Neutron Electric Dipole Moment from Beyond the Standard Model Physics

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Introduction
Neutron Electric Dipole Moments

$$\frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\bar{\psi}\sigma^{\alpha\beta}F^{\mu\nu}\psi \quad = \quad \bar{\psi}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu}\psi$$

Electric dipole moments of non-degenerate elementary particles violates Parity (P) and Time-reversal (T) symmetries.

Violation of time-reversal necessary for baryon-asymmetry in standard cosmological scenario.

Standard model of particle physics has too little violation of T.

Most extensions of the standard model allow P and T violation.

Many of them give rise to electric dipole moments.

Introduction
Standard Model CP Violation

Two sources of CP violation in the Standard Model.

- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM

Dar `arXiv:hep-ph/0008248`.

- Effective $\Theta G\tilde{G}$ interaction from QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$

Crewther *et al., Phys. Lett.* **B88** (1979) 123.

Effective Field Theory

Dimension four

In QCD written in the mass basis, three kinds of T violating pieces up to dimension four are allowed.

$$\begin{aligned} \mathcal{S}^{(4)} &= \mathcal{S}_{QCD}^{\text{massless,CP Even}} - \int d^4x \, i\, \Theta \frac{g^2}{16\pi^2} \epsilon_{\alpha\beta\mu\nu} G^{\alpha\beta\, A} G^{\mu\nu\, A} \\ &+ \bar{\psi}(m + i\, m_5 \gamma_5)\psi \\ &+ \bar{\psi}(m^I + i\, m_5^I \gamma_5)\tau_3 \psi \,, \end{aligned}$$

where τ_3 represents a diagonal $SU(N_f)$ generator.

Theta term corresponding to $U(1)$ electromagnetic factor do not contribute for any finite action configuration, and for the weak $SU(2)$ can be rotated away since they couple only to left chiral fields.

Under field redefinitions $\psi \to \exp(i\gamma_5\alpha)\psi$ and $\psi \to \exp(i\gamma_5\beta\tau_3)\psi$

$$\begin{aligned}
m &\to m\, c_\alpha\, c_\beta - m_5\, s_\alpha\, c_\beta - m_5^I\, c_\alpha\, s_\beta - m^I\, s_\alpha\, s_\beta\,, \\
m_5 &\to m_5\, c_\alpha\, c_\beta + m_5\, s_\alpha\, c_\beta + m^I\, c_\alpha\, s_\beta - m_5^I\, s_\alpha\, s_\beta\,, \\
m^I &\to m^I\, c_\alpha\, c_\beta - m_5^I\, s_\alpha\, c_\beta - m_5\, c_\alpha\, s_\beta - m\, s_\alpha\, s_\beta\,, \\
m_5^I &\to m_5^I\, c_\alpha\, c_\beta + m^I\, s_\alpha\, c_\beta + m\, c_\alpha\, s_\beta - m_5\, s_\alpha\, s_\beta\,, \\
\theta &\to \theta + \alpha\,,
\end{aligned}$$

where $c, s \equiv \cos, \sin$.

Manifold of parameters $C^{N_f} \times S^1/SU(N_f) \times U(1)$ has conical singularity when more than one mass is zero, regular everywhere else.

Field rotations leave $S_{QCD}^{\mathrm{massless,CP-even}}$ invariant.
Only one CP violation parameter.

Effective Field Theory
Dimension five

At dimension five, there are two operators QEDM and QCEDM

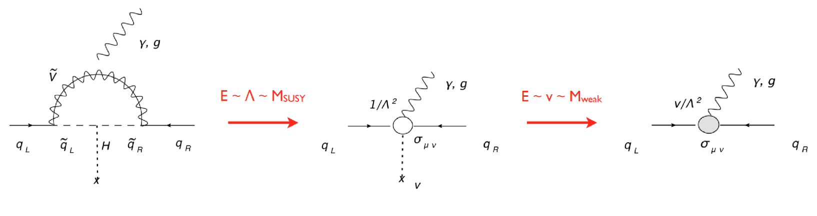
$$\begin{aligned}\mathcal{S}^{(5)} = \mathcal{S}^{(4)} &+ \frac{i\,e}{\Lambda^2_{\rm BSM}}\left(d_u^\gamma \bar{Q}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\tilde{H}U + d_d^\gamma \bar{Q}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu} H D\right)\\ &+ \frac{i\,g_3}{\Lambda^2_{\rm BSM}}\left(d_u^G \bar{Q}\sigma_{\mu\nu}\gamma_5 \lambda^A G^{\mu\nu\,A}\tilde{H}U + d_d^G \bar{Q}\sigma_{\mu\nu}\gamma_5 \lambda^A G^{\mu\nu\,A} H D\right)\end{aligned}$$

where $H, \tilde{H}$ are mass scales breaking $SU(2)_W$ symmetry.
Arise from dimension six operators at the weak scale.
Left-Right operators: often proportional to masses.

