$B$ decay to radially excited $D$

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• Excited $D$ mesons states in questions
• $B \to D'$ non leptonic decay
• Lattice measurement of $f_{D'}/f_D$
• Outlook

D. Becirevic, BB, A. Gérardin, A. Le Yaouanc and F. Sanfilippo,

Excited $D$ meson states in questions

Recently the BaBar Collaboration claimed to have isolated a couple of excited $D$ states [BaBar Collaboration, ’11]

$D\pi$ distribution: a clear peak is observed for $D_2^*(2460)$, “enhancements” are seen and correspond to $D^*(2600)$ and $D^*(2760)$

$D^*\pi$ distribution: a peak is visible for $D_1(2420)$ and 2 structures are observed, interpreted as $D(2550) \equiv D'$ and $D(2750)$

A fit gives $m(D') = 2539(8)$ MeV and $\Gamma(D') = 130(18)$ MeV
Excited $D$ meson states in questions

Recently the BaBar Collaboration claimed to have isolated a couple of excited $D$ states [BaBar Collaboration, ’11]

Question: is this interpretation correct? Popular quark models obtain roughly the same mass (2580 MeV) but a much smaller width (70 MeV) [F. Close and E. Swanson, ’05; Z. Sun et al, ’10]

😊 Radial excitations are very sensitive to the position of the nodes of wave functions, that depends strongly on the model.
\[ J = \frac{1}{2} \oplus j_l, \ H = B, D \]

<table>
<thead>
<tr>
<th>( j_l^P )</th>
<th>( J^P )</th>
<th>ground state</th>
<th>radial excitation</th>
</tr>
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<tbody>
<tr>
<td>( \frac{1}{2}^- )</td>
<td>0–</td>
<td>( H )</td>
<td>( H' )</td>
</tr>
<tr>
<td>1–</td>
<td>( \mathbb{H}^* )</td>
<td>( \mathbb{H}^* )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2}^+ )</td>
<td>0+</td>
<td>( \mathbb{H}^0_0 )</td>
<td>( \mathbb{H}^0_0' )</td>
</tr>
<tr>
<td>1+</td>
<td>( \mathbb{H}^*_1 )</td>
<td>( \mathbb{H}^*_1' )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{2}^+ )</td>
<td>1+</td>
<td>( \mathbb{H}^1_1 )</td>
<td>( \mathbb{H}^1_1' )</td>
</tr>
<tr>
<td>2+</td>
<td>( \mathbb{H}^2_2 )</td>
<td>( \mathbb{H}^2_2' )</td>
<td></td>
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</tbody>
</table>

[F. Bernlochner et al, '12]:
Assume a large \( B(B \to D'\ell\nu) \)
\[ \Gamma(D' \to D_{1/2}\pi) \] much larger than \( \Gamma(D' \to D_{3/2}\pi) \)
\( \implies \) Excess of \( B \to D_{1/2}(\pi)\ell\nu \) events with respect to \( B \to D_{3/2}(\pi)\ell\nu \)

Question: could this potentially large \( B \to D'\ell\nu \) width explain the “1/2 vs. 3/2 puzzle”?
\[ [\Gamma(B \to D_{1/2}\ell\nu) \simeq \Gamma(B \to D_{3/2}\ell\nu)]^{\text{exp}} \text{ while } [\Gamma(B \to D_{1/2}\ell\nu) \ll \Gamma(B \to D_{3/2}\ell\nu)]^{\text{th}} \]
[V. Moréna et al, '01; N. Uraltsev, '04]

In the OPE formalism, a large \( B \to D' \) form factor is going together with a small suppression of \( B \to D(\ast) \) form factors [P. Gambino et al, '12]

Question: can a large \( B \to D' \) form factor solve the tension of \( \sim 3\sigma \) observed between \( V_{cb}^{\text{excl}} \) and \( V_{cb}^{\text{incl}} \)? [M. Bona et al, '10; J. Charles et al, '11]
**Non leptonic $B \to D'$ decay**

Our proposal: let's look at $B \to D'\pi$ non leptonic decay to check the largeness of $\mathcal{B}(B \to D'\ell\nu)$

**Class I non leptonic decay**

![Diagram of the decay process](image)

