Two-Nucleon Systems in a Finite Volume

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From QCD to Nuclear Physics

LQCD + EFT

More parameters: $m_u, m_d, m_s$

Complements experiment

Chiral Effective Field Theory for nuclear forces

Separation of scales: low momenta breakdown scale $\sim 500$ MeV

include long-range pion physics

few short-range couplings, fit to experiment once systematic: can work to desired accuracy and obtain error estimates

Open problems: Power counting and renormalization.

Can lattice QCD provide guidance?

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meissner, …
LQCD Finite Euclidean Spacetime

- \( a \to 0 \)
- \( T \to \infty \)

- Discretized momentum: \( p = \frac{2\pi n}{L} \)
- Maiani & Testa (1990): No-go theorem
- Lüscher (1991): scalar bosons \( E_L, m \to \delta(q^*) \)
- Beane et al. (2003): S-wave NN-system (CM-frame)
- Ishizuka (2009): NN-system (CM-frame)

neutron-proton in a 4D torus
NN-Systems

- Exact Isospin symmetry
- $L=2n$, $I=1$, $S=0$:
  $p p \leftrightarrow n n$
- $L=2n$, $I=0$, $S=1$:
  $n p \leftrightarrow p n$
- $L=2n+1$, $I=0$, $S=0$, $L=2n+1$, $I=1$, $S=1$

Deuteron channel ($^3S_1-^3D_1$)

$$S_1 = \begin{pmatrix} c_{\epsilon_1} & -s_{\epsilon_1} \\ s_{\epsilon_1} & c_{\epsilon_1} \end{pmatrix} \begin{pmatrix} e^{i2\delta(^3S_1)} & 0 \\ 0 & e^{i2\delta(^3D_1)} \end{pmatrix} \begin{pmatrix} c_{\epsilon_1} & s_{\epsilon_1} \\ -s_{\epsilon_1} & c_{\epsilon_1} \end{pmatrix}$$

- Can also have ($L=2n+1$, $I=0$, $S=0$), ($L=2n+1$, $I=1$, $S=1$)
Quantization Condition

\[
\det \left( (M^\infty)^{-1} + \delta G^V \right) = 0
\]

Scattering amplitude Diagonal in J-basis mixes l-states

Kinematic function of (L, E_L) Mixes angular momentum

Diagonal in Spin & Isospin

Diagonal in Spin & Isospin

RB, Zohreh Davoudi (UW) & Tom Luu (LLNL)

Clebsch–Gordan coefficients matter!
Boots & Symmetry

- **O**: Cubic
  \[ \mathbf{d} = (2n_1, 2n_2, 2n_3) \]

- **D_4**: Tetragonal
  \[ \mathbf{d} = (2n_1, 2n_2, 2n_3 + 1) \]

- **D_2**: Orthorhombic
  \[ \mathbf{d} = (2n_1 + 1, 2n_2 + 1, 2n_3) \]

- Isospin, Spin, Parity, \( l_{\text{max}} = 3 \)

**49 QC\texts for 16 scattering parameters!**
T_1 Spectrum

\[d = (0, 0, 0)\]

\[m_\pi \sim 140 \text{ MeV}\]

“J=1, S-Wave state”

“J=1+3, D-Wave state”

3-D quantization condition, mixes J=1+3 channels

t-channel cut

Deuteron

Input physical phase shifts [Nijmegen partial wave analysis http://nn-online.org]
T$_1$ Excited states

Can we expect to extract such small mixing angle?

$\delta E_{NR}^*(T_1)$ [keV]

$\epsilon_1$ [degrees]

$\mathbf{d} = (0, 0, 0)$

S-Wave

D-Wave

Can we expect to extract such small mixing angle?
T_1 Excited bound state \( \mathbf{d} = (0,0,0) \)

Infinite volume deuteron

Practically all S-wave!
Boosted deuteron \( d = (0,0,1) \)

If \( \delta^{(3D_1)} = \delta^{(3D_2)} = \delta^{(3D_3)} = \epsilon_1 = 0 \)
Boosted deuteron \( d = (0, 0, 1) \)

\[
\begin{align*}
L &= \infty \\
A_2, \delta(3D_1) &= \delta (3D_2) = \delta (3D_3) = 0 \\
E_{-irrep}, \delta(3D_1) &= \delta (3D_2) = \delta (3D_3) = 0 \\
E_{-irrep}, \delta(3D_1) &= \delta (3D_2) = \delta (3D_3) = 0
\end{align*}
\]
Boosted deuteron \( d = (0, 0, 1) \)
Boosted deuteron \[ d = (0, 0, 1) \]

Energy gap, strongly depends on magnitude & sign of epsilon!

![Graph showing energy gap vs. L (fm)]
Extraction Estimates

6 input energies, 1 volume, 4 different boosts $d^2 < 4$

$\sigma_{E^*} = 1\%$

$\sigma_{E^*} = 10\%$

Experiment

No extrapolation!
Extraction Estimates

6 input energies, 1 volume, 4 different boosts $d^2 < 4$

$\epsilon_1(i\kappa_0^\infty) [\text{degrees}]$

$L [\text{fm}]$

$\sigma_{E^*} = 1\%$

$\sigma_{E^*} = 10\%$

Experiment

$1\%$, comparable to experiment!
Final remarks

- Generalized dimer formalism: scalar & nuclear
- Master QC for NN systems
  - All J, S, I, L, and boosts
  - 49 QCs: L < 4 and 3 boosts
- Can study the tensor force from QCD!
- No ambiguity on the sign S-matrix elements
- Thanks to collaborators: Zohreh Davoudi, Tom Luu, Martin Savage
Thanks!
Back-up slides
Quantization Condition
sanity check

$$\det \left( (\mathcal{M}_\infty)^{-1} + \delta \mathcal{G}^V \right) = 0$$

Isospin indices suppressed

$$[\delta \mathcal{G}^V]_{Jm_J;J'm'_J} = \sum_{m_L,m'_L,m_S} \langle Jm_J|Lm_L,Sm_S\rangle \langle L'm'_L,Sm_S|J'm'_J\rangle \left[ \delta \tilde{\mathcal{G}}^V \right]_{Lm_L;L'm'_L}$$

S = 0 limit

Agrees with Luscher, Gottlieb & Rummukainen, Sharpe et al., Christ et al.

S = 1/2 limit

Agrees with Bernard et al. & Göckeler et al.

S = 0 $\oplus$ 1 limit

Agrees with P=0 case by N. Ishizuka (proceeding) & Beane et al.

$$c_{Lm} \sim Z_{Lm}$$
Fits to Nijmegen partial wave analysis

[http://nn-online.org]
Extrapolation to negative non-relativistic energies