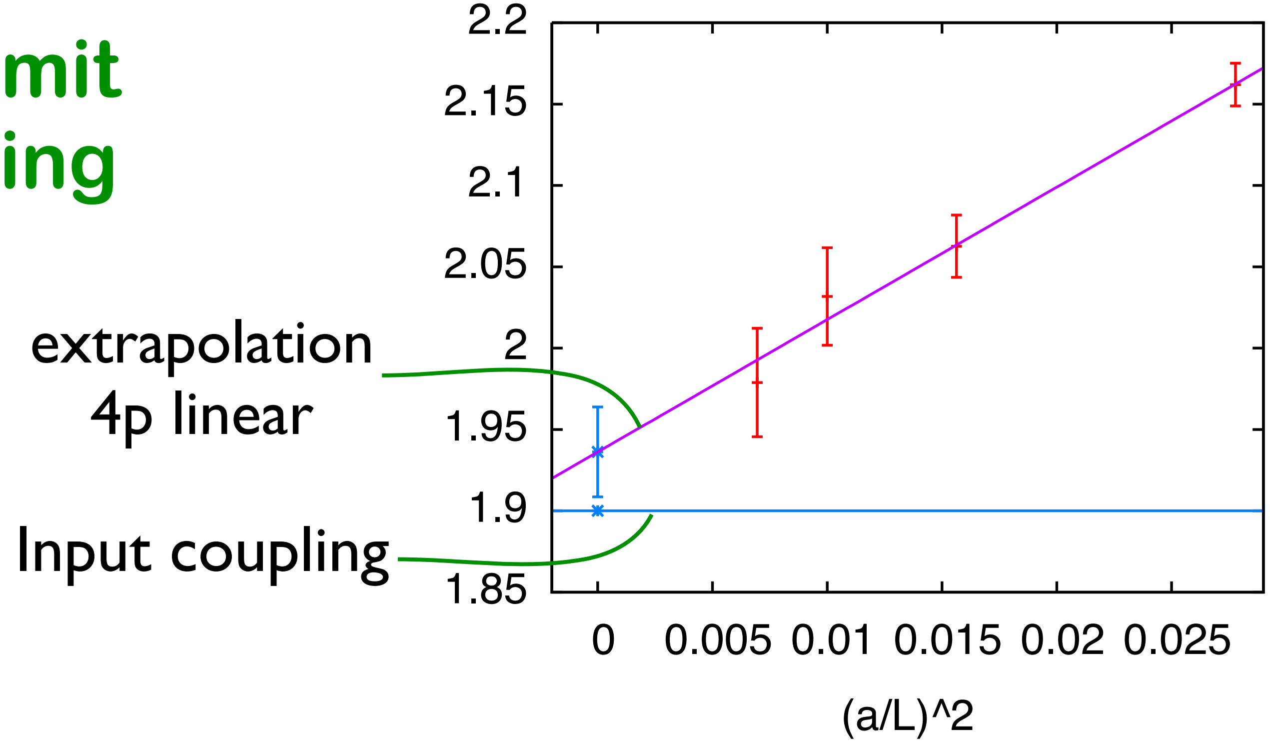


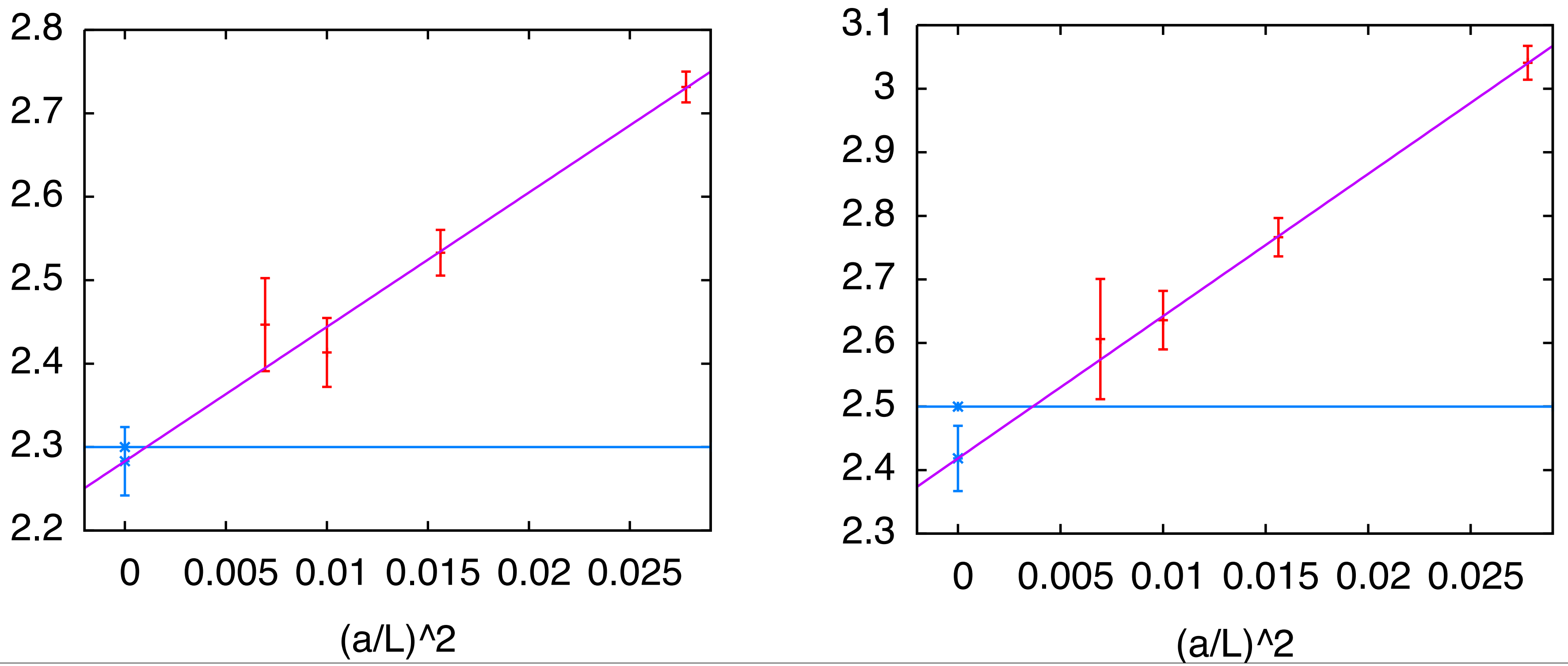


Continuum Limit
Medium Coupling

