Neutral B meson mixing with static heavy and domain-wall light quarks

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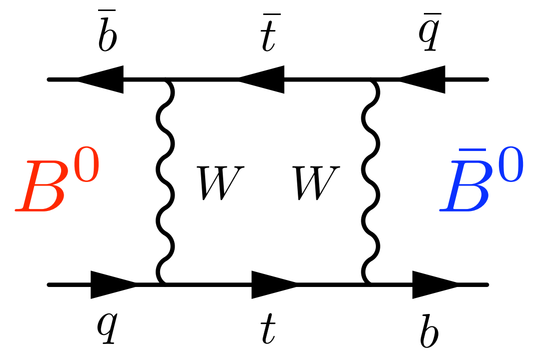
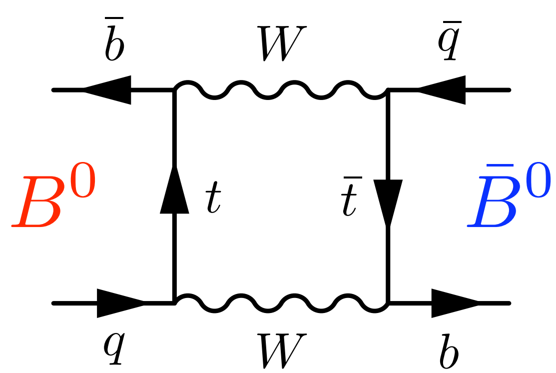
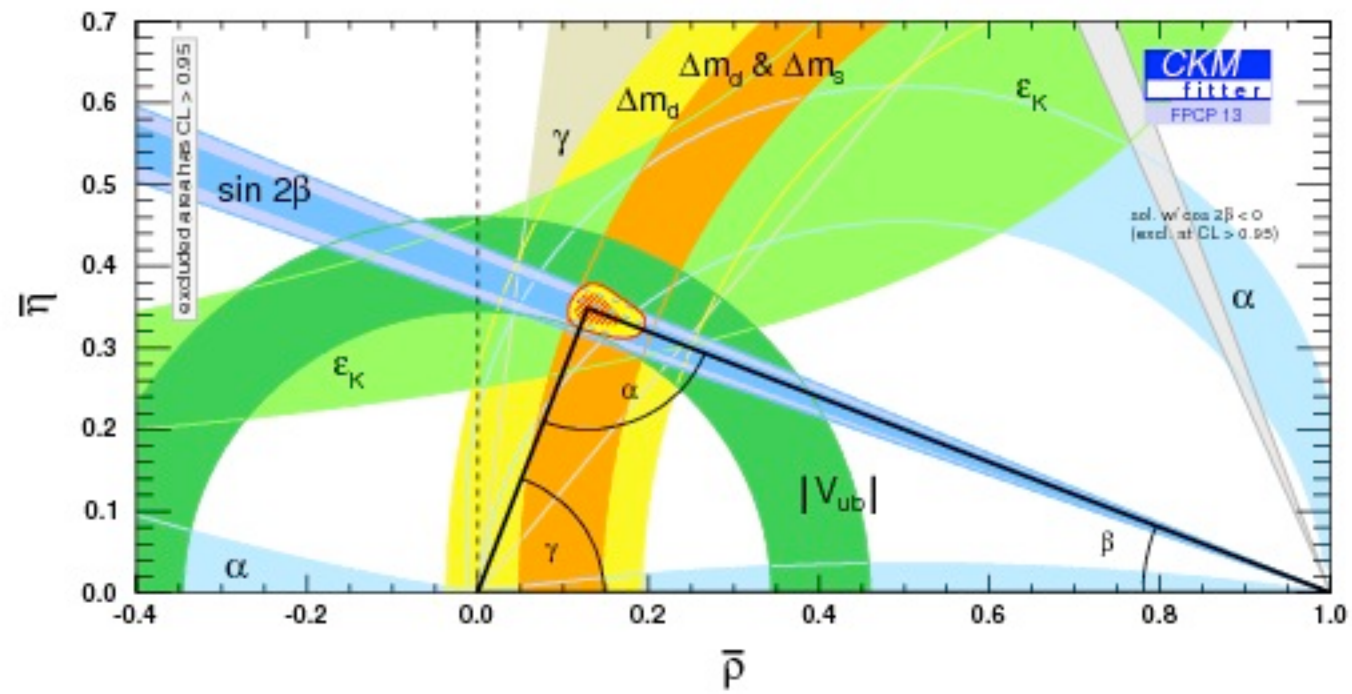
$B^0 - \bar{B}^0$ mixing and CKM

- Constraints to CKM triangle
  - Oscillation frequency





$q = \{d, s\}$



$$\Delta m_q = (\text{known factors}) \times \left|V_{tq}^* V_{tb}\right|^2 \frac{1}{m_{B_q}} \mathcal{M}_{B_q}$$

⇨ constraints on V_{td}, V_{ts}

  - Hadronic matrix elements

$$\mathcal{M}_{B_q} = \langle \overline{B}_q^0 | [\bar{b}\gamma_\mu P_L q][\bar{b}\gamma_\mu P_L q] | B_q^0 \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}$$

$B^0 - \bar{B}^0$ mixing and CKM

Constraints to CKM triangle

- SU(3) breaking ratio ξ

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi \sqrt{\frac{\Delta m_d}{\Delta m_s}\frac{m_{B_s}}{m_{B_d}}} \qquad \xi = \frac{m_{B_d}}{m_{B_s}}\sqrt{\frac{\mathcal{M}_{B_s}}{\mathcal{M}_{B_d}}}$$

  - The most attractive quantity in $B^0 - \bar{B}^0$ phenomena
  - Many of the uncertainties cancel in the ratio.
  - In the lattice calculation, the error is reduced due to correlation between denominator and numerator.

Other important quantities

- B meson decay constants f_{B_d}, f_{B_s}
- Bag parameters $B_q = \frac{3}{8}\frac{\mathcal{M}_{B_q}}{m_{B_q}^2 f_{B_q}^2}$

RBC/UKQCD Static B Physics

- V. Gadiyak and O. Loktik, *Lattice calculation of SU(3) flavor breaking ratios in* $B^0 - \bar{B}^0$ *mixing*, Phys. Rev. D 72 (2005) 114504.
- O. Loktik and T. Izubuchi, *Perturbative renormalization for static and domain-wall bilinears and four-fermion operators with improved gauge actions*, Phys. Rev. D 75 (2007) 034504.
- C. Albertus, Y. Aoki, P. A. Boyle, N. H. Christ, T. T. Dumitrescu, J. M. Flynn, T. I, T. Izubuchi, O. Loktik, C. T. Sachrajda, A. Soni, R. S. Van de Water, J. Wennekers and O. Witzel, *Neutral B-meson mixing from unquenched lattice QCD with domain-wall light quarks and static b-quarks*, Phys. Rev. D 82 (2010) 014505.
- T. I, Y. Aoki, J. M. Flynn, T. Izubuchib, and O. Loktik, *One-loop operator matching in the static heavy and domain-wall light quark system with O(a) improvement*, JHEP 05 (2011) 040.
- Y. Aoki, T. I, T. Izubuchi, C. Lehner and A. Soni (on-going project).

