Phase shifts in $I = 2$ $\pi\pi$ heavy-pion-scattering from two approaches

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Introduction

- Goal:
  - explain nuclear physics by dynamics of QCD (*binding energies, scattering phases, dense matter EoS, etc.*)
- Well-known approaches (from ab-initio QCD):
  - Lüscher's finite volume method (Commun.Math.Phys. 105 (1986))
  - non-zero chemical potential
- new approach:
  - potential method (arXiv:1203.3642 (HAL-QCD 2012))
  - untested, potential systematic drawbacks

☞ test and compare potential method to the Lüscher method (Commun.Math.Phys. 105 (1986)) in case of less complicated $I=2$ $\pi\pi$-scattering problem

Ingredients

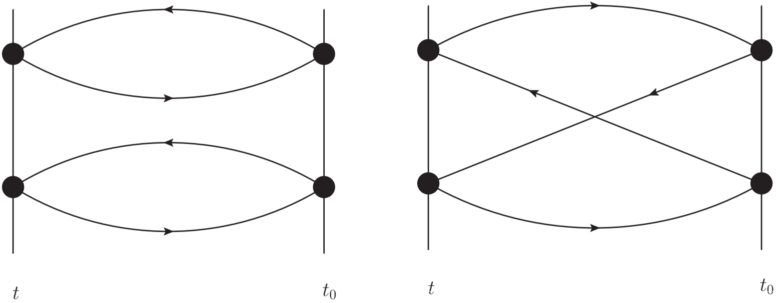
- compute two- and single-pion correlators:

$$C_{\pi\pi}(t,\mathbf{r},\mathbf{P}) \equiv \sum_{\mathbf{R}} e^{-i\mathbf{RP}} \langle \pi^+(t,(\mathbf{R}+\mathbf{r})/2)\pi^+(t,(\mathbf{R}-\mathbf{r})/2) J_{\pi^-}(0,\mathbf{P}) J_{\pi^-}(0,\mathbf{0})\rangle,$$

$$C_{\pi}(t,\mathbf{P}) \equiv \sum_{\mathbf{R}} e^{-i\mathbf{RP}} \langle \pi^+(t,\mathbf{R}) J_{\pi^-}(0,\mathbf{P})\rangle$$

- J_{π^-}: (random) wall or Gaussian sources (Coulomb gauge)
- AP-BC or Dirichlet-BC ($\bar{\psi}(N_T/2,\mathbf{x}) = \psi(N_T/2,\mathbf{x}) = 0$)

Contractions



- contractions for $C_{\pi\pi}(t, \mathbf{r}, \mathbf{P})$

✓ no disconnected contributions in I=2 channel

Time dependent potential method I

- Nambu-Bethe-Salpeter (NBS) wave-function with asymptotic momentum **k** given by (arXiv:1203.3642 (HAL-QCD 2012))

$$\psi_{\mathbf{k}}(\mathbf{r}) \equiv \sum_{\mathbf{x}} \langle 0|\pi^+(\mathbf{x})\pi^+(\mathbf{x}+\mathbf{r})|\pi^-(\mathbf{k})\pi^-(-\mathbf{k})\rangle$$

☞ NBS-WF satisfies Schroedinger-equation for $E < E_{\mathrm{thres}}$

$$\left(\mathbf{k}^2 + \nabla^2\right)\psi_{\mathbf{k}}(\mathbf{r}) = m_\pi \int_{\mathbb{R}^3} \mathrm{d}^3 r' \, U(\mathbf{r},\mathbf{r}')\, \psi_{\mathbf{k}}(\mathbf{r}')$$

✓ asymptotic behavior

$$\psi_{\mathbf{k}}(\mathbf{r}) \xrightarrow{|\mathbf{r}|=r\to\infty} e^{i\delta(k)} \frac{\sin\left(kr + \delta(k)\right)}{kr} + \ldots$$

Time dependent potential method II

- define

$$R(t,\mathbf{r}) \equiv C_{\pi\pi}(t,\mathbf{r},0)/C_\pi^2(t,0)$$

☞ decompose to

$$R(t,\mathbf{r}) = \sum_{\mathbf{k}} \psi_{\mathbf{k}}(\mathbf{r})\, a_{\mathbf{k}}\, e^{-t\Delta E(\mathbf{k})}$$

where $\Delta E(\mathbf{k}) = 2\sqrt{\mathbf{k}^2 + m_\pi^2} - 2m_\pi$.

