Scale hierarchy in high-temperature QCD

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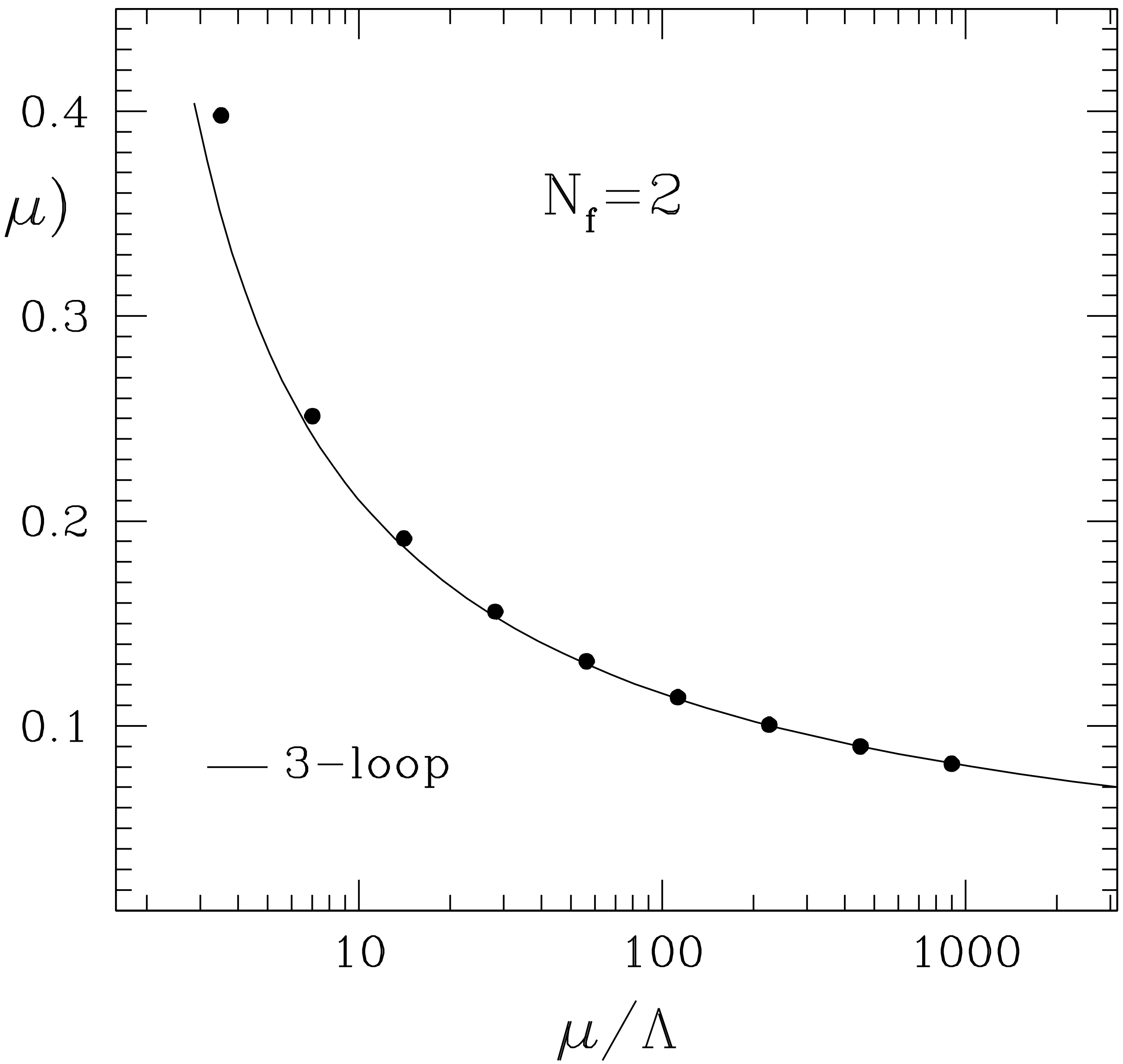
QCD is asymptotically free

$$\frac{g^2}{4\pi} = \alpha(\mu)$$

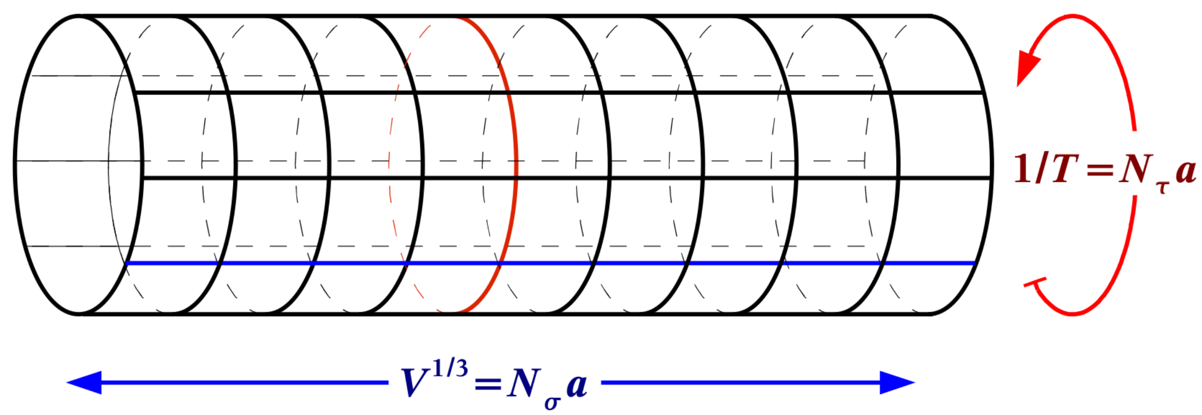


- High temperature: $g(T) \to 0$, deconfinement for $T > T_c$
- Perturbative treatment OK for "sufficiently high" T

Dimensional reduction: $4d \longrightarrow 3d$

- Thermal boundary conditions:

$$\phi(x+1/T) = \pm\phi(x)$$ bosons / fermions



- Fourier decomposition: $\tilde{\phi}_n(x) = \int_0^{1/T} dt\ e^{i2\pi(n+\mathbf{q})t}\phi(x,t)$ $\quad q = \{0, 1/2\}$

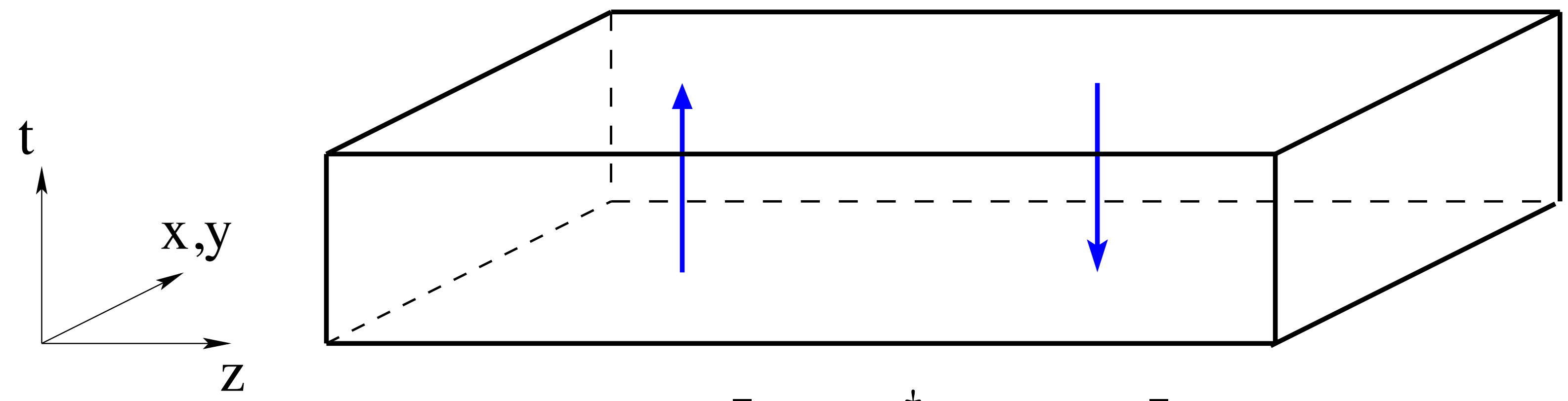
- Tower of states: $E_n^2 = |\vec{k}|^2 + [(2\pi T(n+\mathbf{q}))^2 + m^2] = |\vec{k}|^2 + (m_{\text{eff}}^{3d})^2$

- $|\vec{k}| << T \longrightarrow$ static $(n=0)$ modes for bosons; fermions decouple

Same for gauge fields

- Effective d.o.f.: $\bar{A}_i \equiv A_{i,n=0}$ and $\bar{A}_0 \equiv A_{0,n=0}$ or Polyakov loop L

$$\bar{A}_0 \equiv A_{0,n=0}(\vec{x}) = \int_0^{1/T} dt \ A_0^a(\vec{x},t)\tau_a$$



$L{=}\exp(+i\bar{A}_0)$ $L^\dagger{=}\exp(-i\bar{A}_0)$

- Effective action: $S^{4d} = \int d^3x\, dt\ \mathrm{Tr} F_{\mu\nu}^2 \longrightarrow$

$$S_{\rm eff}^{3d} = \int d^3x \ [\mathrm{Tr}\bar{F}_{ij}^2 + m^2 \bar{A}_0^2 + (D_i \bar{A}_0)^2 + \lambda \bar{A}_0^4 + \cdots]$$

ie. 3d Yang-Mills + adjoint Higgs

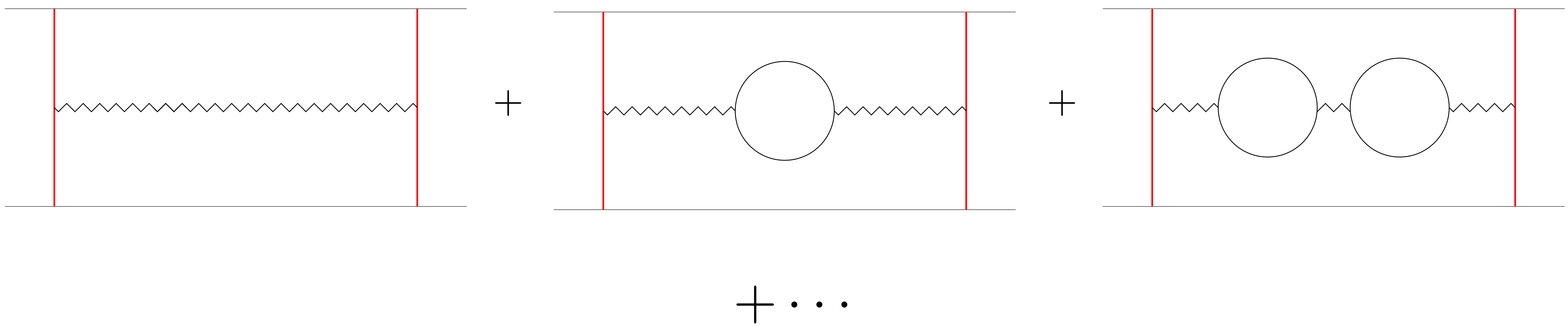
3d couplings by integrating out non-static modes. Tree-level: $(g_{\rm eff}^{3d})^2 = g(T)T$

Note: 3d theory is confining, ie. non-perturbative in IR (spatial string tension, $3d$ glueball, ...)