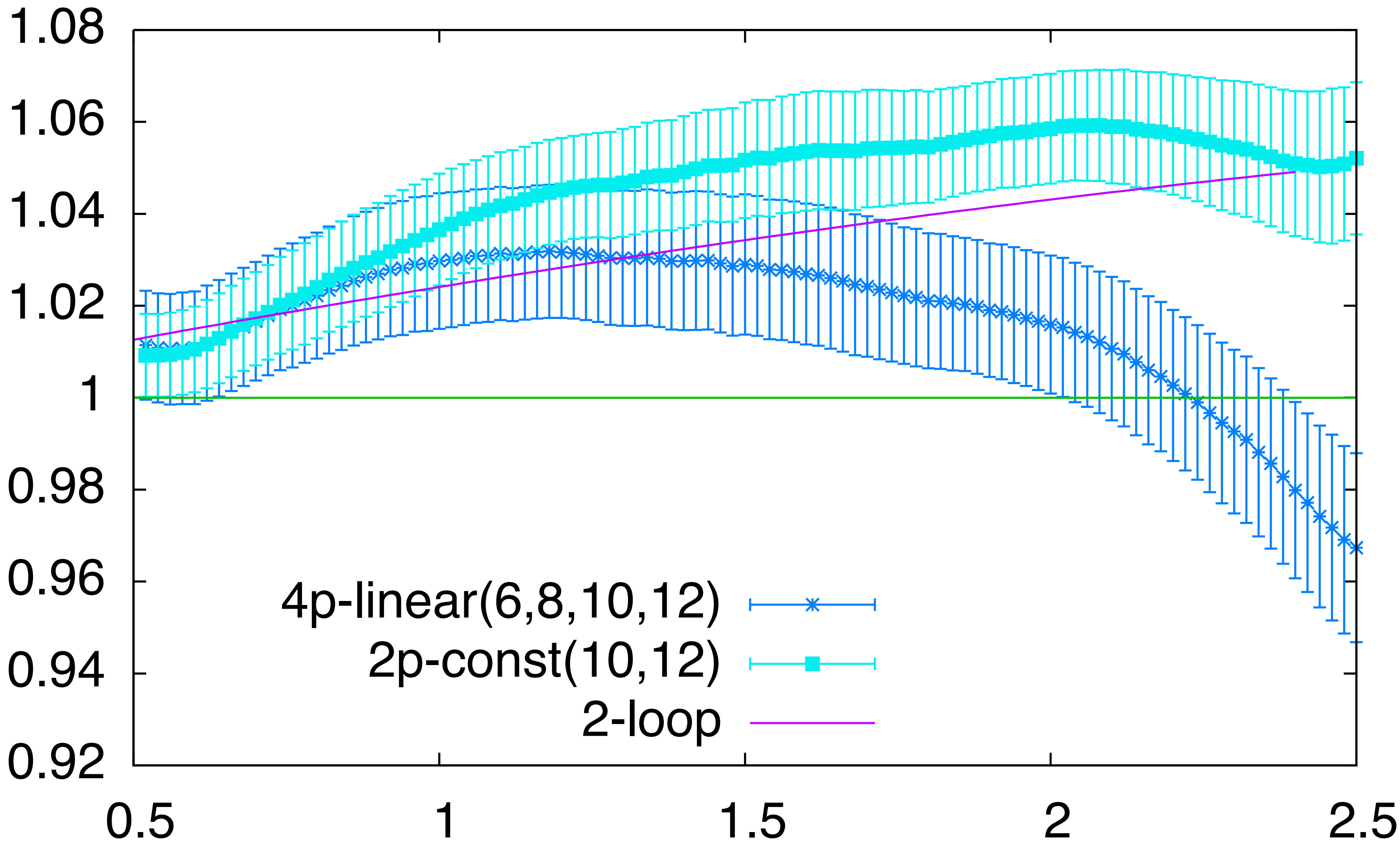


Continuum Limit
Larger Coupling



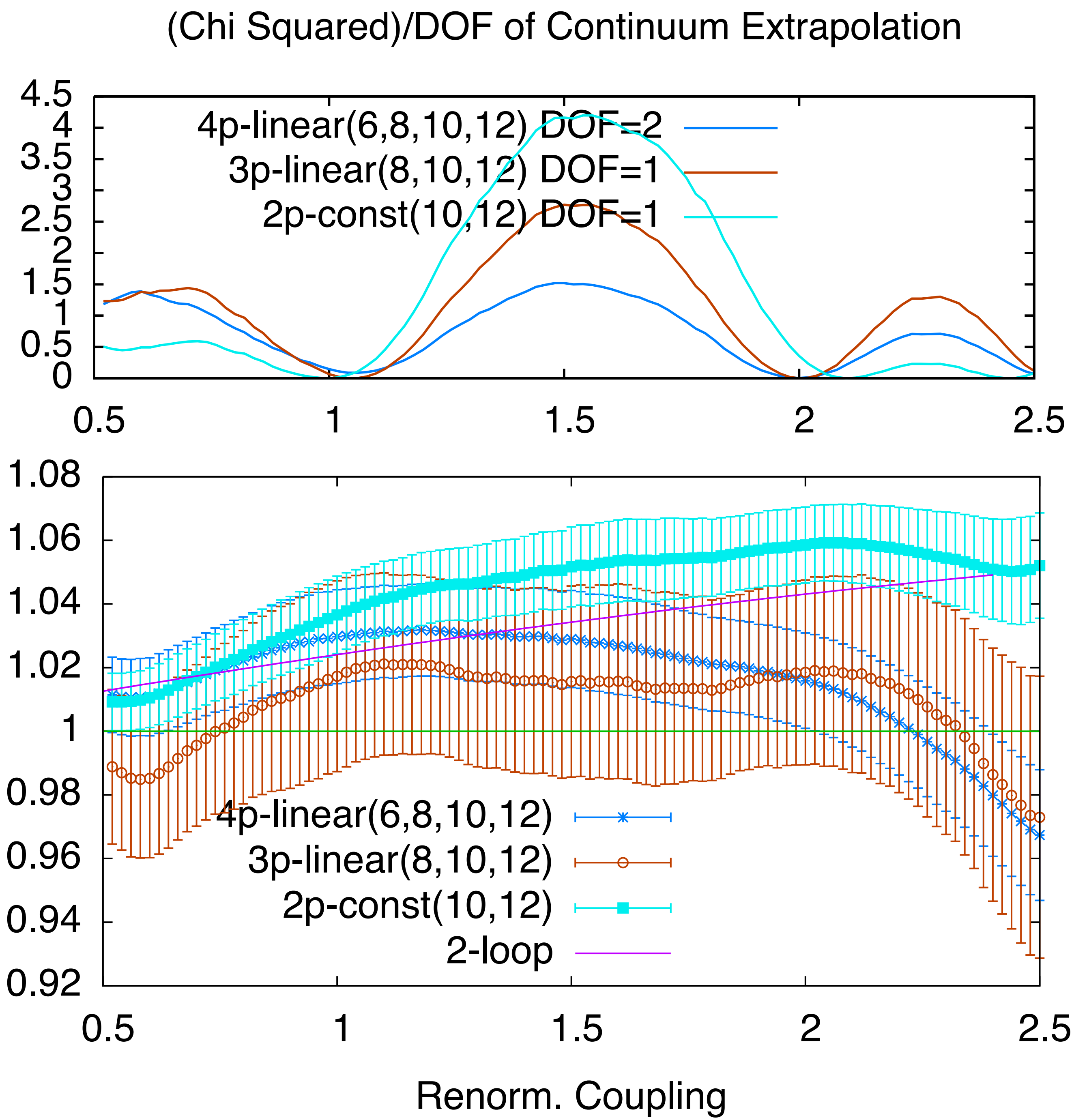