- it is

$$\left(-\frac{\partial}{\partial t} + \frac{1}{4m_\pi}\frac{\partial^2}{\partial t^2}\right) R(t,\mathbf{r}) = \sum_{\mathbf{k}} \frac{\mathbf{k}^2}{m_\pi} \psi_{\mathbf{k}}(\mathbf{r})\, a_{\mathbf{k}}\, e^{-t\Delta E(\mathbf{k})}$$

Time dependent potential method III

- this yields

$$\left(-\frac{\partial}{\partial t}+\frac{1}{4m_\pi}\frac{\partial^2}{\partial t^2}+\frac{\nabla^2}{m_\pi}\right)R(t,\mathbf{r})=\int_{\mathbb{R}^3}\mathrm{d}^3r'\,U(\mathbf{r},\mathbf{r}')R(t,\mathbf{r}')$$

- expand non-local potential for I=2 case:

$$U(\mathbf{r},\mathbf{r}')\to V_C(\mathbf{r})\,\delta(\mathbf{r}-\mathbf{r}')+\mathcal{O}(\nabla^2)$$

- this allows us to compute LO potential V_C:

$$V_C(\mathbf{r})=\frac{\nabla^2 R(t,\mathbf{r})}{m_\pi R(t,\mathbf{r})}-\frac{(\partial/\partial t)R(t,\mathbf{r})}{R(t,\mathbf{r})}+\frac{1}{4m_\pi}\frac{(\partial/\partial t)^2 R(t,\mathbf{r})}{R(t,\mathbf{r})}$$

Cooking recipe

- ✓ compute $R(t, r)$ on the lattice
- ✗ extract LO potential $V_C(r)$
- ✗ model potential, $V \to \infty$ limit
- ✓ solve SE for arbitrary k (ext. parameter) and obtain $\psi_k(r)$
- ✓ compute scattering phases using $\beta \equiv [r \,\mathrm{d} \ln \psi_k / \mathrm{d}r]_{r=R}$

$$\tan \delta(k) = \frac{kR\, j_0'(kR) - \beta\, j_0(kR)}{kR\, n_0'(kR) - \beta\, n_0(kR)}$$

where $j_0(\rho) = \sin \rho/\rho$ and $n_0(\rho) = -\cos \rho/\rho$

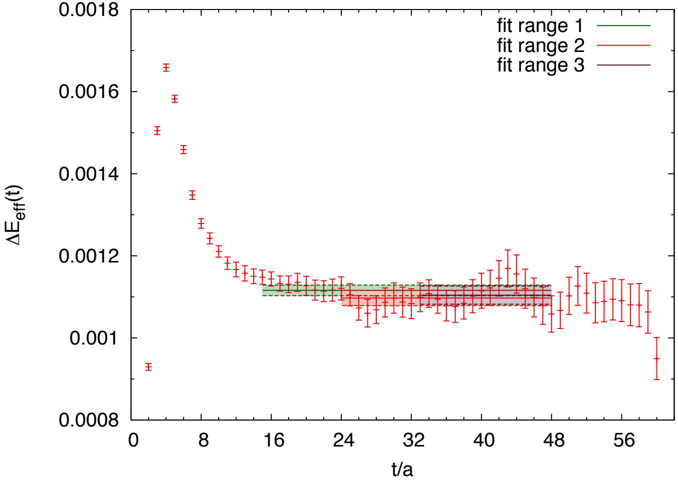
- ☞ compute low energy observables by fitting $\delta(k)$ to ERE:

$$\frac{k \cot \delta(k)}{m_\pi} = \frac{1}{m_\pi a_{\pi\pi}^{I=2}} + \frac{1}{2} m_\pi r \left(\frac{k^2}{m_\pi^2}\right) + P (m_\pi r)^3 \left(\frac{k^2}{m_\pi^2}\right)^2 + \dots$$