Linde: non-perturbative scale is $g^2(T)T$

Debye screening: QED

- QED: e^+e^- thermal pair creation screens static charges



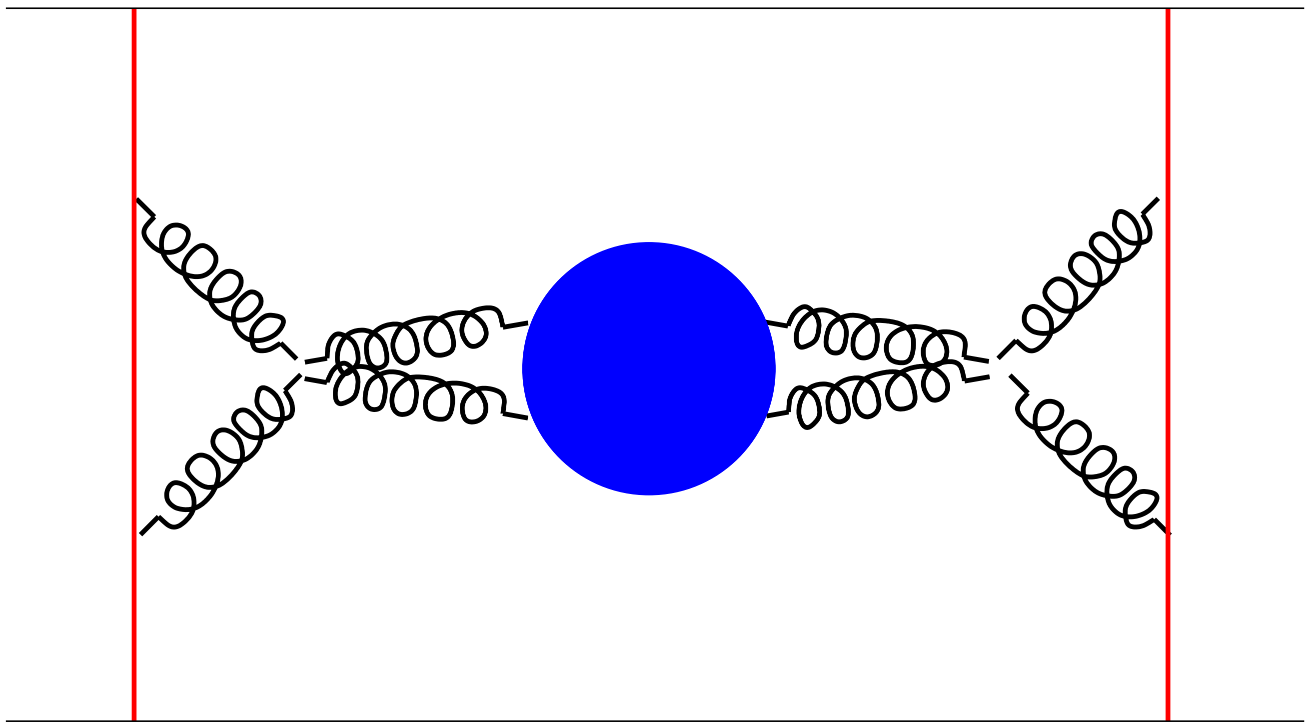
Resum all orders: $$\sum_k \mathcal{O}(e^2)^k \to \mathcal{O}(e)$$

Coulomb potential $\propto \frac{1}{r}$ at $T = 0$ $\longrightarrow$ Yukawa $\propto \frac{\exp(-m_D r)}{r}$ at $T > 0$

$$m_D = \frac{eT}{\sqrt{3}}(1 + \mathcal{O}(e^2))$$

Debye screening: QCD

- QCD: $\mathrm{Tr(Polyakov\ loop)}$ is color singlet, so at least 2 gluons emitted
- to lowest order, similar to QED: $m_D = gT\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$
- higher order: 3- and 4-gluon vertex couples to $3d$ glueball: $m_G \sim \mathcal{O}(g^2 T)$



Confining potential $\propto \sigma r$ at $T = 0$ $\longrightarrow$ $\propto \frac{\exp(-m_D r)}{r}$ then $r \nearrow$ $\propto \frac{\exp(-m_G r)}{r}$

Recap: 3 scales in high-T QCD

- "hard" scale $2\pi T$ non-static modes
- "soft" scale $g(T)T$ Debye mass ("electric")
- "ultrasoft" scale $g^2(T)T$ 3d glueball mass ("magnetic")

Hierarchy as $T \rightarrow \infty, \quad g(T) \rightarrow 0$

- integrate out "hard" scale $\longrightarrow$ effective theory EQCD (electric)

  3d Yang-Mills + adjoint Higgs

- integrate out "soft" scale $\longrightarrow$ effective theory MQCD (magnetic)

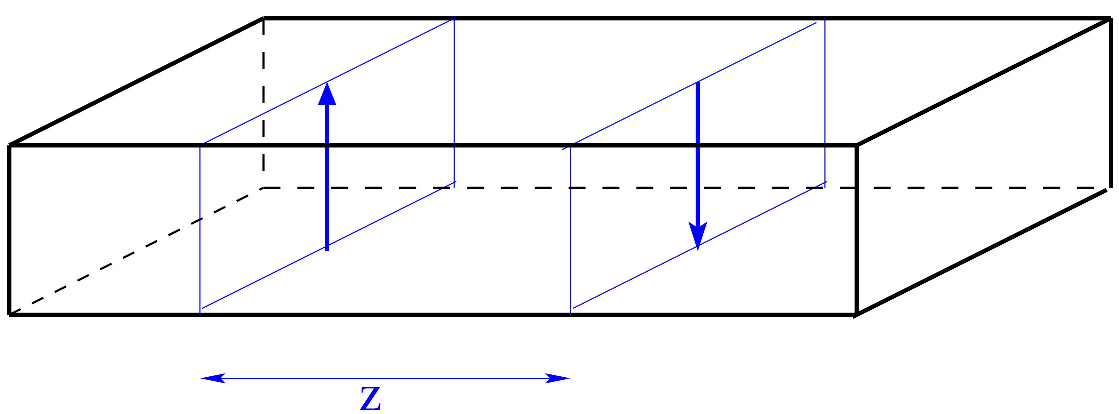
  3d Yang-Mills

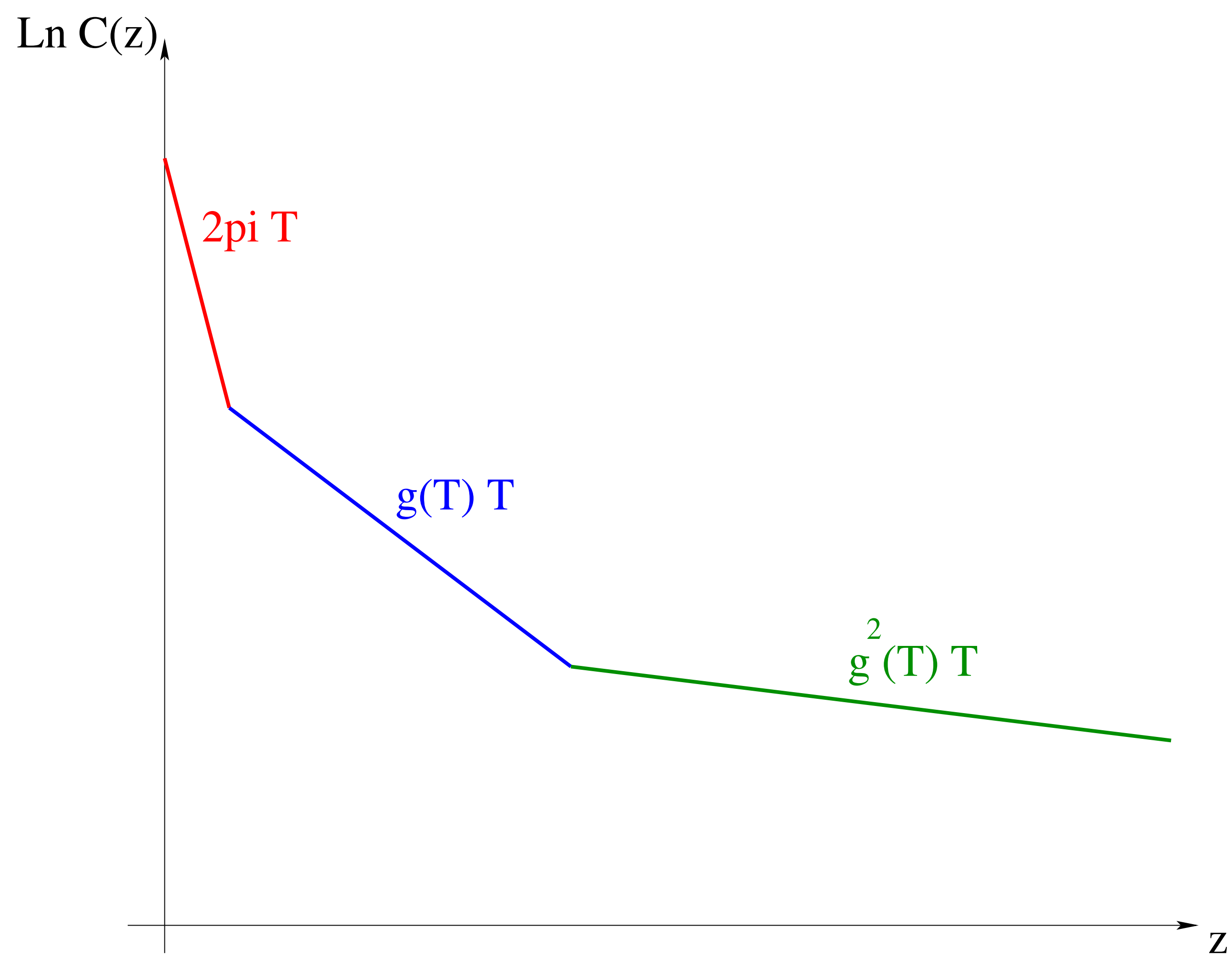
..., Kajantie et al, ...

How high should T be ?