Static limit

- Static approximation (leading order of HQET)
  - Easy to implement (Static quark propagator is almost free.)
  - Symmetries (HQ spin symmetry + chiral symmetry)

    reduced operator mixing
  - Continuum limit exists even in the perturbative renormalization.
  - But, we always have the error coming from static approx.

$$O(\Lambda_{\mathrm{QCD}}/m_b) \sim 10\%$$

  - Always sitting as an anchor point for other HQ approach.

- Ratio quantities (ξ , f_{B_s}/f_{B_d}) in the static limit
  - Error coming from static approximation is reduced to:

$$O\left(\frac{m_s - m_d}{\Lambda_{\mathrm{QCD}}} \times \frac{\Lambda_{\mathrm{QCD}}}{m_b}\right) \sim 2\%$$

Lattice action setup

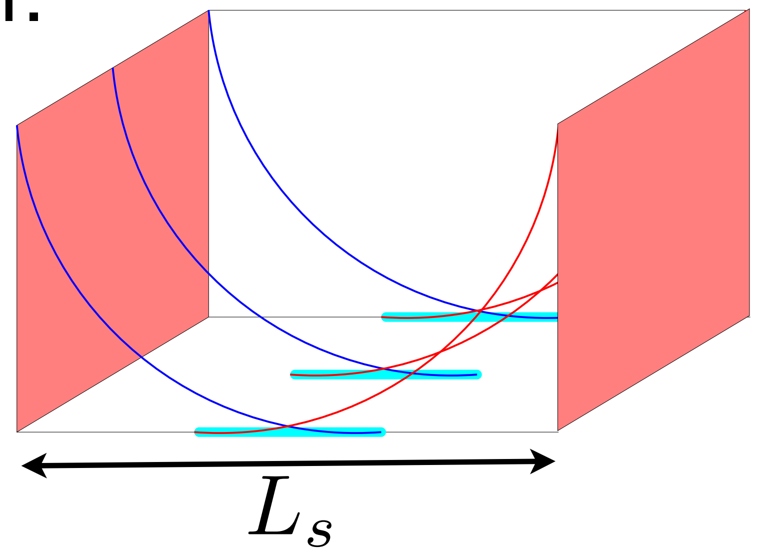
- Standard static action with link smearing

$$S_{\rm stat} = \sum_{\vec{x},t} \overline{\Psi}_h(\vec{x},t) \left[\Psi_h(\vec{x},t) - U_0^\dagger(\vec{x},t-a)\Psi_h(\vec{x},t-a) \right]$$

  - Reduced 1/a power divergence.
    - HYP1 [Hasenfratz and Knechtli, 2001]
    - HYP2 [Della Morte et al.(ALPHA), 2004]

- Domain-wall light quark action
  - 5 dimensional, controllable approximate chiral symmetry
  - Unphysical operator mixing does not occur.

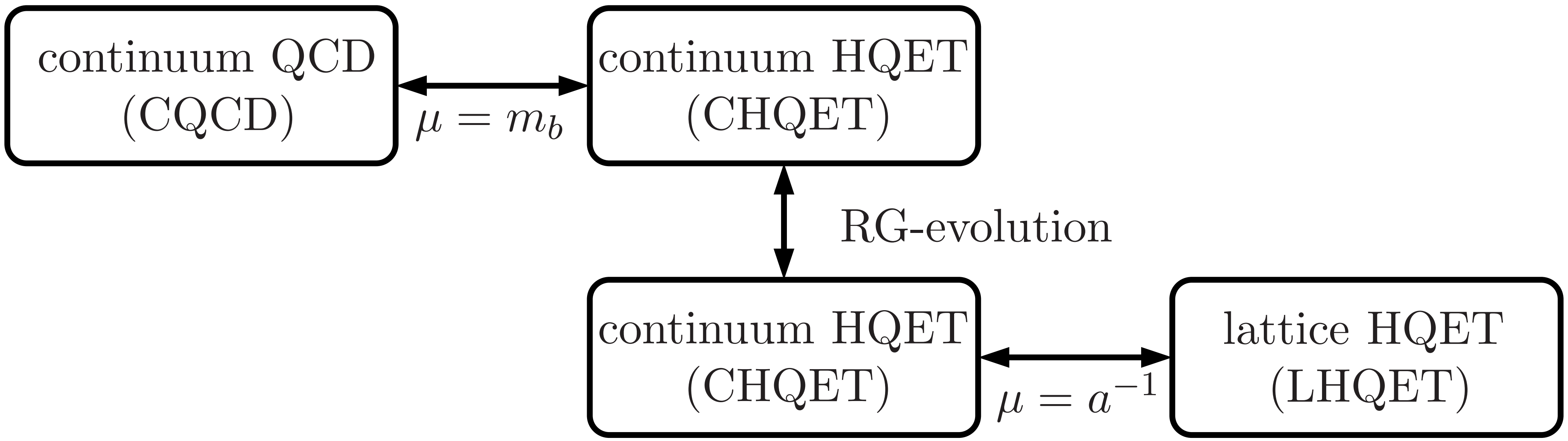
- Iwasaki gluon action



Matching procedure

▸ Operator matching

- Different renormalization ⟶ Operator matching is needed.



- Matching between continuum QCD and continuum HQET at $\mu = m_b$
- 2-loop RG running from $\mu = m_b$ to $\mu = a^{-1}$ in continuum HQET
- PT matching between continuum HQET and lattice HQET at $\mu = a^{-1}$

  Static with link smearing + DWF
  O(a) disc error is taken into account.

