Lattice 2013
July 30, 2013

Two-Nucleon Systems in a Finite Volume

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In collaboration with:
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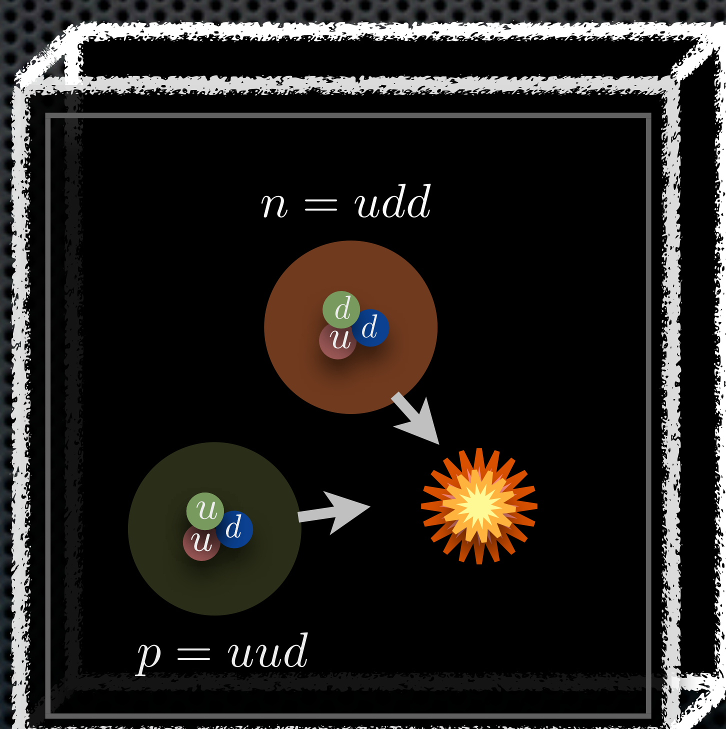
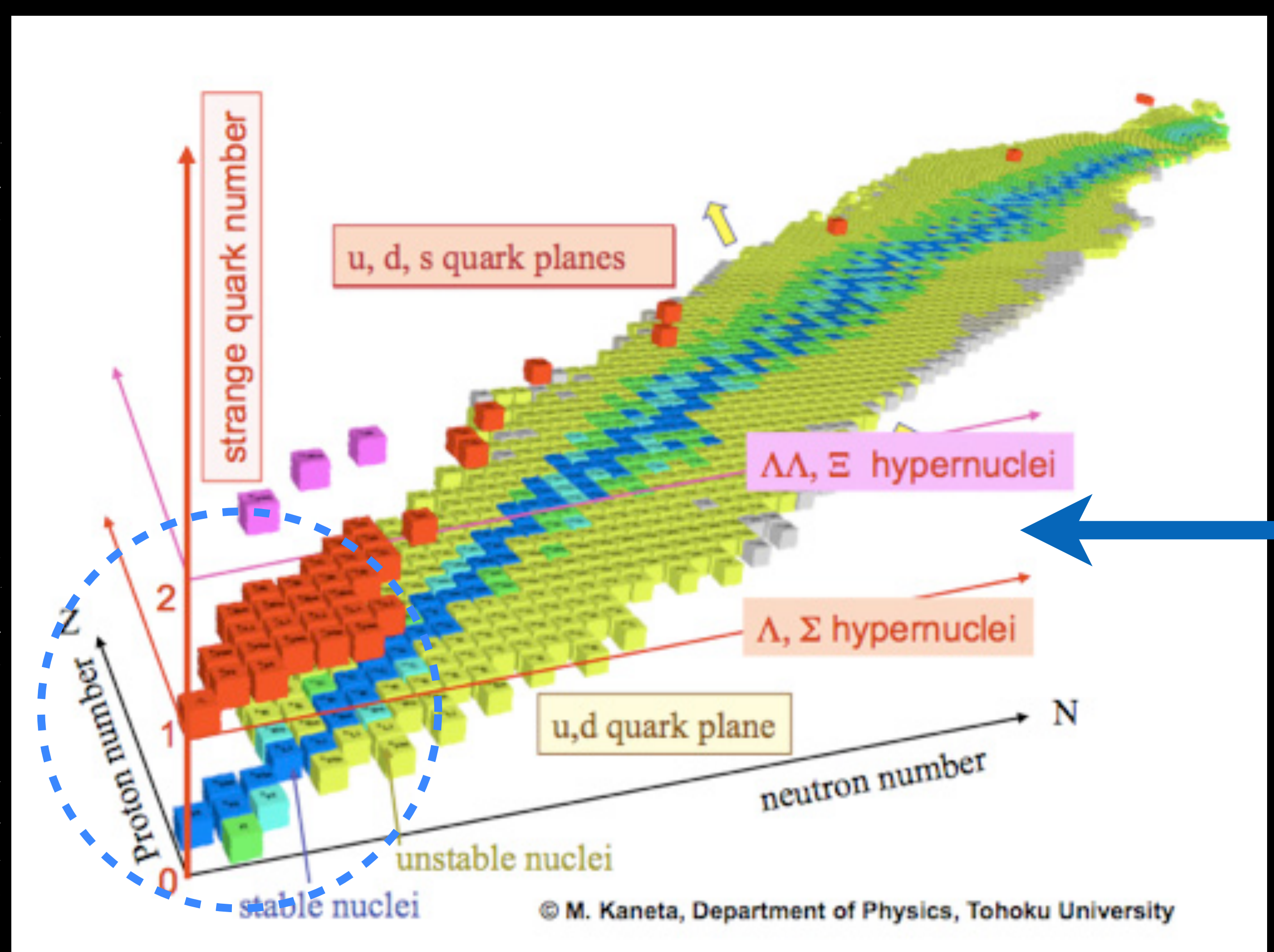
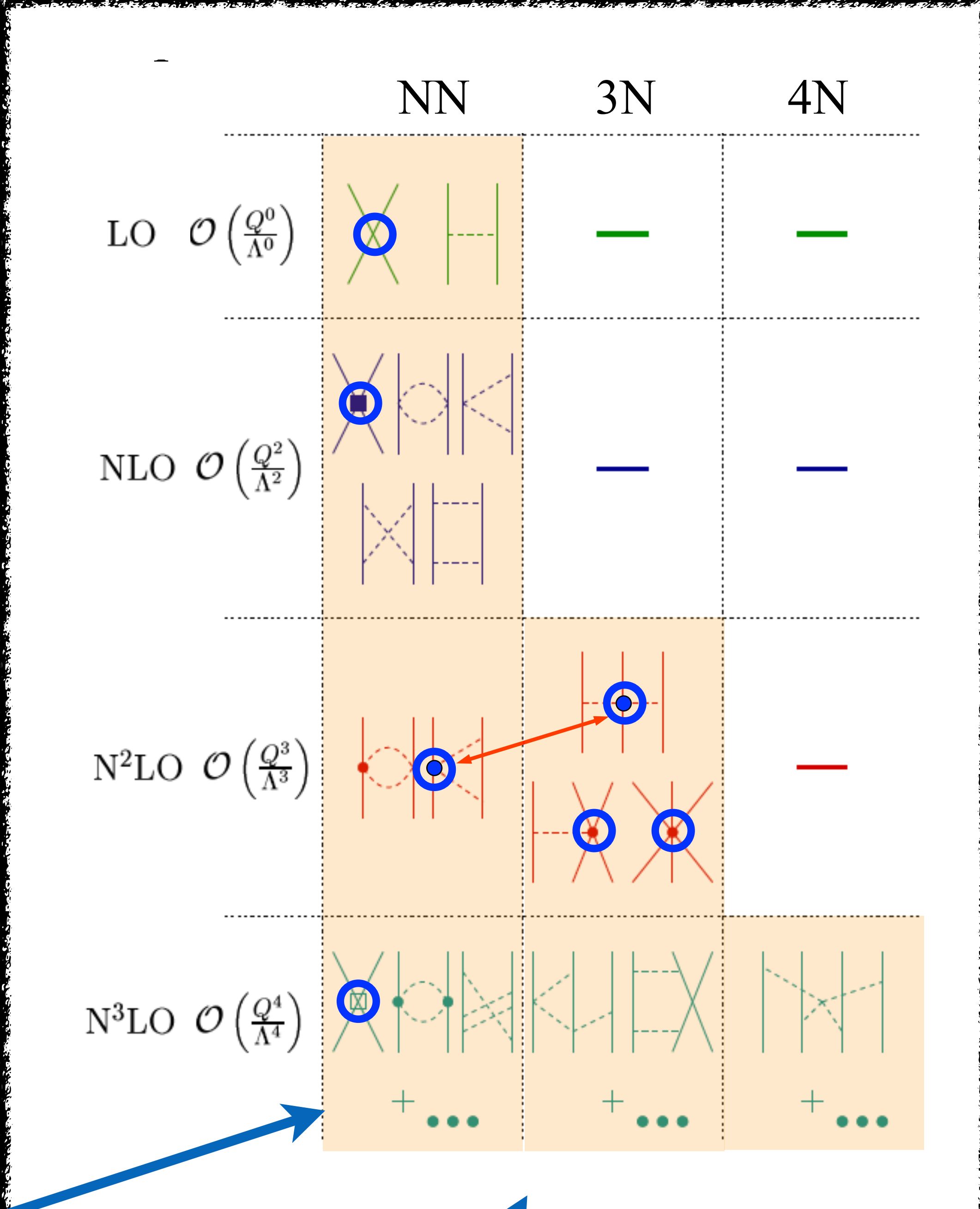
Tom Luu
Martin Savage



