UNIVERSITY OF Southampton

LATTICE 13

The $B^*B\pi$ coupling with relativistic heavy quarks

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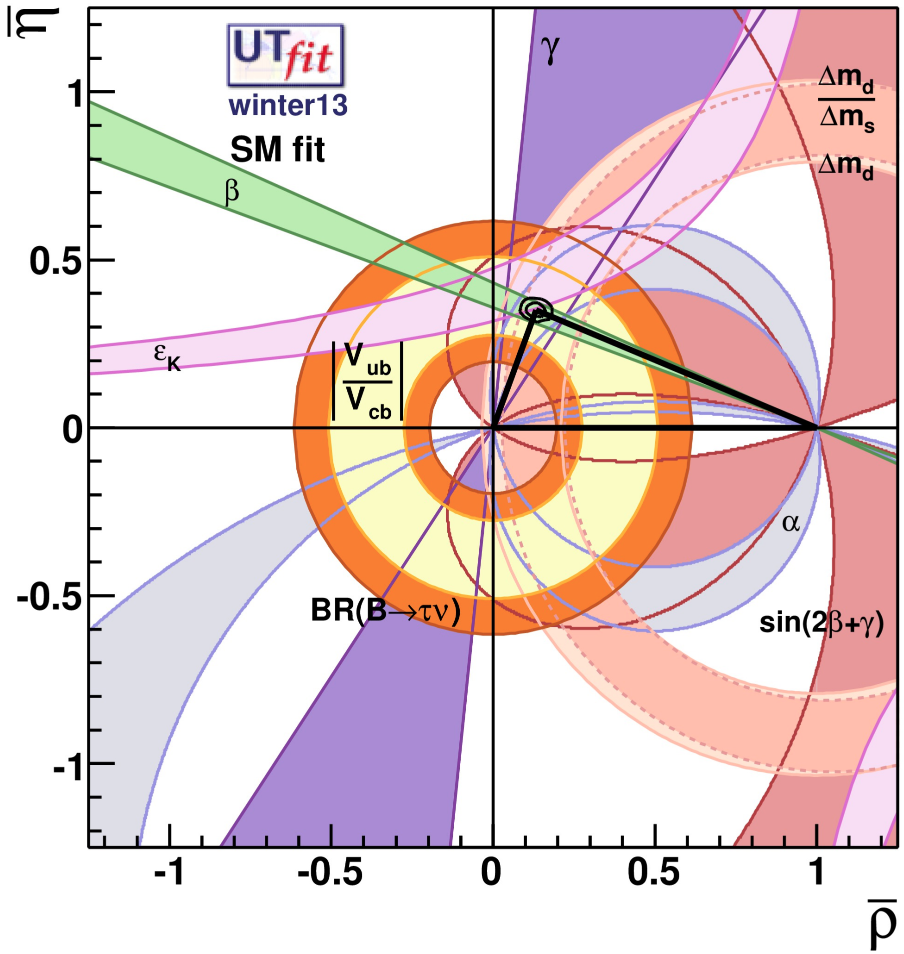
Heavy-quark physics



B-physics useful to test Standard Model / constrain CKM matrix

Lots of experimental progress

- LHCb (+ Atlas, CMS)
- BaBar
- Belle

Theoretical Input also needed

- Perturbation theory
- Lattice QCD

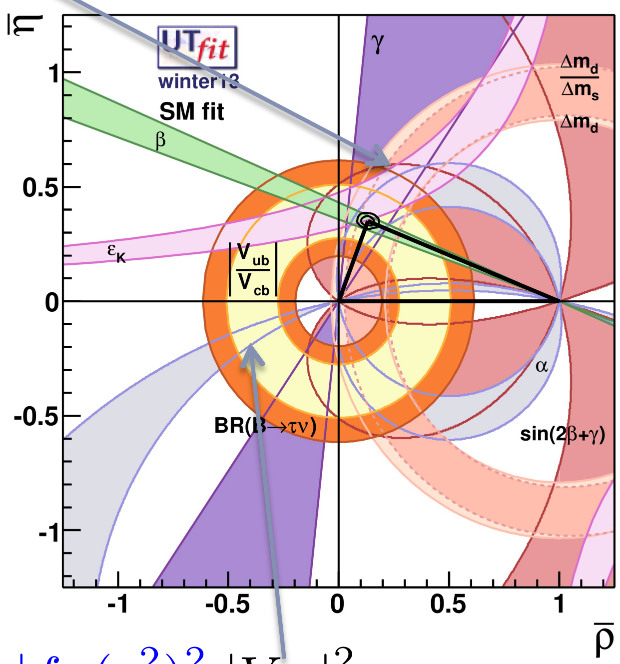
Heavy-quark physics

Neutral B-meson mixing

$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 m_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2$$

[Oliver Witzel's tallk here earlier for f_{Bd} and f_{Bs}]

- Inami-Lim function, S_0 and η_B accessible through perturbation theory
- Decay constant and bag parameter are non-perturbative
- Experimental uncertainties on Δm_q are < 1%
- Current lattice uncertainties for ξ are ~ 3%



B → π l ν form factor

[See Taichi Kawanai's talk here earlier]

- Allows determination of $|V_{ub}|$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3} \left[(m_B^2 + m_\pi^2 - q^2) - 4m_B^2 m_\pi^2\right]^{3/2} |f_+(q^2)^2 \, |V_{ub}|^2$$

Heavy Meson Chiral Perturbation Theory

Light-quark masses: $m_u, m_d, m_s << \Lambda_{QCD}$ *(maybe not strange…)*

- Chiral symmetry: write EFT in terms of pseudo-goldstone bosons from SSB.
- Chiral Perturbation Theory (χPT)

Heavy-quark masses: $m_c, m_b, m_t >> \Lambda_{QCD}$ *(maybe not charm…)*

- For large m_q heavy quarks become like static colour source
- Spin-flavour symmetry
- Heavy quark effective theory (HQET)

Heavy Meson χPT: $$\mathcal{L}^{int}_{HM\chi PT} = gTr\left(\bar{H}_a H_b \mathcal{A}^{ba}_\mu \gamma^\mu \gamma 5\right)$$

$$H = \frac{1+\not{v}}{2}\left(B^*_\mu \gamma^\mu - B\gamma_5\right) \qquad \mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger\right)$$