[T.I, Aoki, Flynn, Izubuchi, Loktik (2011)]

Measurement

Gluon ensemble

- Nf=2+1 dynamical DWF + Iwasaki gluon (RBC-UKQCD)

[Phys. Rev. D 83, 074508 (2011)]

label	β	$L^3 \times T \times L_s$	a^{-1} [GeV]	a [fm]	$am_{\rm res}$	m_l/m_h	m_π [MeV]	$m_\pi aL$
24c1	2.13	$24^3 \times 64 \times 16$	1.729(25)	0.114	0.003152(43)	0.005/0.04	327	4.54
24c2						0.01/0.04	418	4.79
32c1	2.25	$32^3 \times 64 \times 16$	2.280(28)	0.0864	0.0006664(76)	0.004/0.03	289	4.05
32c2						0.006/0.03	344	4.83
32c3						0.008/0.03	393	5.52

Measurement parameters

label	am_q	Measured MD traj	# of data	# of src	Δt
24c1	0.005, 0.034, 0.040	900–8980 every 40	203	4	20
24c2	0.010, 0.034, 0.040	1460–8540 every 40	178	2	
32c1	0.004, 0.027, 0.030	520–6800 every 20	315	1	24
32c2	0.006, 0.027, 0.030	1000–7220 every 20	312	1	
32c2	0.008, 0.027, 0.030	520–5540 every 20	252	1	

- Gaussian smearing on fermion field (width ~ 0.45 fm)

Measurement

- Operators
  - 2PT correlation functions

$$
\begin{aligned}
C^{\tilde{L}S}(t) &= \sum_{\vec{x}} \langle A_0^L(\vec{x},t) A_0^S(\vec{x}_0,0)^\dagger \rangle, \\
C^{\tilde{S}S}(t) &= \sum_{\vec{x}} \langle A_0^S(\vec{x},t) A_0^S(\vec{x}_0,0)^\dagger \rangle, \\
C^{SS}(t) &= \langle A_0^S(\vec{x}_0,t) A_0^S(\vec{x}_0,0)^\dagger \rangle.
\end{aligned}
$$

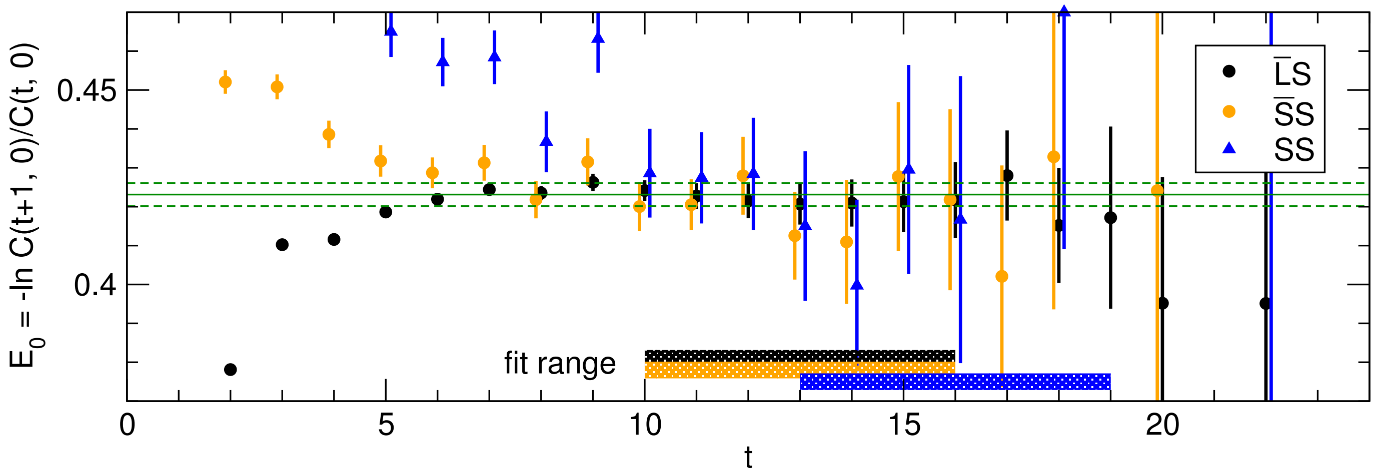
  - 3PT correlation functions

$$
\begin{aligned}
C_L(t_f,t,t_0) &= \sum_{\vec{x}} \langle A_0^S(\vec{x}_0,t_f)^\dagger O_{VV+AA}(\vec{x},t) A_0^S(\vec{x}_0,t_0)^\dagger \rangle, \\
C_S(t_f,t,t_0) &= \sum_{\vec{x}} \langle A_0^S(\vec{x}_0,t_f)^\dagger O_{SS+PP}(\vec{x},t) A_0^S(\vec{x}_0,t_0)^\dagger \rangle.
\end{aligned}
$$

$$
\begin{aligned}
A_0^L(\vec{x},t) &: \text{local} \\
A_0^S(\vec{x},t) &: \text{smeared both on heavy and light} \\
A_0^L(\vec{x},t),\ O_{VV+AA}(\vec{x},t) &: O(a) \text{ improved operators}
\end{aligned}
$$

Data extraction

Correlator fitting

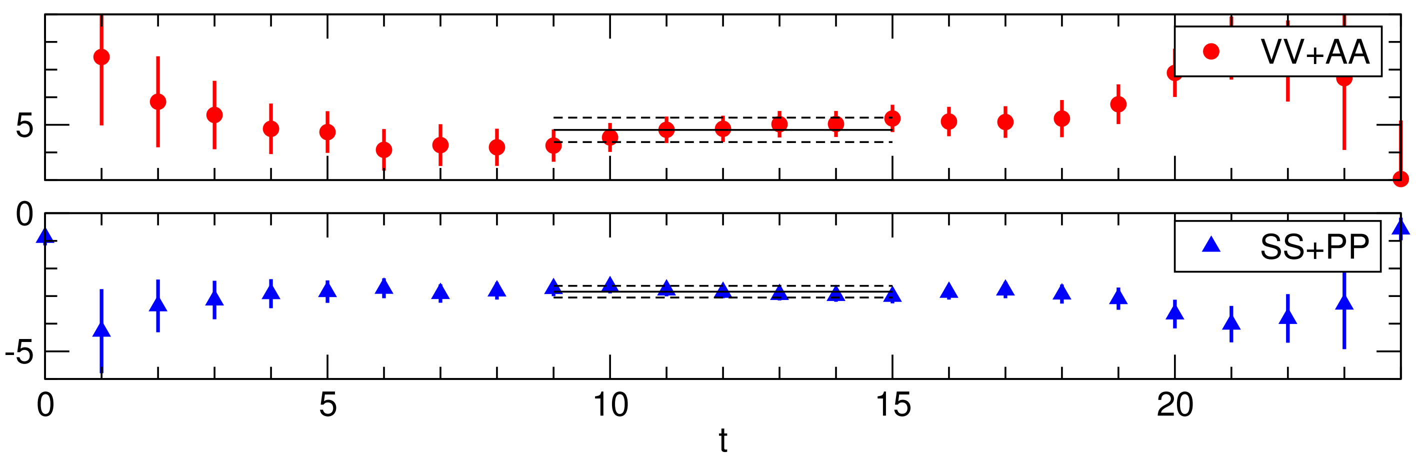


32c, HYP1

(m_h, m_l, m_q)

$= (0.03, 0.004, 0.004)$

$\chi^2/\text{d.o.f.} = 0.4$



$\chi^2/\text{d.o.f.} = 0.5$

$\chi^2/\text{d.o.f.} = 0.5$

$$C_{2\text{PT}}(t) = A(e^{-E_0 t} + e^{-E_0(T-t)}), \quad C_{3\text{PT}}(t_f, t, t_0) = A_{3\text{PT}} \longrightarrow \Phi^{\text{lat}}_{B_q}, \quad M^{\text{lat}}_{B_q}$$

