Neutron and proton EDM in $N_f = 2+1$ domain-wall fermion

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T. Blum and T Izubuchi and A. Soni for RBC/UKQCD collaboration
Outline

- Motivation of lattice calculation of EDM in $\theta$ term
- Strategy and method
- (Preliminary) results
- Summary and future work
1. Introduction

Motivation

- CP violation (CPV) in QCD and BSM
  - EDM is sensitive to CPV in BSM
    - SM contribution is extremely small, $10^{-33} \text{ e} \cdot \text{cm}$
  - Strong CP problem
- There are many experimental plans of EDM
  - pEDM experiment @ BNL,
  - nEDM experiment @ ORNL, ILL, FRM-2, FNAL, PSI/KEK/TRIUMF, …
  - Charged particle (d, p)EDM @ COSY
  - Lepton EDM @ J-PARC, FNAL
  - aim to $10^{-29} \text{ e} \cdot \text{cm}$ (present nEDM limit: $|d_N^{\text{exp}}| < 2.9 \times 10^{-26} \text{ e} \cdot \text{cm}$)
- BSM (like SUSY) predicts $\sim 10^{-27} \text{ e} \cdot \text{cm}$
- Large uncertainty is hadron effect in low-energy scale.
- Lattice QCD provides important information of BSM prediction.
1. Introduction

Nucleon EDM in lattice QCD

- $\theta$ term contribution
  - Renormalizable
  - Feasible study toward the BSM (quark EDM and chromo EDM) calculation
    Bhattacharya et al, Lattice 2012, Lattice 2013
  - Nice check of uncertainties in quark model, BChPT

- Some difficulties
  - Statistical noise
    - Gauge background (topological charge, sea quark) which are intrinsically noisy.
    - Disconnected diagram (flavor singlet) should include (In SU(3) limit this is vanishing. Study including disc. diagram is future work).
  - Systematic study
    - Finite volume effect may be significant. (e.g. BChPT discussion)
      O’Connell, Savage, PLB633, 319(2006), Guo, Meissner, 1210.5887
    - Chiral behavior($d_N=O(m)$) is also important check.
2. EDM calculation on the lattice

Strategy

- Precise calculation of EDM in $\theta$ term
2. EDM calculation on the lattice

Strategy

- Precise calculation of EDM in $\theta$ term
  - Use of domain-wall fermion (DWF)
    - Chiral symmetry on the lattice, and control the $O(a)$ lattice artifact.
    - Good control of chiral behavior without counterterm (cf. Wilson-clover)
2. EDM calculation on the lattice

Strategy

- Precise calculation of EDM in $\theta$ term
  - Use of domain-wall fermion (DWF)
    - Chiral symmetry on the lattice, and control the $O(\alpha)$ lattice artifact.
    - Good control of chiral behavior without counterterm (cf. Wilson-clover)
  - Error reduction techniques
    - All-mode-averaging (AMA)
    - Efficient way to reduce the statistical error of correlator
      
      Blum, Izubuchi, ES, arXiv:1208.4349 [hep-lat], ES (lattice 2012)
2. EDM calculation on the lattice

**Strategy**

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    - Blum, Izubuchi, ES, arXiv:1208.4349 [hep-lat], ES (lattice 2012)
- **EDM form factor**
  - Extraction from matrix element including CPV of $\theta$ angle
  - Extrapolation into physical kinematics, $-q^2 = 0$ and $m = m_{\text{phy}}$
2. EDM calculation on the lattice

Domain-wall fermion

- Chiral symmetry
  - L, R fermion is localized on boundaries
  - \( \Rightarrow \) Chiral symmetry is realized (if \( L_s \rightarrow \infty \)).
  - Remaining good chiral symmetry
    - \( m_{\text{res}} \sim \exp(-L_s) \)
  - Reasonable computational cost

- RBC/UKQCD collaboration
  - Generation of \( N_f = 2+1 \) configurations.
  - Various lattice size, lattice cut-off, quark mass are available.
  - Many studies of Keon physics, nucleon physics, finite temperature , …

[Blum Soni, (97), CP-PACS(99), RBC(00), RBC/UKQCD. (05--) ]
2. EDM calculation on the lattice

Error reduction techniques

- Covariant approximation averaging (CAA)

  For original correlator $O$, (unbiased) improved estimator is defined as

  $$O^{(imp)} = O^{(rest)} + \frac{1}{N_G} \sum_{g \in G} O^{(appx),g}, \quad O^{(rest)} = O - O^{(appx)}$$