B*B π coupling definition

- Defined by strong matrix element

$$\langle B(p)\pi(q)|B^*(p',\lambda)\rangle = -g_{B^*B\pi}\, q\cdot\epsilon^{\lambda}(p')$$

- Equivalent quantity in HM χ PT

$$\langle B(p)\pi(q)|B^*(p',\lambda)\rangle = -\frac{2m_B}{f_\pi} g_b\, q\cdot\epsilon^{\lambda}(p')$$

$$\rightarrow \quad g_{B^*B\pi} = \frac{2m_B}{f_\pi} g_b$$

Chiral extrapolations

- Cannot perform lattice simulation at physical light-quark mass
- Perform extrapolations guided by NLO HM χ PT

$$f_{B_d} = F\left(1 + \frac{3}{4}(1 + 3g_b^2)\frac{m_\pi^2}{(4\pi f_\pi)^2}\log(m_\pi^2/\mu^2)\right) + \cdots$$

$$B_{B_d} = B\left(1 + \frac{3}{4}(1 - 3g_b^2)\frac{m_\pi^2}{(4\pi f_\pi)^2}\log(m_\pi^2/\mu^2)\right) + \cdots$$

- Knowledge of g_b will decrease the systematic uncertainties

Relativistic Heavy-Quark Action

[N. Christ, M. Li and H.w. Lin, Physical Review D 76 (2007)]

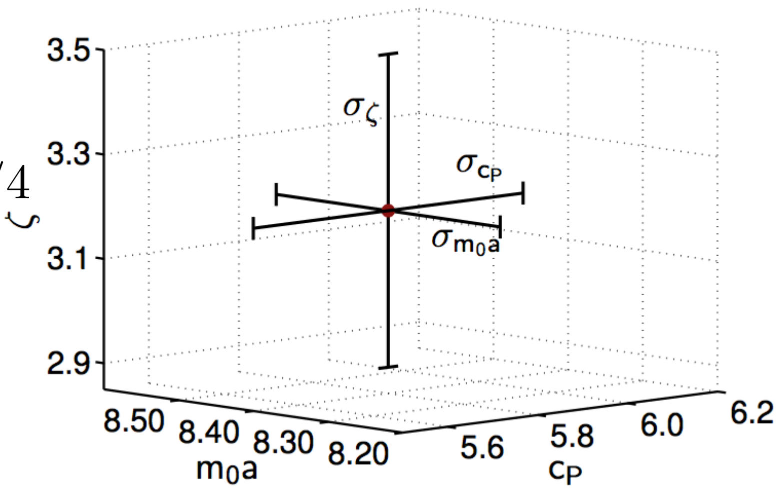
$$S_{RHQ} = a^4 \sum_{x,y} \bar{\psi}(y) \left(m_0 + \gamma_0 D_0 + \xi \vec{\gamma} \cdot \vec{D} - \frac{a}{2}(D_0)^2 - \frac{a}{2}\xi(\vec{D})^2 + \sum_{\mu\nu} \frac{ia}{4} c_p \sigma_{\mu\nu} F_{\mu\nu} \right)_{y,x} \psi(x)$$

- Only 3 unknown parameters: m_0, ξ, c_p
- Improved to $\mathcal{O}(ma)^n$ for all n, and to $\mathcal{O}(\underline{p}a)$.
- Parameters have been tuned non-perturbatively

[Y. Aoki et al., Phys Rev D 86 (2012)]

Calculate and match to PDG values.

- Spin averaged mass $\bar{M} = (M_{B_s} + 3M_{B_s^*})/4$
- Mass splitting $\Delta M = M_{B_s^*} - M_{B_s}$
- Ratio $M_1/M_2 = M_{\text{rest}}/M_{\text{kinetic}}$



Computing the coupling

LSZ reduction, PCAC relation

$$g_{B^*B\pi}(q^2)\epsilon^\lambda \cdot q = \frac{m_\pi^2 - q^2}{f_\pi m_\pi^2} \int_x \langle B(p)|q_\mu A^\mu(x)|B^*(p',\lambda)\rangle$$

Form factor decomposition

$$\langle B(p)|A^\mu|B^*(p',\lambda)\rangle = 2m_{B^*}A_0(q^2)\frac{\epsilon\cdot q}{q^2}q^\mu$$
$$+ (m_{B^*}+m_B)A_1(q^2)\left[\epsilon^\mu - \frac{\epsilon\cdot q}{q^2}q^\mu\right]$$
$$+ A_2(q^2)\frac{\epsilon\cdot q}{m_{B^*}+m_B}\left[p^\mu + p'^\mu - \frac{m_{B^*}^2 - m_B^2}{q^2}q^\mu\right]$$

$$g_{B^*B\pi} = \frac{2m_{B^*}A_0(0)}{f_\pi}$$ at q^2=0

Computing the coupling

- We need $A_0(0)$, but cannot simulate at $q^2=0$
- A_0 has pole at $q^2=m^2_\pi$ so difficult to extrapolate
- Use relation

$$g_{B^*B\pi} = \frac{1}{f_\pi}\left[(m_{B^*}+m_B)A_1(0)+(m_{B^*}-m_B)A_2(0)\right]$$

- In static limit:

$$g_{B^*B\pi} = \frac{2m_B}{f_\pi}A_1(0)$$

Lattice correlation functions

Three point function:

$$C^{(3)}_{\mu\nu}(t_x, t_y; \bar{p}, \bar{p}') = \sum_{\bar{x}\bar{y}} e^{-\imath\bar{p}\cdot\bar{x}} e^{-\imath\bar{p}'\cdot\bar{y}} \langle B(y) A_\nu(0) B^*(x)\rangle_{t_x<0<t_y}$$

$$\approx \sum_{\lambda} \frac{Z_B^{1/2} Z_{B^*}^{1/2}}{2E_B 2E_{B^*}} \langle B(p')|A_\nu|B^*(p,\lambda)\rangle (\epsilon^\lambda)_\mu e^{-E_B t_y} e^{-E_{B^*}(T-t_x)}$$