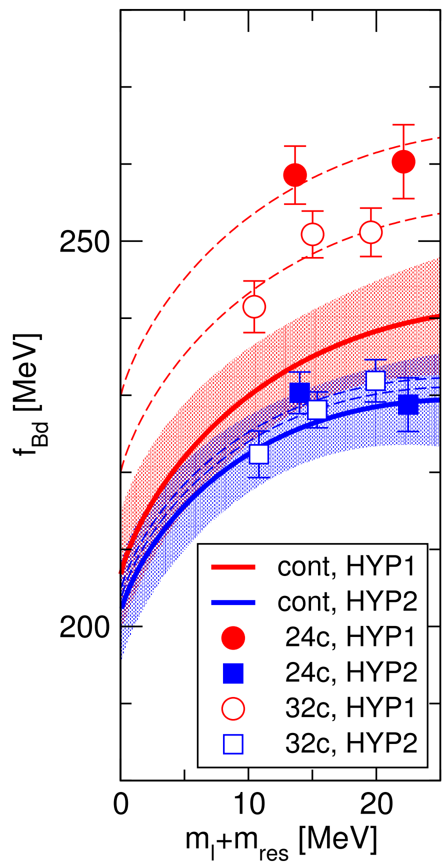
Matching (continuum QCD and lattice HQET)

$$f_{B_q} = (\text{matching factor}) \times \frac{\Phi^{\text{lat}}_{B_q}}{\sqrt{m_B}}, \quad \mathcal{M}_{B_q} = (\text{matching factor}) \times m_B M^{\text{lat}}_{B_q}$$