Blum, Izubuchi, ES, 1208.4349, ES (lattice 2012), Chulwoo, plenary on Fri
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2. EDM calculation on the lattice

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    O^{\text{imp}} = O^{\text{rest}} + \frac{1}{N_G} \sum_{g \in G} O^{\text{appx}, g}, \quad O^{\text{rest}} = O - O^{\text{appx}}
    \]
  - $\langle O \rangle = \langle O^{\text{imp}} \rangle$ if approximation is **covariant under lattice symmetry** $g$
  - **Improved error** $\text{err}^{\text{imp}} \simeq \text{err} / \sqrt{N_G}$

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  - $\langle O \rangle = \langle O^{(imp)} \rangle$ if approximation is covariant under lattice symmetry $g$
  - Improved error $\text{err}^{imp} \approx \text{err}/\sqrt{N_G}$
  - Computational cost of $O^{(imp)}$ is cheap.
2. EDM calculation on the lattice

Error reduction techniques

- **Covariant approximation averaging (CAA)**
  - For original correlator $O$, (unbiased) improved estimator is defined as
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- **All-mode-averaging (AMA)**
  - Relaxed CG solution for approximation
    \[
    O^{\text{appx}} = O[S_l], \quad S_l = \sum_{\lambda=1}^{N_\lambda} v_\lambda v_\lambda^\dagger \frac{1}{\lambda} + P_n(\lambda)|\lambda|>N_\lambda
    \]
2. EDM calculation on the lattice

Error reduction techniques

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    O^{\text{appx}} = O[S_l], \quad S_l = \sum_{\lambda=1}^{N_\lambda} v_{\lambda}v_{\lambda}^\dagger \frac{1}{\lambda} + P_n(\lambda)|_{|\lambda|>N_\lambda}
    \]
  - $P_n(\lambda)$ is polynomial approximation of $1/\lambda$
    - Low mode part : # of eigen mode
    - Mid-high mode : degree of poly.

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Blum, Izubuchi, ES, 1208.4349, ES (lattice 2012), Chulwoo, plenary on Fri
2. EDM calculation on the lattice

EDM Form factor

- EM Matrix element

\[ \langle n(P_1) | J^\text{EM}_\mu | n(P_2) \rangle_\theta = \bar{u}_N^\theta \left[ \frac{F_3^\theta(Q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} Q_\nu + F_1 \gamma_{\mu} + \frac{F_2}{2m_N} \sigma_{\mu\nu} Q_\nu + \cdots \right] u_N^\theta \]

P,T-odd

P,T-even

2. EDM calculation on the lattice

EDM Form factor

- **EM Matrix element**

\[
\langle n(P_1) | J_{\mu}^{EM} | n(P_2) \rangle_{\theta} = \bar{u}_{N}^\theta \left[ \frac{F_3^\theta(Q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} Q_{\nu} + F_1 \gamma_{\mu} + \frac{F_2}{2m_N} \sigma_{\mu\nu} Q_{\nu} + \cdots \right] u_{N}^\theta
\]

\[
\sum_s u_N^\theta (s) \bar{u}_N^\theta (s) = \frac{ip \cdot \gamma + m_N e^{i\alpha_N^\theta} \gamma_5}{2E_N}
\]

CPV phase \( \alpha_N \) in nucleon propagator

2. EDM calculation on the lattice

**EDM Form factor**

- **EM Matrix element**

\[
\langle n(P_1)|J_{\mu}^{\text{EM}}|n(P_2)\rangle_{\theta} = \bar{u}_{N}^{\theta} \left[ \frac{F_3^\theta(q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} Q_\nu + F_1 \gamma_{\mu} + \frac{F_2}{2m_N} \sigma_{\mu\nu} Q_\nu + \cdots \right] u_{N}^{\theta}
\]

\[
\sum_{s} u_{N}^{\theta}(s) \bar{u}_{N}^{\theta}(s) = \frac{ip \cdot \gamma + m_N e^{i\alpha_N^\theta} \gamma_5}{2E_N} \quad \text{CPV phase } \alpha_N^\theta \text{ in nucleon propagator}
\]

\[
\langle \theta|\eta_N J_{\mu}^{\text{EM}} \bar{\eta}_N |\theta\rangle = \langle 0|\eta_N J_{\mu}^{\text{EM}} \bar{\eta}_N |0\rangle + i\theta \langle 0|\eta_N J_{\mu}^{\text{EM}} Q \bar{\eta}_N |0\rangle
\]