Two point functions:

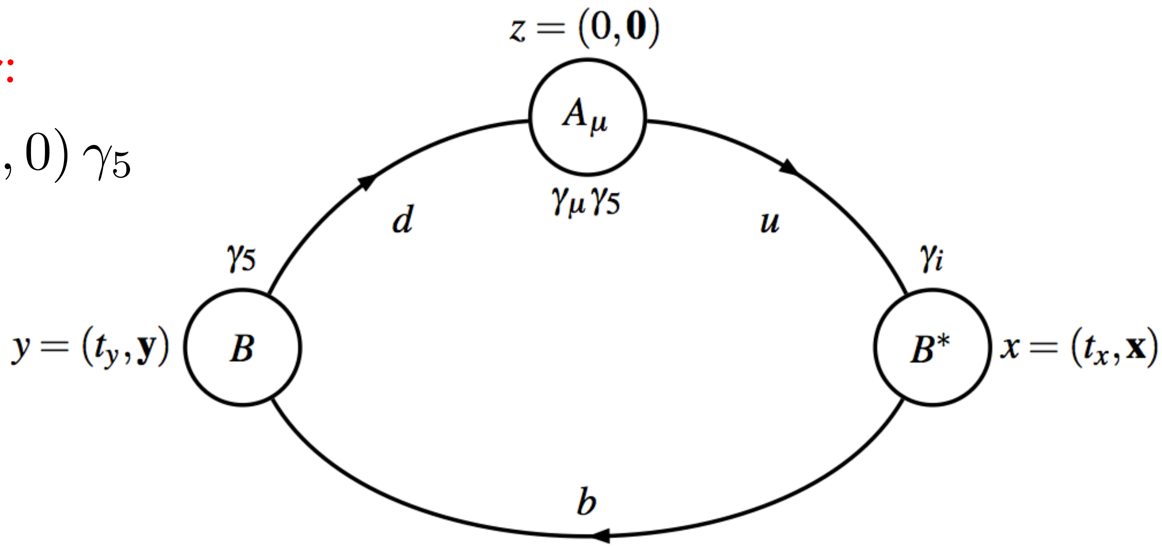
$$C^{(2)}_{BB}(t; \bar{p}) = \sum_{\bar{x}} e^{-\imath\bar{p}\cdot\bar{x}} \langle B(x) B(0)\rangle \approx Z_B \frac{e^{-E_B t}}{2E_B}$$

$$C^{(2)}_{B^*_\mu B^*_\nu}(t; \bar{p}) = \sum_{\bar{x}} e^{-\imath\bar{p}\cdot\bar{x}} \langle B^*_\nu(x) B^*_\mu(0)\rangle \approx Z_{B^*} \frac{e^{-E_{B^*} t}}{2E_{B^*}} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2},\right)$$

Lattice correlation functions

Sequential heavy-quark propagator:

$$D_{RHQ}\left(y,x\right)\Sigma\left(x,0\right)=\gamma_5 S_l\left(y,0\right)\gamma_5$$



$$C^{(3)}_{\mu\nu}\left(t_x,t_y;\bar{p},\bar{p}'\right)=\sum_{\bar{x}\bar{y}}e^{-\imath\bar{p}\cdot\bar{x}}e^{-\imath\bar{p}'\cdot\bar{y}}Tr\left[\gamma_5\Sigma^\dagger\left(x,0\right)\gamma_5\gamma_\nu S_l\left(x,0\right)\gamma_\mu\right]\Big|_{t_y>t_x}$$

Correlator ratios

[Abada, A. et al. *Physical Review D (2002)*]

Set $\bar{p} = \bar{p}' = 0$ such that $q_0^2 = (m_{B^*} - m_B)^2 \approx 0$

$$R_1 = \frac{C^{(3)}_{i,i}(t_x, t_y; \bar{p}, \bar{p}')\, Z_B^{1/2} Z_{B^*}^{1/2}}{C^{(2)}_{BB}(t_y; \bar{p})\, C^{(2)}_{B_i^* B_i^*}(T - t_x; \bar{p})} = (m_{B^*} + m_B) A_1(q_0^2)$$

To extract A_2 inject one unit of momentum $\bar{q} = \bar{p} = (1, 0, 0) \times 2\pi/L$

$$R_2 = \frac{C^{(3)}_{1,0}(t_x, t_y; \bar{p}, \bar{p}')\, Z_B^{1/2} Z_{B^*}^{1/2}}{C^{(2)}_{BB}(t_y; \bar{p})\, C^{(2)}_{B_2^* B_2^*}(T - t_x; \bar{p})} \qquad R_3 = \frac{C^{(3)}_{1,1}(t_x, t_y; \bar{p}, \bar{p}')\, Z_B^{1/2} Z_{B^*}^{1/2}}{C^{(2)}_{BB}(t_y; \bar{p})\, C^{(2)}_{B_2^* B_2^*}(T - t_x; \bar{p})}$$

$$\frac{A_2}{A_1} = \frac{(m_{B^*} + m_B)^2}{2 m_B^2 q_1^2} \left[-q_1^2 + E_{B^*}(E_{B^*} - m_B) - \frac{m_{B^*}^2 (E_{B^*} - m_B)}{E_{B^*}} \frac{R_3}{R_4} - \mathrm{i} \frac{m_{B^*}^2 q_1}{E_{B^*}} \frac{R_2}{R_4} \right]$$