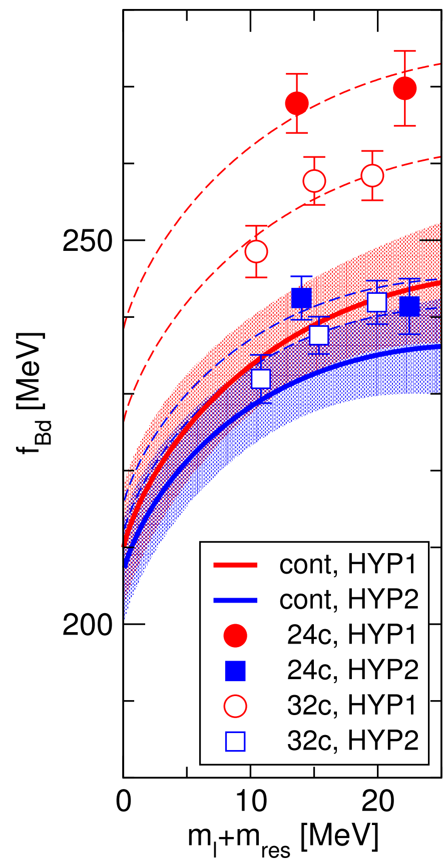
Chiral and continuum extrapolation

- a^2 scaling and consistency check

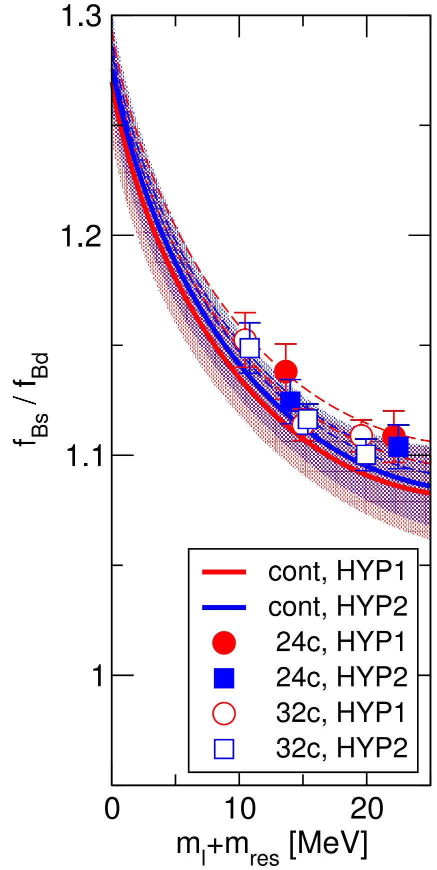
O(a) unimp



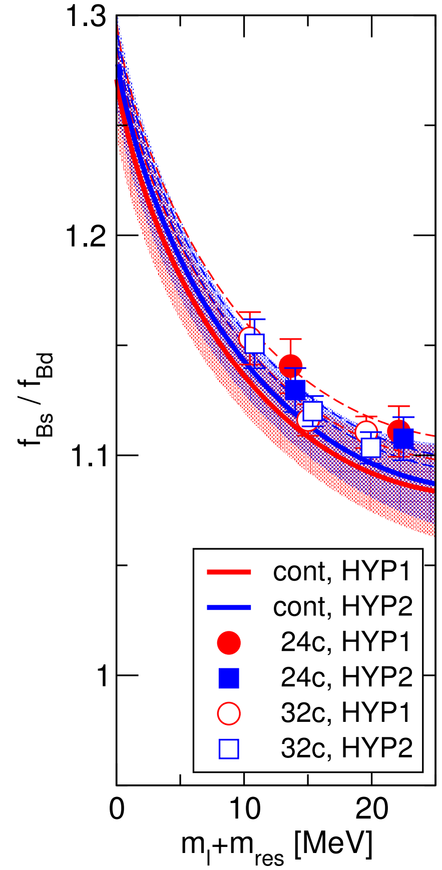
O(a) imp



O(a) unimp



O(a) imp



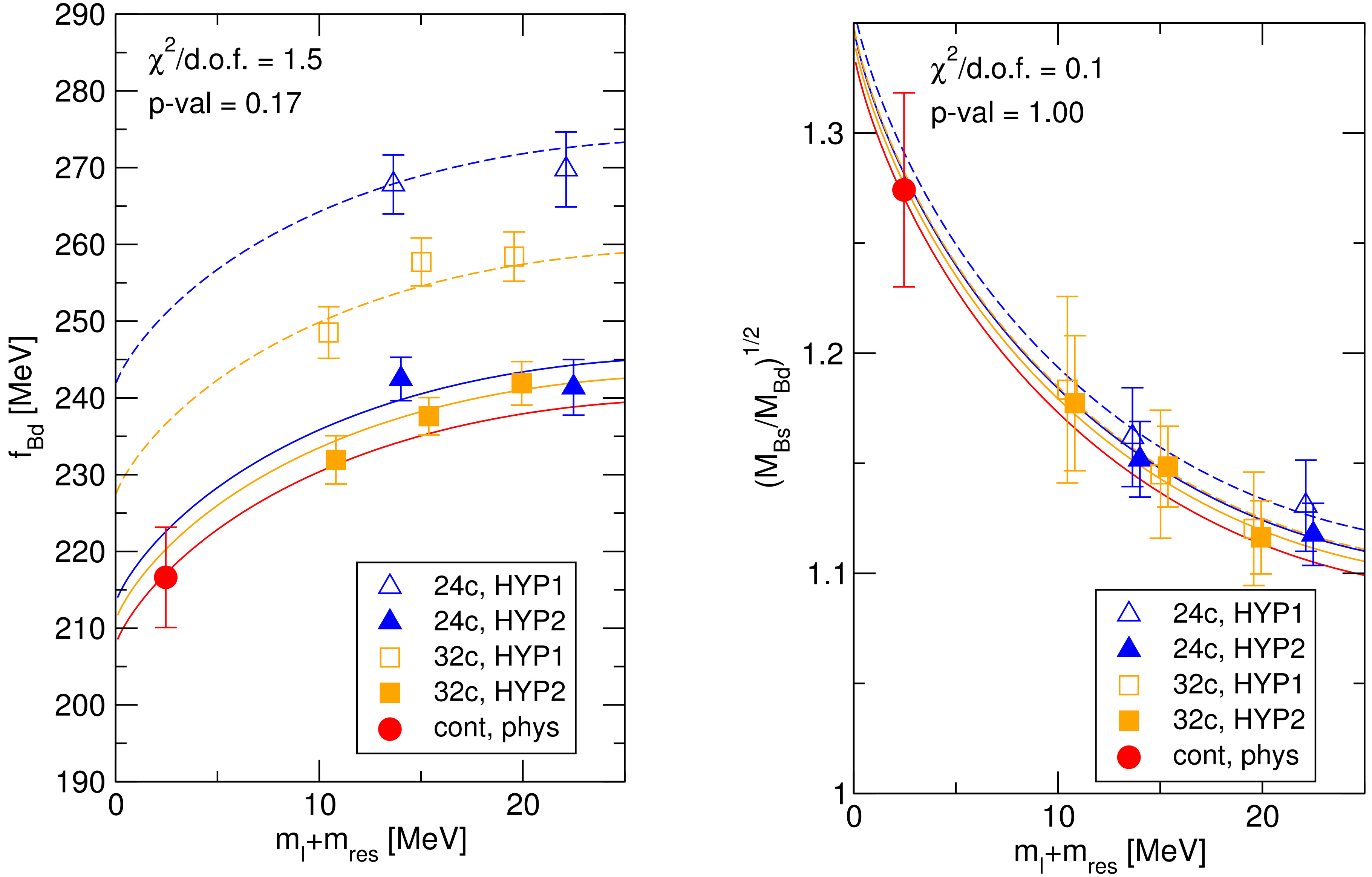
NLO SU(2) HMChPT

- HYP1 shows larger scaling violation than HYP2.
- HYP1 and HYP2 look consistent in the continuum.
- In the ratio, scaling violation is quite small.

Chiral and continuum extrapolation

- Combined fits

NLO SU(2) HMChPT



Linear fits are also used to estimate an uncertainty from chiral fits.

