Wave functions of the Nucleon and the $N^*(J^P = 1/2^-)$

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Nucleon Wave Functions

- Consider the three-quark Fock-state in the infinite-momentum frame
- In leading twist
  - with transverse momentum components integrated out
  - the nucleon wave function can be written as

$$|N,\uparrow\rangle = f_N \int \frac{[dx]\varphi(x_i)}{2\sqrt{24x_1x_2x_3}}\{|u^{\uparrow}(x_1)u^{\downarrow}(x_2)d^{\uparrow}(x_3)\rangle - |u^{\uparrow}(x_1)d^{\downarrow}(x_2)u^{\uparrow}(x_3)\rangle\}$$

  - where
    - x_i: longitudinal momentum fractions
    - $\int[dx] = \int_0^1 dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$
    - f_N: Leading-twist normalization constant, “Wave function at the Origin”
    - $\varphi(x_i)$: Nucleon Distribution Amplitude

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  - Expand wave function in multiplicatively renormalizable terms [Braun, Manashov, Rohrwild]:

$$\begin{aligned}\varphi(x_i;\mu^2) =& 120x_1x_2x_3\Big\{1 + c_{10}(x_1 - 2x_2 + x_3)L^{\frac{8}{3\beta_0}} + c_{11}(x_1 - x_3)L^{\frac{20}{9\beta_0}} \\ &+ c_{20}\left[1 + 7(x_2 - 2x_1x_3 - 2x_2^2)\right]L^{\frac{14}{3\beta_0}} + c_{21}(1-4x_2)(x_1-x_3)L^{\frac{40}{9\beta_0}} \\ &+ c_{22}\left[3 - 9x_2 + 8x_2^2 - 12x_1x_3\right]L^{\frac{32}{9\beta_0}} + \dots\Big\}\end{aligned}$$

    - where $L \equiv \alpha_s(\mu)/\alpha_s(\mu_0)$
    - c_{ij}: "shape parameters"

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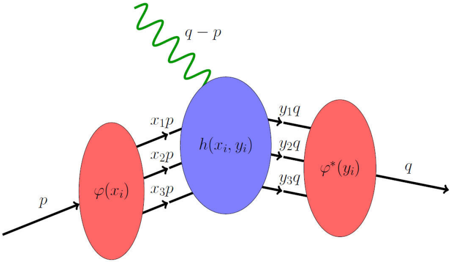
- In next-to-leading twist
  - obtain information about the orbital angular momentum of the quarks
  - λ_1, λ_2: Next-to-leading twist normalization constants

Motivation

- The wave functions of quarks in hadrons by themselves are interesting quantities
- Calculation of form factors from first principles is a challenge
- Light-Cone Sum Rules relate form factors to distribution amplitudes
- Post- and prediction of form factor data is possible once wave functions are known
- Compare theory results to experiment: JLab, FAIR, ...



Distribution Amplitudes from Lattice QCD

The calculation of Distribution Amplitudes from LQCD requires the following steps:

- Use operators that transform according to irreducible representations of the spinorial hypercubic group $\overline{H(4)}$ [Kaltenbrunner *et al.*, Eur.Phys.J.C55(2008)387]
- Calculate non-perturbative renormalization constants for these operators
  - Non-perturbative renormalization and 1-loop-conversion RI-MOM $\rightarrow$ $\overline{\text{MS}}$ have been performed by [Göckeler *et al.* [QCDSF], Nucl.Phys.B812 (2009) 205]
  - 2-loop-conversion factors are in progress
- Compute matrix elements of the operators on the lattice
  - Calculate two-point functions of the form

$$\langle \mathcal{O}(x)_{\alpha\beta\gamma} \bar{\mathcal{N}}(y)_{\tau} \rangle$$