From QCD to Nuclear Physics



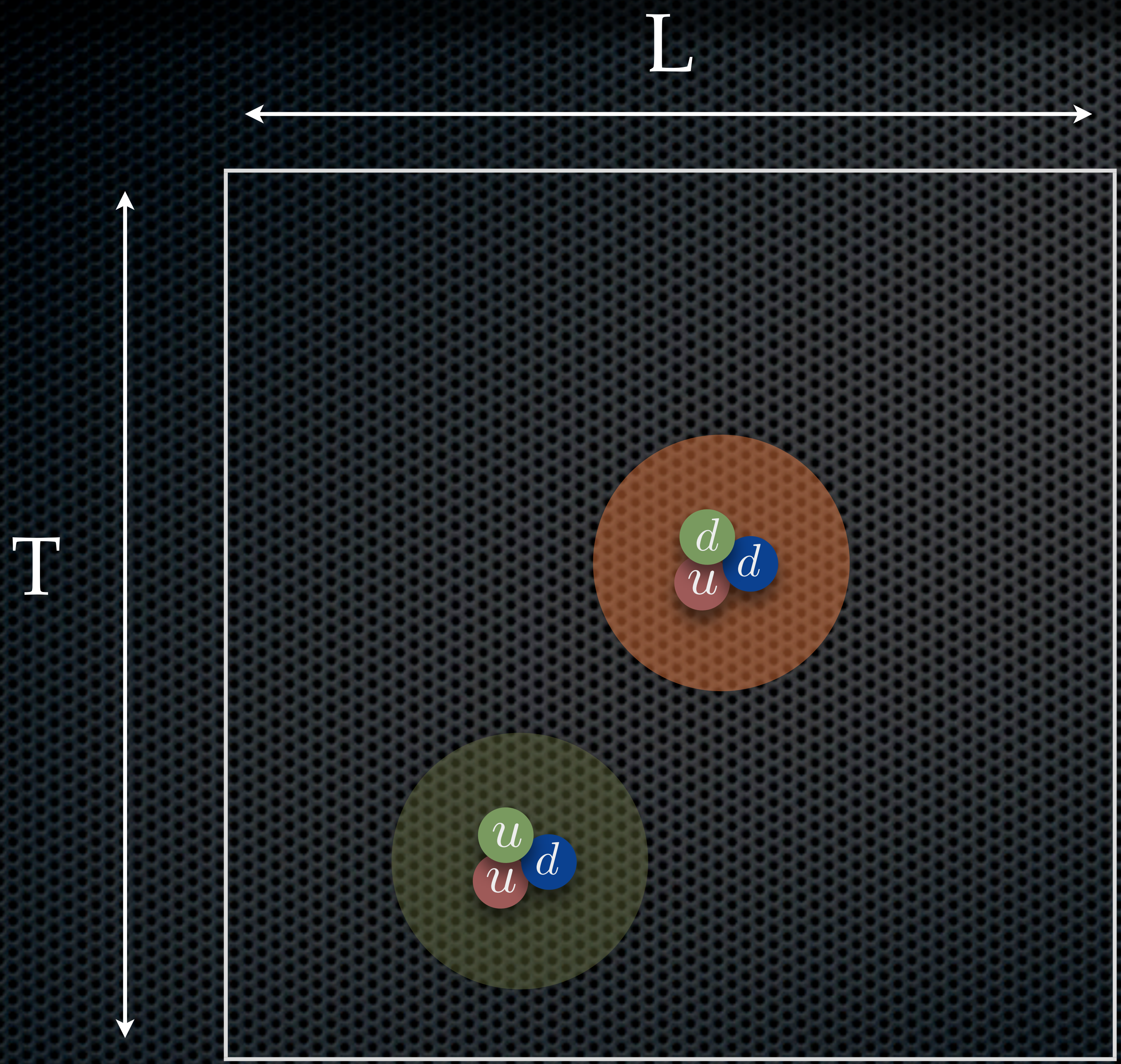


LQCD + EFT

More parameters: m_u, m_d, m_s

Complements experiment

LQCD Finite Euclidean Spacetime



neutron-proton in a 4D torus

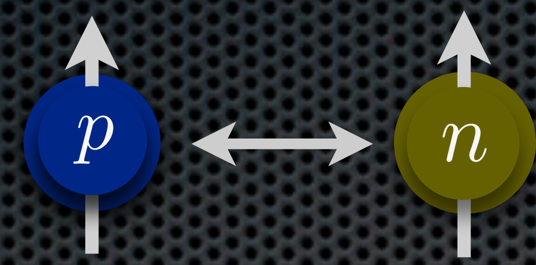
$a \to 0$

$\mathrm{T} \to \infty$

- Discretized momentum: $p = \frac{2\pi n}{L}$
- Maiani & Testa (1990):
  No-go theorem
- Lüscher (1991) : scalar bosons
  $E_L, m \to \delta(q^*)$
- Beane *et al.* (2003):
  S-wave NN-system (CM-frame)
- Ishizuka (2009):
  NN-system (CM-frame)

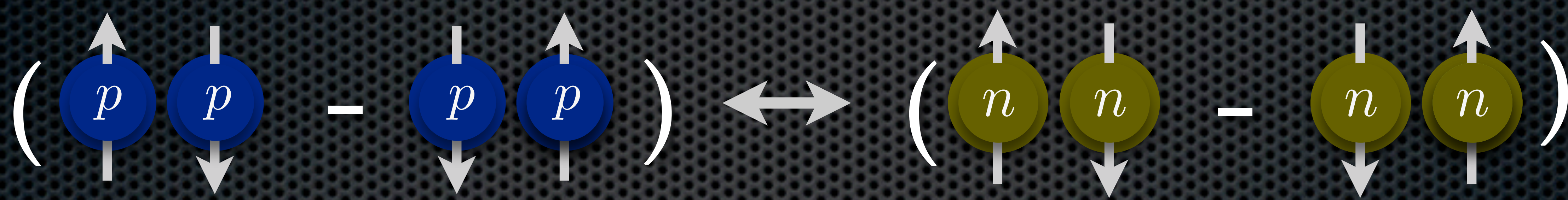
NN-Systems

- Exact Isospin symmetry

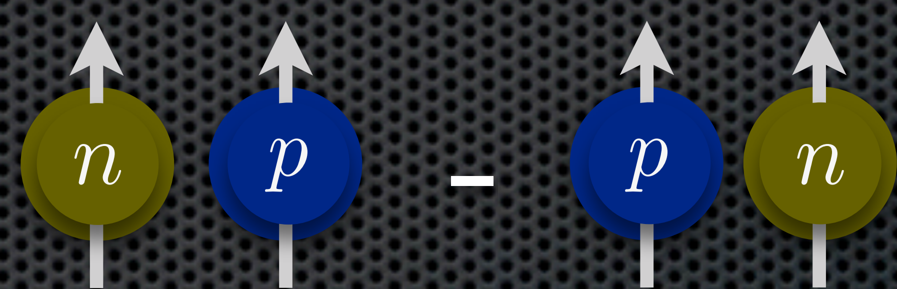


$m_N \sim 940$ MeV

- L=2n, I=1, S=0 :



- L=2n, I=0, S=1:



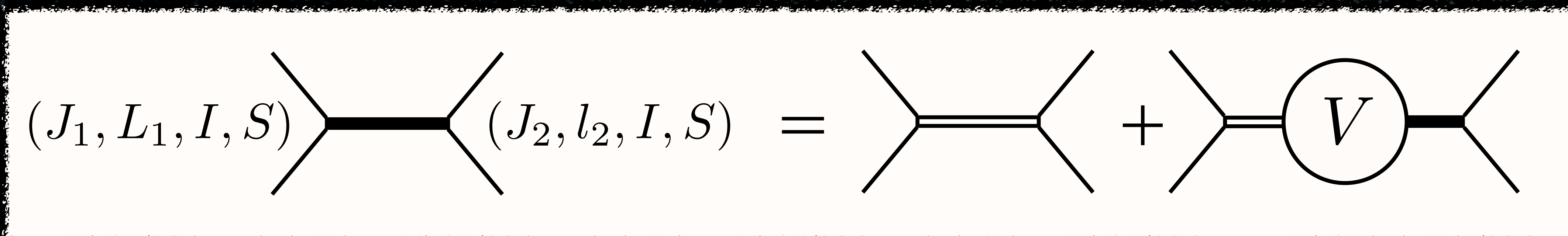
Deuteron channel ($^3S_1$-$^3D_1$)

$$S_1 = \begin{pmatrix} c_{\epsilon_1} & -s_{\epsilon_1} \\ s_{\epsilon_1} & c_{\epsilon_1} \end{pmatrix} \begin{pmatrix} e^{i2\delta^{(^3S_1)}} & 0 \\ 0 & e^{i2\delta^{(^3D_1)}} \end{pmatrix} \begin{pmatrix} c_{\epsilon_1} & s_{\epsilon_1} \\ -s_{\epsilon_1} & c_{\epsilon_1} \end{pmatrix}$$

$$c_{\epsilon_1} = \cos(\epsilon_1)$$
$$s_{\epsilon_1} = \sin(\epsilon_1)$$

- Can also have (L=2n+1, I=0, S=0), (L=2n+1, I=1, S=1)

Quantization Condition



$$\det\left(\left(\mathcal{M}^{\infty}\right)^{-1} + \delta\mathcal{G}^{V}\right) = 0$$

Scattering amplitude
Diagonal in J-basis
mixes l-states

Diagonal in
Spin & Isospin

Kinematic function of (L, E_L)
Mixes angular momentum

Diagonal in
Spin & Isospin

arXiv:1305.4903 [hep-lat] (accepted in PRD):
RB, Zohreh Davoudi (UW) & Tom Luu (LLNL)

Clebsch–Gordan coefficients matter!

Boosts & Symmetry

- O : Cubic

  $\mathbf{d} = (2n_1, 2n_2, 2n_3)$

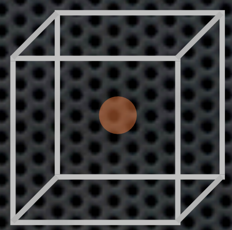
- D_4 : Tetragonal

  $\mathbf{d} = (2n_1, 2n_2, 2n_3+1)$

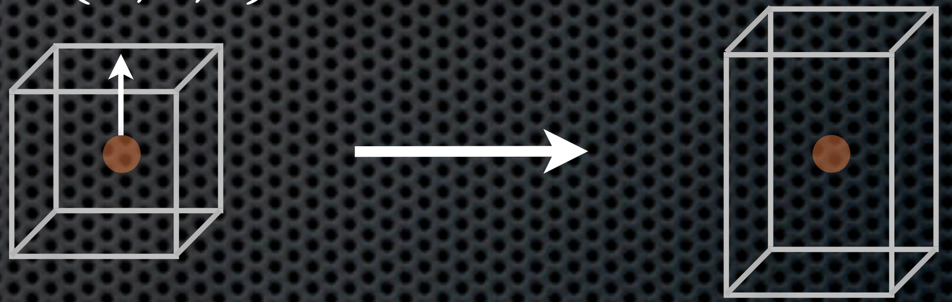
- D_2 : Orthorhombic

  $\mathbf{d} = (2n_1+1, 2n_2+1, 2n_3)$

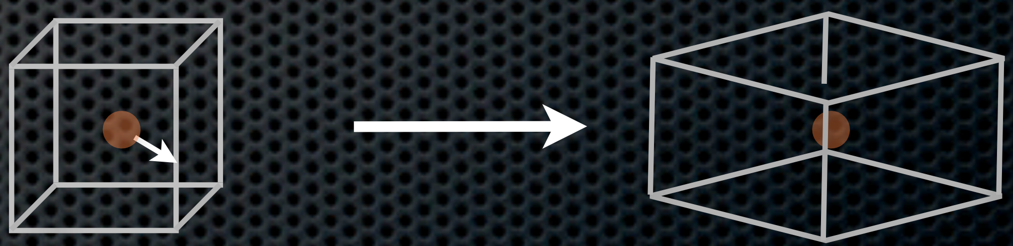
d = (0,0,0)



d = (0,0,1)



d = (1,1 ,0)