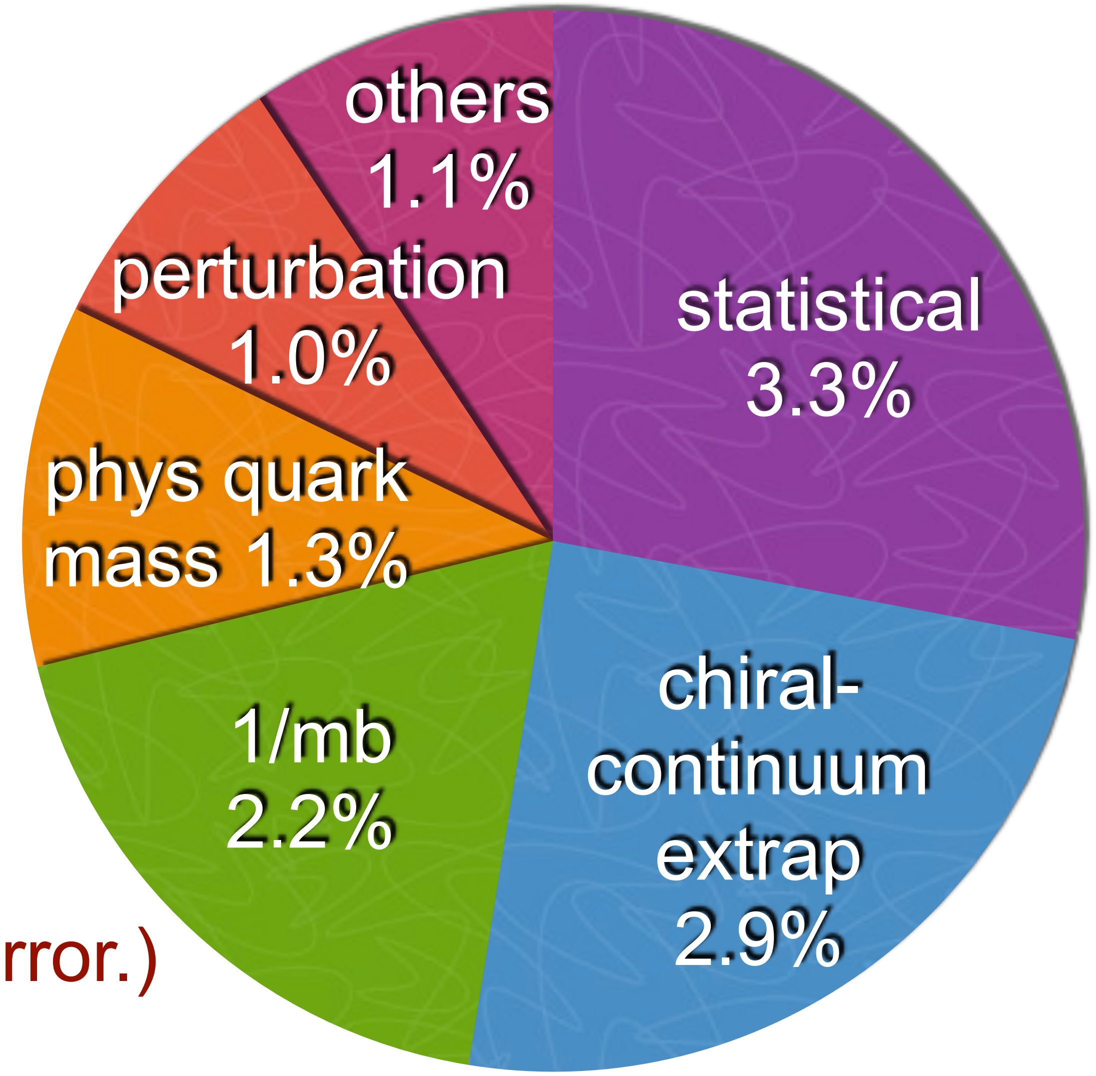
Results

Preliminary results

	not incl 1/mb uncertainty	incl 1/mb uncertainty
f_{B_d} [MeV] =	222(17),	222(31),
f_{B_s} [MeV] =	265(19),	265(37),
f_{B_s}/f_{B_d} =	1.192(43),	1.192(51),
$\mathcal{M}_{B_d}$ [(GeV)^4] =	2.79(44),	2.79(56),
$\mathcal{M}_{B_s}$ [(GeV)^4] =	4.34(46),	4.34(69),
$\sqrt{\mathcal{M}_{B_s}/\mathcal{M}_{B_d}}$ =	1.238(59),	1.238(66),
$\hat{B}_{B_d}$ =	1.15(13),	1.15(19),
$\hat{B}_{B_s}$ =	1.22(10),	1.22(18),
B_{B_s}/B_{B_d} =	1.047(65),	1.047(80),
ξ =	1.218(58),	1.218(65).

(Systematic errors are included in the error.)

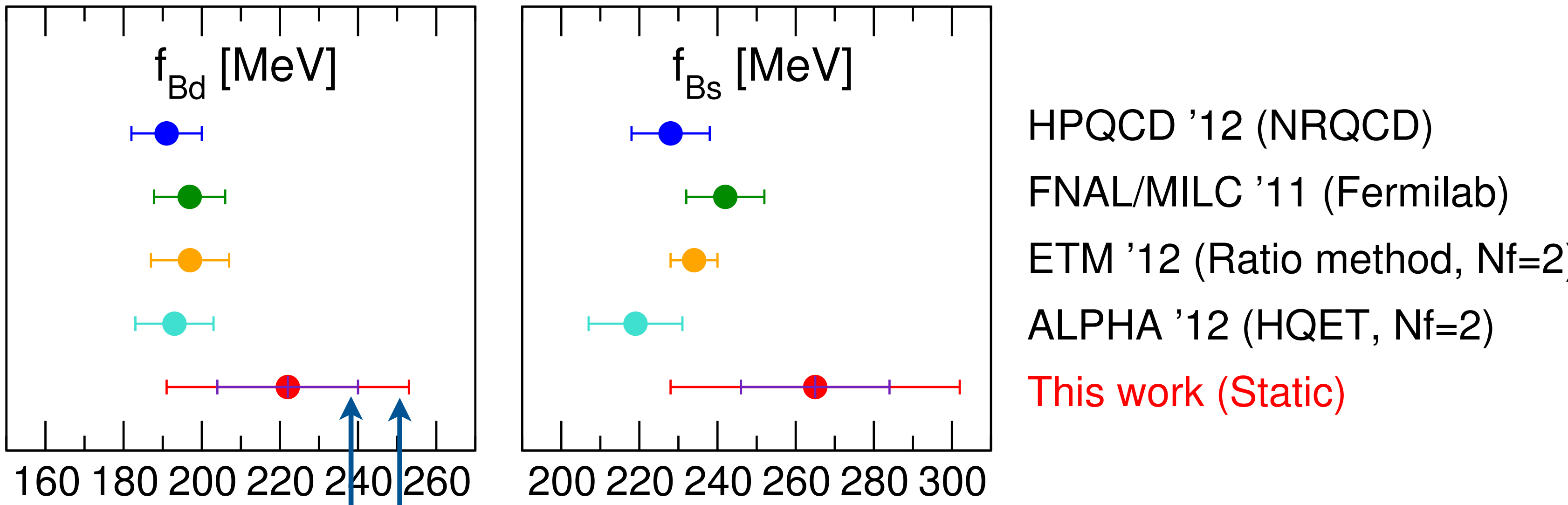
Error budget on ξ
(total 5.3% error)



Reducing statistical and
chiral/continuum extrapolation errors important.

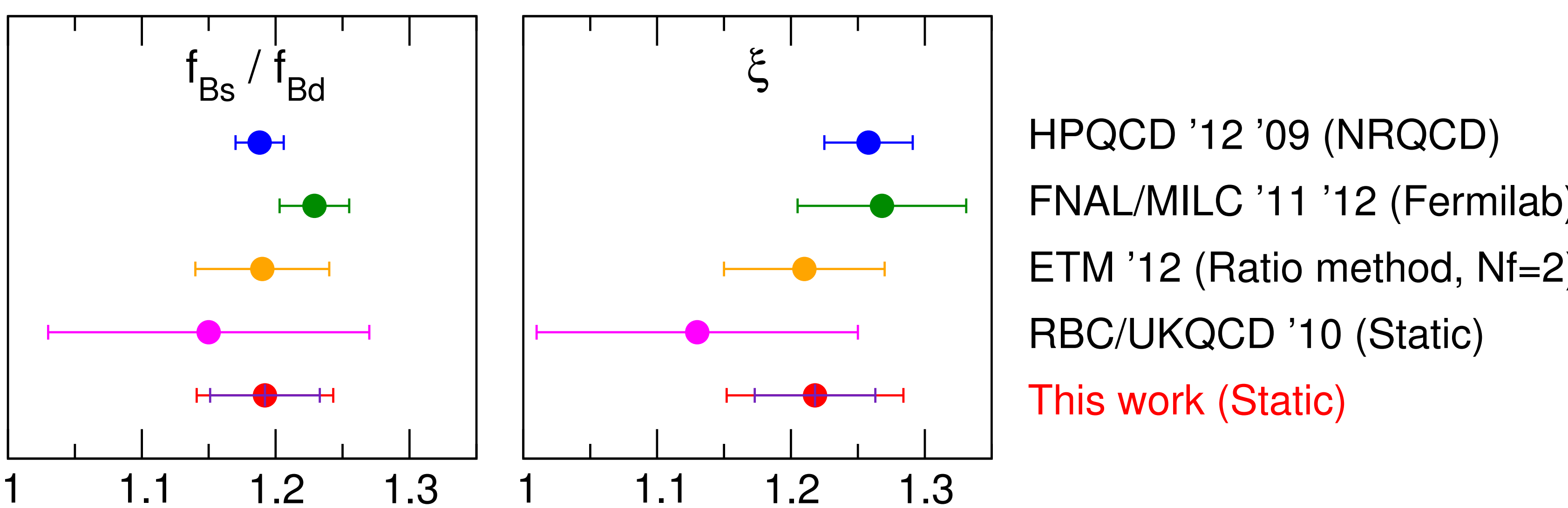
Results

Comparison



1/mb uncertainty is included in the error.

1/mb uncertainty is not included in the error.