May be generated from corresponding anomalous magnetic dipole moments by field rotation.

These quark dipole moments are generated at 3-loops in the standard model and give tiny nEDM ($\sim 10^{-34}$ e-cm).

They are generated at one loop in BSM.



Expected contribution is around experimental limit $\sim 2.9 \times 10^{-26}$e-cm.

Baker *et al.*, *Phys. Rec. Lett.* **97** (2006) 131801.

Effective Field Theory

Dimension six

At dimension six, we have

Weinberg three gluon operator: $\frac{1}{3} f^{ABC} \epsilon_{\nu\alpha\beta\phi} G^{A\,\mu\nu} G^{B\,\alpha\beta} G^{C\,\phi}{}_{\mu}$
Gauge electric dipole moment operator: $\frac{1}{2} \epsilon_{\nu\alpha\beta\phi} W^{\dagger\mu\nu} F^{\alpha\beta} W^{\phi}{}_{\mu}$
Four-fermion operators: *e.g.*,

$$\frac{i}{4} \epsilon^{jk}\, \bar{\psi}^j (1+\gamma_5) u\, \bar{\psi}^k (1+\gamma_5) d$$
$$\frac{i}{4} \epsilon^{jk}\, \bar{\psi}^j T^a (1+\gamma_5) u\, \bar{\psi}^k T^a (1+\gamma_5) d$$
$$\frac{i}{2} \bar{d} \gamma^\mu (1-\gamma_5) u\, \bar{u} (1-\gamma_5) \gamma_\mu d - (u \leftrightarrow d)$$

Engel *et al.*, arXiv:1303.2371 [nucl-th]
Grzadkowski *et al.*,008.4884 [hep-ph]
Aguilar-Saavedra, arXiv:1008.3562 [hep-ph]

Operator Mixing
Classification

$$\begin{array}{lcll} \text{Dimension 3} & : & \bar{\psi}\gamma_5\psi, & \bar{\psi}\gamma_5\tau_3\psi \\ \text{Dimension 4} & : & \epsilon_{\alpha\beta\mu\nu}G^{\alpha\beta}G^{\mu\nu} & \\ \text{Dimension 5} & : & \epsilon_{\alpha\beta\mu\nu}\bar{\psi}F^{\alpha\beta}\sigma^{\mu\nu}\psi, & \epsilon_{\alpha\beta\mu\nu}\bar{\psi}G^{\alpha\beta}\sigma^{\mu\nu}\psi \end{array}$$

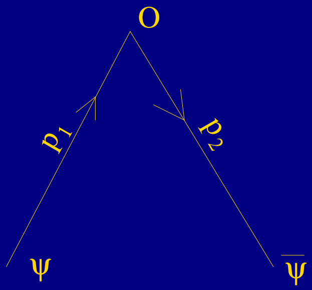
In $\overline{\mathrm{MS}}$ scheme, at zeroth order in α_{EM}, all except QCEDM are multiplicatively renormalized in the chiral limit:

- Operators of different dimensions don't mix,
- Dimension three operators differ under chiral $SU(N_f)$,
- Loops of photons bring α_{EW},
- QCEDM can induce QEDM.

At finite mass, QCEDM, Θ, and Dim 3 operators mix.

Operator Mixing
RI-SMOM

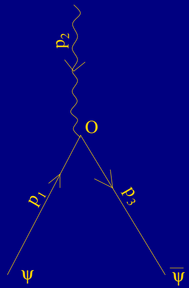
We need to define RI-SMOM with two and three external states. With two external states, we need $p_1^2,\ p_2^2,\ (p_2-p_1)^2$ to be in the deep Euclidean.



Standard choice $p_1^2 = p_2^2 = (p_2-p_1)^2 \equiv \Lambda^2$.

With three external states, we need $p_1^2,\ p_2^2,\ p_3^2,\ (p_3 - p_1 - p_2)^2,$ $(p_1 + p_2)^2,\ (p_3 - p_1)^2,\ (p_3 - p_2)^2$ all in the deep Euclidean. But these are not independent:

$$p_1^2 + p_2^2 + p_3^2 + (p_3 - p_1 - p_2)^2 = (p_1 + p_2)^2 + (p_3 - p_1)^2 + (p_3 - p_2)^2\,.$$



Choose
$$p_1^2 = p_2^2 = p_3^2 = (p_3 - p_1 - p_2)^2 \equiv \Lambda^2$$
$$(p_1 + p_2)^2 = (p_3 - p_1)^2 = (p_3 - p_2)^2 = \frac{4}{3}\Lambda^2$$

Operator Mixing
Matrix Elements

RI-SMOM scheme needs calculation of matrix elements in momentum space in fixed gauge. Projections of truncated matrix elements set to tree-level values.

