Improved Lattice Renormalization Group Techniques

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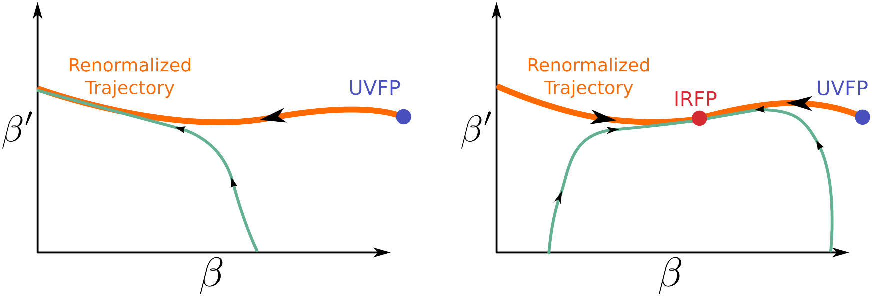
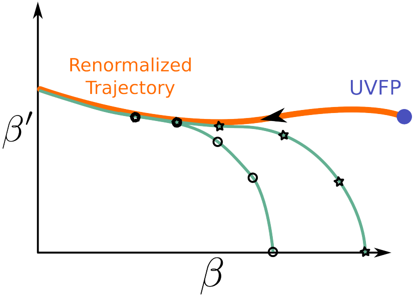
Wilson Renormalization Group

RG transformation integrates out high momentum modes



- *flow* to renormalized trajectory in irrelevant directions
- *flow* along renormalized trajectory in relevant directions
- *flow* away from ultraviolet fixed points
- *flow* to infrared fixed points

Monte Carlo Renormalization Group



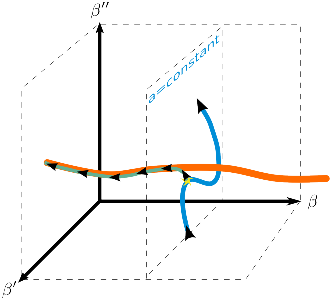
- block (integrate out UV)
  - $a \to 2a$
  - $\Lambda_{\text{cut}} \to \frac{\Lambda_{\text{cut}}}{2}$
- match $(\beta_1, n_b) \equiv (\beta_2, n_{b-1})$
- optimize

Two ways of Optimizing

- Traditional Weakness of MCRG
  1. finite number of blocking steps not enough to reach RT
  2. optimization of block transformation to reach RT faster leads to ‘composite’ step scaling function
- Solution: optimization of Wilson flow before blocking
  1. reaches RT quickly
  2. get a unique step scaling function

Optimization with Wilson Flow

Wilson flow integrates infinitesimal smearing steps.



- Wilson flow removes UV fluctuation
- Wilson flow does not change lattice spacing
- Moves system toward the RT
- Proceed with MCRG

SU(3) $N_f = 12$

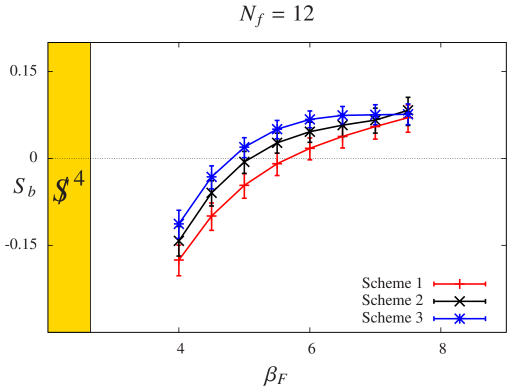
- fermion mass exactly zero
- APBC lattices
- fundamental adjoint gauge action
- nHYP smeared staggered fermions

Volume	Bare Coupling (β_F)
6^4	3.4, 3.6, ... 7.8, 8.0
8^4	3.0, 3.2, ... 7.8, 8.0
12^4	3.4, 3.6, ... 7.8, 8.0
16^4	3.0, 3.2, ... 7.8, 8.0
24^4	4.0, 4.5, ... 7.5, 8.0
32^4	4.0, 6.0
48^4	6.0

6^4 - 12^4 - 24^4 Matching

Errors indicate the spread in s_b predicted by each observable.