Decay constants have ~10% deviation from other works.
Ratio quantities do not have such a significant deviation.

All-mode-averaging (AMA)

[Blum, Izubuchi and Shintani (2012)]

⟶ Plenary session 8/2 9:30AM Chulwoo Jung

- Example (32c, lightest quark mass parameter)
  - Translational invariance as a symmetry $g \in G$
    - Many source points $N_g = N_{\rm src} = 64 \quad (2 \times 2 \times 2 \times 8)$

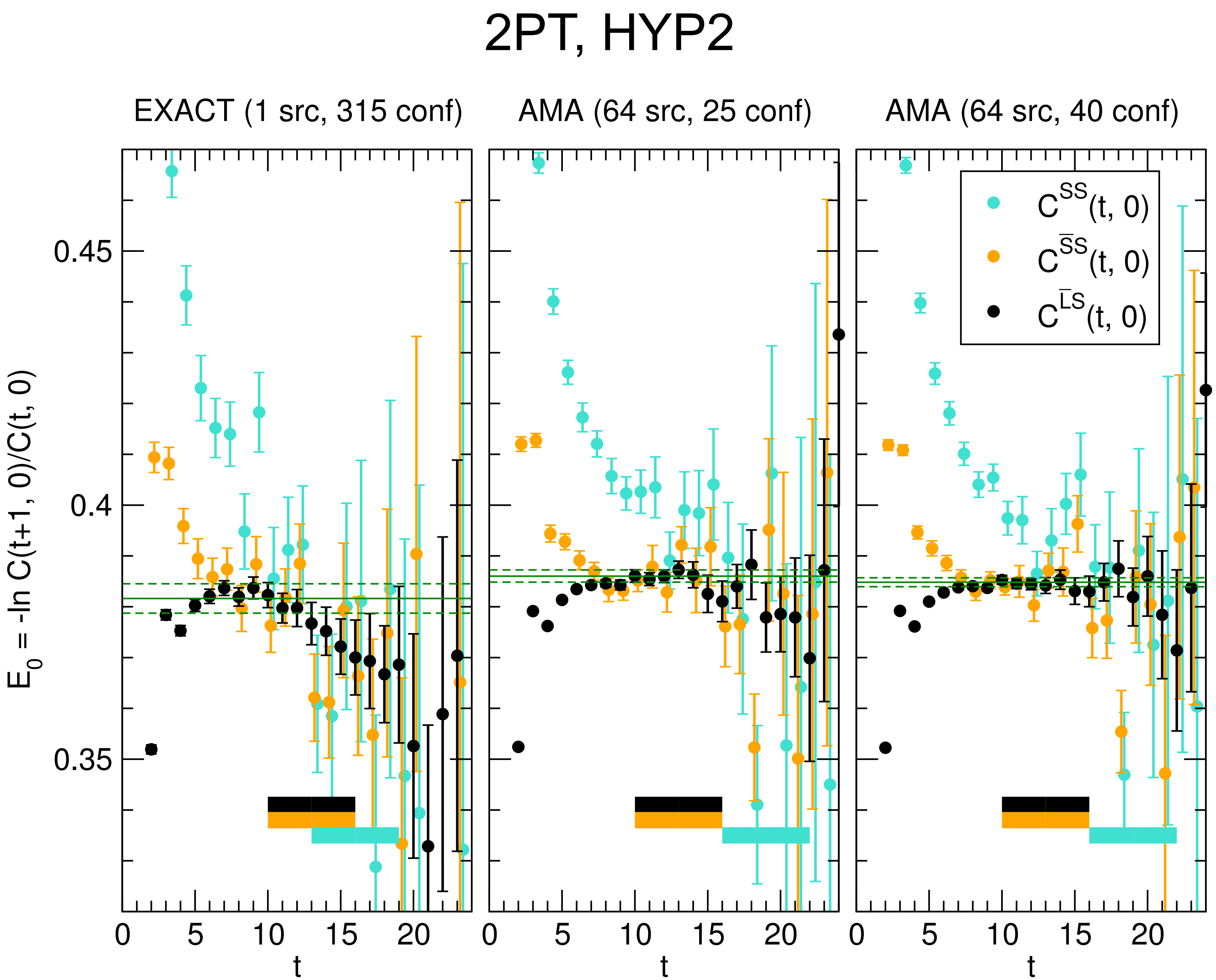
      divide $32 \times 32 \times 32 \times 64$ lattice by $2 \times 2 \times 2 \times 8$

  - Sloppy CG as an approximation
    - deflation with 130 lowest eigenvectors
    - CG iter = 120 to achieve res = 3e-3

      CG iter ~ 750 to achieve res = 1e-8

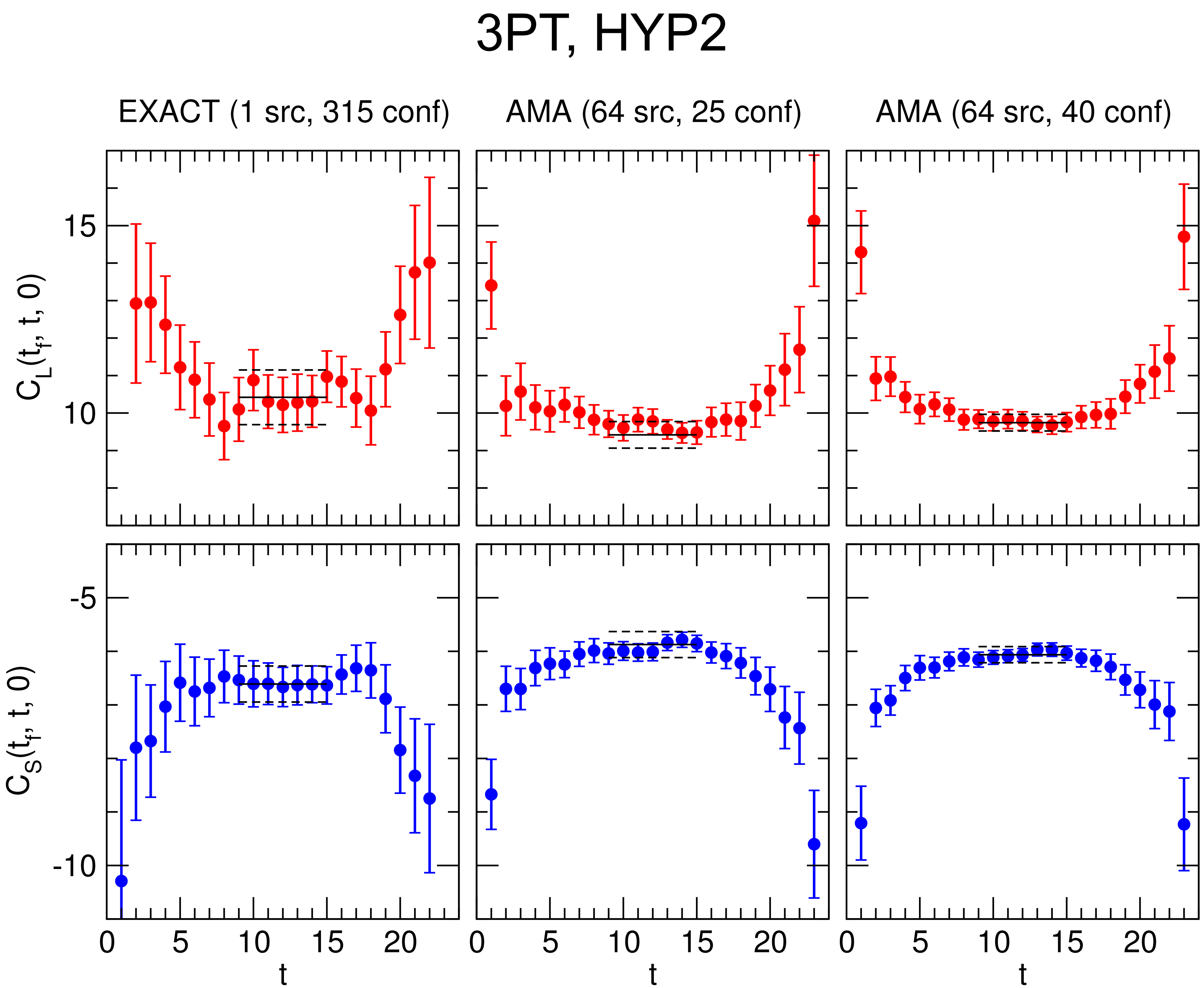
      CG iter ~ 4000 to achieve res = 1e-8 (no deflation)