To match between schemes, we need to add equation of motion and gauge-variant operators.

Extra terms do not contribute to matrix elements between physical states, but gives finite contributions to off-shell Green's functions.

BRST symmetry restricts allowed counterterms.

Operator Mixing

Operator Basis

Off-shell BRST symmetry allows adding:

- Dimension five: $\bar{\psi}_E \not{A} \frac{1\pm\gamma_5}{2} \psi, \qquad \bar{\psi}_E \not{\partial} \frac{1\pm\gamma_5}{2} \psi,$
  $\bar{\psi} \not{A} \frac{1\pm\gamma_5}{2} \psi_E, \qquad \bar{\psi} \not{\partial} \frac{1\pm\gamma_5}{2} \psi_E,$
  $\partial_\mu (D_\nu \bar{\psi} \sigma^{\mu\nu} \frac{1\pm\gamma_5}{2} \psi),$ and $\partial_\mu (\bar{\psi} \sigma^{\mu\nu} D_\nu \frac{1\pm\gamma_5}{2} \psi)$.
- Dimension four: $\bar{\psi}_E \frac{1\pm\gamma_5}{2} \psi, \qquad \bar{\psi} \frac{1\pm\gamma_5}{2} \psi_E,$
  $(G_E^\mu - g[\partial^\mu \bar{c}, c]) A_\mu,$
  $(\partial^\mu \bar{c}) D_\mu c,$ and $(D_\mu \partial^\mu \bar{c}) c,$

where $\psi_E = (i\not{D} - m)\psi$ and $G_E^\mu = D_\nu G^{\nu\mu} + g\bar{\psi}\gamma^\mu\psi$.

Deans and Dixon, PRD 18:4(1978)1113–1129.

On the basis of this, we choose the following operator basis

$$\bar\psi i\gamma_5\psi,$$

$$\bar\psi_E i\gamma_5\psi + \bar\psi i\gamma_5\psi_E, \qquad \epsilon_{\alpha\beta\mu\nu}G^{\alpha\beta}G^{\mu\nu}$$

$$\bar\psi_E i\gamma_5\psi_E, \qquad \partial_\mu[\bar\psi_E i\gamma^\mu\gamma_5 + \psi i\gamma^\mu\gamma_5\psi_E],$$

$$\bar\psi_E \not{A} i\gamma_5\psi + \bar\psi \not{A} i\gamma_5\psi_E, \qquad \partial_\mu\epsilon^{\mu\nu\alpha\beta}\bar\psi\sigma_{\alpha\beta}\overleftrightarrow{D}_\nu\psi,$$

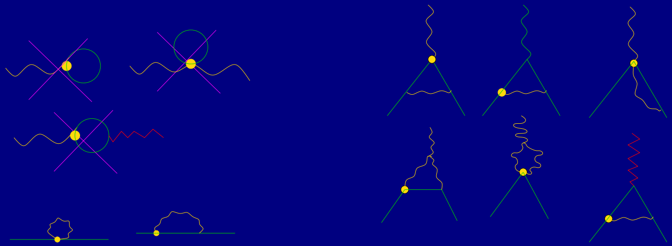
$$\bar\psi\epsilon_{\mu\nu\alpha\beta}\sigma^{\mu\nu}G^{\alpha\beta}\psi, \qquad \bar\psi\epsilon_{\mu\nu\alpha\beta}\sigma^{\mu\nu}F^{\alpha\beta}\psi,$$

$$(\partial^\mu\bar c)D_\mu c, \qquad \partial_\mu[(\partial^\mu\bar c)c].$$

Operator Mixing

Regularization and renormalization

Put conditions on the truncated Green's functions: $\langle G|O|\Omega\rangle$, $\langle G|O|G\rangle$, $\langle G|OJ_\mu|\Omega$, $\langle\psi|O|\psi\rangle$, $\langle\psi,g|O|\psi\rangle$, and $\langle\psi|J_\mu O|\psi\rangle$.



1-particle reducible diagrams also need to be considered.

Conclusions

Mixing Structure

Without EM loops, QEDM operator multiplicatively renormalized.

In the chiral theory, QCEDM does not mix with the Θ term, but it does need a m_5 counterterm. This can change the choice of field basis.

Need one-loop $\overline{\mathrm{MS}}$ calculation of the truncated Green's function of QCEDM between quark states, between gluon states, and its qqg and qqγ three point functions.

Tree-level contributions of the other operators needed.

Ghost operators do not contribute to this order.