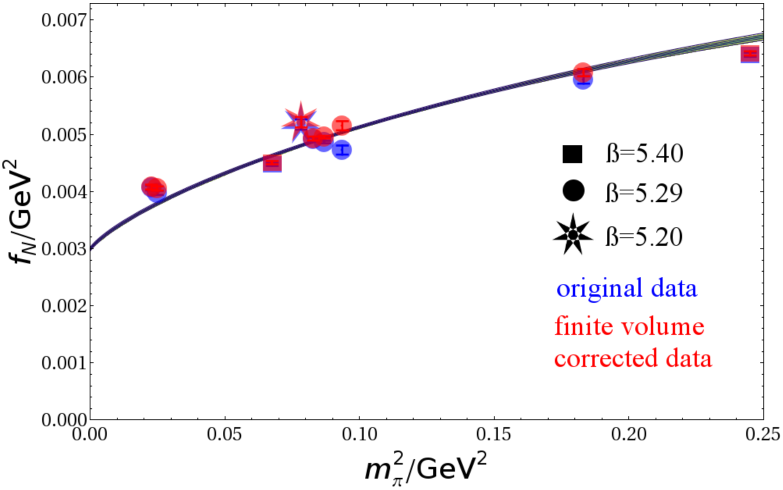
- Extrapolate $m_\pi \rightarrow m_\pi^{\text{phys}}$, $V \rightarrow \infty$ and $a \rightarrow 0$
  - Leading-one-loop baryon χPT formulae including finite volume correction terms for the nucleon have been worked out by Wein *et al.* [Eur.Phys.J.A47 (2011) 149]
  - Several pion masses, volumes and lattice spacings available

Lattices used ($N_f = 2$ Clover Wilson Fermions)

κ	m_π / MeV	Size	# Configurations
$\beta = 5.20, a = 0.0815$ fm			
0.13596	280	$32^3 \times 64$	1079^*
$\beta = 5.29, a = 0.0715$ fm			
0.13620	428	$24^3 \times 48$	1170
0.13632	306	$24^3 \times 48$	540
0.13632	295	$32^3 \times 64$	950
0.13632	288	$40^3 \times 64$	2026
0.13640	158	$48^3 \times 64$	3499
0.13640	151	$64^3 \times 64$	1080
$\beta = 5.40, a = 0.0605$ fm			
0.13640	495	$32^3 \times 64$	1124
0.13660	260	$48^3 \times 64$	2178

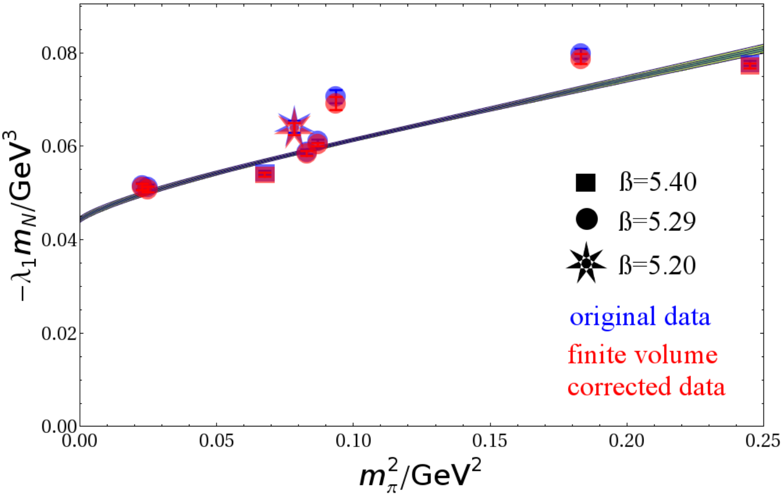
*Thanks to W. Söldner for simulating this in “record time”