Factorisation approximation works well

$$\frac{\mathcal{B}(B^0 \to D'^+\pi^-)}{\mathcal{B}(B^0 \to D^+\pi^-)} = \left( \frac{m_B^2 - m_{D'}^2}{m_B^2 - m_D^2} \right)^2 \left[ \frac{\lambda(m_B, m_{D'}, m_\pi)}{\lambda(m_B, m_D, m_\pi)} \right]^{1/2} \left| \frac{f_{+}^{B \to D'}(0)}{f_{+}^{B \to D}(0)} \right|^2$$

$$\lambda(x, y, z) = [x^2 - (y + z)^2][x^2 - (y - z)^2] \quad f_{+}^{B \to D'}(m_\pi^2) \sim f_{+}^{B \to D}(0)$$

[BaBar Collaboration, '10]

![Graph](image)

$$V_{cb}f_{+}^{B \to D}(0) = 0.02642(8) \text{ and } |V_{cb}| = 0.0411(16): \quad f_{+}^{B \to D}(0) = 0.64(2)$$

$$m_{D'} = 2.54 \text{ GeV}: \quad \frac{\mathcal{B}(\bar{B}^0 \to D'^+\pi^-)}{\mathcal{B}(B^0 \to D^+\pi^-)} = (1.65 \pm 0.13) \times \left| f_{+}^{\bar{B} \to D'}(0) \right|^2$$

$$\mathcal{B}(\bar{B}^0 \to D^+\pi^-) = 0.268(13)\%: \quad \mathcal{B}(\bar{B}^0 \to D'^+\pi^-) = \left| f_{+}^{\bar{B} \to D'}(0) \right|^2 \times (4.7 \pm 0.4) \times 10^{-3}$$

Theoretical estimates of $f_{+}^{\bar{B} \to D'}(0)$ are in the range $[0.1, 0.4]$

[F. Bernlochner *et al*, '12; D. Ebert *et al*, '99; N. Faustov and V. Galkin, '12; Z. Wang *et al*, '12]

$$\mathcal{B}(\bar{B}^0 \to D'^+\pi^-)^{\text{th}} \sim 10^{-4}: \text{ it can be measured with the B factories samples and at LHCb.}$$
Class III decay

\[
A_{\text{III fact}} = -i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[ a_1 f_\pi [m_B^2 - m_{D'}^2] f_{B \to D'} (m_\pi^2) + a_2 f_{D'} [m_B^2 - m_\pi^2] f_{B \to \pi} (m_{D'}^2) \right]
\]

Normalisation by the Class I decay

\[
\frac{B(B^- \to D^{0}_{(\pi^-)}}{B(B^0 \to D^{+}_{(\pi^-)} = \frac{\tau_{B^-}}{\tau_{B^0}} \left[ 1 + \frac{a_2}{a_1} \times \frac{m_B^2 - m_\pi^2}{m_B^2 - m_{D'}^2} \times \frac{f_{0 \to \pi}(m_{D'}^2)}{f_{+ \to D'}(0) f_D} \frac{f_{D'}}{f_\pi} \right]^2
\]

\(a_2/a_1\) determined from \(\frac{B(B^- \to D^{0}_{(\pi^-)}}{B(B^0 \to D^{+}_{(\pi^-)}),\) known experimentally

\(f_{D'}/f_D\) and \(f_D/f_\pi\) extracted from lattice QCD simulations
Lattice measurement of $f_{D'}/f_D$

Lattice set up

<table>
<thead>
<tr>
<th>$N_f$</th>
<th>action</th>
<th>action parameter</th>
<th>$m_{\text{sea}}$</th>
<th>configurations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>MtmQCD</td>
<td>[0.055, 0.1] fm</td>
<td>$[m_s/6, m_s/2]$</td>
<td>ETMC ensembles</td>
</tr>
<tr>
<td>2</td>
<td>Wilson-Clover</td>
<td>0.065 fm</td>
<td>$\sim m_s$</td>
<td>QCDSF ensembles</td>
</tr>
<tr>
<td>0</td>
<td>Wilson-Clover</td>
<td>0.065 fm</td>
<td></td>
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</tbody>
</table>

\[ C_{DqDq}(t) = \langle \sum_{\vec{x}} P_{Dq}(\vec{x}; t) P^\dagger_{Dq}(0; 0) \rangle \]

\[ t \gg 0 \quad |Z_{Dq}|^2 \frac{\cosh[m_{Dq}(T/2-t)]}{m_{Dq}} e^{-m_{Dq}T/2} \]

Subtraction of the ground state contribution:

\[ C'_{DqDq}(t) = C_{DqDq}(t) - |Z_{Dq}|^2 \frac{\cosh[m_{Dq}(T/2-t)]}{m_{Dq}} e^{-m_{Dq}T/2} \]