$$g^2 T \ll gT \ll 2\pi T$$

Look for 3 scales in decay rate of correlator of Polyakov loop L





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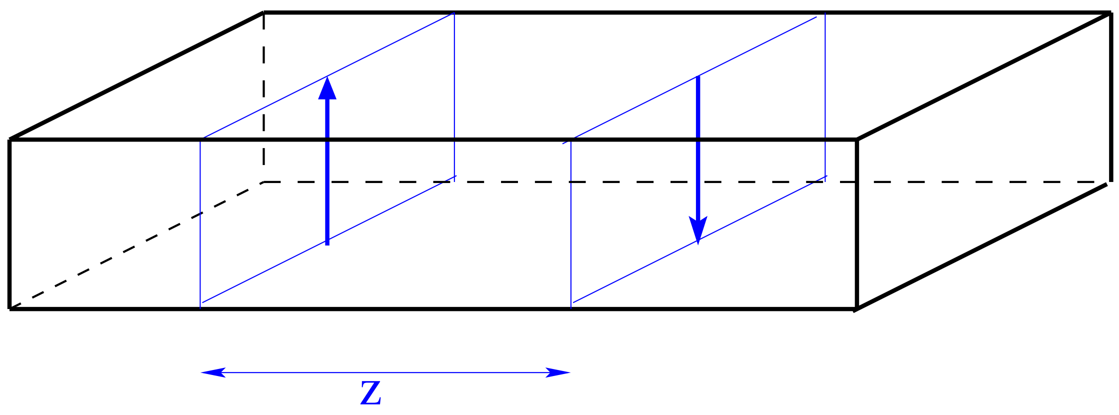
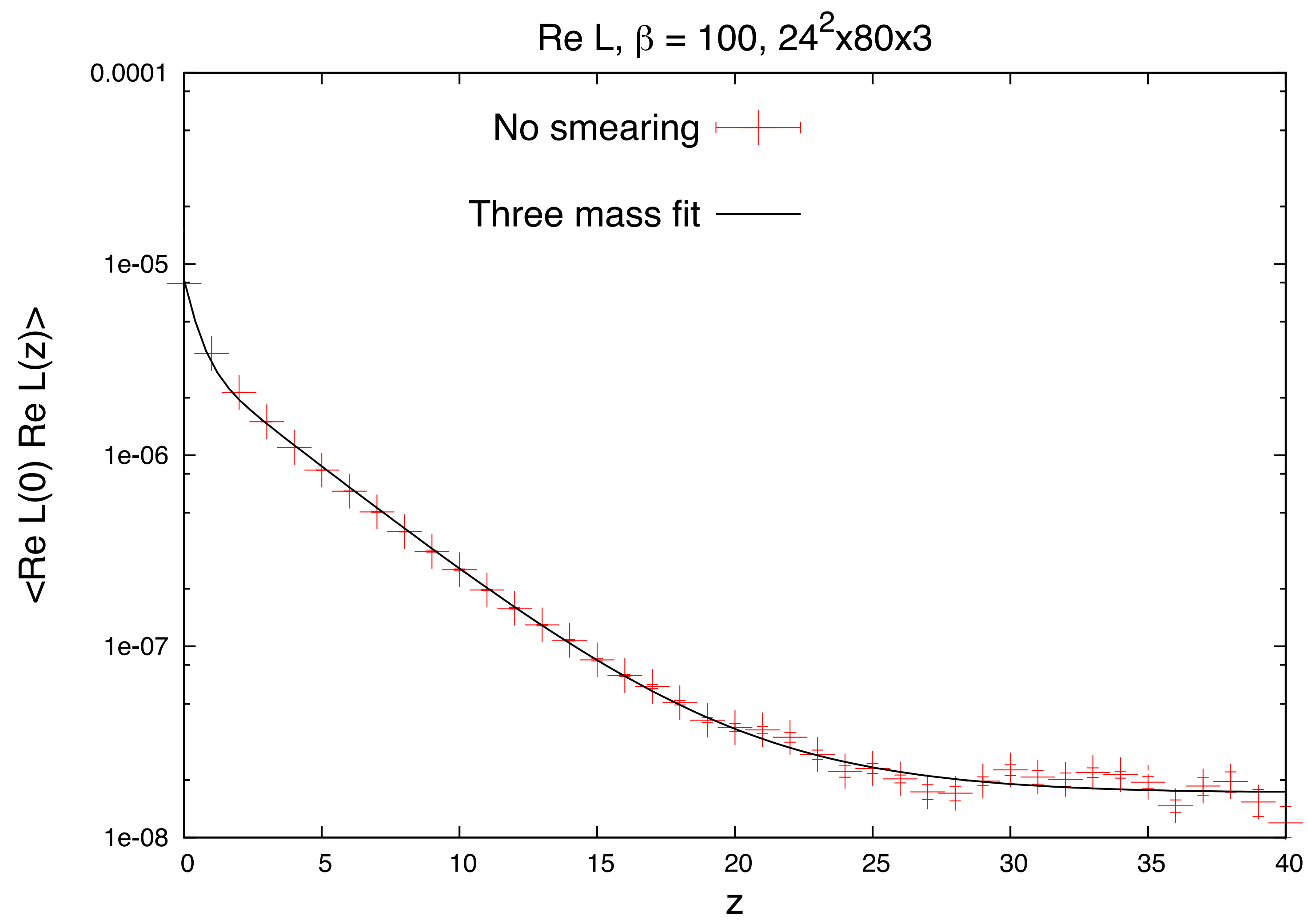
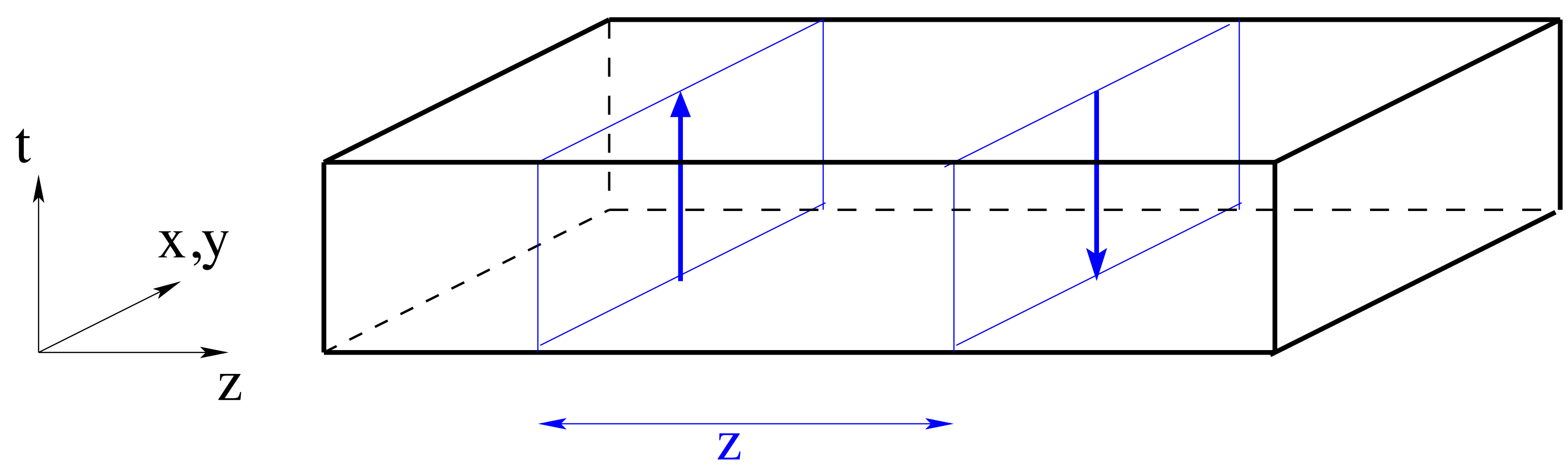




Symmetries

- Reversal of Euclidean time "R": $t \to -t, \quad A_0 \to -A_0, \ \mathrm{Tr}L \to \mathrm{Tr}L^\dagger$

- MQCD (3d effective theory, no A_0) is "R"-even $\to$ scale g^2T in "R"-even observables only



Define $\mathrm{Re}L = \dfrac{L + L^\dagger}{2}$ (R-even), $\mathrm{Im}L = \dfrac{L - L^\dagger}{2}$ (R-odd)

$$\langle [\sum_{xy} \mathrm{Tr}\mathrm{Re}L(x,y,0)][\sum_{xy} \mathrm{Tr}\mathrm{Re}L(x,y,z)]\rangle_c$$ projects on $\{2\pi T, gT, g^2T\}$

$$\langle [\sum_{xy} \mathrm{Tr}\mathrm{Im}L(x,y,0)][\sum_{xy} \mathrm{Tr}\mathrm{Im}L(x,y,z)]\rangle$$ projects on $\{2\pi T, gT\}$

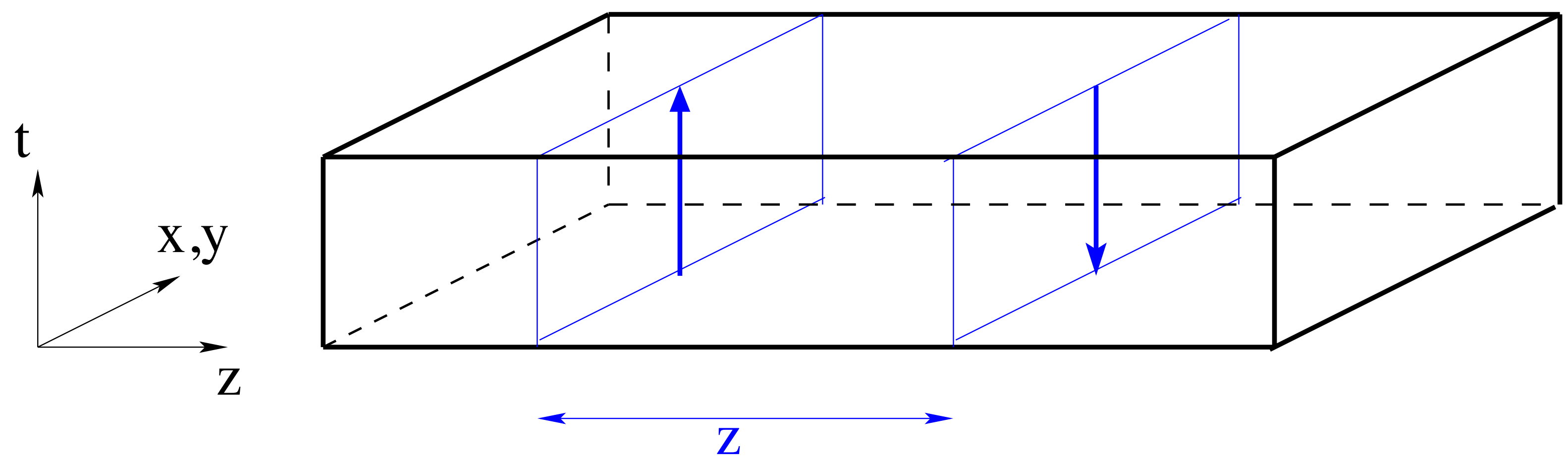
Expectations

- Polyakov loop: $L = \exp(i\bar{A}_0) \underset{\bar{A}_0 \ll 1}{\approx} 1 + i\bar{A}_0 - \frac{1}{2}\bar{A}_0^2 - \frac{i}{6}\bar{A}_0^3 + \cdots$ with $\mathrm{Tr}\bar{A}_0 = 0$

- At $\mathcal{O}(gT)$:

$$\mathrm{TrRe}L \sim \bar{A}_0^2 \longrightarrow \text{mass } 2m_E$$

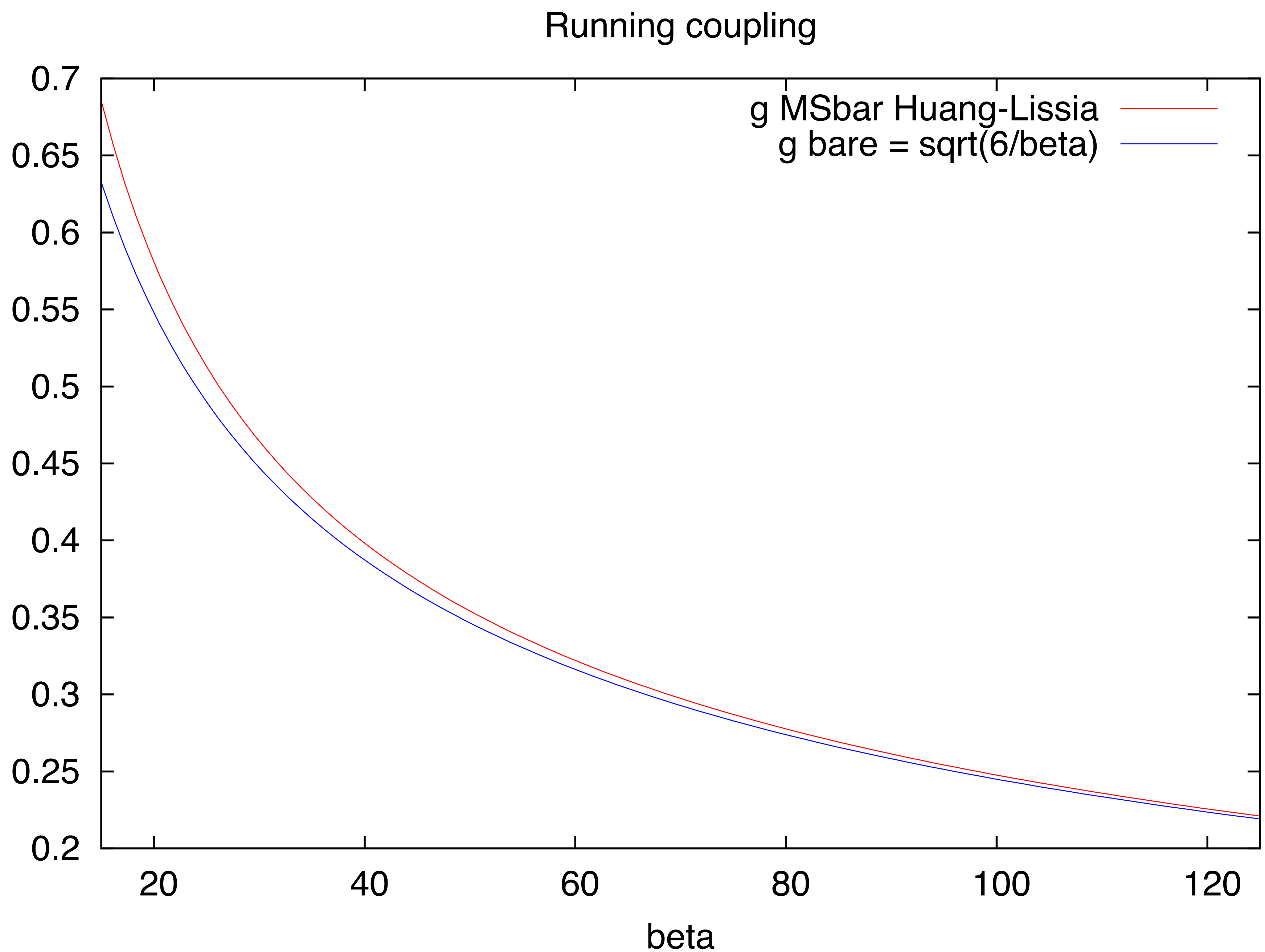
$$\mathrm{TrIm}L \sim \bar{A}_0^3 \longrightarrow \text{mass } 3m_E$$