Nucleon: Leading twist normalization constant f_N



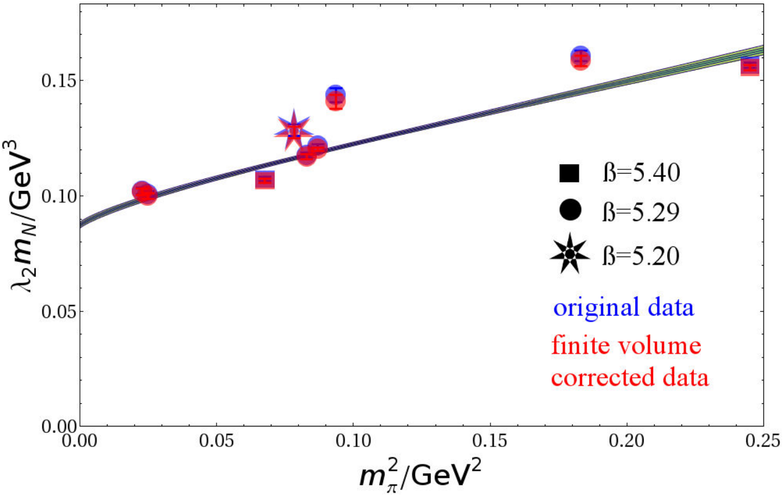
(preliminary result)

Nucleon: Next-to-leading twist normalization constant λ_1



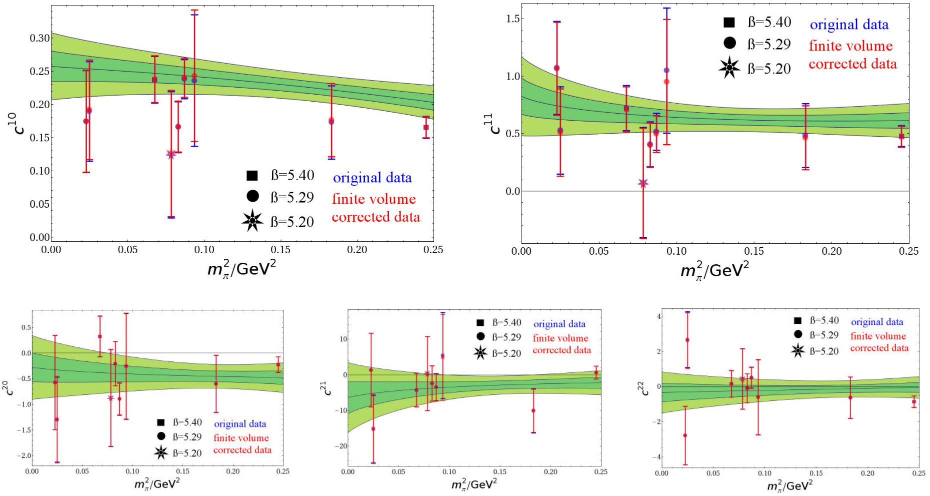
(preliminary result)

Nucleon: Next-to-leading twist normalization constant λ_2



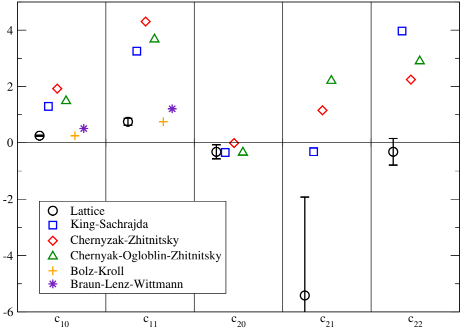
(preliminary result)

Nucleon: Shape parameters



(preliminary results)

Nucleon Shape Parameters: Lattice vs. QCD Sum Rules and data-driven wave functions

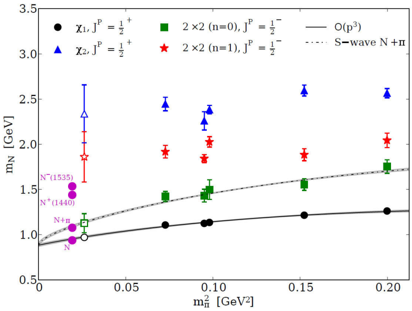


(preliminary result)