Effective mass:

\[ \frac{\cosh \left[ m_{Dq}'(t) \left( \frac{T}{2} - t \right) \right]}{\cosh \left[ m_{Dq}'(t) \left( \frac{T}{2} - t - 1 \right) \right]} = \frac{C'_{DqDq}(t)}{C'_{DqDq}(t+1)} \quad f_{D'} \text{ is related to } Z_{Dq} \]
Alternative approach: Generalised Eigenvalue Problem
Interpolating fields $P_{D_q i} \equiv \bar{\psi} c i \gamma^5 \psi_{q i}$, different Gaussian smearing levels $i$ are used to build a matrix of correlators $C_{D_q D_q i j}(t)$

$$\psi_{q i} = \left(1 + \frac{\kappa_G}{1+6\kappa_G} \Delta \right)^i, \kappa_G = 4.0, i = \{0, 2, 10, 32\}$$

Solving the GEVP: $C_{D_q D_q i j}(t) v_j^{(n)}(t, t_0) = \lambda^{(n)}(t, t_0) C_{D_q D_q i j}(t_0) v_j^{(n)}(t, t_0)$

$$\tilde{P}^{(n)}_{D_q}(t, t_0) = \sum_i v_i^{(n)}(t, t_0) P_{D_q i} \quad \langle D_q^{(m)} | \tilde{P}^{(n)}_{D_q} | 0 \rangle = A_n \delta_{mn}$$

Effective mass: $m_{D_q}^{\text{eff}}(t) = \text{arccosh} \left[ \frac{\lambda^{(n)}(t+1, t_0) + \lambda^{(n)}(t-1, t_0)}{2 \lambda^{(n)}(t, t_0)} \right]$}

Decay constant: $Z_{D_q}^{(n)} = \lim_{t \to \infty} \frac{\sqrt{A_n}}{\sum_{i j} C_{D_q D_q 0 i}(t) v_i^{(n)}(t, t_0) C_{D_q D_q i j}(t) v_j^{(n)}(t, t_0)}$

[ETMC ensemble ($a = 0.085 \text{ fm} \quad m_\pi = 350 \text{ MeV}$)]
Combined fit of ETMC data at different $a$ and $m_{\text{sea}}$:

\[ F^{\text{latt.}} = A_F \left[ 1 + B_F m_q + C_F \left( \frac{a}{a_\beta=3.9} \right)^2 \right] \]

\[ a_\beta=3.9 = 0.085 \text{ fm} \]

\[ \frac{m_{D'_s}}{m_{D_s}} = 1.53(7) \quad \frac{f_{D'_s}}{f_{D_s}} = 0.59(11) \]

\[ \frac{m_{D'_D}}{m_{D}} = 1.55(9) \quad \frac{f_{D'_D}}{f_{D}} = 0.57(16) \]

\[ f_{D'_q}/f_{D_q} < 1 \quad \text{while} \quad f_{B'_q}/f_{B_q}\big|_{\text{static HQET}} > 1 \]

[T. Burch et al, '09; B. Blossier et al, '10]

\[(m_{D'_D}/m_D)^\text{exp} = 1.36: \text{ is there any issue?}\]

- MtmQCD breaks parity at finite lattice spacing \(\implies\) calculation made with Wilson-Clover fermions at \(a \sim 0.065\) fm:

\[
\begin{align*}
\text{MtmQCD:} & \quad \left. \frac{m_{D'_s}}{m_{D_s}} \right|_{a=0.065\text{ fm}} = 1.55(6) \quad \left. \frac{f_{D'_s}}{f_{D_s}} \right|_{a=0.065\text{ fm}} = 0.69(5) \\
\text{Clover:} & \quad \left. \frac{m_{D'_s}}{m_{D_s}} \right|_{a=0.065\text{ fm}} = 1.48(7) \quad \left. \frac{f_{D'_s}}{f_{D_s}} \right|_{a=0.065\text{ fm}} = 0.77(9)
\end{align*}
\]
Combined fit of ETMC data at different $a$ and $m_{\text{sea}}$:

$$F^{\text{latt.}} = A_F \left[ 1 + B_F m_q + C \left( \frac{a}{a_\beta=3.9} \right)^2 \right] \quad a_\beta=3.9 = 0.085 \text{ fm}$$

![Graph showing the fit of $F^{\text{latt.}}$ against $m_q$.

\[
\begin{align*}
\frac{m_{D_s'}}{m_{D_s}} &= 1.53(7) \quad \frac{f_{D_s'}}{f_{D_s}} = 0.59(11) \\
\frac{m_{D_s'}}{m_{D_s}} &= 1.55(9) \quad \frac{f_{D_s'}}{f_{D_s}} = 0.57(16)
\end{align*}
\]

$f_{D_q'}/f_{D_q} < 1 \text{ while } f_{B_q'}/f_{B_q}\bigg|_{\text{static HQET}} > 1$

[T. Burch et al, ’09; B. Blossier et al, ’10]

$\left( \frac{m_{D_s'}}{m_{D_s}} \right)^{\text{exp}} = 1.36$: is there any issue?