Gauge configurations

RBC/UKQCD 2+1 flavour Domain-wall fermion / Iwasaki gauge action

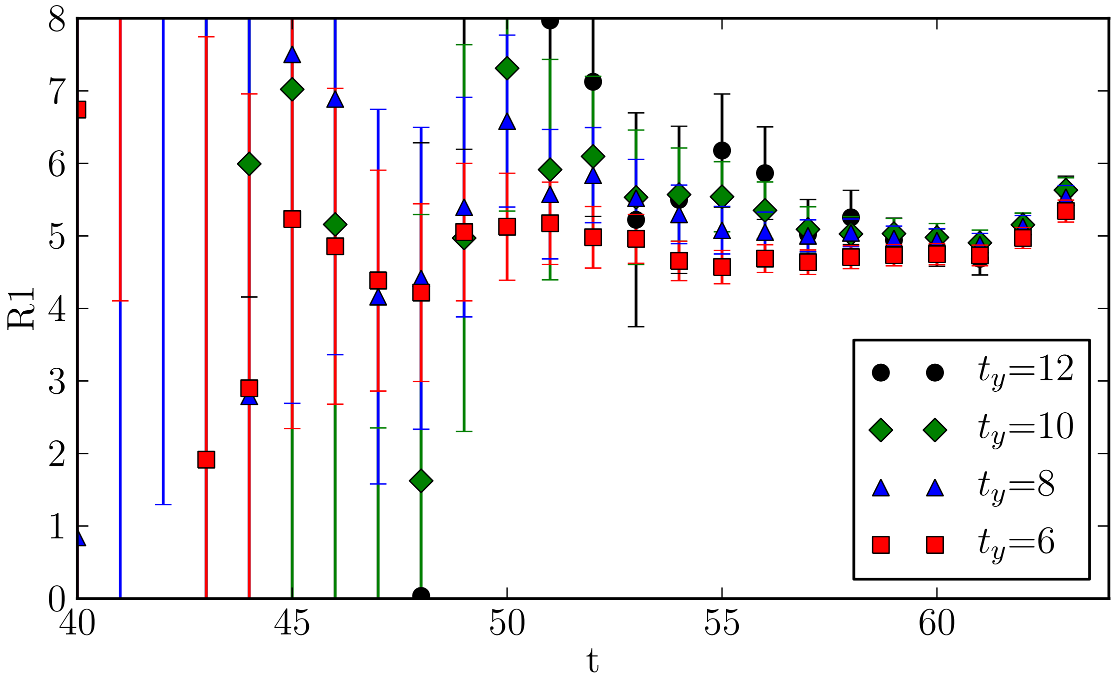
[Aoki, Y, et al. *Physical Review D (2011)*]

$L^3 \times T$	a(fm)	$m_l a$	$m_s a$	m_π(MeV)	#Configs	Sources
$24^3 \times 64$	0.11	0.005	0.04	329	1636	1
$24^3 \times 64$	0.11	0.010	0.04	422	1419	1
$24^3 \times 64$	0.11	0.020	0.04	558	345	1
$32^3 \times 64$	0.08	0.004	0.03	289	628	2
$32^3 \times 64$	0.08	0.006	0.03	345	889	2
$32^3 \times 64$	0.08	0.008	0.03	394	544	2

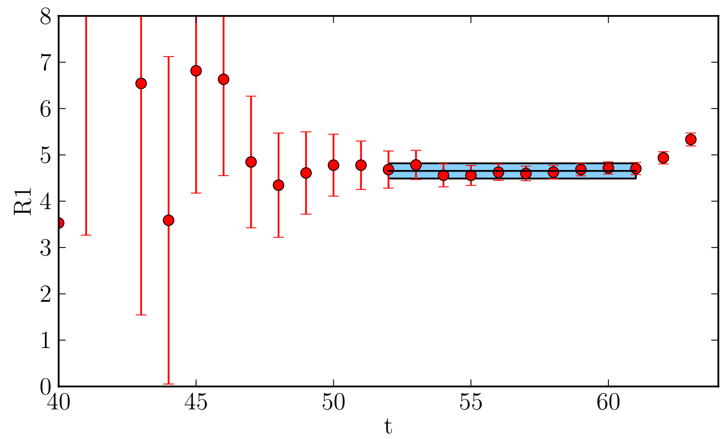
- Physical volume ~2.6fm
- Pions from 290 – 560 MeV

Preliminary results

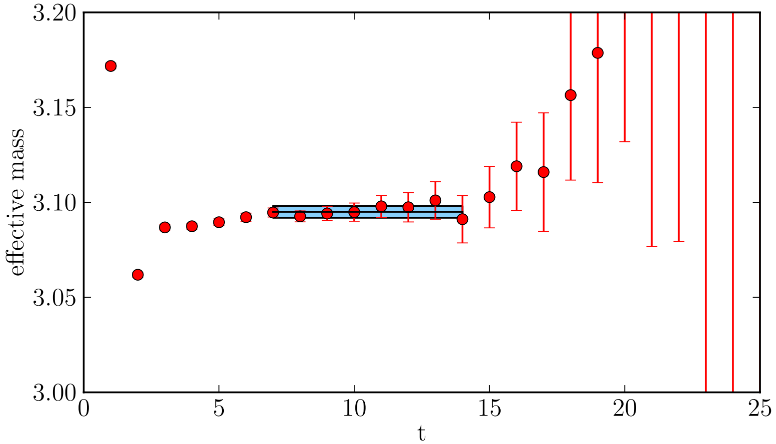
Ratio R1



Ratio R1 (t_y = 6)