$$\langle[\sum_{xy}\mathrm{TrRe}L(x,y,0)][\sum_{xy}\mathrm{TrRe}L(x,y,z)]\rangle_c \longrightarrow m_{\mathrm{eff}} = \{\sim 2\pi T,\ 2m_E + \mathrm{corr.},\ m_G(0^+)\}$$

$$\langle[\sum_{xy}\mathrm{TrIm}L(x,y,0)][\sum_{xy}\mathrm{TrIm}L(x,y,z)]\rangle \longrightarrow m_{\mathrm{eff}} = \{\sim 2\pi T,\ 3m_E + \mathrm{corr.}\}$$

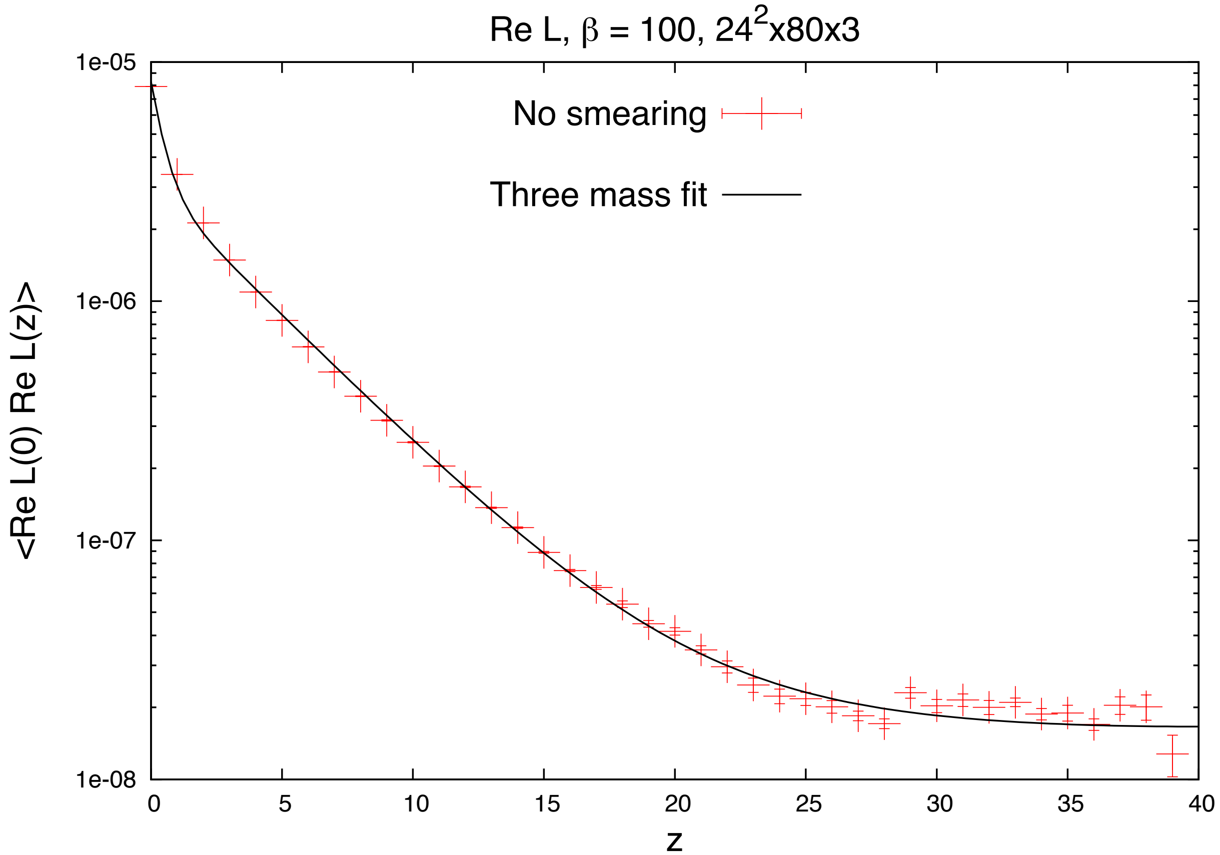
Results: $SU(3), 24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20..120]$

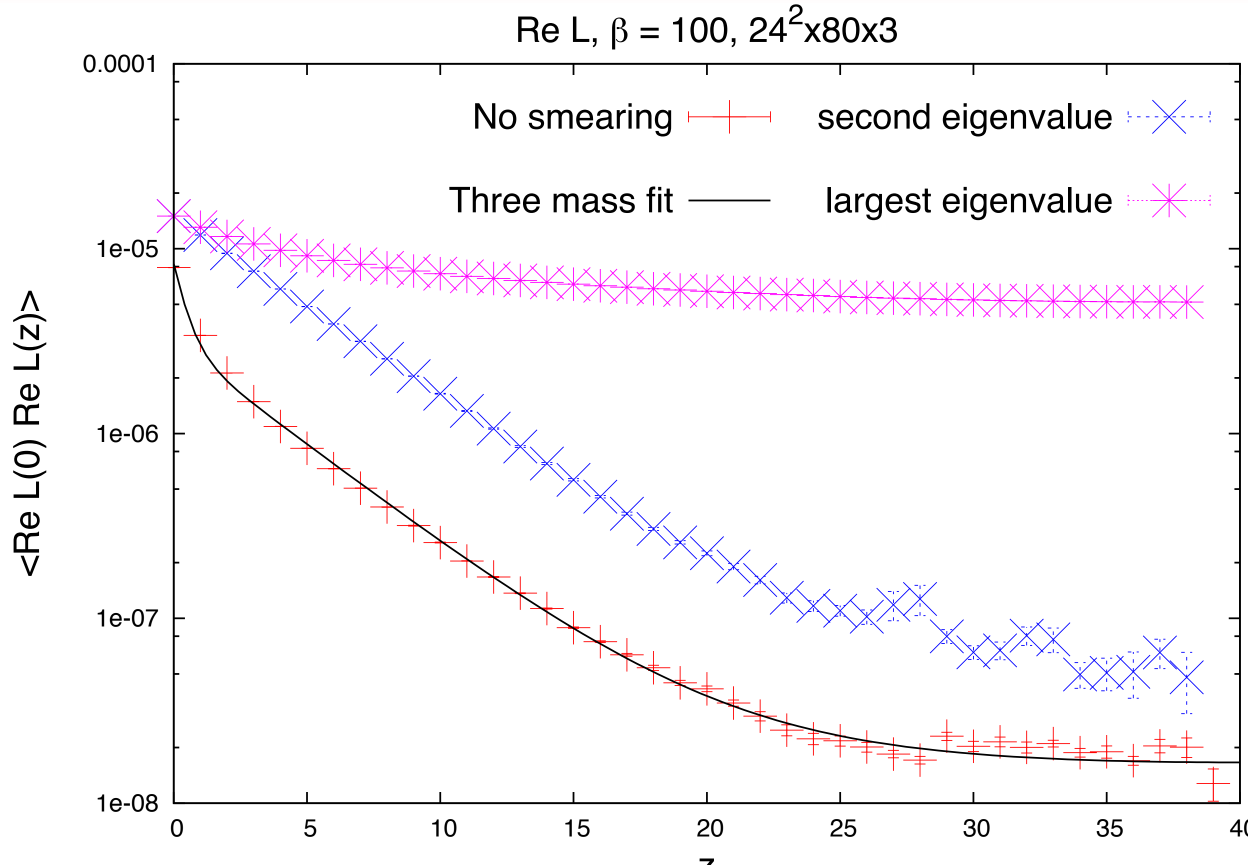


g chosen suitably small (0.2 - 0.6) for a clear scale hierarchy -- *hopefully*

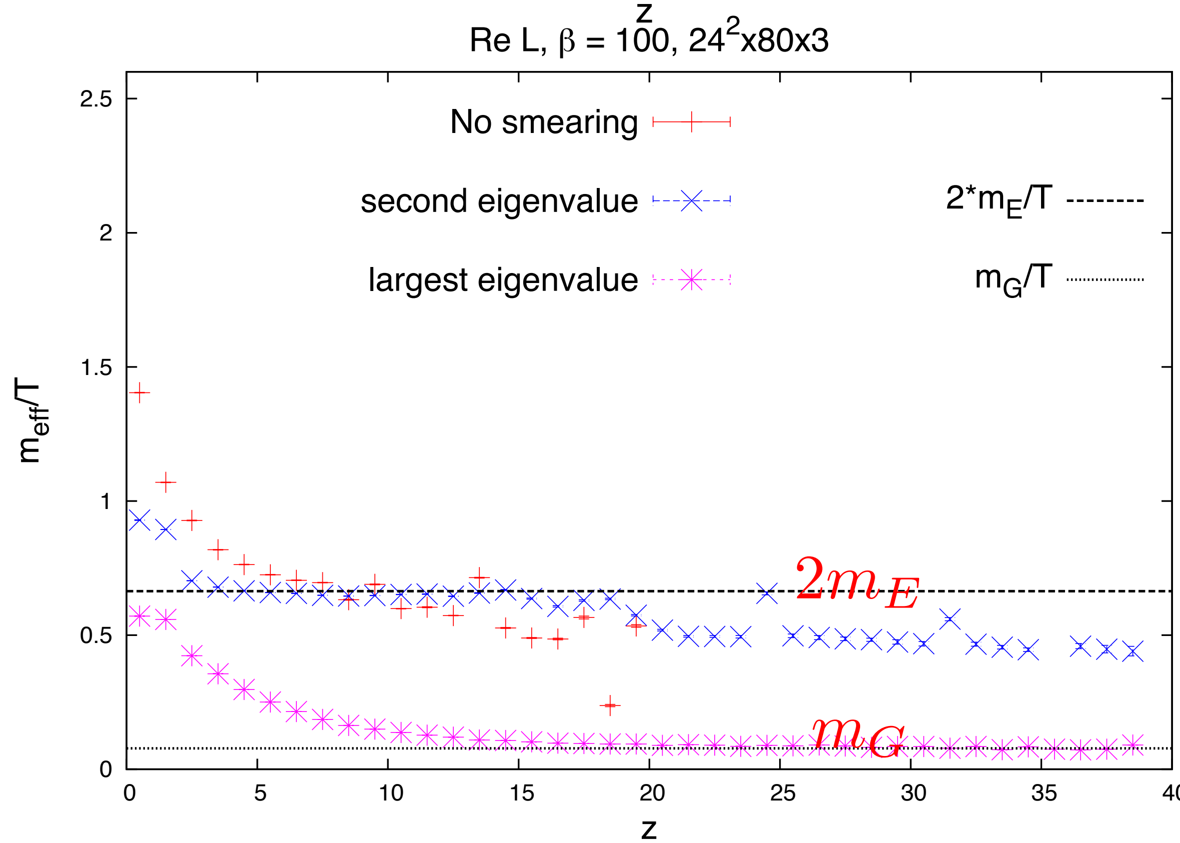
But $T \sim 10^9 - 10^{69} T_c$!!!