- QCD sum rule wave functions of King-Sachrajda, Chernyak-Zhitnitsky and Chernyak-Ogloblin-Zhitnitsky ruled out
- Data-driven wave functions of Bolz-Kroll and Braun-Lenz-Wittmann are in reasonable agreement with lattice results

Negative Parity: Separation of states

- Get negative parity with "projection" operator $\frac{1}{2}\left(1 - \frac{m_N}{E_N}\gamma_4\right)$
- Melnitchouk *et al.* [Phys.Rev.D67 (2003) 114506] have shown that two negative parity states can be separated using the interpolators $\mathcal{O}_1 = (uC\gamma_5 d)u$ and $\mathcal{O}_2 = (uCd)(\gamma_5 u)$
- Alexandrou *et al.* [arXiv:1302.4410] have shown that mass of ground state is consistent with $m_N + m_\pi$:



Negative Parity: Three Interpolators

- Verduci and Lang [Phys.Rev.D87 (2013) 054502] (talk on Fri) have extended this study to essentially three interpolating currents: $\mathcal{O}_1$, $\mathcal{O}_2$ and a five-quark interpolator

PHYSICAL REVIEW D **87,** 054502 (2013)

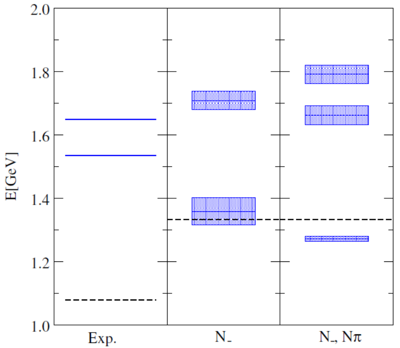


FIG. 4 (color online). Comparison of the energy levels. Left: physical mass values (experiment). Middle: result when using only 3-quark interpolators. Right: result when pion-nucleon interpolators are included. The dashed lines indicate the scattering thresholds.

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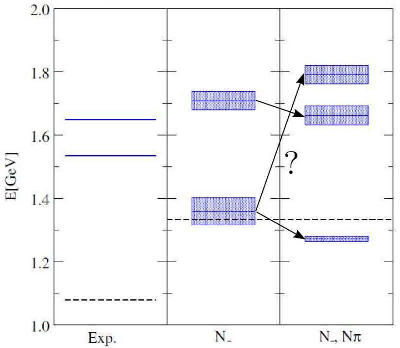
- Inspection of eigenvectors suggested picture on right

PHYSICAL REVIEW D **87,** 054502 (2013)



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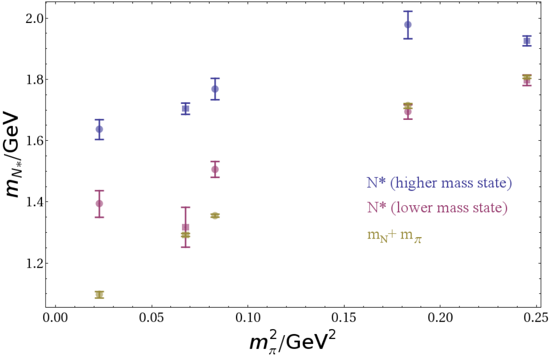
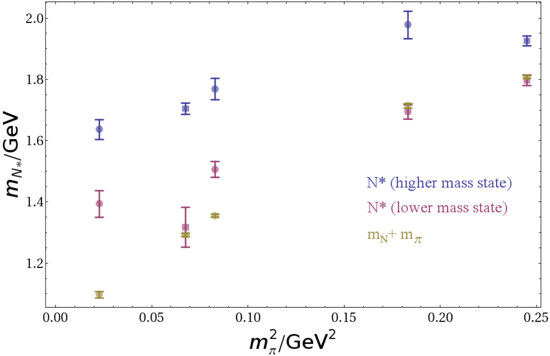
$N^*(J^P = 1/2^-)$: Masses



(preliminary result)

- Mass of ground state not always consistent with $m_N + m_\pi \Leftarrow$ smearing?, fit range?