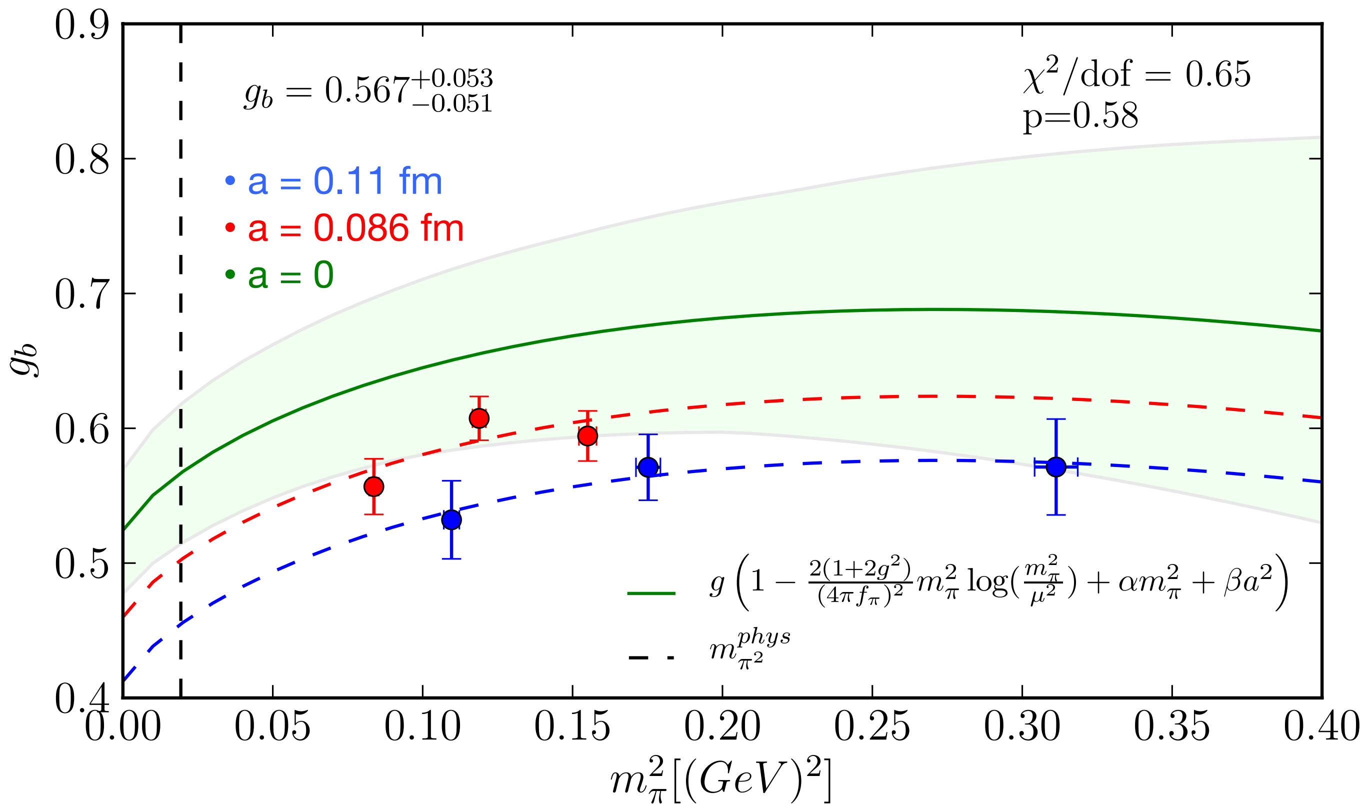


B* meson effective mass



24^3 m_l = 0.005 ensemble

Preliminary results



$$g = g_0 \left(1 - \frac{2(1 + 2g_0^2)}{(4\pi f_\pi)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + \alpha m_\pi^2 + \beta a^2 \right)$$

[W. Detmold, C.J. Lin and S. Meinel, Physical Review D 84 (2011) 094502, 1108.5594]

ICSS

Preliminary error budget

- Chiral extrapolation
- Continuum extrapolation
- Unphysical strange-quark mass
- Uncertainties in the RHQ parameters
- Finite volume corrections
- Lattice spacing uncertainty

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- ✓ Lattice spacing uncertainty 1%

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- ○ Chiral extrapolation
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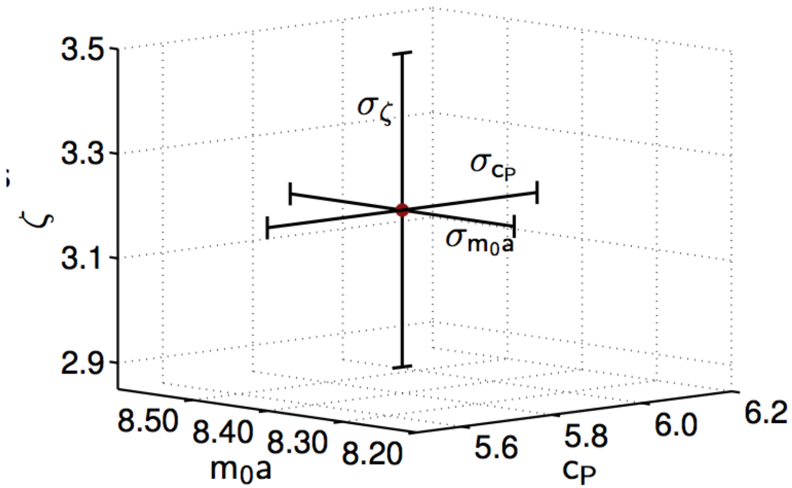
RHQ parameter uncertainties



$$M_{g_b}^{RHQ} = J_M \times \begin{bmatrix} m_o a \\ c_p \\ \xi \end{bmatrix}^{Tuned} + A_M$$

$$J_M = \left[\frac{M_3 - M_2}{2\sigma_{m_0 a}}, \quad \frac{M_5 - M_4}{2\sigma_{c_p}}, \quad \frac{M_7 - M_6}{2\sigma_{\xi}} \right]$$

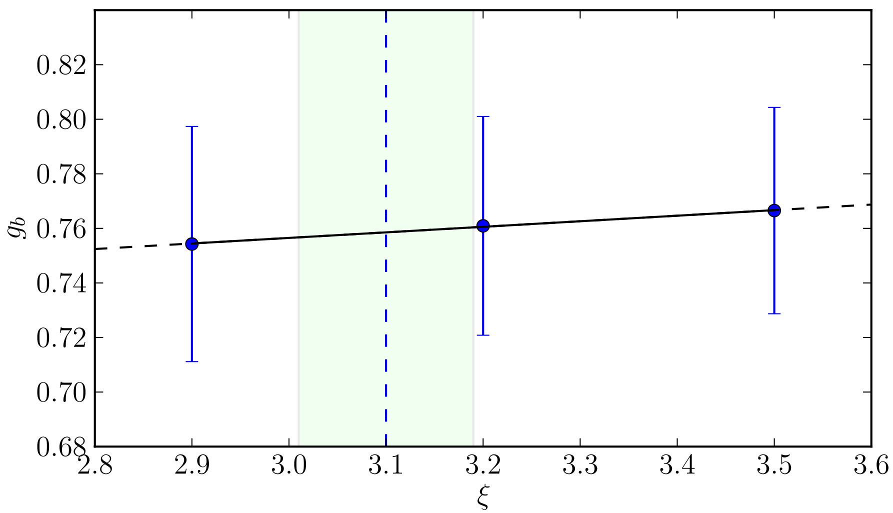
$$\begin{bmatrix} m_0 a \\ c_p \\ \xi \end{bmatrix} \begin{bmatrix} m_0 a - \sigma_{m_0 a} \\ c_p \\ \xi \end{bmatrix} \begin{bmatrix} m_0 a + \sigma_{m_0 a} \\ c_p \\ \xi \end{bmatrix} \begin{bmatrix} m_0 a \\ c_p - \sigma_{c_p} \\ \xi \end{bmatrix} \begin{bmatrix} m_0 a \\ c_p + \sigma_{c_p} \\ \xi \end{bmatrix} \begin{bmatrix} m_0 a \\ c_p \\ \xi - \sigma_\xi \end{bmatrix} \begin{bmatrix} m_0 a \\ c_p \\ \xi + \sigma_\xi \end{bmatrix}$$