Step Scaling Function s=2



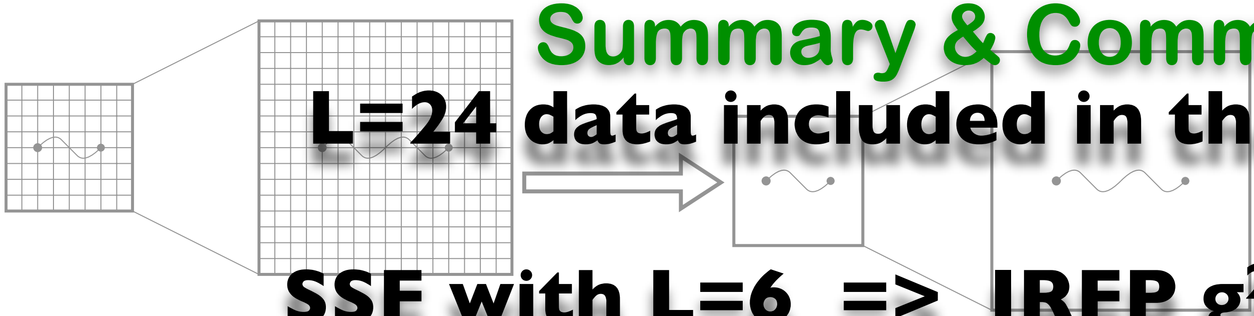
Similar Problem are displayed in **PoS LATTICE2010 (2010) 054** (E Itou, et al)

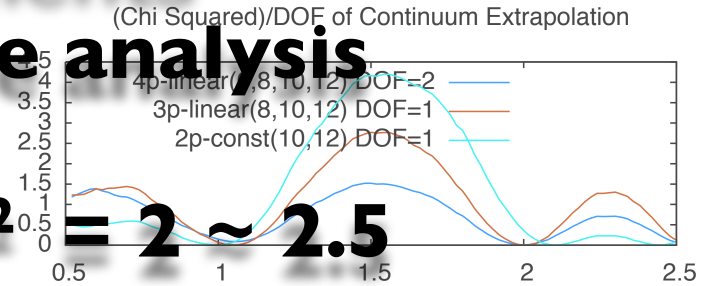
One cannot chose one extrapolation from chi-squared