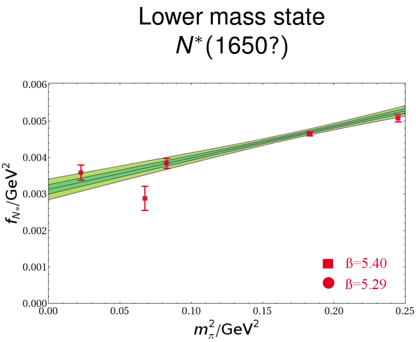
$N^*(J^P = 1/2^-)$: Masses

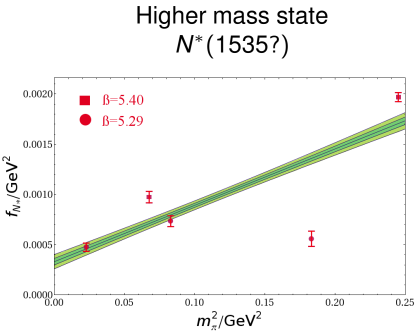


(preliminary result)

- Mass of ground state not always consistent with $m_N + m_\pi \Leftarrow$ smearing?, fit range?
- Following Verduci and Lang, we will label the lower mass state "1650?" and the higher mass state "1535?"

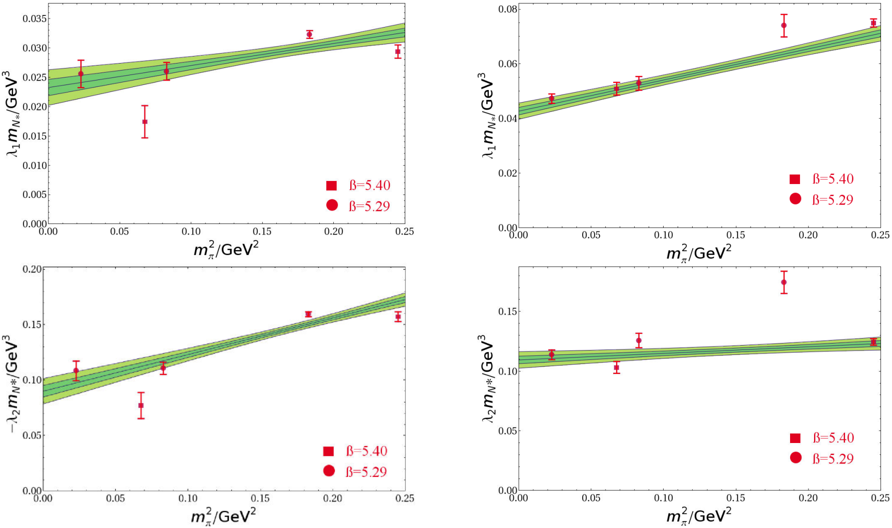
$N^*(J^P = 1/2^-)$: Leading twist normalization constant





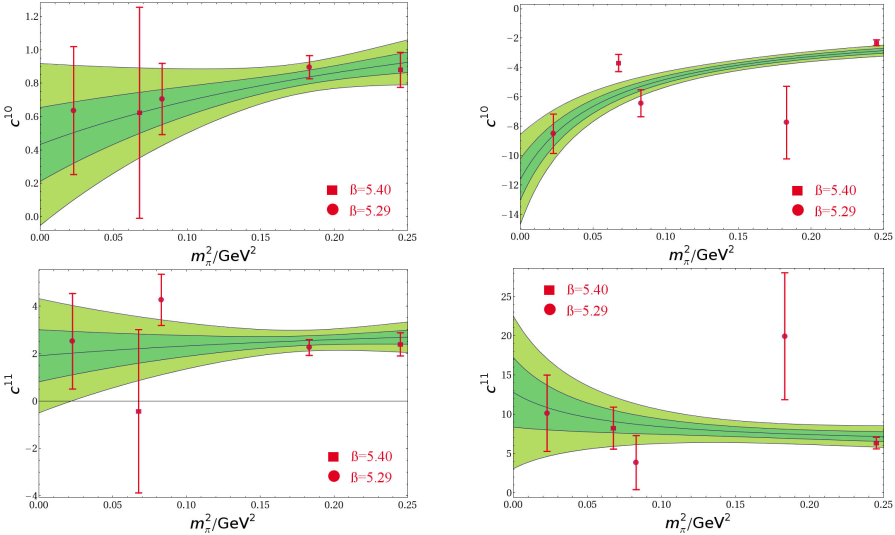
(preliminary results)