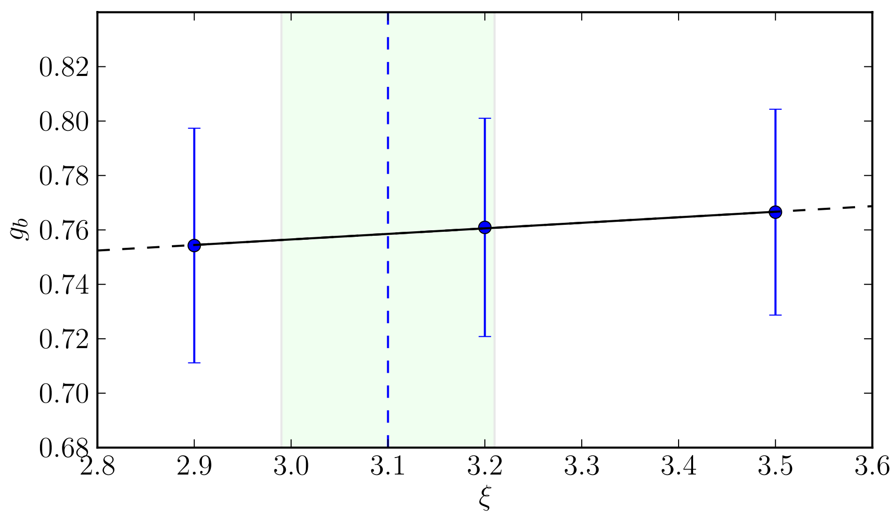
RHQ parameter uncertainties

	$m_o a$	c_P	ξ
a≈0.11 fm	8.45(6)(13)(50)(7)	5.8(1)(4)(4)(2)	3.10(7)(11)(9)(0)
a≈0.056 fm	3.99(3)(6)(18)(3)	3.57(7)(22)(19)(14)	1.93(4)(7)(3)(0)

RHQ parameter uncertainties (statistical, HQ discretisation, lattice spacing, experimental)

[Y. Aoki et al., Phys Rev D 86 (2012)]





Preliminary error budget

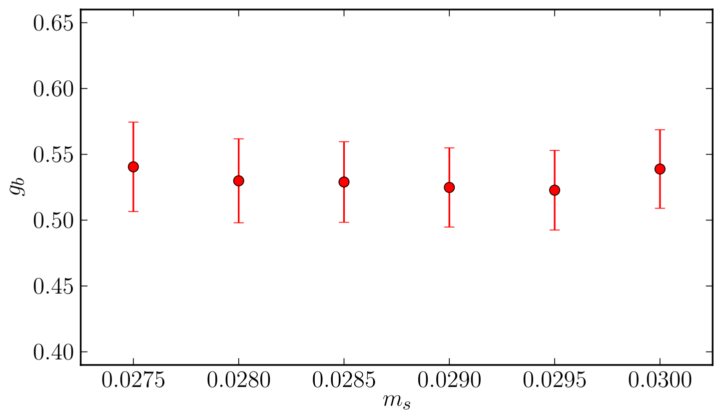
- ○ Chiral extrapolation
- ○ Continuum extrapolation
- ○ Unphysical strange-quark mass
- ✓ Uncertainties in the RHQ parameters 1.5%
- ✓ Finite Volume corrections 1%
- ✓ Lattice spacing uncertainty 1%

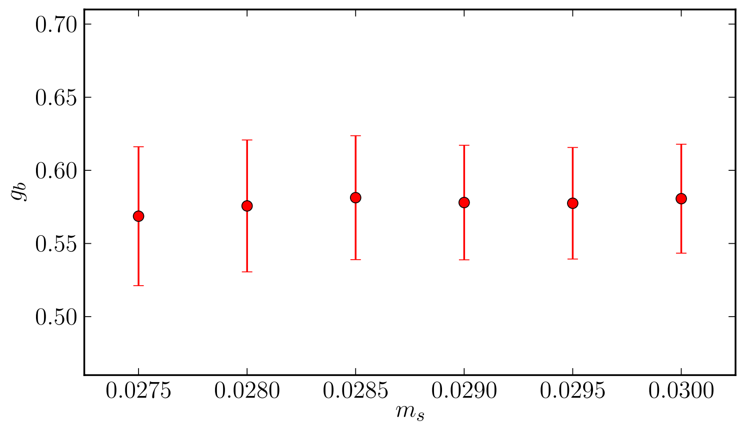
Unphysical strange-quark mass

- $m_{physical}$ differs from $m_{simulated}$ by ~10%
- No valence strange-quarks, only a sea effect

1. Reweighting

$$\langle \mathcal{O} \rangle = \frac{\sum_i w_i \mathcal{O}_i}{\sum_i w_i} \qquad w_i = \frac{D_2[U_i]}{D_1[U_i]}$$





Cannot discern any effect within statistics

Using partially quenched HMχPT: ~1.5% effect

Preliminary error budget

- ○ Chiral extrapolation
- ○ Continuum extrapolation
- ✓ Unphysical strange-quark mass 1.5%
- ✓ Uncertainties in the RHQ parameters 1.5%
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Heavy-quark discretisation errors