Results: $SU(3), 24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20..120]$





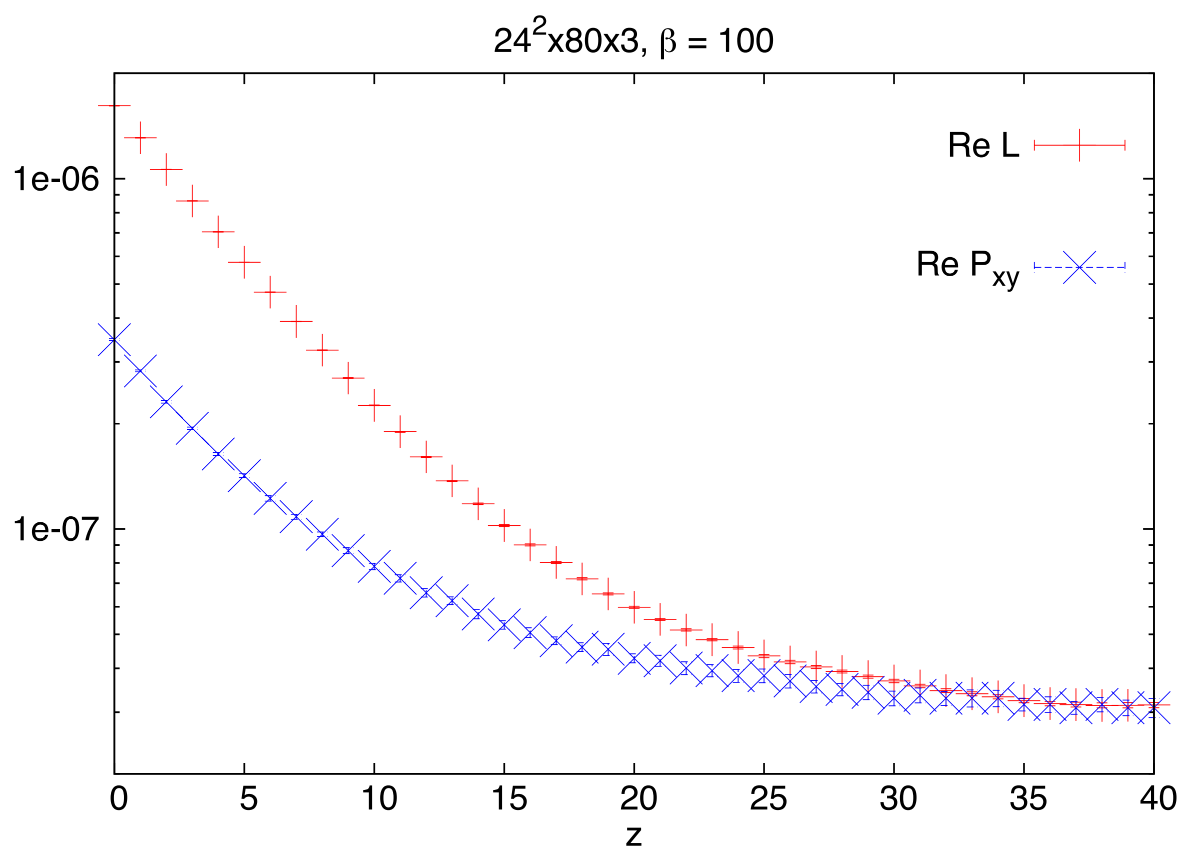
- $\mathrm{Re}L \longrightarrow m_{\mathrm{eff}} = \{\sim 2\pi T,\ 2m_E + \mathrm{corr.},\ m_G(0^+)\}$



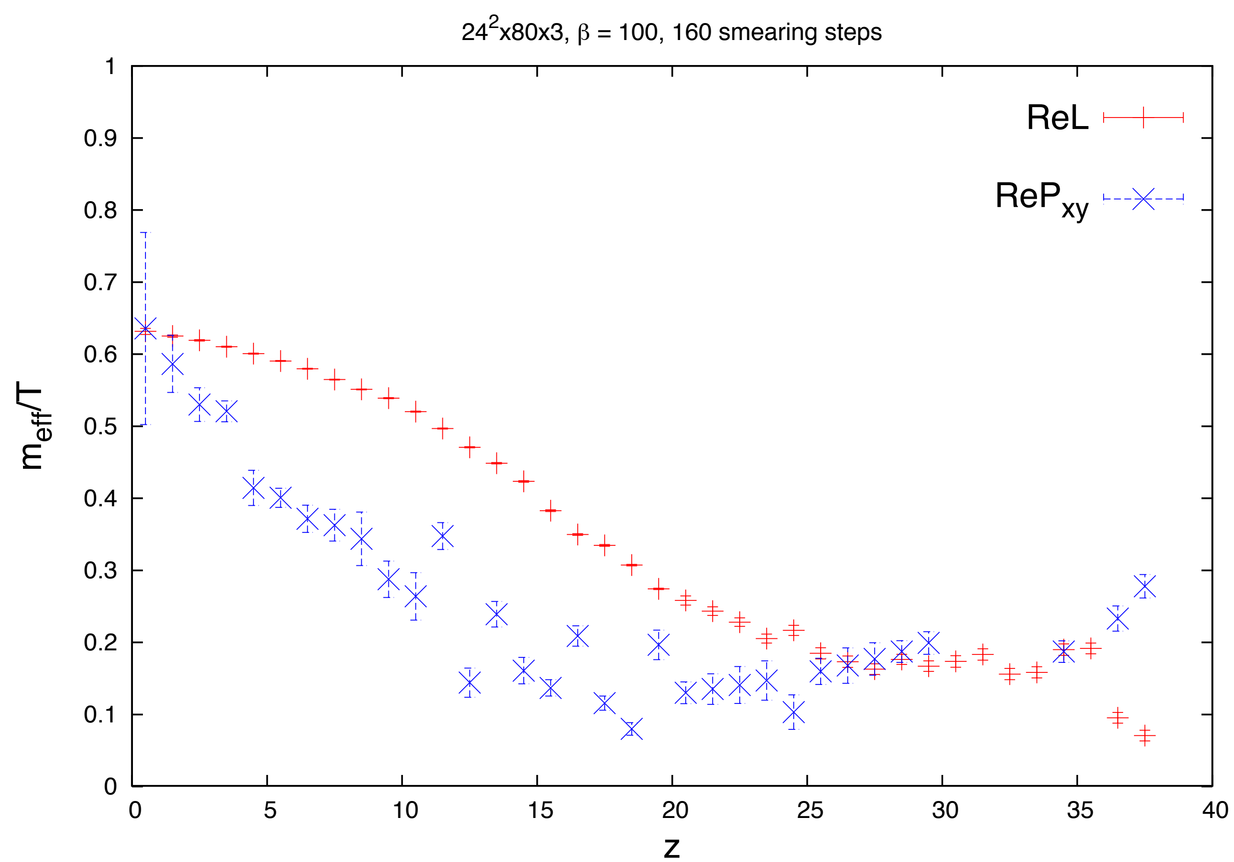
Results: $SU(3), 24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20..120]$

- $\mathrm{Re}L \longrightarrow m_{\mathrm{eff}} = \{\sim 2\pi T,\ 2m_E + \mathrm{corr.},\ m_G(0^+)\}$

- Crosscheck $m_G(0^+)$ by measuring correlator of $\mathrm{ReTr\ Plaq}_{xy} \sim F_{xy}^2$