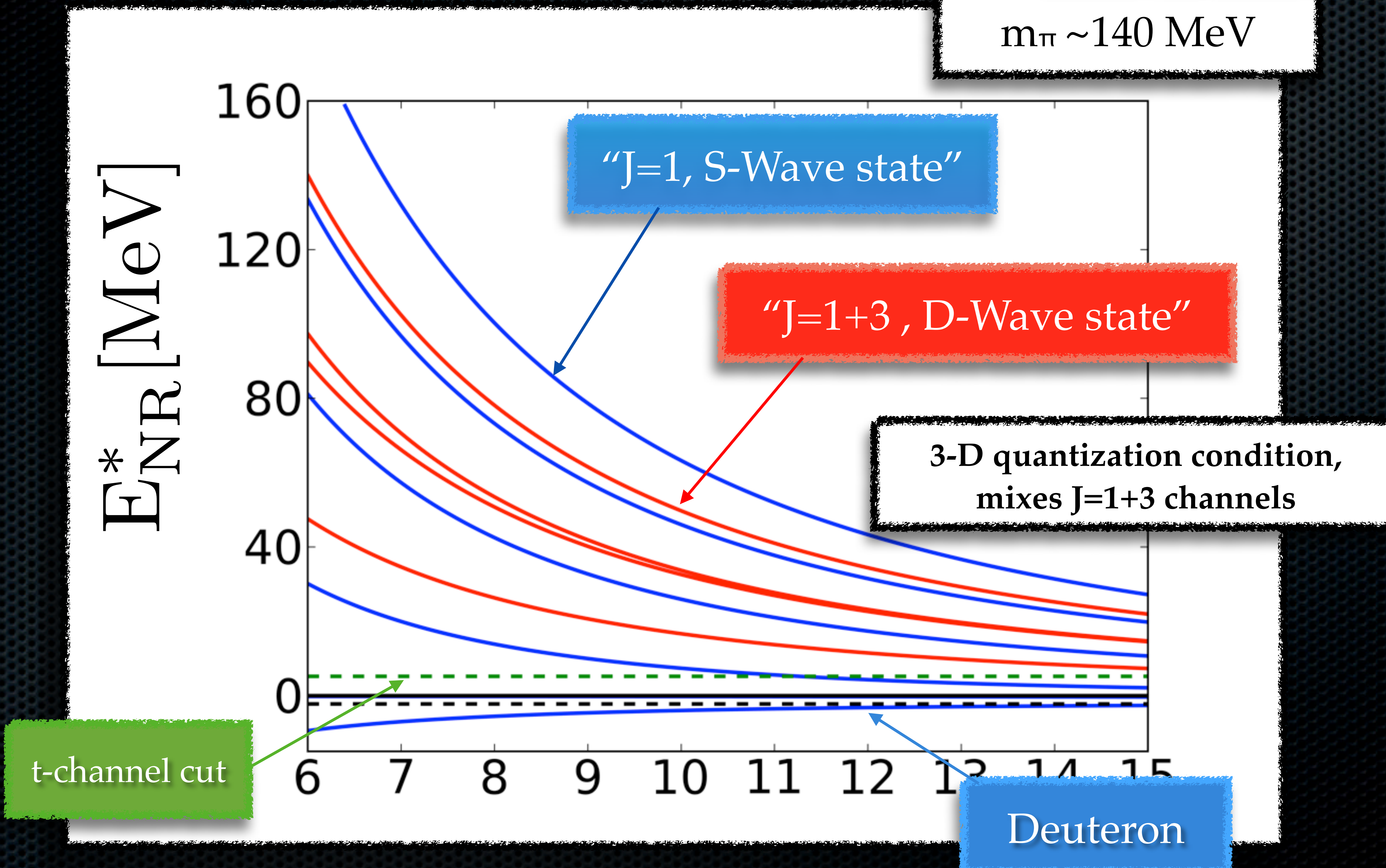
- Isospin, Spin, Parity, $l_{max}=3$

49 QCs for 16 scattering parameters!

T_1 Spectrum

$\mathbf{d} = (0, 0, 0)$

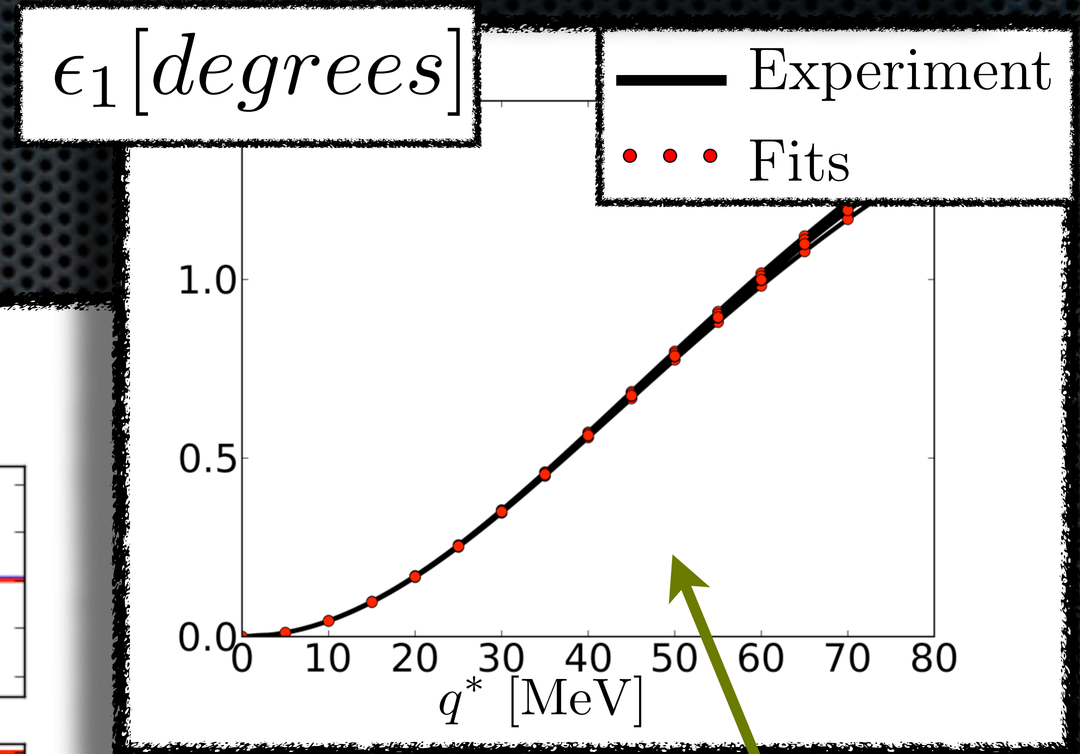
m_π ~140 MeV



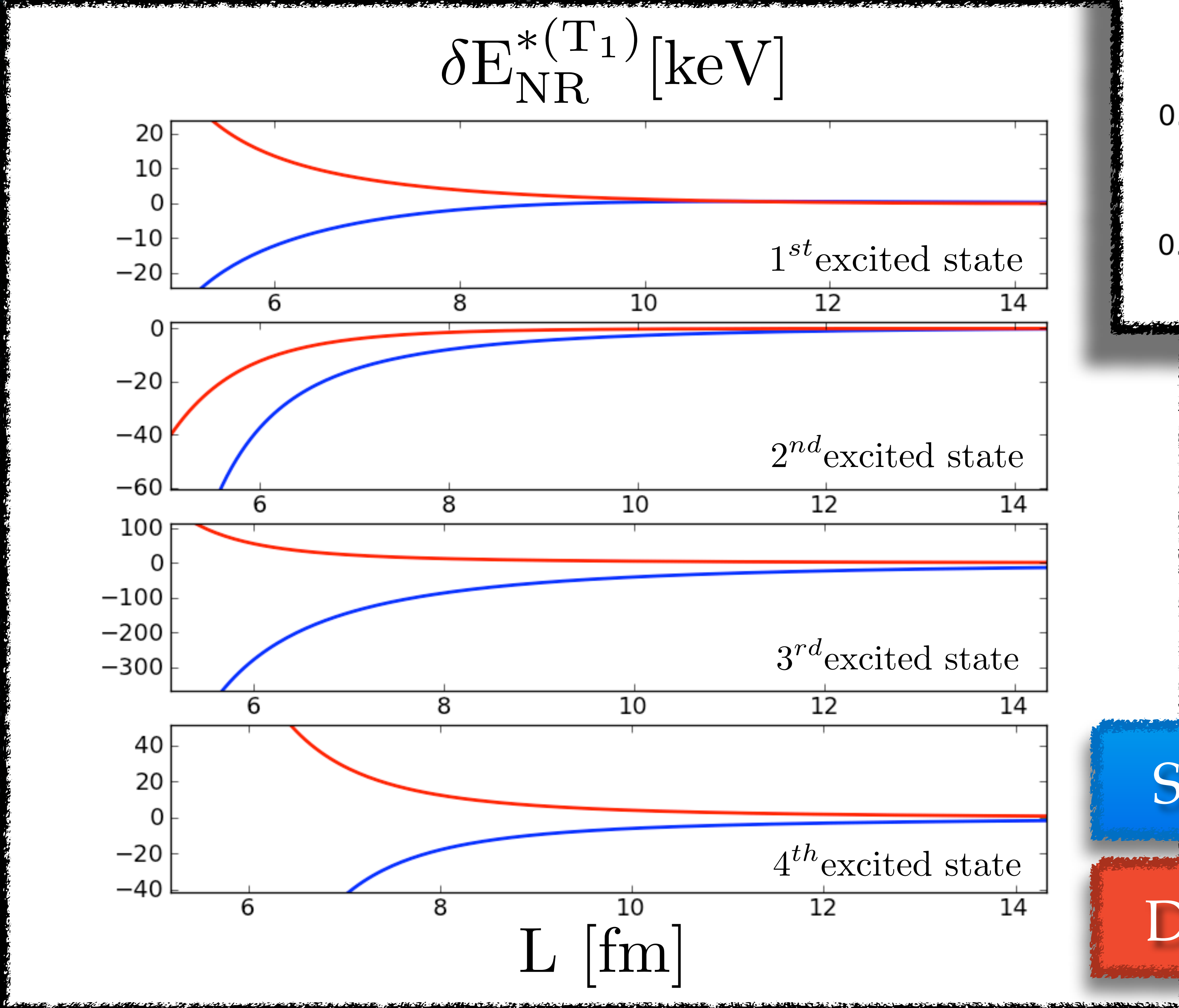
Input physical phase shifts [Nijmegen partial wave analysis http://nn-online.org]

T₁ Excited states

$\mathbf{d} = (0,0,0)$



Can we expect to extract such small mixing angle?