- nHYP block transformation
- $n_b = 3$
- $V_f = 3^4$



Scheme 1: 0.6 0.2 0.2 Scheme 2: 0.6 0.3 0.2

Scheme 3: 0.65 0.3 0.2

6^4 - 12^4 - 24^4 Matching

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- nHYP block transformation
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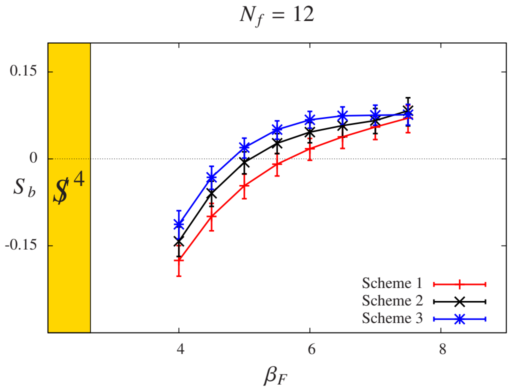
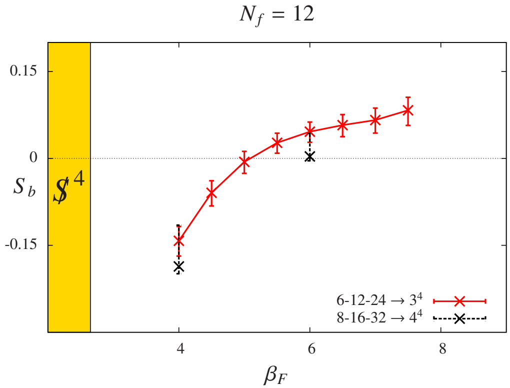


Now let's look at Scheme 2 and change n_b, V_f.

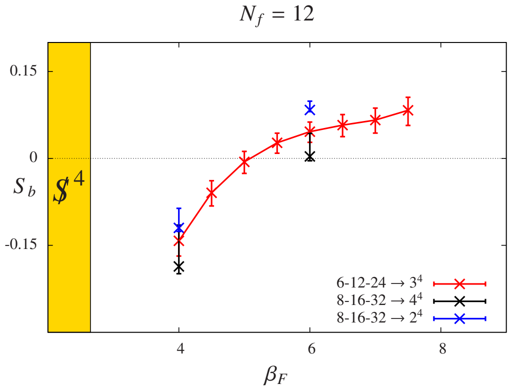
8^4 - 16^4 - 32^4 Matching

- Scheme 2
- $n_b = 3$
  $V_f = 4^4$



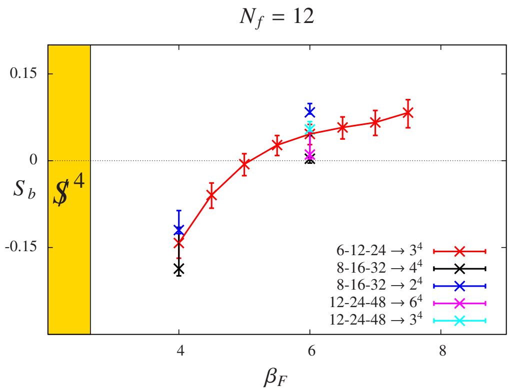
8^4 - 16^4 - 32^4 Matching

- Scheme 2
- $n_b = 3$
  $V_f = 4^4$
- $n_b = 4$
  $V_f = 2^4$



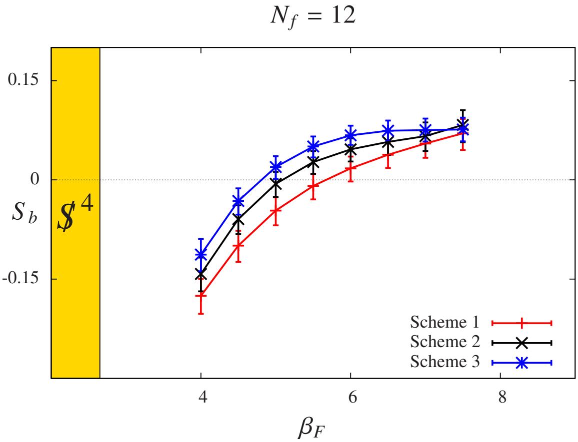
12^4 - 24^4 - 48^4 Matching

- Scheme 2
- $n_b = 3$
  $V_f = 4^4$
- $n_b = 4$
  $V_f = 2^4$
- $n_b = 3$
  $V_f = 6^4$
- $n_b = 4$
  $V_f = 3^4$



Gauge Coupling Scaling Dimension y_g

The slope of the step scaling function at the fixed point is related to y_g.