$N^*(J^P = 1/2^-)$: Next-to-leading twist normalization constants



(preliminary results)

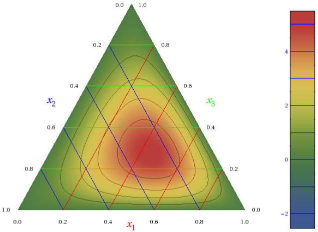
$N^*(J^P = 1/2^-)$: First order shape parameters



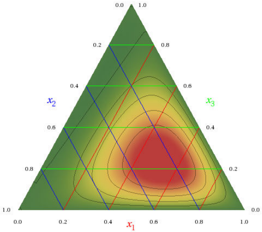
(preliminary results)

Barycentric plot of the wave functions

- Only first moments used for this plot



Nucleon



$N^*(1650?)$

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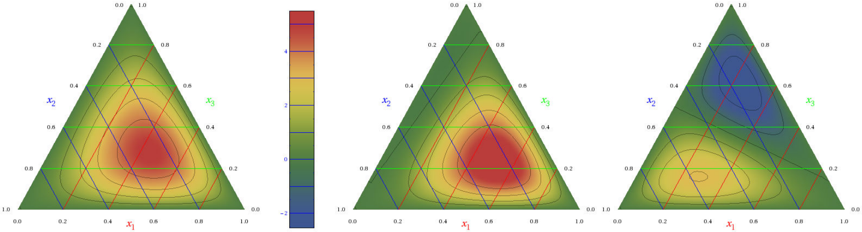

Nucleon $\qquad N^*(1650?) \qquad N^*(1535?)$

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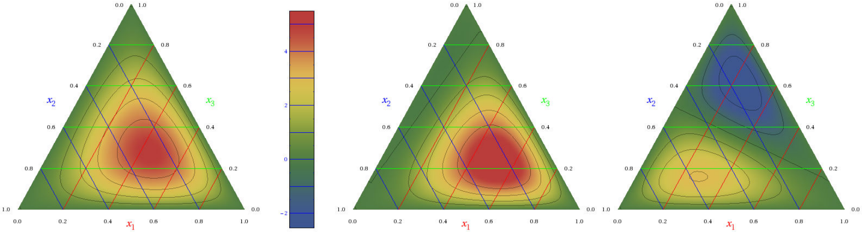

Nucleon $\qquad$ $N^*(1650?)$ $\qquad$ $N^*(1535?)$

(preliminary results)

- Difference between the two N^*s not surprising [PDG]:
  - $N^*(1535) \to N\pi(\ 45\%), \to N\eta(\ 40\%)$
  - $N^*(1650) \to N\pi(\ 70\%), \to N\eta(\ 10\%), \to N\pi\pi(\ 15\%)$

Conclusions and Outlook

- f_N, f_{N^*}, λ_1, λ_2 and the first moments of the nucleon and $N^*(J^P = 1/2^-)$ distribution amplitudes are "in good shape"
- Second moments need yet higher statistics
- Further investigation of discretization effects required
- Identification of negative parity states?
- $N_f = 2 + 1$ might give an answer!