2. EDM calculation on the lattice

**EDM Form factor**

- **EM Matrix element**

\[
\langle n(P_1)|J^\text{EM}_\mu|n(P_2)\rangle_\theta = \bar{u}_N^\theta \left[ \frac{F_3^2(Q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} Q_{\nu} + \frac{F_1}{2m_N} \gamma_\mu + \frac{F_2}{2m_N} \sigma_{\mu\nu} Q_{\nu} + \cdots \right] u_N^\theta
\]

- **CPV phase** \(\alpha_N\) in nucleon propagator

\[
\sum_s u_N^\theta(s) \bar{u}_N^\theta(s) = \frac{i p \cdot \gamma + m_N e^{i \alpha_N} \gamma_5}{2E_N}
\]

- **Computation**

\[
\langle \theta|\eta_N J^\text{EM}_\mu \bar{\eta}_N|\theta \rangle = \langle 0|\eta_N J^\text{EM}_\mu \bar{\eta}_N|0 \rangle + i\theta \langle 0|\eta_N J^\text{EM}_\mu Q \bar{\eta}_N|0 \rangle
\]

- **Subtraction**

\[
\langle 0|\eta_N(t_1) J^\text{EM}_\mu(t) Q \bar{\eta}_N(t_0)|0 \rangle = \frac{\alpha_N}{2} \left( F_1 \gamma_\mu + F_2 \frac{q_\nu \sigma_{\mu\nu}}{2m_N} \right) \frac{i p \cdot \gamma + m_N}{2E_N} + \frac{1 + \gamma_4}{2} \left[ F_1 \gamma_\mu + F_2 \frac{q_\nu \sigma_{\mu\nu}}{2m_N} \right] \frac{\alpha_N}{2} \gamma_5
\]

- **Extraction**

\[
+ \frac{1 + \gamma_4}{2} \left[ F_3 \frac{q_\nu \gamma_5 \sigma_{\mu\nu}}{2m_N} + F_A(iq^2 \gamma_\mu \gamma_5 - 2m_Nq_\nu \gamma_5) \right] \frac{i p \cdot \gamma + m_N}{2E_N}
\]

2. EDM calculation on the lattice

**EDM Form factor**

\[ \langle n(P_1)|J_{\mu}^{EM}|n(P_2)\rangle_{\theta} = \bar{u}_N^\theta \left[ \frac{F_3^2(Q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} Q_{\nu} + F_1 \gamma_{\mu} + \frac{F_2}{2m_N} \sigma_{\mu\nu} Q_{\nu} + \cdots \right] u_N^\theta \]

\[ \sum_s u_N^\theta(s) \bar{u}_N^\theta(s) = \frac{ip \cdot \gamma + m_N e^{i\alpha_N^N \gamma_5}}{2E_N} \]

**CPV phase** \( \alpha_N^N \) in nucleon propagator

\[ \langle \theta|\eta_N J_{\mu}^{EM} \bar{\eta}_N |\theta \rangle = \langle 0|\eta_N J_{\mu}^{EM} \bar{\eta}_N |0 \rangle + i\theta \langle 0|\eta_N J_{\mu}^{EM} Q \bar{\eta}_N |0 \rangle \]

\[ \langle 0|\eta_N(t_1) J_{\mu}^{EM} (t) Q \bar{\eta}_N(t_0) |0 \rangle \]

\[ = \frac{\alpha_N^N}{2} \gamma_5 \left[ F_1 \gamma_{\mu} + \frac{F_2}{2m_N} \frac{q_{\nu} \sigma_{\mu\nu}}{2m_N} \right] \frac{ip \cdot \gamma + m_N}{2E_N} + \frac{1 + \gamma_4}{2} \left[ \frac{F_1 \gamma_{\mu} + \frac{F_2}{2m_N} \frac{q_{\nu} \sigma_{\mu\nu}}{2m_N}}{2} \right] \frac{\alpha_N}{2} \gamma_5 \]

\[ + \frac{1 + \gamma_4}{2} \left[ \frac{F_3}{2m_N} \frac{q_{\nu} \gamma_5 \sigma_{\mu\nu}}{2m_N} + F_A(iq^2 \gamma_{\mu} \gamma_5 - 2m_N q_{\mu} \gamma_5) \right] \frac{ip \cdot \gamma + m_N}{2E_N} \]

- **Subtraction of CP-odd phase**, \( \alpha_N^N \), in n propagator and CP-even part \( F_{1,2} \)

\[ d_N = \lim_{Q^2 \to 0} \frac{F_3(Q^2)}{2m_N} \]