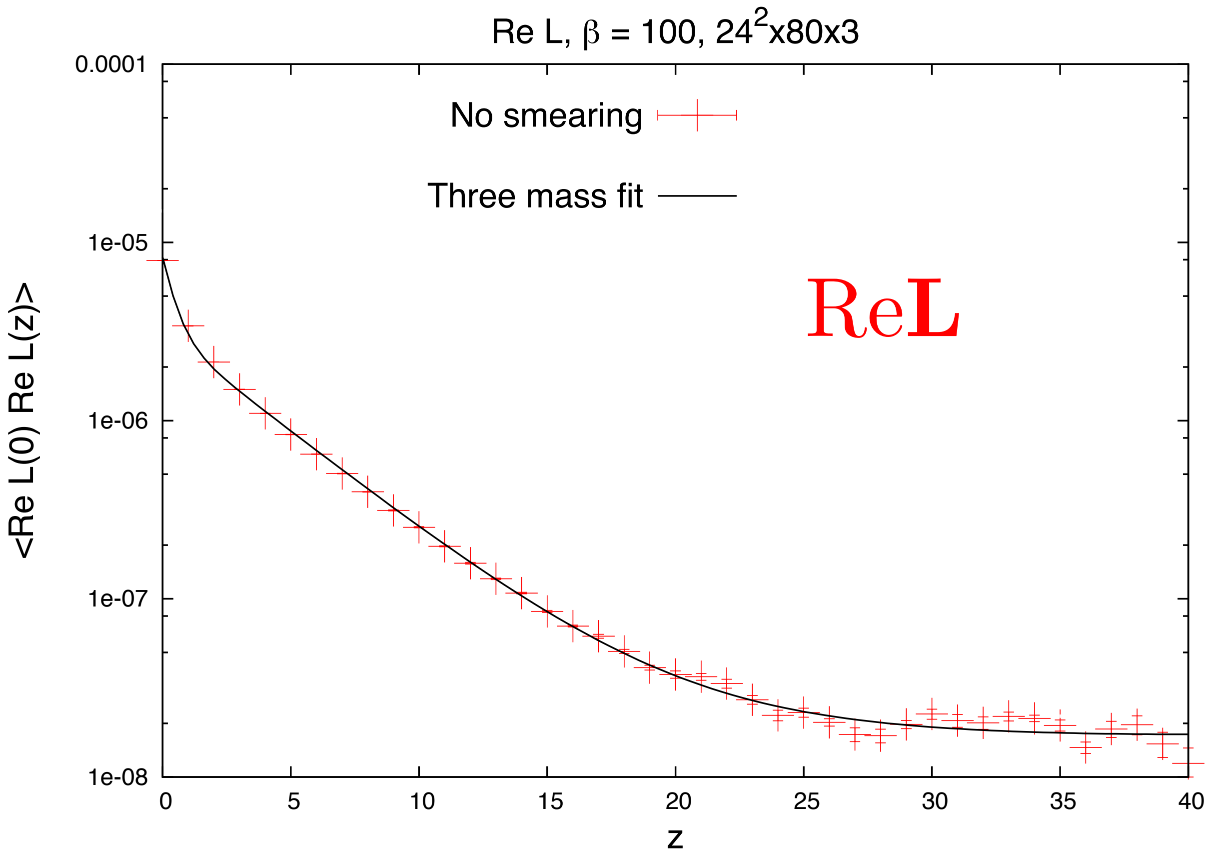


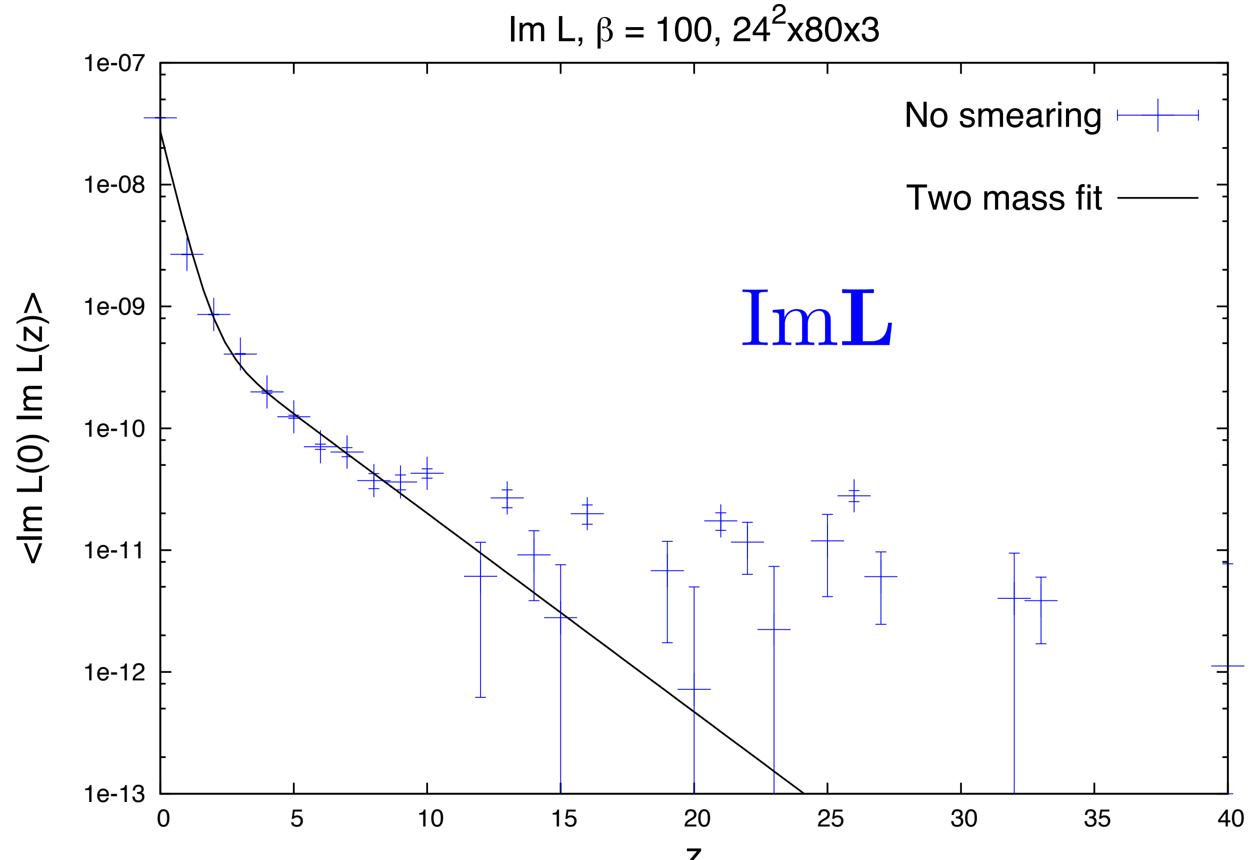
Compare correlators

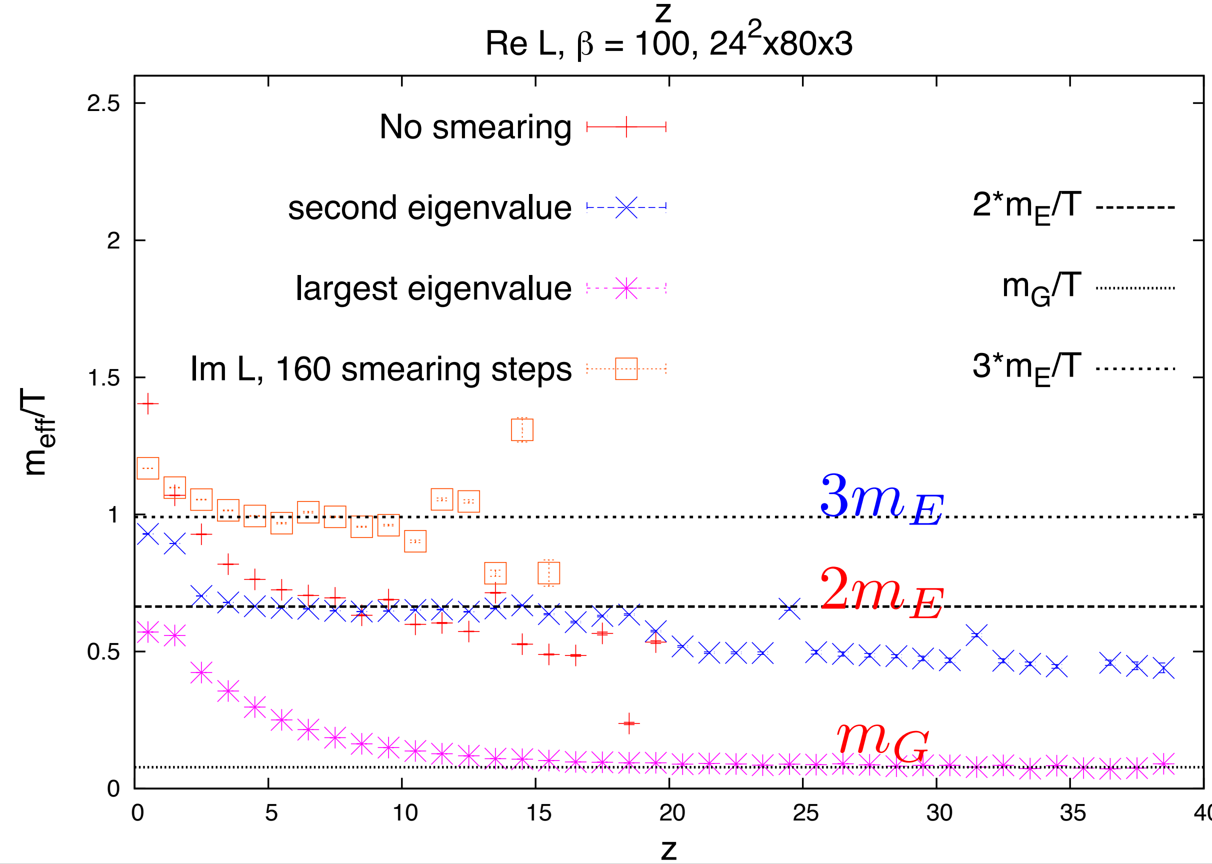


Compare effective masses

Results: $SU(3), 24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20..120]$



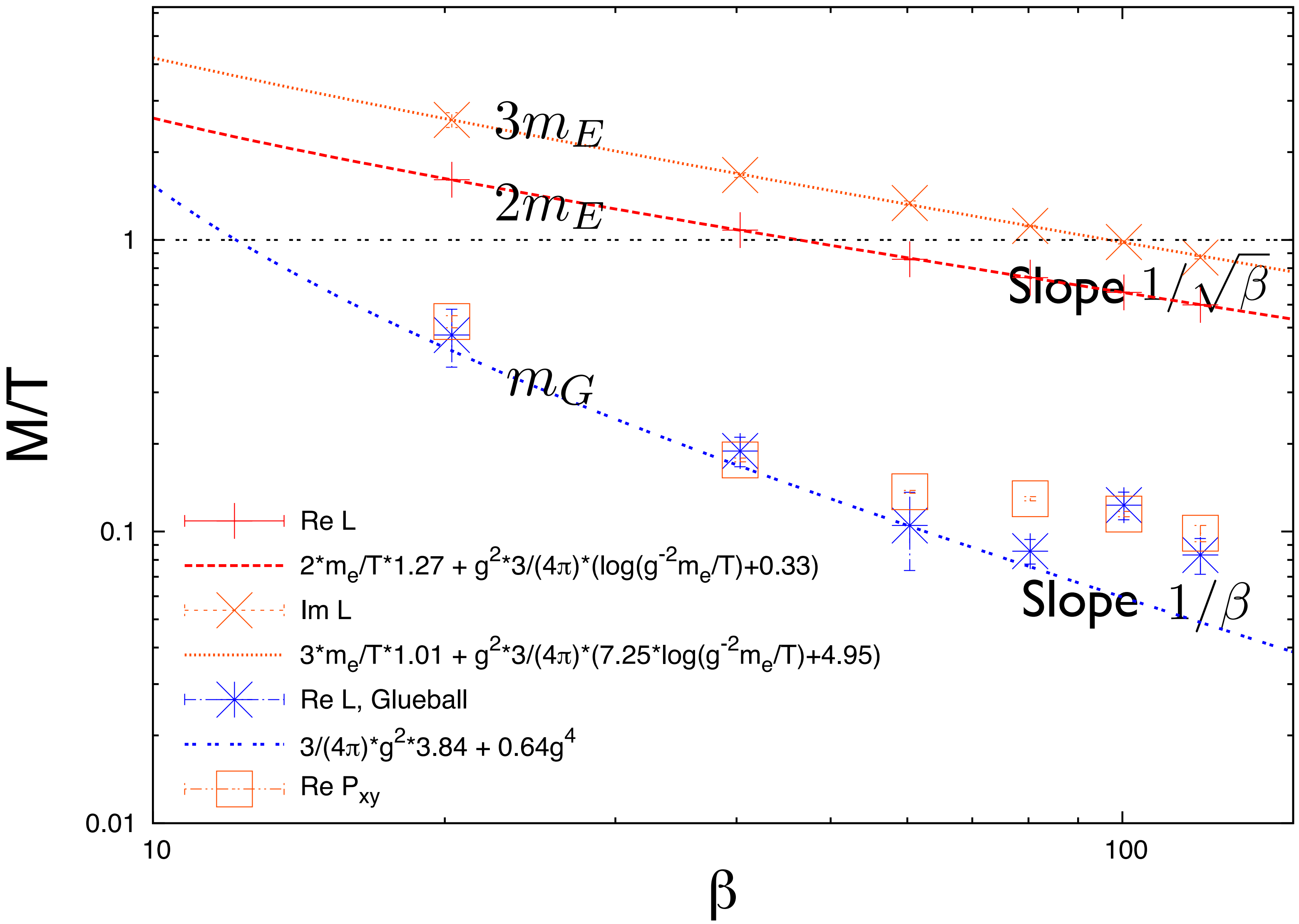




- $\mathrm{Im}L \longrightarrow m_{\text{eff}} = \{\sim 2\pi T,\ 3m_E + \text{corr.}\}$

Results: $SU(3), 24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20..120]$

- Masses versus β: - check $m_E \sim gT,\ m_G \sim g^2 T$
  - fit non-perturbative corrections to m_E
  - finite-size effects on m_G at high β $(m_G\, L \lesssim 1)$
  + discretization errors $\mathcal{O}(1/\beta)$