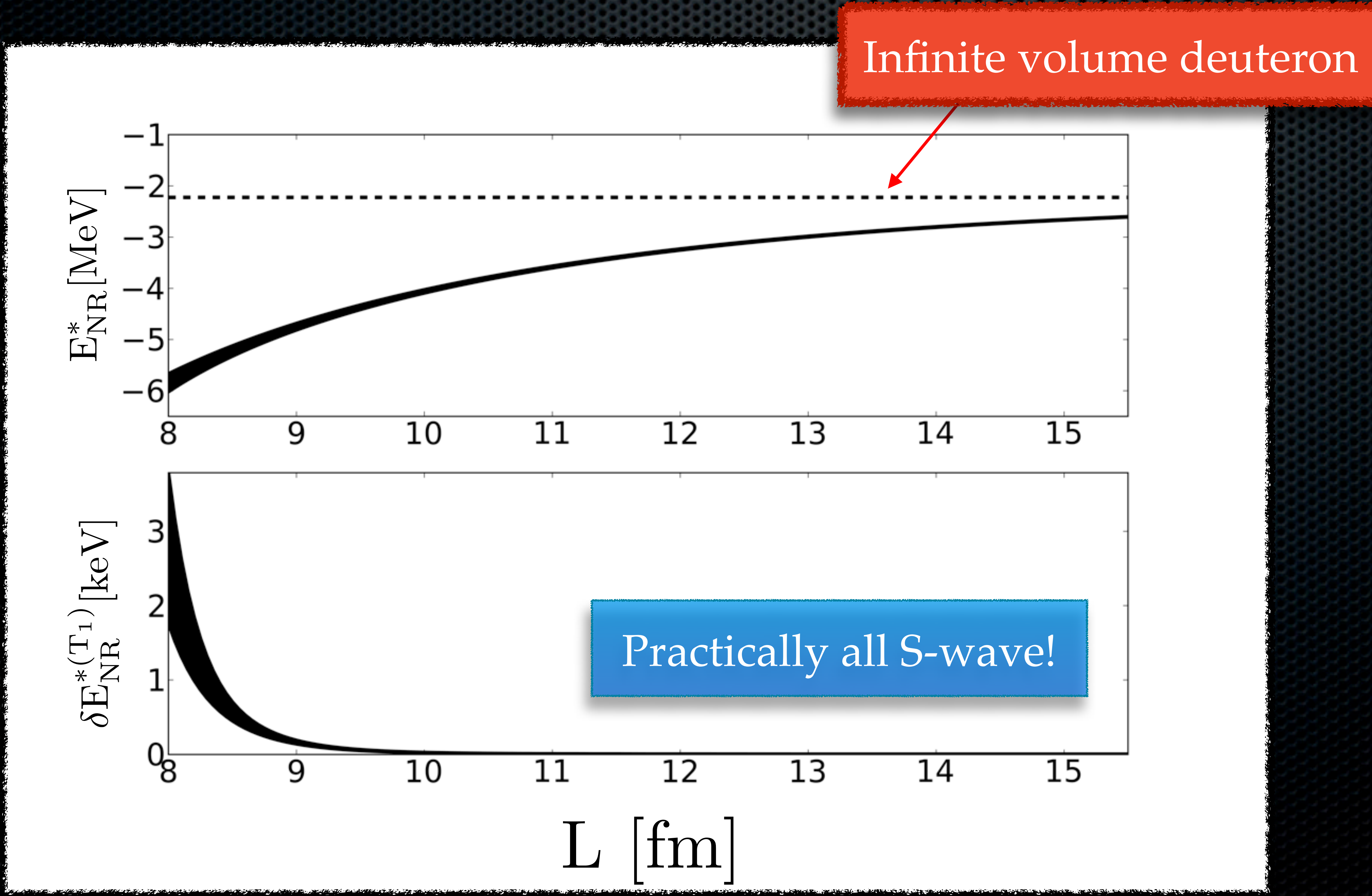


S-Wave

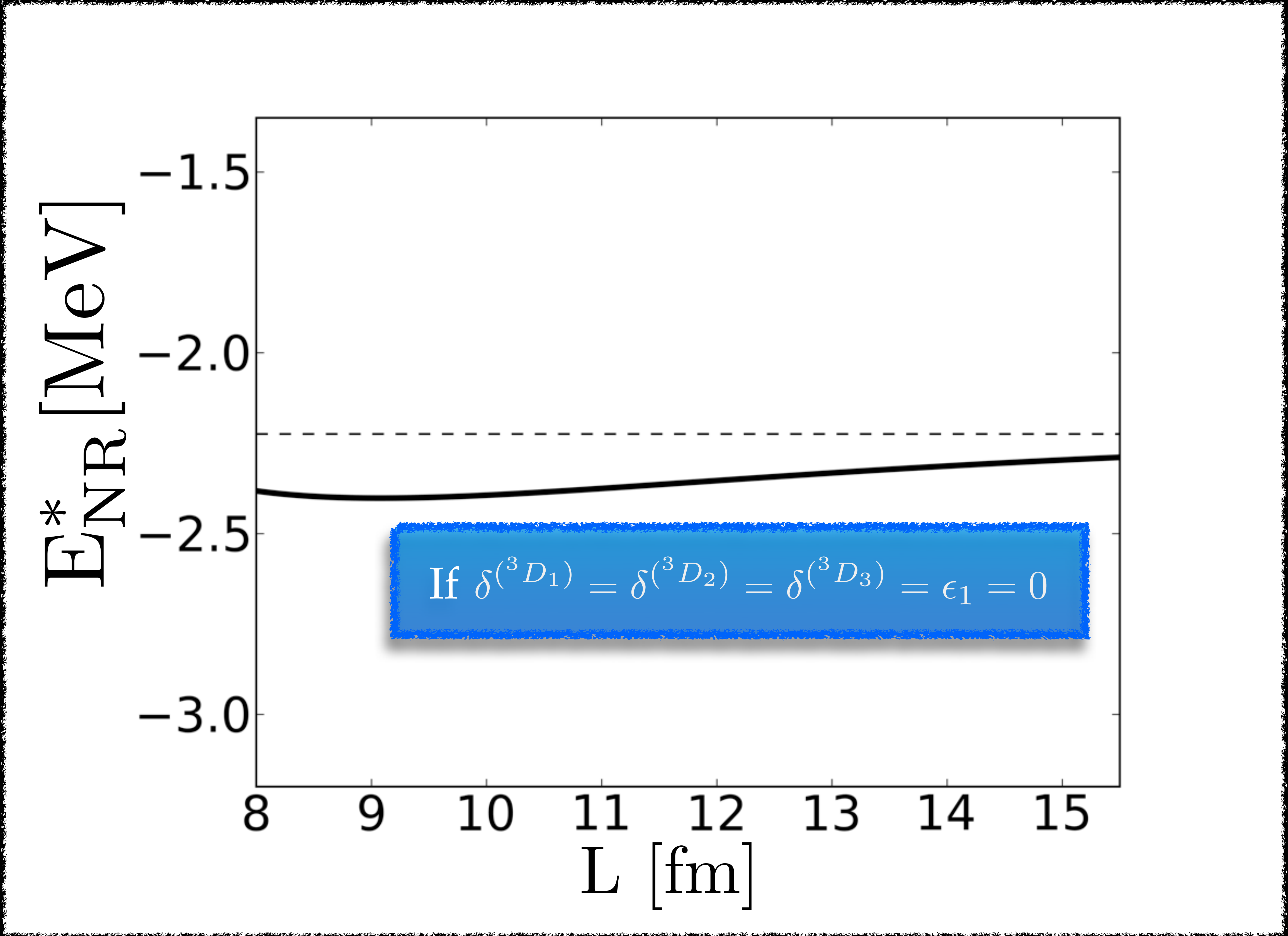
D-Wave

T_1 Excited bound state

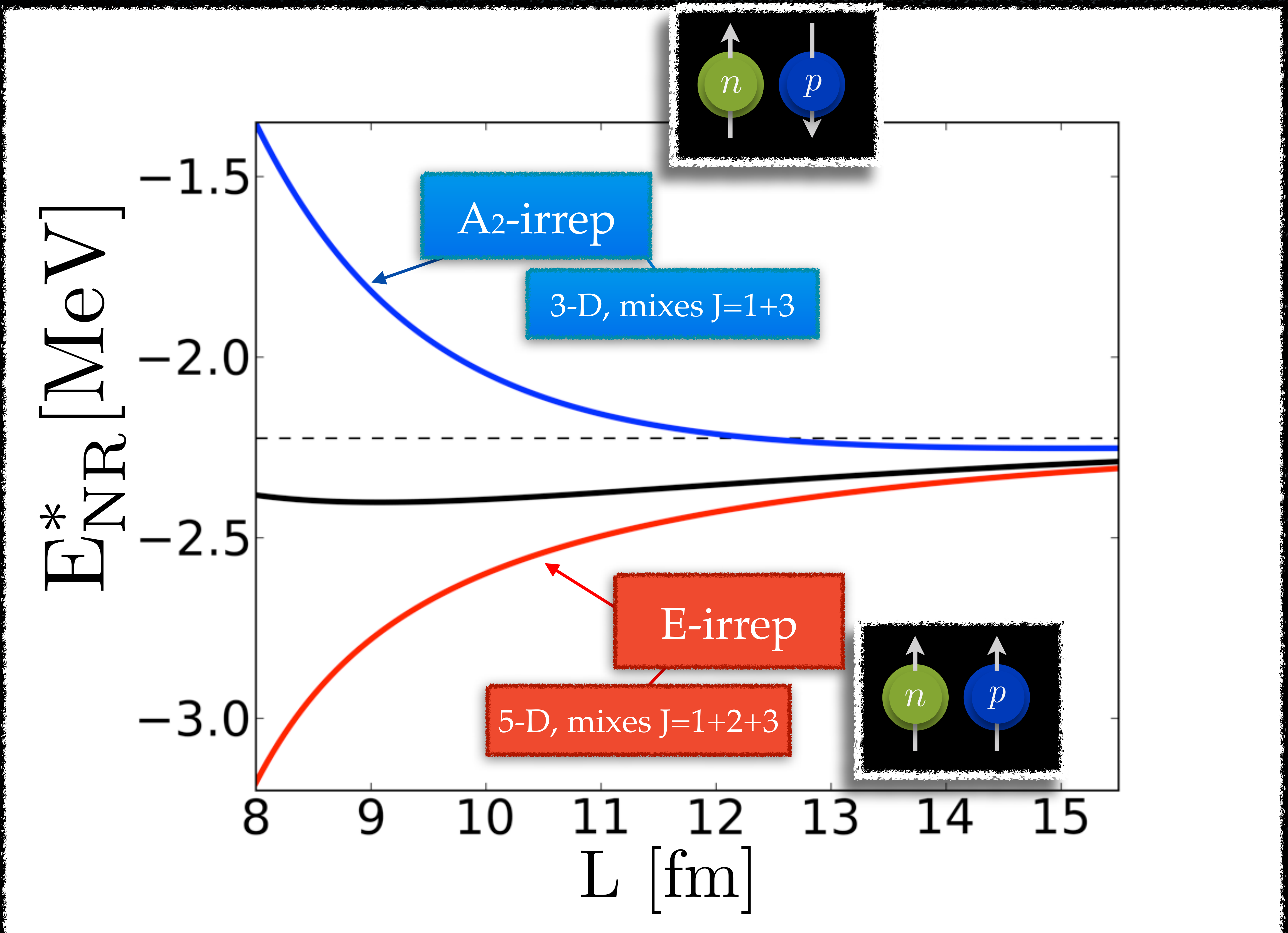
$\mathbf{d} = (0,0,0)$