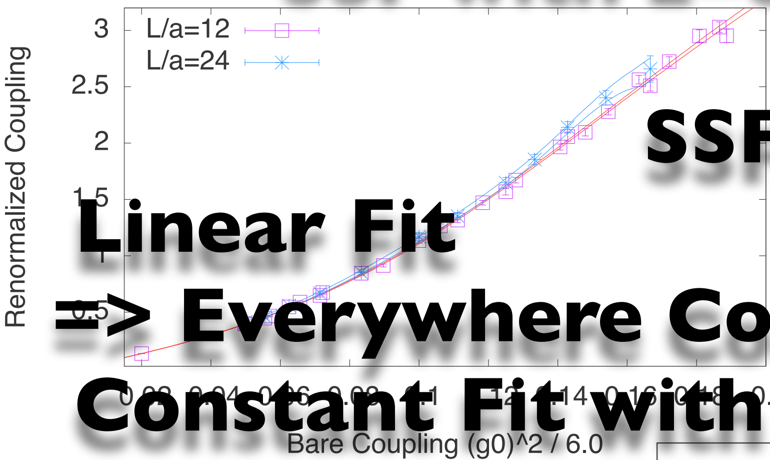
Summary & Comments

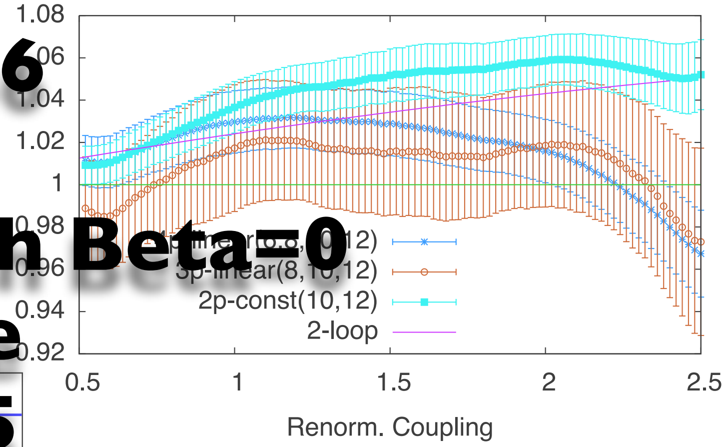
L=24 data included in the analysis





SSF with L=6 => IRFP g^2 = 2 ~ 2.5



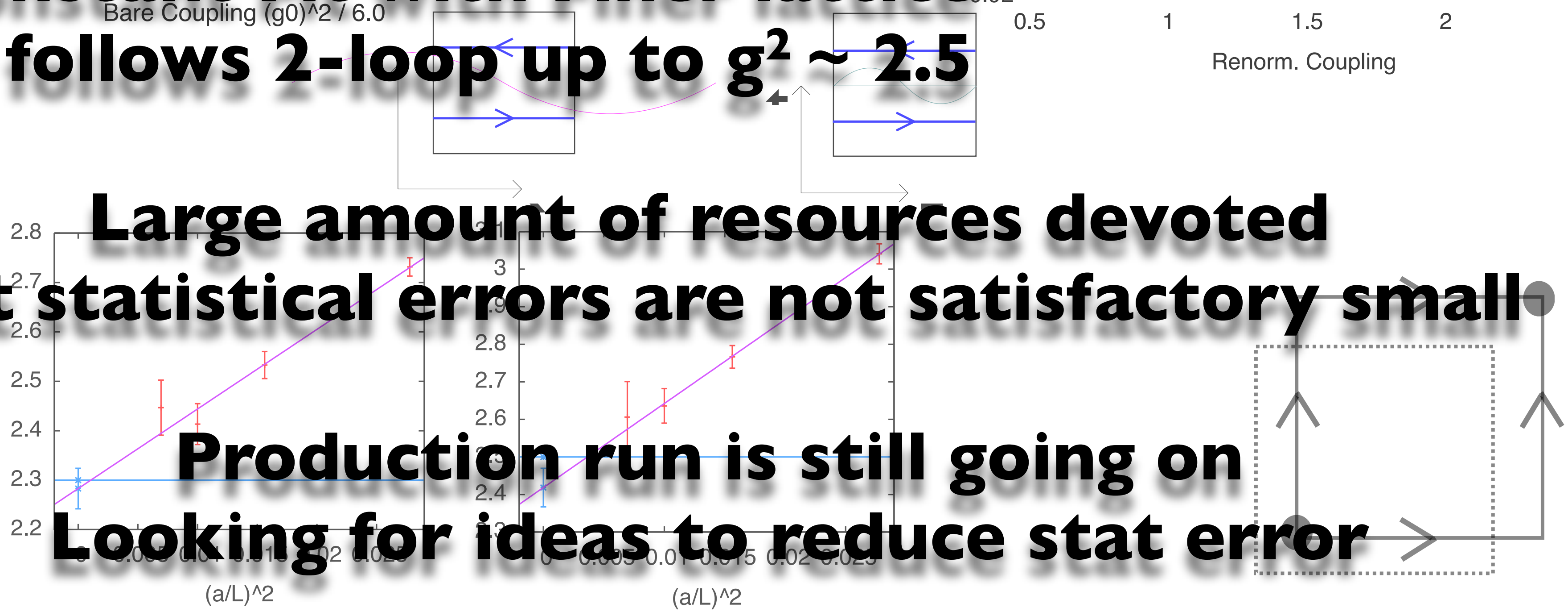


SSF without L=6

Linear Fit
=> Everywhere Consistent with Beta=0

Constant Fit with Finer lattice
=> follows 2-loop up to g^2 ~ 2.5

Large amount of resources devoted
but statistical errors are not satisfactory small



Production run is still going on
Looking for ideas to reduce stat error