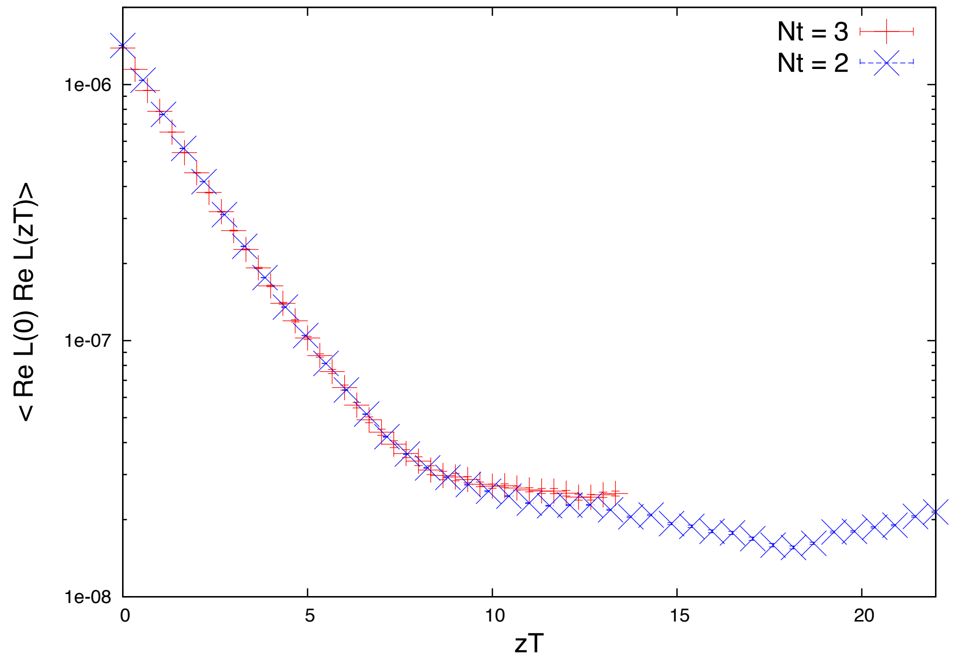


$$3m_E \to 3m_E \times 1.01(14) + \frac{3g^2}{4\pi}[\log \frac{m_E}{g^2 T} \times 7.25(50) + 4.95(80)]$$

$$2m_E \to 2m_E \times 1.27(1) + \frac{3g^2}{4\pi}[\log \frac{m_E}{g^2 T} + 0.33(12)]$$

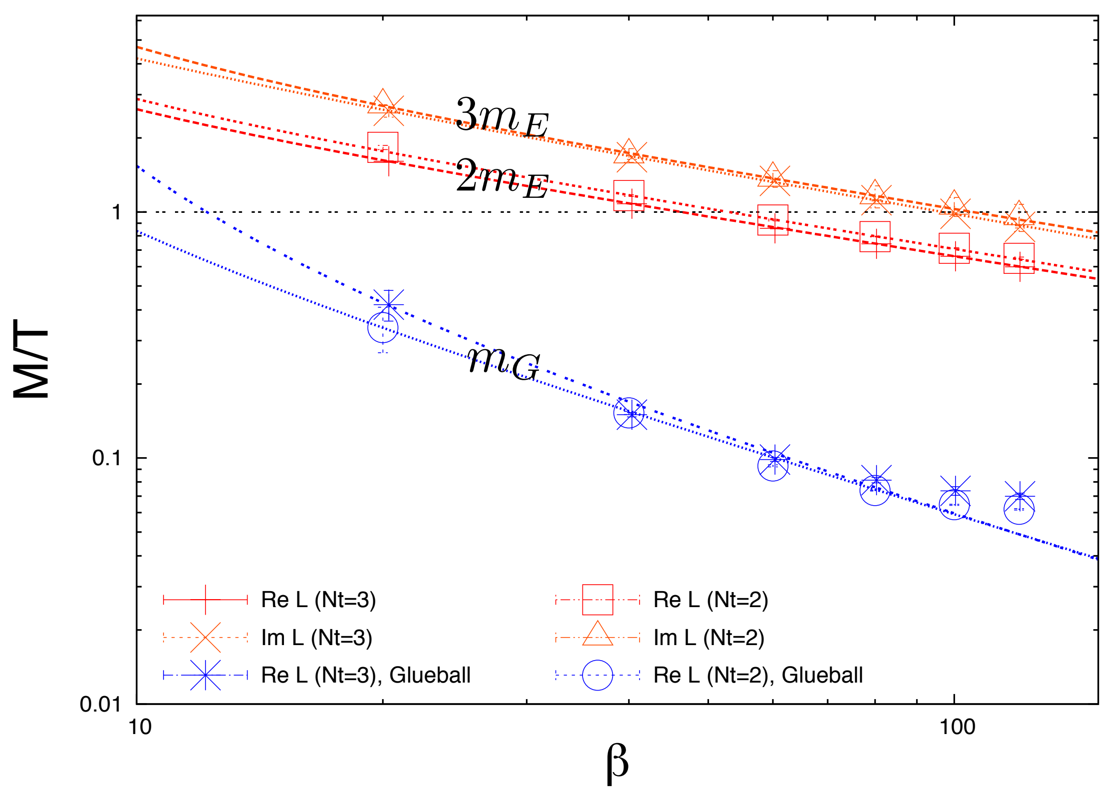
Results: $SU(3), 24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20..120]$

- Continuum limit: compare $(N_t = 2, \beta)$ and $(N_t = 3, \beta + \Delta\beta)$



Correlators match after rescaling coordinates!

Small spectrum corrections, consistent with $1/N_t^2$

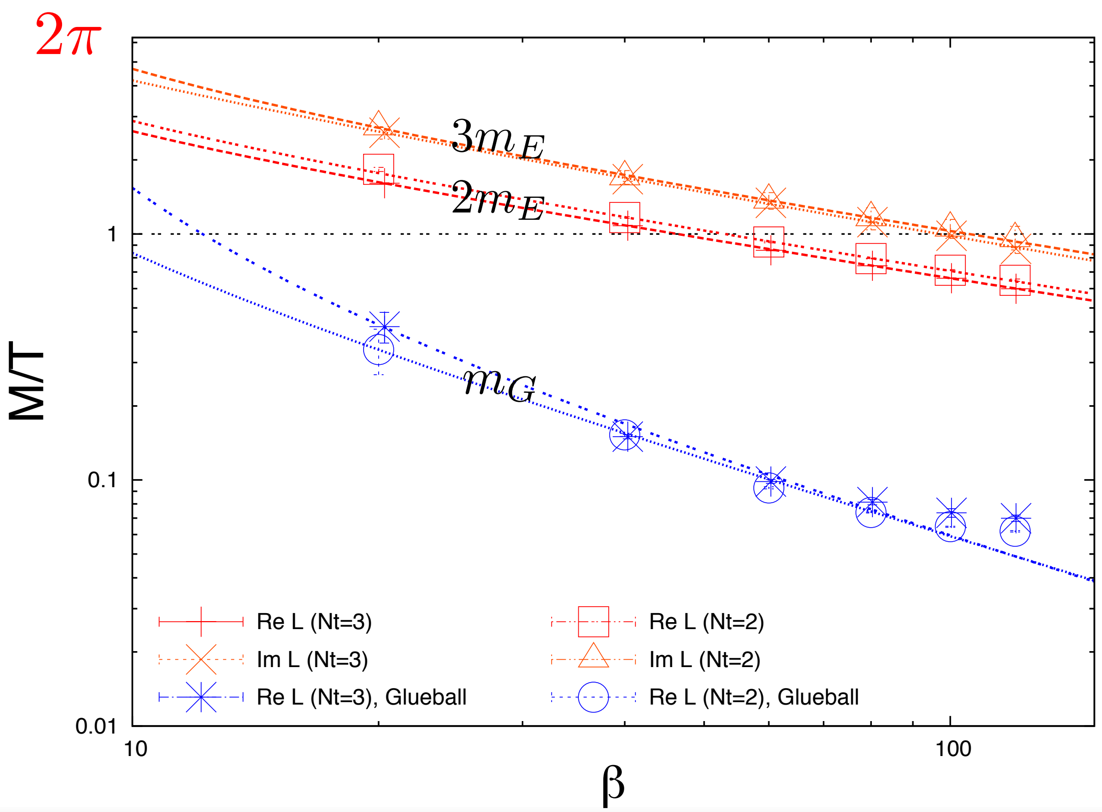


$$3m_E \to 3m_{E,\text{cont}} \times \{1.29(7)[N_t = 2], 1.01(14)[N_t = 3]\}$$

$$2m_E \to 2m_{E,\text{cont}} \times \{1.34(3)[N_t = 2], 1.27(1)[N_t = 3]\}$$

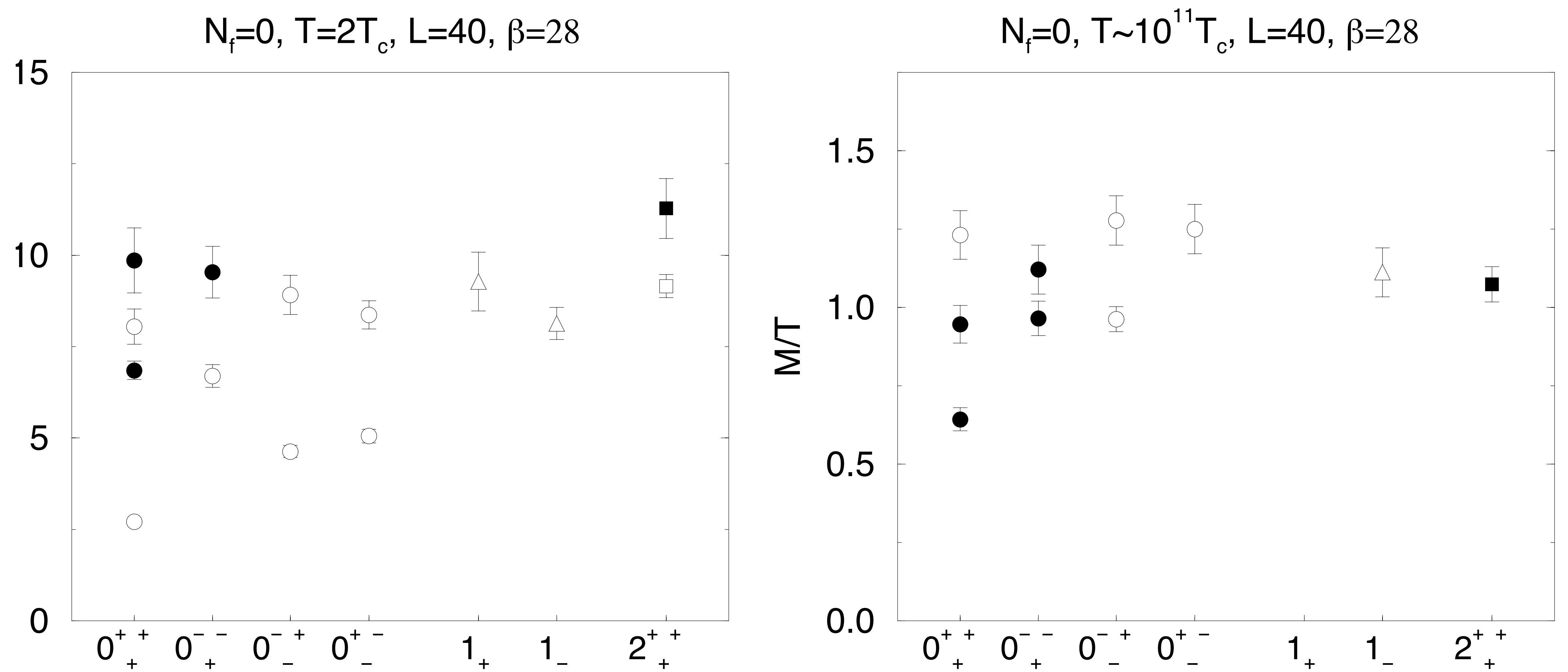
Summary

- $\mathrm{Im}L$: $\dfrac{M(\mathrm{Tr}[A_0^3])}{T} = 3\dfrac{m_E}{T} + \dfrac{g^2 N_c}{4\pi}\left(b_3 \log\dfrac{m_E}{g^2 T} + c_3\right)$, b_3, c_3 non-perturbative
- $\mathrm{Re}L$: $\dfrac{M(\mathrm{Tr}[A_0^2])}{T} = 2\dfrac{m_E}{T} + \dfrac{g^2 N_c}{4\pi}\left(\log\dfrac{m_e}{g^2 T} + c_2\right)$, c_2 non-perturbative and $m_G(0^+)$
- $\dfrac{M}{T} \gtrsim 1$: No clear scale hierarchy even at $T \sim 10^{30} T_c$



Lesson: success of effective 3d description at $T \gtrsim 3 - 10\ T_c$ does not necessarily imply scale hierarchy

Compare with 3d simulations of EQCD: hep-ph/0004060 (Hart et al)



The spectrum of screening masses in various quantum number channels at $N_f = 0, T = 2T_c$ (left), $N_f = 0, T \sim 10^{11} T_c$ (right).