Setup

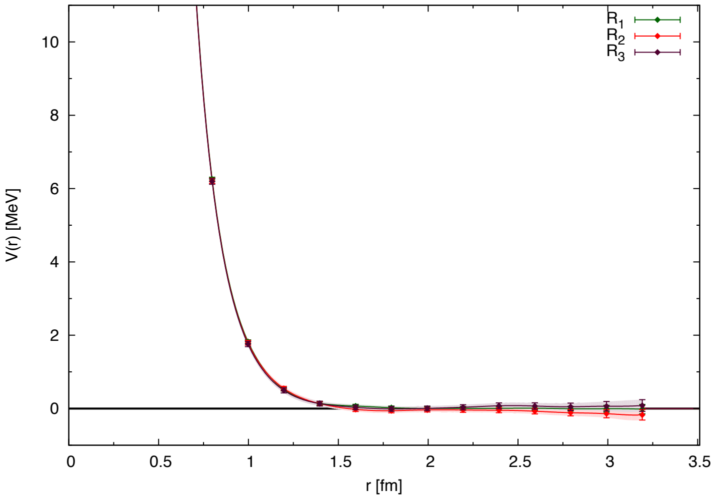
- ☞ test method on quenched setup using $M_\pi \sim (330-940)\,\mathrm{MeV}$ with 2 HEX smeared tree-level improved clover-Wilson quarks
- $a{\approx}0.115\,\mathrm{fm}$ and $L{\approx}3.7\,\mathrm{fm}$ (wall source: $1.84{<}L{<}4.9\,\mathrm{fm}$)
- statistical error ⇒ 2000 bootstrap samples
- systematic uncertainties ⇒ histogram method
  - ✓ rotational invariance breaking ⇒ extract potential along axis, surface-diagonals and cubic diagonal
  - ✓ source dependence ⇒ use wall and gauss sources (radius $r{\approx}0.3\,\mathrm{fm}$)
  - ✓ ground state saturation (energy dependence) ⇒ use different time-slices
  - ✓ potential modeling ⇒ use different potential models (empirical) ☞ (barycentric) rational interpolation
  - ✓ asymptotic regime of ψ_k ⇒ different distances R