3. (Preliminary) Results

Parameters

- **DWF**
  - $24^3 \times 64$ lattice, $\sigma^l = 1.73$ GeV (~3 fm$^3$ lattice)
  - $L_s = 16$ and $am_{\text{res}} = 0.003$
  - $m = 0.005, 0.01$ corresponding to $m_\pi = 0.33, 0.42$ GeV
  - Two temporal separation of N sink and source in 3 pt. function
    - $t_{\text{sep}} = 12$ ($t_{\text{source}} = 0, t_{\text{sink}} = 12$), $t_{\text{sep}} = 8$ ($t_{\text{source}} = 0, t_{\text{sink}} = 8$)
  - # configs = 751 ($m=0.005$), 700 ($m=0.01$) [$t_{\text{sep}} = 12$]
    - # configs = 180 ($m=0.005$) [$t_{\text{sep}} = 8$]

- **AMA**
  - # of low-mode : $N_\lambda = 400$ ($m=0.005$), 180 ($m=0.01$)
  - Stopping condition, $|r| < 0.003$
  - $N_G = 32$ (2 separation for spatial, 4 separation for temporal direction of source location) $\rightarrow$ effectively $O(10^4)$ statistics
3. (Preliminary) Results

$\alpha_N$

- Projection with $\gamma_5$ for 2 pt with Q charge, global fitting with

$$\text{tr} \left[ \gamma_5 \langle N(t) \bar{N}(0) Q \rangle \right] = Z_N \frac{2m_N}{E_N} \alpha_N e^{-E_N t} + O(e^{-E_{N^*}})$$

- By using AMA, this factor is determined within 15% error.
- It does not depend on smearing and momentum, but mass dependence is not so clear.
3. (Preliminary) Results

**Subtraction term and 3pt function**

- Splitting EDM form factor into two parts:
  \[ F_3 = F_Q + F_\alpha, \quad F_Q = C(m_N) \langle N J_{EM}^t \bar{N}Q \rangle, \quad F_\alpha = F(\alpha_N, F_{1,2}) \]
- \( F_\alpha \) is good precision, and fluctuation of \( F_Q \) is large.
3. (Preliminary) Results

Comparison with $\mu = t, z$

- EDM form factor is given from two directions of EM current
- Two signals are consistent, and data in t direction is much stable.
3. (Preliminary) Results

Comparison with different $t_{\text{sep}}$

- The sink and source separation in 3pt function enables us to control the statistical noise and excited state contamination
  - Short: statistical fluctuation $<$ excited state contamination
  - Long: statistical fluctuation $>$ excited state contamination

Comparison

$t_{\text{sep}} = 12$ (blue),
\[ N_{\text{conf}} = 751 \]
$t_{\text{sep}} = 8$ (green)
\[ N_{\text{conf}} = 180 \]

- Good consistency between them.
- Precision in $t_{\text{sep}}=8$ is much better.
3. (Preliminary) Results

- $q^2 = 0$ extrapolation

- Fitting data of EDM form factor at each momenta.

- Open($t_{sep} = 8$), filled ($t_{sep} = 12$)

- Fitting function
  - 3 point linear:
    - $-q^2 < 0.55 \text{ GeV}^2$
  - 2 point linear:
    - $-q^2 < 0.4 \text{ GeV}^2$

- Estimate of systematic error of extrapolation
3. (Preliminary) Results

Mass dependence

- **Comparison with full QCD results**
  - DWF results are in the lightest quark mass.
  - Statistical error is dominant rather than systematic one.
  - Central value is 10 times larger than models.
3. (Preliminary) Results

Statistical error

- Comparison between AMA error reduction and number of configurations.
- Number of configurations: reduce stat. error and relating to $Q$ distribution.
- AMA error reduction: reduce stat. error.

?? %:

<table>
<thead>
<tr>
<th>Error rate</th>
<th>100 config $N_g=32$</th>
<th>200 config $N_g=32$</th>
<th>400 config $N_g=32$</th>
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<tbody>
<tr>
<td>36%</td>
<td>33%</td>
<td>51%</td>
<td></td>
</tr>
<tr>
<td>52%</td>
<td>76%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>73%</td>
<td></td>
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</tr>
</tbody>
</table>

AMA works well

Reduction rate when increase of configs. is slightly better.

Full statistics $\rightarrow$
4. Summary

Summary and future plan

- Nucleon EDM in $N_f = 2+1$ DWF in $\theta$ vacuum
  - Signal of EDM within 40% statistical error using AMA techniques.
  - 3-pt function is still noisy.
  - Short $t_{sep}$ allows us to reduce the statistical error without large excited state contamination effect.
4. Summary

Summary and future plan

- Nucleon EDM in $N_f = 2+1$ DWF in $\theta$ vacuum
  - Signal of EDM within 40% statistical error using AMA techniques.
  - 3-pt function is still noisy.
  - Short $t_{sep}$ allows us to reduce the statistical error without large excited state contamination effect.