Filled symbols denote 3d glueball states, which have become the lightest excitations at $T \sim 10^{11} T_c$

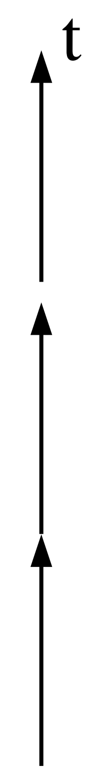
Hierarchy $2\pi T \gg m_E \gg m_G$ more or less OK at $T \sim 10^{11} T_c$

Inverted into $m_E \sim 2\pi T \lesssim m_G$ at $T = 2T_c$

Supplement with center degrees of freedom: 0801.1566 (Kurkela et al)

Subtlety

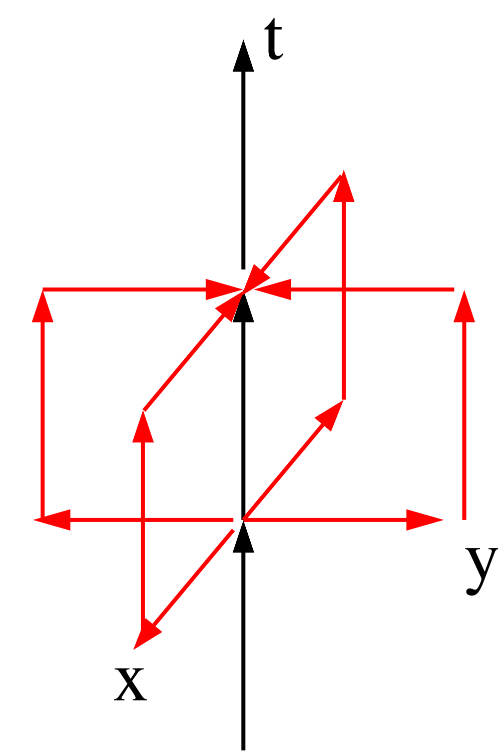
- Without smearing, $\mathrm{Tr}\,\mathrm{Im}L = \mathrm{Tr}\frac{L - L^\dagger}{2}$ is "R"-odd (changes sign under $t \to -t$)



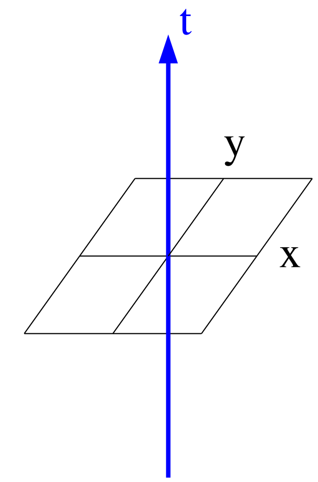
- No longer true after smearing: changes sign under $\{t \to -t, x \to -x, y \to -y\}$ ***together***

$\mathrm{Tr}\mathrm{Im}L$ is neither "R"-odd nor "R"-even

projects onto lightest state ("R"-even glueball)

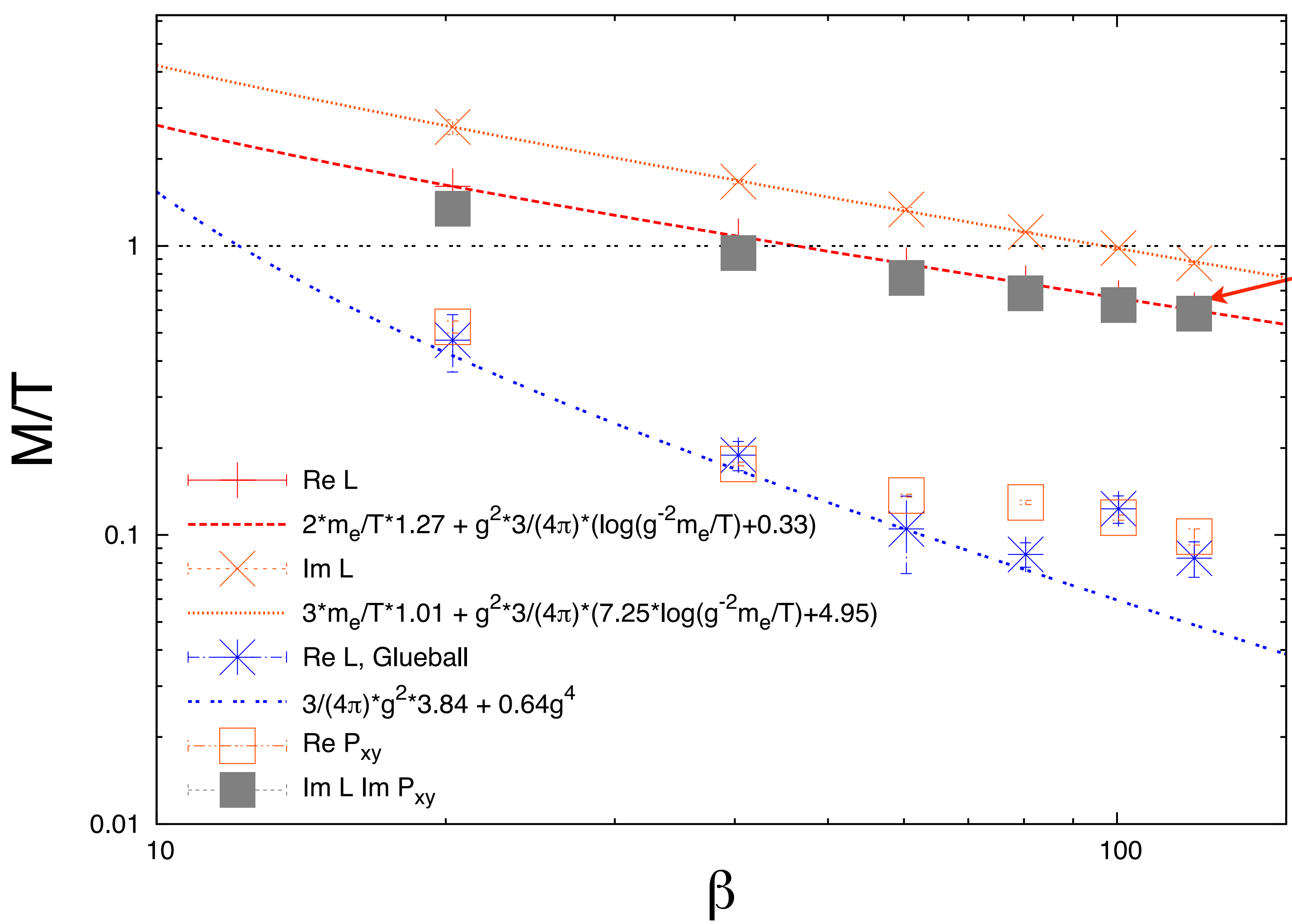


Mystery?



- Measure mass from $\mathrm{Im}L\ \mathrm{ImPlaq}_{xy} \sim \mathrm{ReTr}[A_0 F_{xy}] \sim m_E$

Expect $$\frac{M(\mathrm{Tr}[A_0 F_{xy}])}{T} = \frac{m_E}{T} + \frac{g^2 N_c}{4\pi}\left(\log\frac{m_E}{g^2 T} + c_1\right)$$



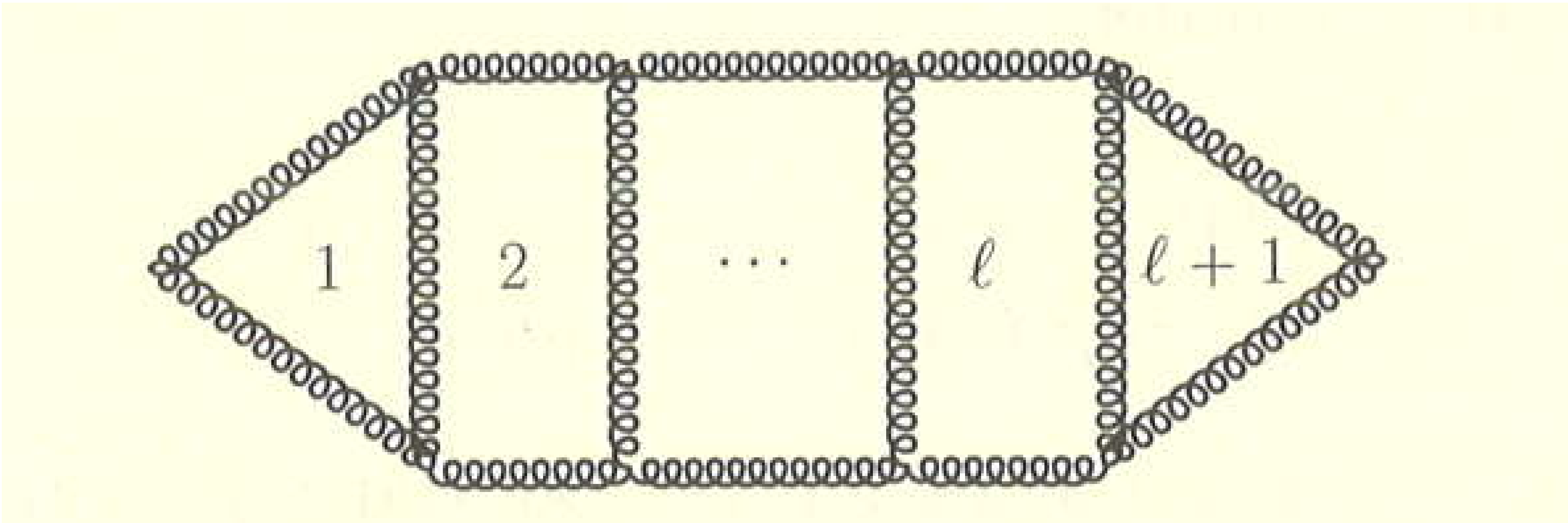
Too heavy?

$"m_E"$ approaches $"2m_E"$

as $\beta \to \infty$ ⁇

Non-perturbative $\mathcal{O}(g^2T)$: Linde problem

- Consider perturbative expansion of free energy (pressure)



$(l+1)$ loops, $2l$ vertices, $3l$ propagators: $$g^{2l}\left(T\int d^3k\right)^{l+1} k^{2l}(k^2+m^2)^{-3l}$$

$m = 0 \;\rightarrow\; \int dk\; k^{3l+2-4l}$ IR-divergent if $l \geq 3$ ie. non-perturbative at $\mathcal{O}(g^6)$

Divergence cured by non-perturbative mass $m_G \sim \mathcal{O}(g^2T)$ mass gap of 3d theory

(3d glueball)