Mass Plateau



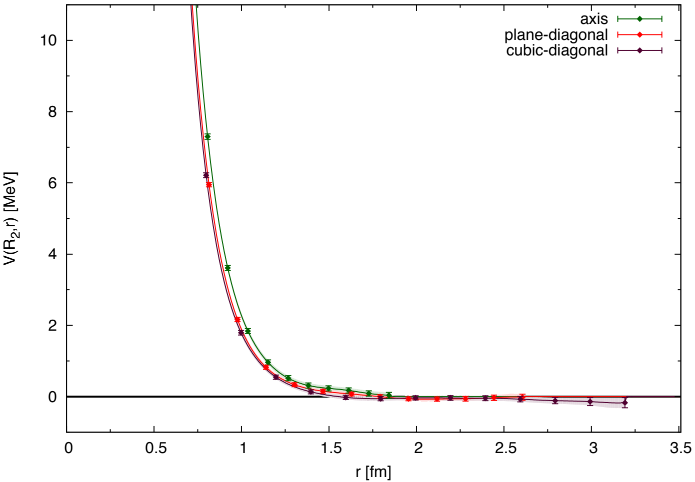
✓ clean plateaus

Potentials and excited state effects



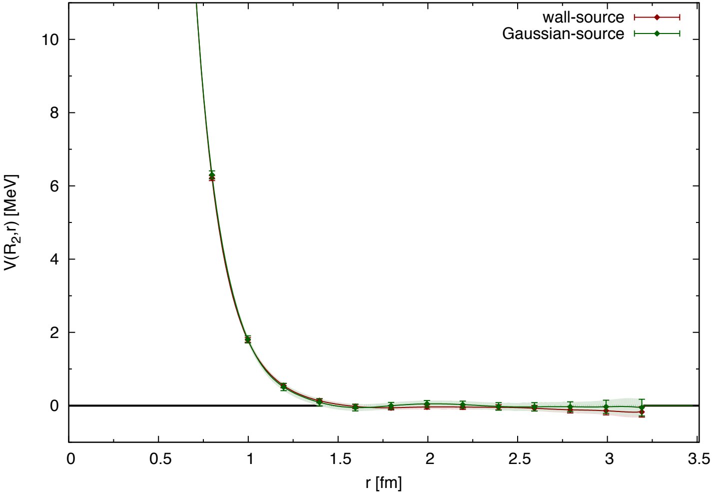
✓ no time-dependence (for $t \gtrsim 1.73\,\mathrm{fm}$)

Direction dependence



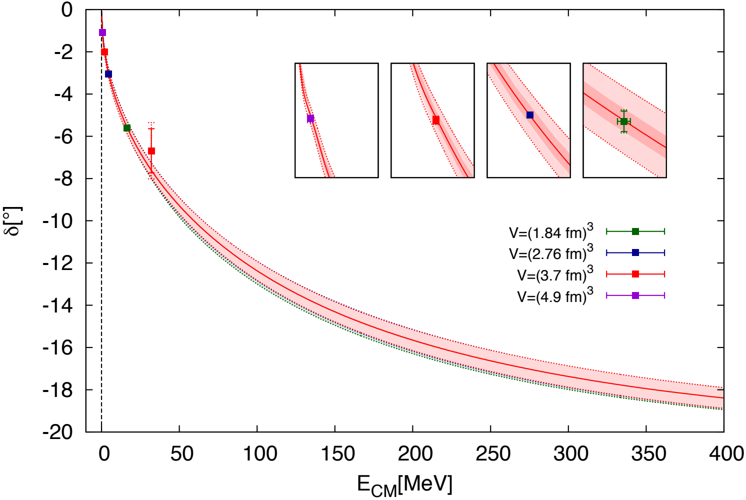
☞ breaking of rotational invariance dominant

Source-dependence



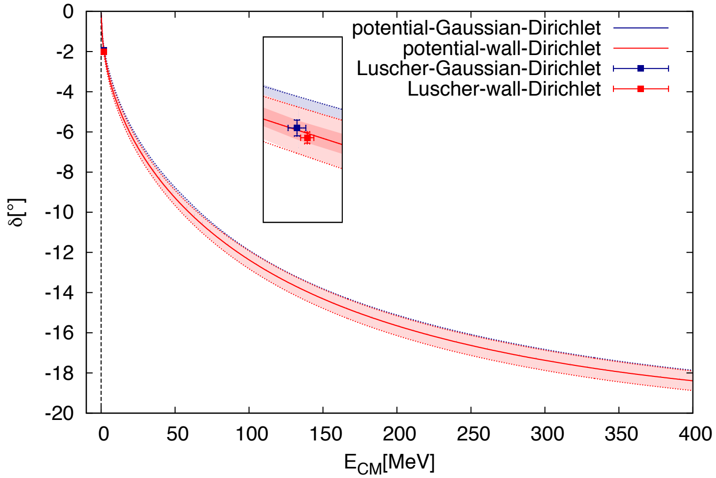
✓ no source-dependence

Scattering phase shifts for different volumes



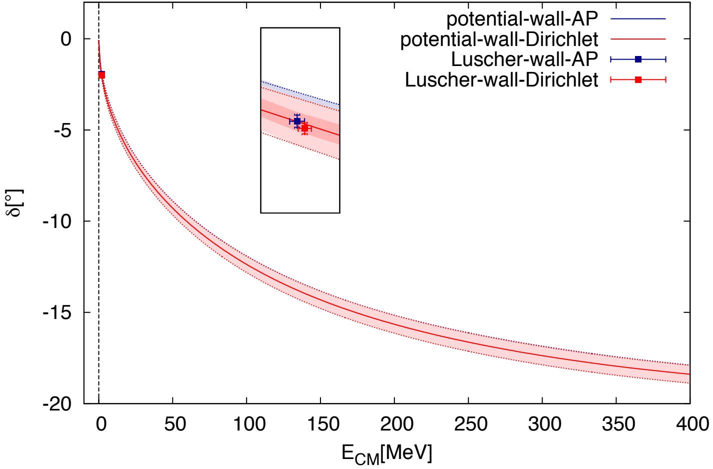
✓ results from Lüscher's and potential method agree

