SU(2) gauge theory with chirally symmetric fermions

- Introduction
- Strategy/setup
- Nf=2 domain-wall simulation
- Overlap eigenvalues
- Summary/outlook

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Introduction

- SU(2) gauge theory with fundamental fermions
  - Pseudoreal, chiral symmetry breaking: SU(2Nf) → Sp(2Nf)
  - Many works on confinement mechanism, finite temperature/density
  - Beyond standard model
    - Technicolor, conformal window (many talks in Lattice 2013)
    - Dark matter Cf. Lewis, Pica, Sannino (2012)
- Chiral dynamics depending on gauge group and fermion repr.
  - Different symmetry breaking pattern

Fundamental	Adjoint
SU(2): SU(2Nf) → Sp(2Nf) SU(N) N>2: SU(Nf)xSU(Nf) → SU(Nf)	SU(N): SU(2Nf) → SO(2Nf)

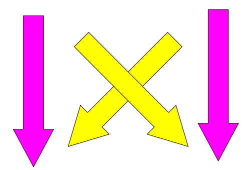
  - Number of flavors
  - Finite temperature/density

strategy

- Chiral symmetric fermion is better device
  - Overalp: best symmetry, high numerical cost, involved setup (Aoki phase, etc.) H.M., Kikukawa, Yamada, Nagai, Lattice 2010, 2009
  - Domain-wall: good properties, numerically feasible
    - approaches to overlap with large Ns
    - Residual mass probes chiral symmetry violation

Configuration — **Domain-wall** | **Ovelap**

Domain-wall:
- Topology changes
- Large lattice is possible



Ovelap:
- Fixed topology
- p-regime/ε-regime

Mesurement — **Domain-wall** | **Ovelap**

- Spectroscopy
- Residual mass/breaking of G-W relation
- Eigenvalue/vectors

Setup

- Iwasaki gauge action
- Nf=2 dynamical standard domain-wall fermions
- Lattice sizes: 8^3x16, 12^3x24, 16^3x32

- HMC
  - Domain-wall/Pauli-Villars
  - Omelyan integrator + multi-time step (2-level)
- Valence fermion: domain-wall/overlap
  - Hadrons correlators
  - Residual mass
  - Eigenvalue/vectors
- Fundamental setup: making basis for further studies
  - many flavors, finite T/μ, adjoint fermions, ...
  - Comparion with improved domain-wall, dynamicsal overlap

Domain-wall fermion

- Standard domain-wall fermion action

$$
\begin{aligned}
S_{DW} &= \sum_{x,s} \bar{\psi}(x,s) D_W(x,y;-M_0)\psi(y,s) \\
&\quad -\frac{1}{2}\sum_{x,s} \bar{\psi}(x,s)\left[(1-\gamma_5)\psi(x,s+1) + (1+\gamma_5)\psi(x,s-1) - 2\psi(x,s)\right] \\
&\quad +m\left[\bar{\psi}(x,1)P_R\psi(x,L_s) + \bar{\psi}(x,L_s)P_L\psi(x,1)\right]
\end{aligned}
$$

$$
D_W(x,y;M) = M\delta_{x,y} - \frac{1}{2}\sum_{\mu=1}^{4}\left\{(1-\gamma_\mu)U_\mu(x)\delta_{x+\hat{\mu},y} + (1+\gamma_\mu)U_\mu^\dagger(x-\hat{\mu})\delta_{x-\hat{\mu},y} - 4\delta_{x,y}\right\}
$$

  - M_0: domain-wall height, m: fermion mass
  - Ls: extent of 5-th direction
  - Boundary conditions: $P_R\psi(s=0) = P_L\psi(s=L_s+1) = 0$
  - 4D fermion field:

$$
\begin{aligned}
q(x) &= P_L\,\psi(x,s=1) + P_R\,\psi(x,s=L_s) \\
\bar{q}(x) &= \bar{\psi}(x,s=1)P_R + \bar{\psi}(x,s=L_s)P_L
\end{aligned}
$$

Resources/environment

- Machines
  - Hitachi SR16000, IBM Blue Gene/Q at KEK
  - φ at KMI, Nagoya Univ.