Write Symanzik-like effective theories for QCD and the lattice theory

$$\mathcal{L}^{QCD} \doteq \mathcal{L}^{Sym} = \cdots - \bar{Q}\left(\gamma_4 D_4 + m_1 + \sqrt{\frac{m_1}{m_2}}\gamma \cdot \mathbf{D}\right) Q + \sum_i \mathcal{C}_i^{Cont}(g^2, m_2 a, \mu a)\mathcal{O}_i$$

$$\mathcal{L}^{Lat} = \cdots - \bar{Q}\left(\gamma_4 D_4 + m_1 + \sqrt{\frac{m_1}{m_2}}\gamma \cdot \mathbf{D}\right) Q + \sum_i \mathcal{C}_i^{Lat}(g^2, m_2 a, \mu a)\mathcal{O}_i$$

Discretisation effects come from mismatch between coefficients $C_i^{lat} - C_i^{cont}$

O_i and C_i have been calculated to tree level

[M.B. Oktay and A.S. Kronfeld, Phys Rev D 78 (2008)]

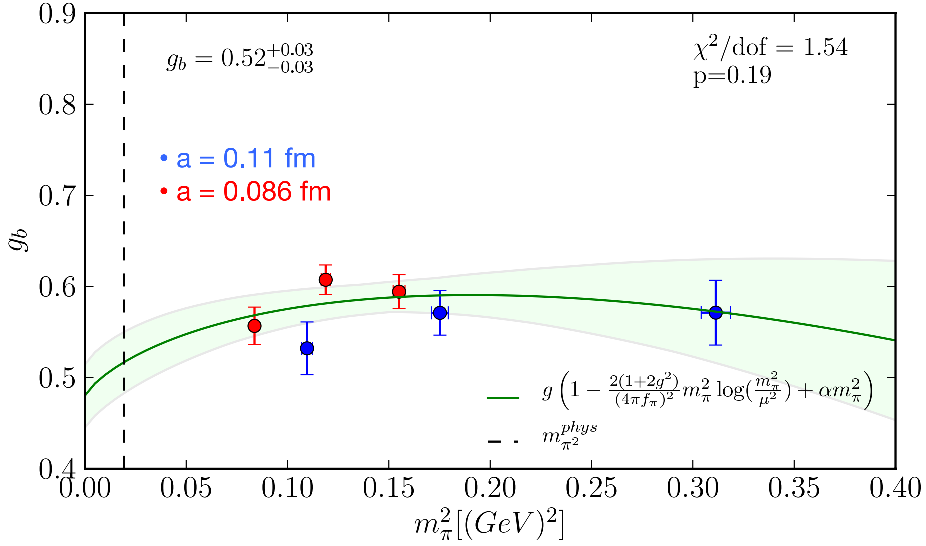
$$g_b^{\mathrm{error}} = g_b \sum_i \left(\mathcal{C}_i^{Cont} - \mathcal{C}_i^{Lat}\right) \sum_i \frac{\langle \mathcal{O}_i \rangle}{2M_B}$$

Estimate $\langle O_i \rangle$ using HQET power-counting $\langle \mathcal{O}_E \rangle^{HQET} \sim a^2 \Lambda^3_{QCD}$

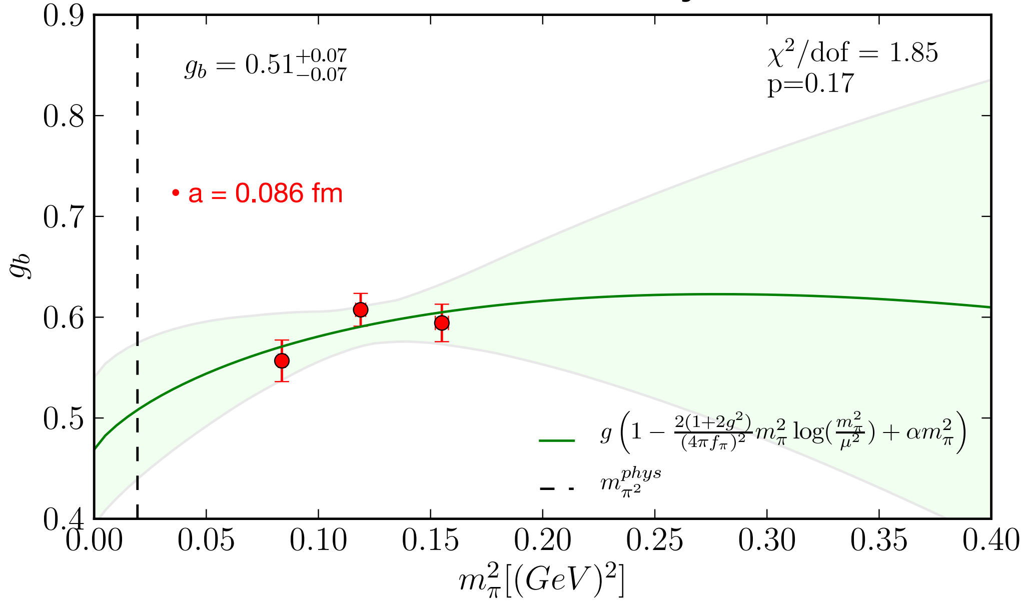
Error negligible

Light-quark and gluon discretisation errors

No cut off effects?