Boosted deuteron $\mathbf{d} = (0, 0, 1)$



Boosted deuteron $\mathbf{d} = (0, 0, 1)$



Boosted deuteron

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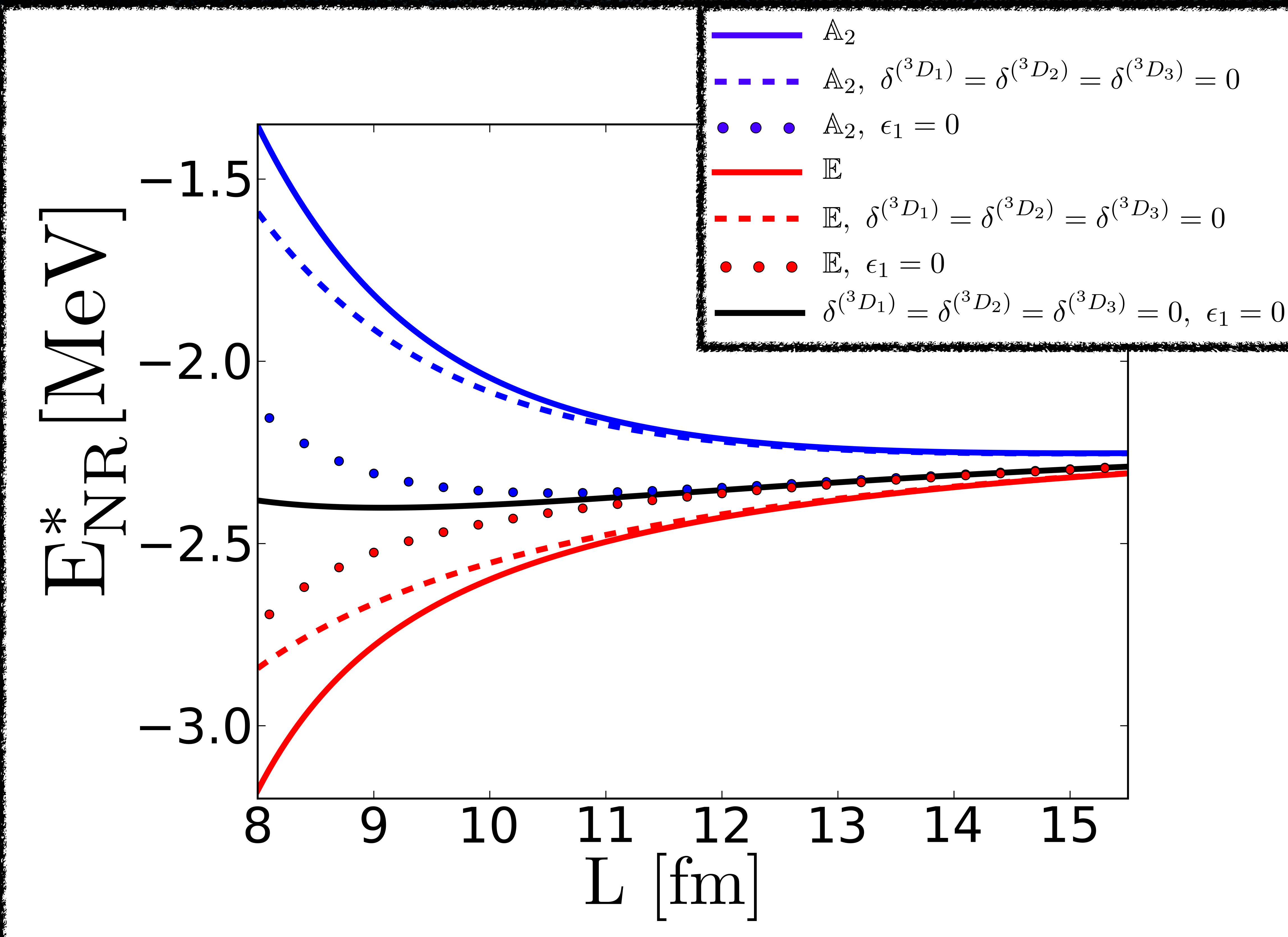