- (Near) physical point of DWF configurations
  - Ensembles near physical points and large volume are available.
  - AMA with Möbius-DWF approximation is helpful.
  - Remove chiral extrapolation $\rightarrow$ less than 10% precision

<table>
<thead>
<tr>
<th>Lattice size</th>
<th>Physical size</th>
<th>$a$</th>
<th>$L_s$</th>
<th>Gauge action</th>
<th>Pion mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$32^3 \times 64$</td>
<td>4.6 fm$^3$</td>
<td>0.135 fm</td>
<td>32</td>
<td>DSDR</td>
<td>171 -- 241 MeV</td>
</tr>
<tr>
<td>$48^3 \times 96$</td>
<td>5.5 fm$^3$</td>
<td>0.115 fm</td>
<td>16</td>
<td>Iwasaki</td>
<td>135 MeV</td>
</tr>
</tbody>
</table>
Thank you for your attention!
Backup
Nucleon EDM in the SM

- CKM phase in EDM
  - No CPV phase in 1-loop ($|V_{dq}|^2$ : no phase) and 2-loop diagram (unitarity).
  - Three-loop order (short) or pion loop correction (long)
    
    \[ d_N^{\text{KM short}} \sim -10^{-34} \, \text{e} \cdot \text{cm}, \quad d_N^{\text{KM long}} \sim 10^{-30} \, 10^{-32} \, \text{e} \cdot \text{cm} \]

  \[ d_N^{\text{KM}} = d_N^{\text{KM short}} + d_N^{\text{KM long}} \sim 10^{-30} \, 10^{-32} \, \text{e} \cdot \text{cm} \]

  6-order magnitude below the experimental upper limit.

- $\theta$ term in the QCD Lagrangian
  
  Renormalizable and CP-violating from in topological charge density.
  
  \[ \mathcal{L}_\theta = \frac{1}{64 \pi^2} \bar{G} \tilde{G}, \quad \bar{\theta} = \theta + \arg \det M, \quad \arg \det M \sim \eta \frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_s m_u} \]

  - $\theta$ term has been estimated as $\bar{\theta} < 10^{-9\pm 1}$ Crewther, et al. (1979), Ellis, Gaillard (1979)

  - Unnatural cancellation (strong CP problem)
Nucleon EDM in the BSM

- Possible higher dimension operators
  - In supersymmetry (SUSY) model there is CPV phase from 1-loop \((\text{Im}(g_L g_R^*) \neq 0)\)
  - CPV effective Hamiltonian with higher dimension than \(\theta\) term
    \[
    H_{CP} = \sum_k C_k(\mu) \mathcal{O}_k
    \]
    \[
    \mathcal{O}_{qEDM} = d_q \bar{q}(\sigma \cdot F)\gamma_5 q \quad \text{: Quark-photon (5-dim)}
    \]
    \[
    \mathcal{O}_{cEDM} = d_c^c \bar{q}(\sigma \cdot G)\gamma_5 q \quad \text{: Quark-gluon (5-dim)}
    \]
    \[
    \mathcal{O}_{\text{Weinberg}} = d^G G \bar{G} \quad \text{: Pure gluonic (6-dim)}
    \]
  - Contribution to nEDM in low energy model

BChPT:

\[
\begin{align*}
 d_N &= d_N^{QCD} \bar{\theta} + d_N(d_q, d^c_q) + d_N(d^G) \\
 &\sim 10^{-17}[\text{e} \cdot \text{cm}] \bar{\theta} + (1.4-0.47)d_d - (0.12-0.35)d_u + O(10^{-2})d^c_q \sim O(10^{-25}-10^{-27}) \text{ e} \cdot \text{cm}
\end{align*}
\]

QCD sum rule:

Hisano, Shimizu (04), Ellis, Lee, Pilaftsis (08), Hisano, Lee, Nagata, Shimizu (12)

Hisano, Shimizu (04), Ellis, Lee, Pilaftsis (08), Hisano, Lee, Nagata, Shimizu (12)
Lattice methods on EDM

- **Spectrum method**

- **Form factor**

- **Imaginary θ**
EM form factor

\[ q^2 \text{ GeV}^2 \]

\[ F_1(q^2) \]

\[ F_2(q^2) \]
Volume effect?

- **BChPT analysis**

In LO, NLO BChPT analysis, there may be more than 20% finite size effect.

- CP violating coupling