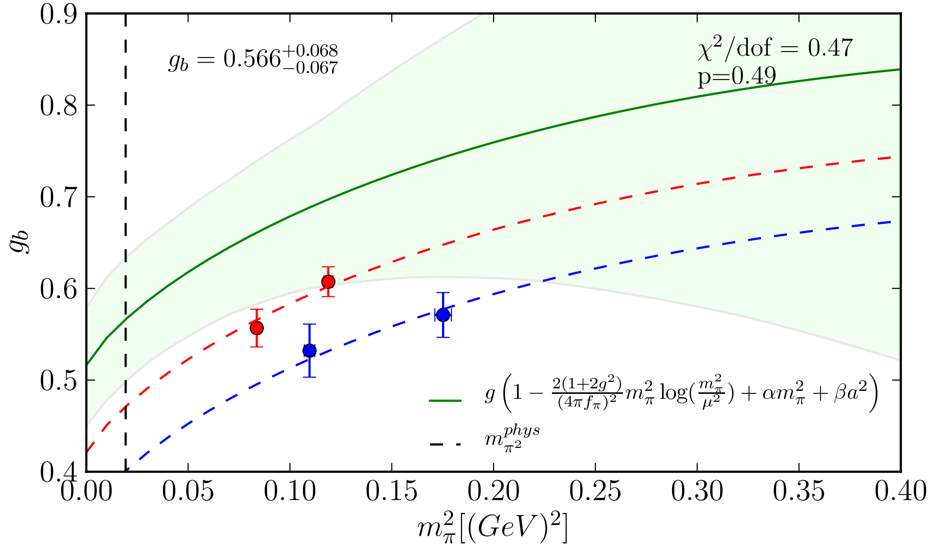


Finer lattice only

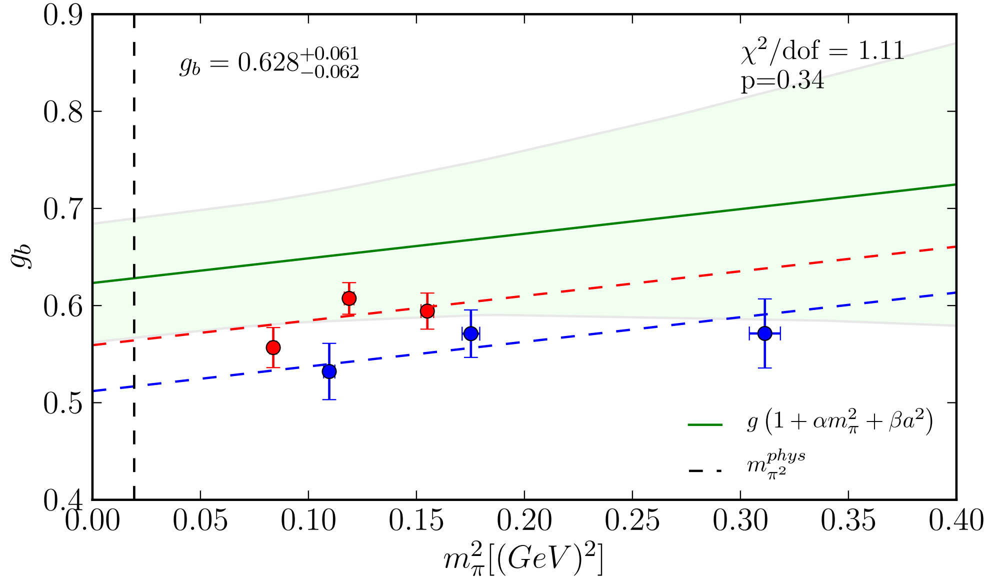


Chiral extrapolation

Drop heaviest masses



Linear fit



Preliminary error budget

- ✓ Chiral extrapolation
- ✓ Continuum extrapolation

] 10%

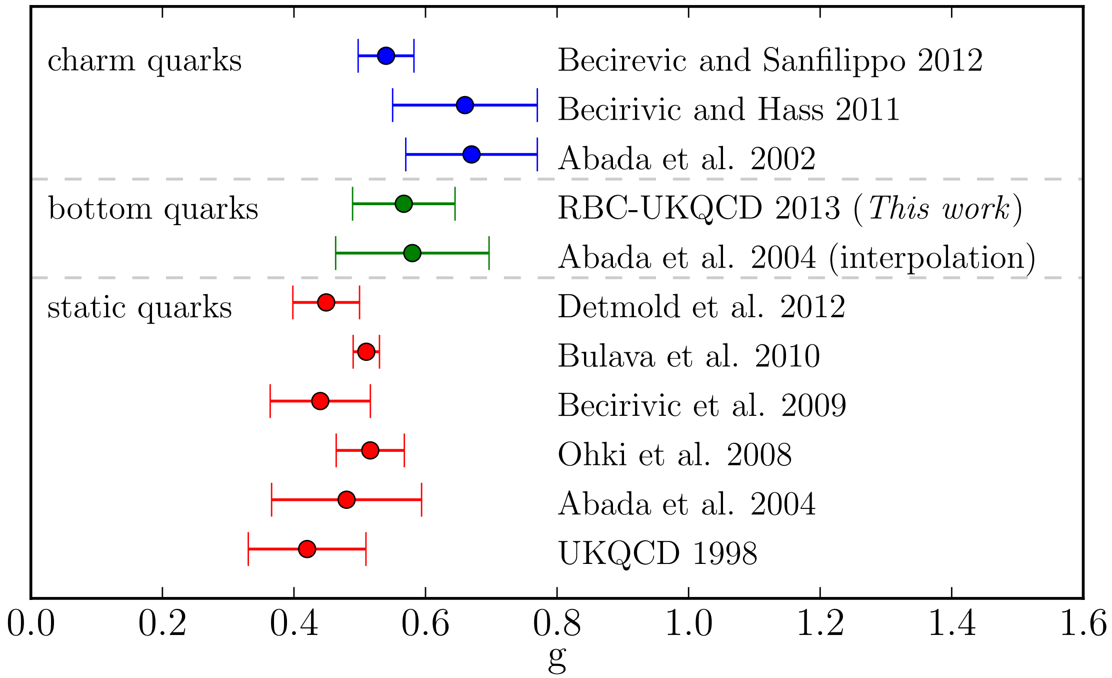
- ✓ Unphysical strange-quark mass 1.5%
- ✓ Uncertainties in the RHQ parameters 1.5%
- ✓ Finite Volume corrections 1%
- ✓ Lattice spacing uncertainty 1%

Total systematic uncertainties 10.5%

$$g_b = 0.567(52)(58)$$

Conclusions

- We have determined the coupling g_b and considered all sources of systematic errors
- This is the first result directly at the b-quark mass
- The result is consistent with other determinations, and between the average value of g_c and the average value of g_∞
- This result will prove useful in ongoing B-physics analyses by RBC/UKQCD and other collaborations



Thank you for listening!