Boosted deuteron

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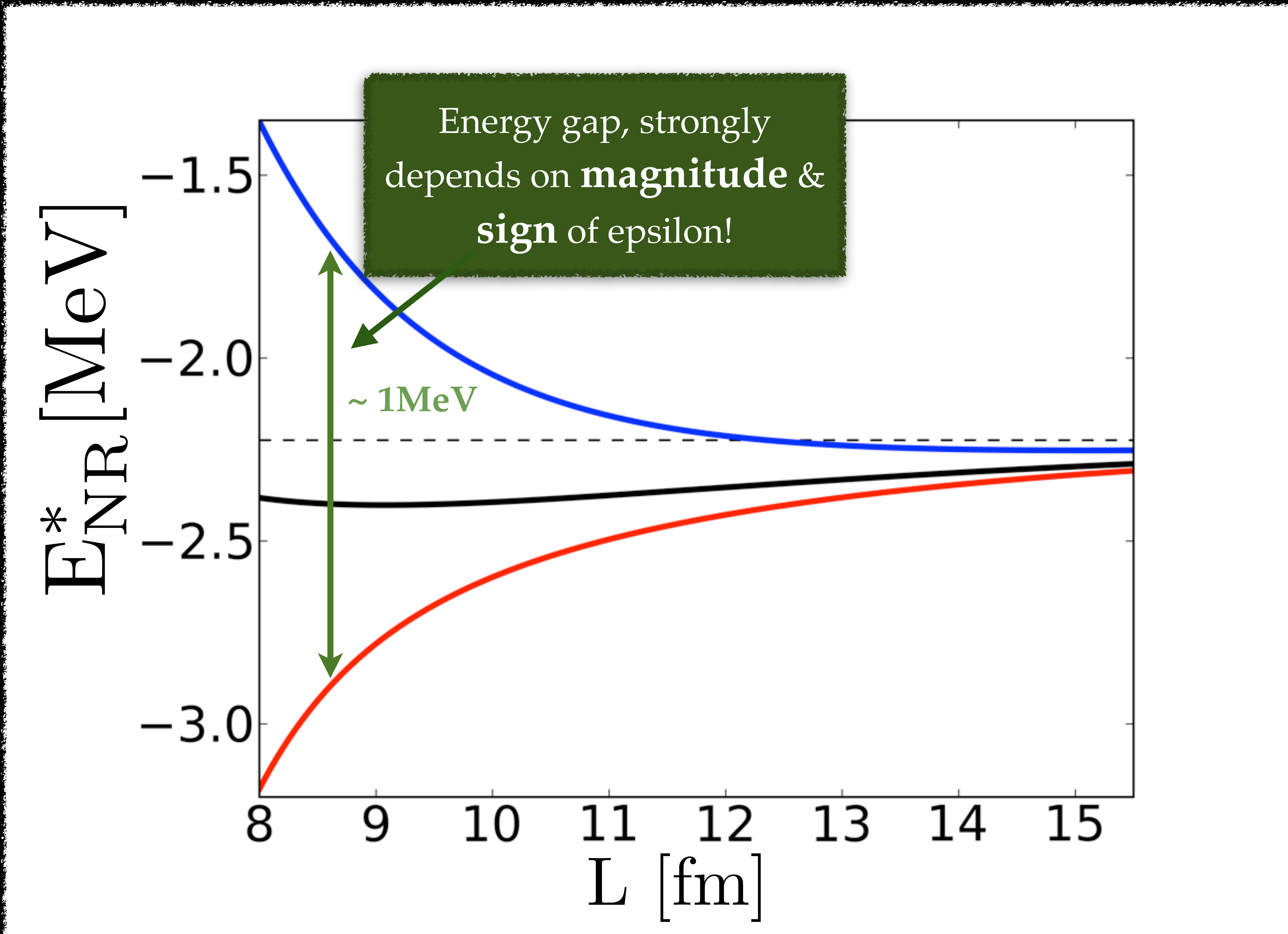
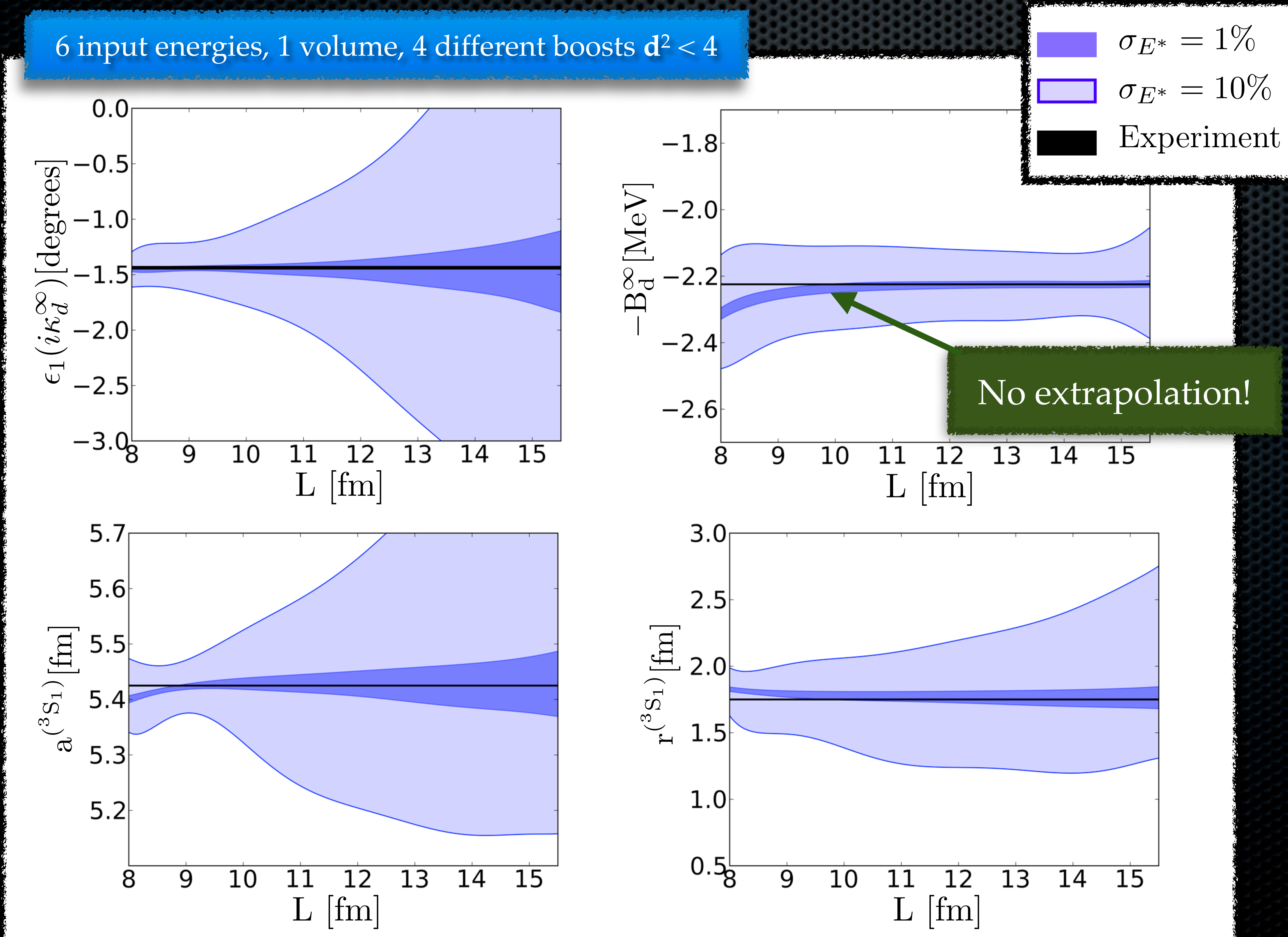
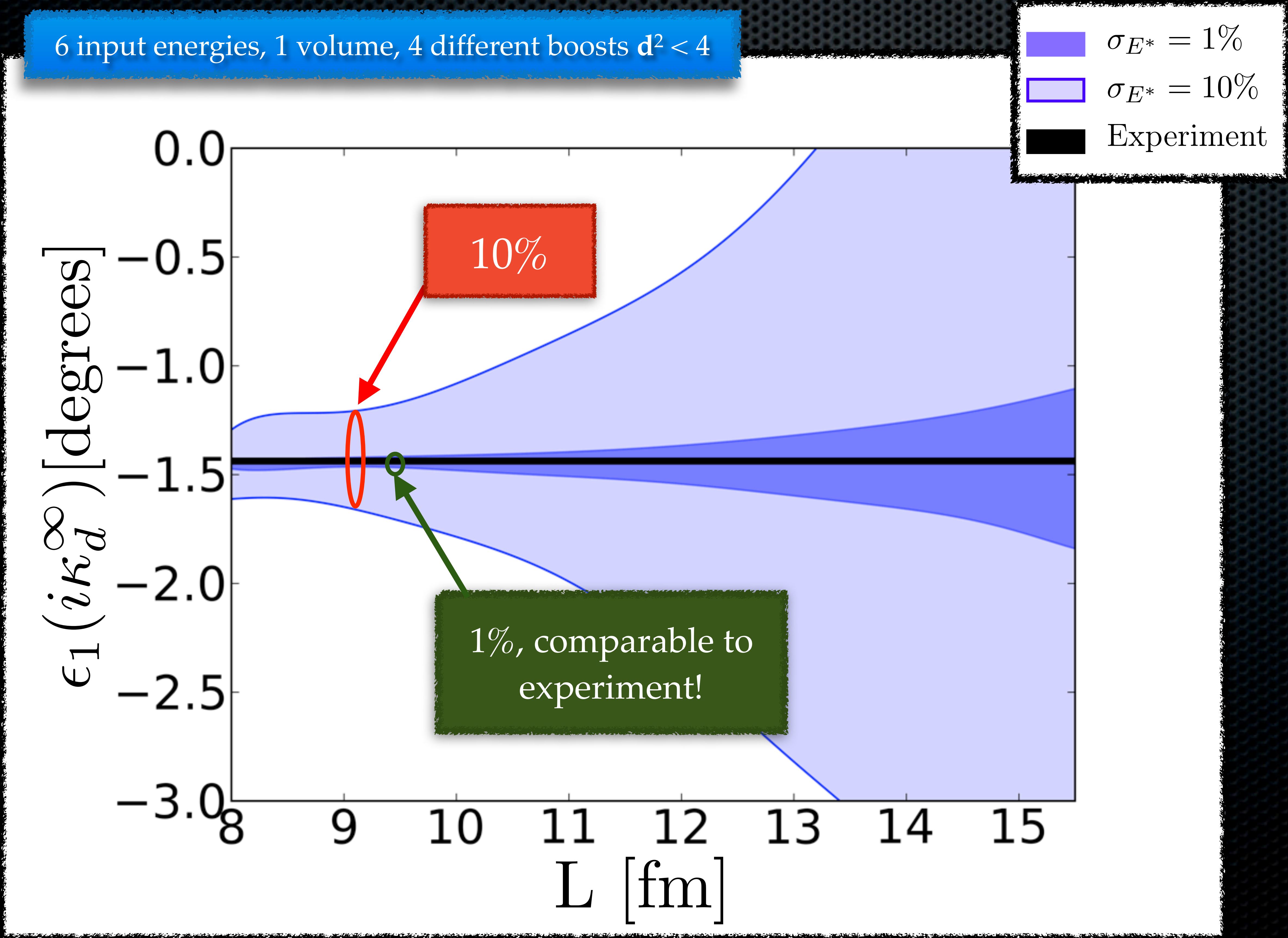


Extraction Estimates

6 input energies, 1 volume, 4 different boosts $\mathbf{d}^2 < 4$



Extraction Estimates



Final remarks

- Generalized dimer formalism: scalar & nuclear
- Master QC for NN systems
  - All J, S, I, L, and boosts
  - 49 QCs: $L < 4$ and 3 boosts
- Can study the tensor force from QCD!
- No ambiguity on the sign S-matrix elements
- Thanks to collaborators: Zohreh Davoudi, Tom Luu, Martin Savage

Thanks!

Back-up slides

Quantization Condition

sanity check

$$\det\left(\left(\mathcal{M}^{\infty}\right)^{-1}+\delta\mathcal{G}^{V}\right)=0$$

Isospin indices suppressed

$c_{Lm} \sim Z_{Lm}$

$$\left[\delta\mathcal{G}^{V}\right]_{Jm_J;J'm'_J}=\sum_{m_L,m'_L,m_S}\langle Jm_J|Lm_L,Sm_S\rangle\langle L'm'_L,Sm_S|J'm'_J\rangle\left[\delta\tilde{\mathcal{G}}^{V}\right]_{Lm_L;L'm'_L}$$

S = 0 limit

Agrees with
Luscher, Gottlieb &
Rummukainen,
Sharpe et al., Christ et al.