$y_g = -0.11(4)$

Another Approach

New RG approaches have been proposed that use Wilson Flow to find the renormalized step scaling function.

MCRG

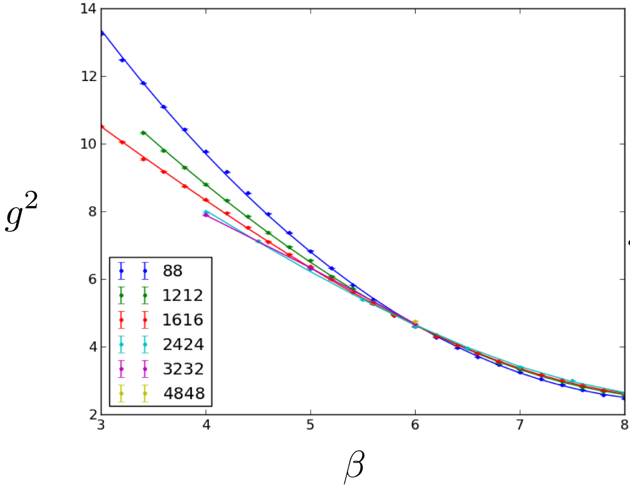
- does not depend on perturbation theory
- does not require special b.c.
- requires a scale factor of 2

Wilson Flow

- relies on perturbation theory
- does not require special b.c.
- does not require a scale factor of 2

Fodor, Holland, Kuti, Nogradi, Wong: arXiv:1208.1051
Fritzsch, Ramos: arXiv: 1301.4388

Wilson Flow - Preliminary



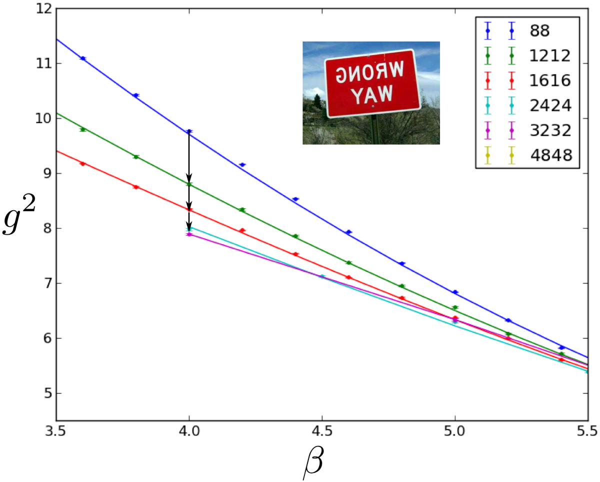
$c = \frac{\sqrt{8t}}{L} = 0.3$

$g_c^2 = \frac{128\pi^2 \langle t^2 E(t) \rangle}{3(N^2-1)(1+\delta_c)}$

$1 + \delta_c \approx 0.97$

Continuum Extrapolation - Preliminary

Very different from asymptotic freedom.

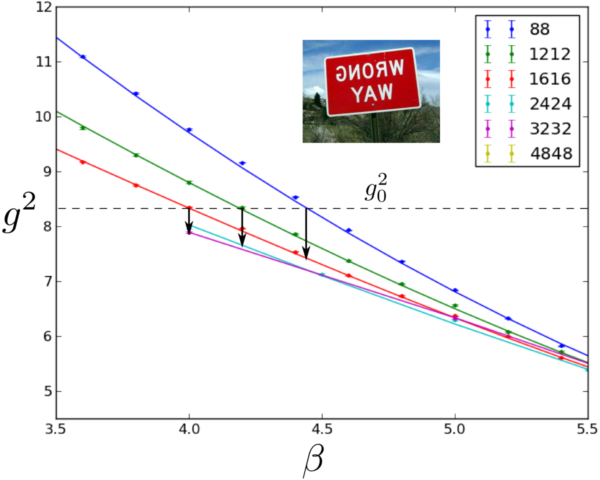


at fixed β:

$g^2(L)$ decreases as L increases

Continuum Extrapolation - Preliminary

This is not $\beta = \infty$ continuum limit.



fix $g^2(L) = g_0^2$

compare
$g^2(2L) - g^2(L)$

$L \to \infty$
requires
decreasing β

Conclusions

- New work on an established method
- With Wilson flow MCRG we can find a unique step scaling function
- Wilson flow MCRG predicts an IRFP and $y_g = -0.11(4)$
- Wilson flow MCRG is computationally inexpensive; does not rely on perturbation theory

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