- At $N_f = 2$, channels with a pion or a kaon in the final state may open: $D'$ can decay into $D^*\pi$ (P wave) and $D_{0}^*\pi$ (S wave), $D_s'$ into $D^*K$ and $D_{s0}^*K$. No such a “danger” in the quenched approximation!

\[
\begin{align*}
N_f = 0 : \quad \frac{m_{D_s'}}{m_{D_s}} \bigg|_{a=0.065\text{fm}} &= 1.41(9) \quad \frac{f_{D_s'}}{f_{D_s}} \bigg|_{a=0.065\text{fm}} &= 0.67(12) \\
N_f = 2 : \quad \frac{m_{D_s'}}{m_{D_s}} \bigg|_{a=0.065\text{fm}} &= 1.48(7) \quad \frac{f_{D_s'}}{f_{D_s}} \bigg|_{a=0.065\text{fm}} &= 0.77(9)
\end{align*}
\]
Back to phenomenology

Lattice inputs: \(m_{D'}/m_D = 1.55(9)\) \(f_{D'}/f_D = 0.57(16)\) \(f_D/f_\pi = 1.56(3)(2)\)

Phenomenological inputs: \(a_2/a_1 = 0.368\) \(\tau_{B^0}/\tau_{B^-} = 1.079(7)\) \(f^{B\to D'}_+(0) = 0.64(2)\) \(f^{B\to \pi}_0(m_D^2) = 0.29(4)\)

\[
\frac{\mathcal{B}(B^-\to D'^0\pi^-)}{\mathcal{B}(B^0\to D'^+\pi^-)} = \frac{\tau_{B^-}}{\tau_{B^0}} \left[ 1 + \frac{0.14(4)}{f^{B\to D'}_+(0)} \right]^2
\]

\[
\frac{\mathcal{B}(B^0\to D'^0\pi^-)}{\mathcal{B}(B^0\to D^0\pi^-)} \bigg|_{(m_{D'}/m_D)^{\text{exp}}} = (1.65 \pm 0.13) \times \left| f^{B\to D'}_+(0) \right|^2
\]

This ratio does not depend so much on \(m_{D'}\)

Fixing \(f^{B\to D'}_+(0) = 0.4\) and taking \((m_{D'}/m_D)^{\text{exp}}\) we have also:

\[
\frac{\mathcal{B}(B^0\to D'^+\pi^-)}{\mathcal{B}(B^0\to D_{2}^*\pi^-)} = 1.6(3)
\]

\[
\frac{\mathcal{B}(B^-\to D'^0\pi^-)}{\mathcal{B}(B^-\to D_{2}^{*0}\pi^-)} = 1.4(3)
\]

Conclusion: if \(f^{B\to D'}_+\) is large, as claimed by many authors, the measurement of \(\mathcal{B}(B \to D'\pi)\) should be as feasible as \(\mathcal{B}(B \to D_{2}^{*}\pi)\)
Outlook

• Excited meson states are massively produced in experiments. To exploit fruitfully the numerous data at $B$ factories and LHCb, theorists do have to put an important effort in confronting their models predictions with measurements, by proposing to experimentalists unambiguous observables to look at.

• The case of the radially excited $D'$ meson is illuminating: understanding its physics will help to control systematics on $V_{cb}$.

• We have proposed to check the claim that $\mathcal{B}(B \to D')$ is large in semileptonic decays by analysing non leptonic decays. In particular, we have computed $f_{D'/D}$ that enters the Class III factorised amplitude.

• We have shown that the largeness of $\mathcal{B}(B \to D'l\nu)$ implies a good feasibility to measure $\mathcal{B}(B \to D'\pi)$.

• The possibility to extract the $B \to D'l\nu$ form factor directly on the lattice has not been explored yet. A first step is to study the $B^{*'} \to B$ transition in static HQET. [talk by A. Gérardin]

A pretty similar discussion can be made for the $B \to D^*_0$ decay: it is an on-going project.