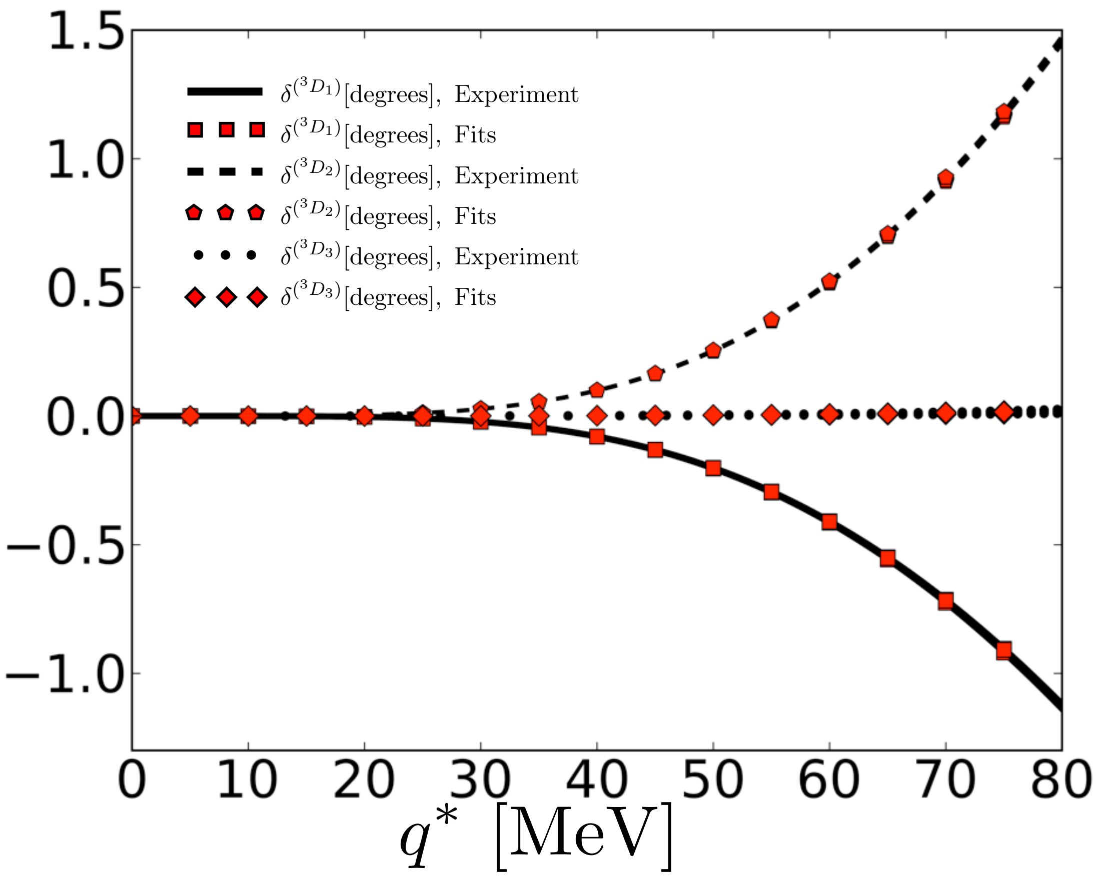
S = 1/2 limit

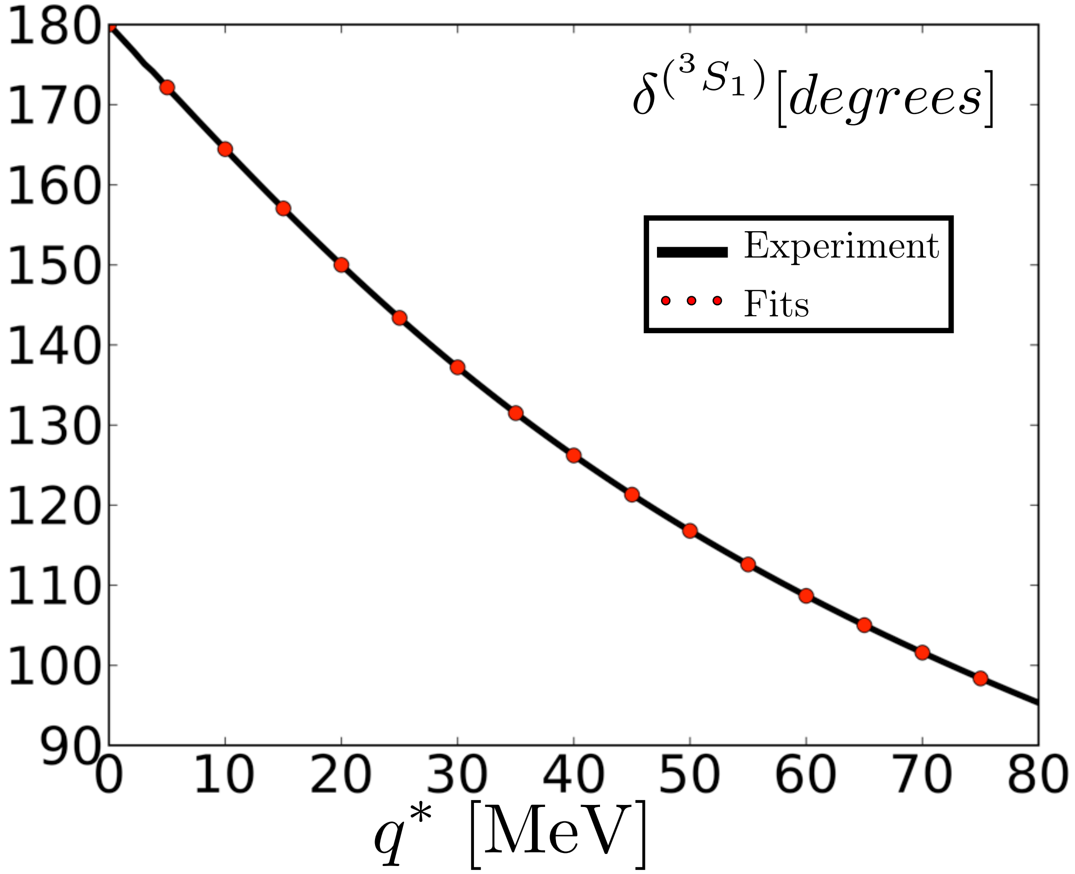
Agrees with
Bernard et al.
& Göckeler et al.

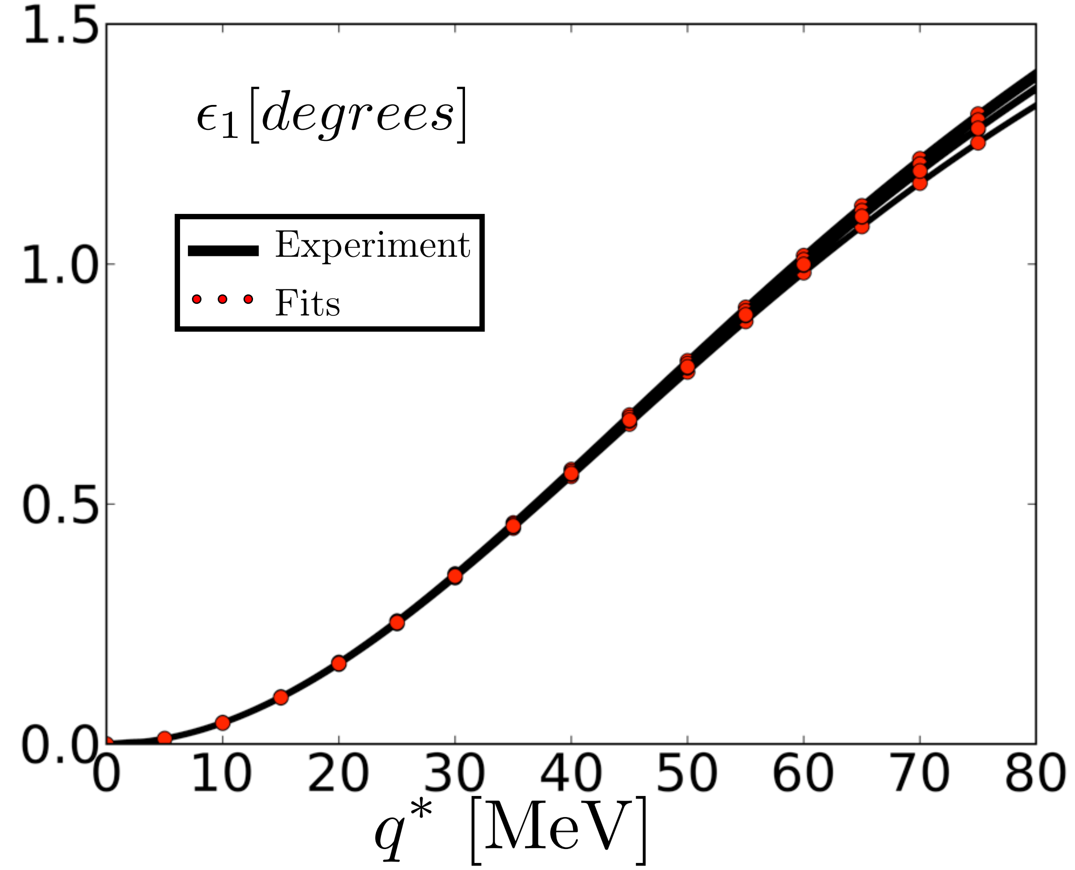
S = 0 ⊕ 1 limit

Agrees with
P=0 case by
N. Ishizuka (proceeding)
& Beane et al.

Fits to Nijmegen partial wave analysis [http://nn-online.org]







Extrapolation to negative non-relativistic energies