Different sources



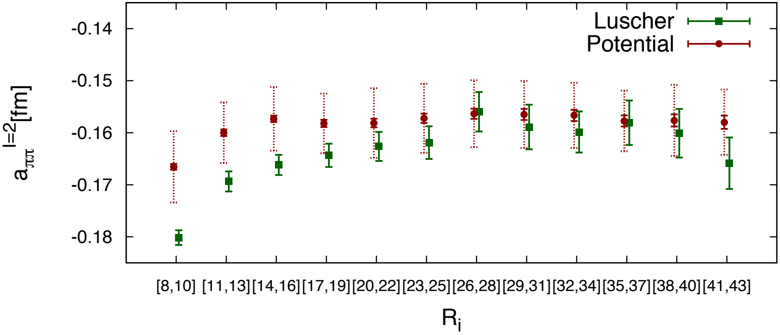
✓ phases independent of source-type

Different BC



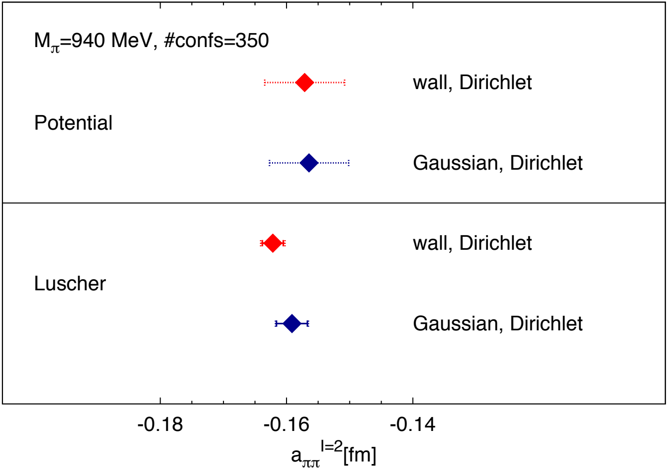
✓ phases independent of BC

Phases V



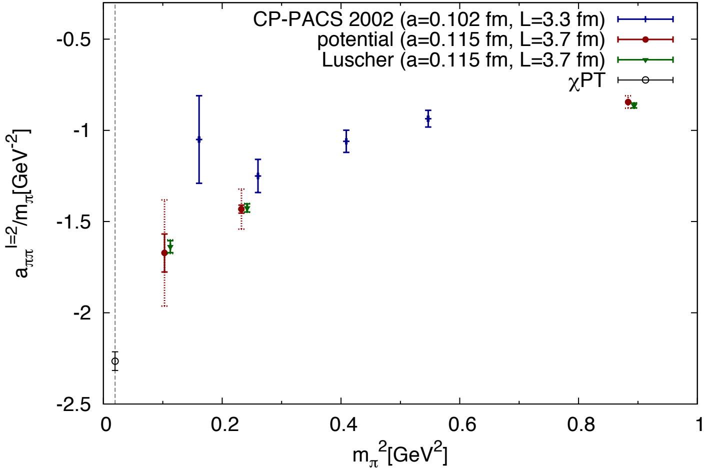
✓ potential method less sensitive to excited states contaminations

Scattering lengths



✓ scattering lengths agree for $M_\pi \approx 940\,\mathrm{MeV}$

Mass dependence of scattering lengths



✓ agreement with previous quenched calculations

Summary

- ☞ compared potential method to Lüscher's approach to scattering problems
- ✓ both allow for extracting scattering phases and lengths
- ✓ compatible results
- ✗ Lüscher method: mapping out $\delta(k)$ requires use of different volumes, excited state extractions or boosted frames
- ✓ potential method: k^2 is free parameter
- ✓ potential method can use earlier time-slices $\Rightarrow$ advantageous for multi-baryon scattering