All-mode-averaging (AMA)



[Cost] EXACT(315 conf) : AMA(25 conf) : AMA(40 conf) ~ 1 : 1 : 1.6

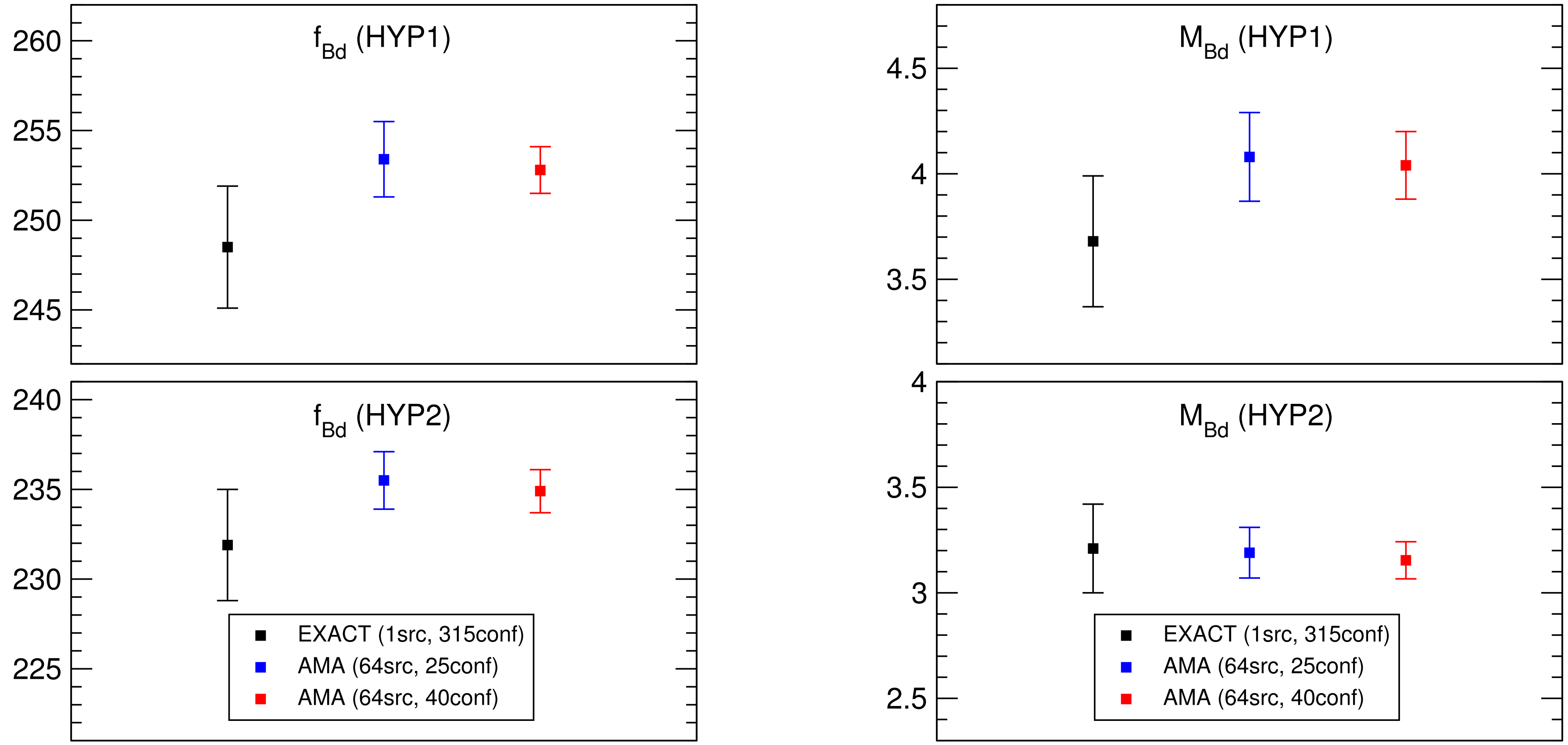
All-mode-averaging (AMA)



[Cost] EXACT(315 conf) : AMA(25 conf) : AMA(40 conf) ~ 1 : 1 : 1.6

All-mode-averaging (AMA)

Impact on physical quantities



- Gain (compared with deflated exact CG)
  - Decay constant : x2.6 (HYP1), x3.8 (HYP2)
  - Matrix element : x2.2 (HYP1), x3.1 (HYP2)
- Approximation is only used for light sector, then the gain in our heavy-light system would be smaller than other light-light simulations.

Summary and outlook

- B meson decay constants and neutral B meson mixing matrix elements in the continuum limit are obtained using static approximation.
- Two kinds of link smearing in the static action are used (HYP1 and HYP2). They give consistent results in the continuum limit.
- Decay constants has ~10% deviation from other works, possibly due to 1/mb error.
- Ratio quantities does not have significant deviation from other work, because 1/mb error is largely suppressed.
- Reducing statistical and chiral extrapolation error is important as a next step.
- AMA can largely reduce statistical error.
- Considering calculations at physical pion mass point.
- Considering non-perturbative matching.