Code:
  - Bridge++ (C++)
    - Cf. S.Ueda's talk (Fri pm)
  - Fortran code

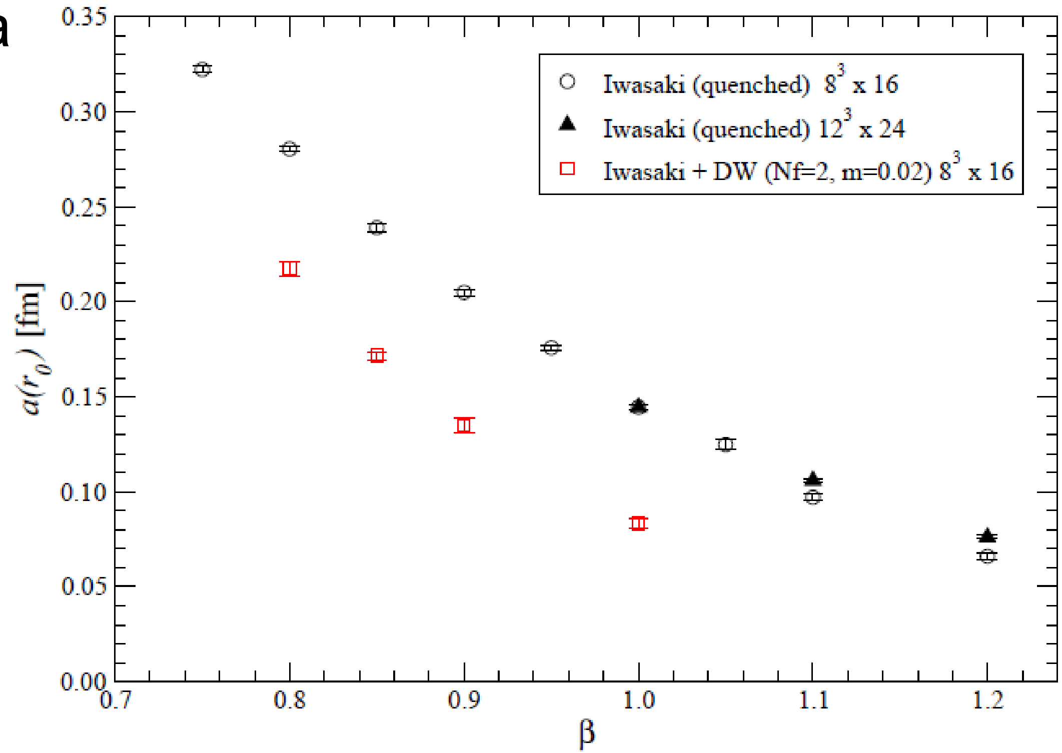
- JLDG (Japan Lattice Data Grid)
  - for fast data transfer

Japan Lattice Data Grid

Parameter and scale

- Lattice sizes: 8^3x16, 12^3x24, 16^3x32
- β=0.85, 0.90, (also 0.80, 1.0)
- Ns=16, M_0=1.6
- Sea fermion mass: m=0.02, 0.05, 0.10, 0.20

• Sommer's scale determined from static potential
  - Physical units only for rough idea

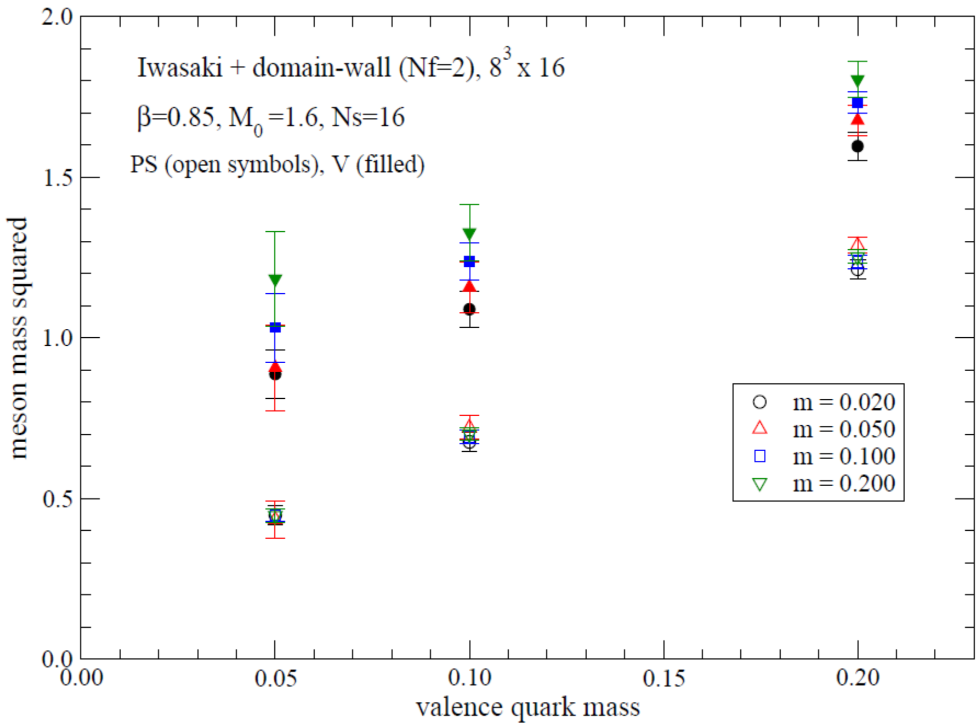


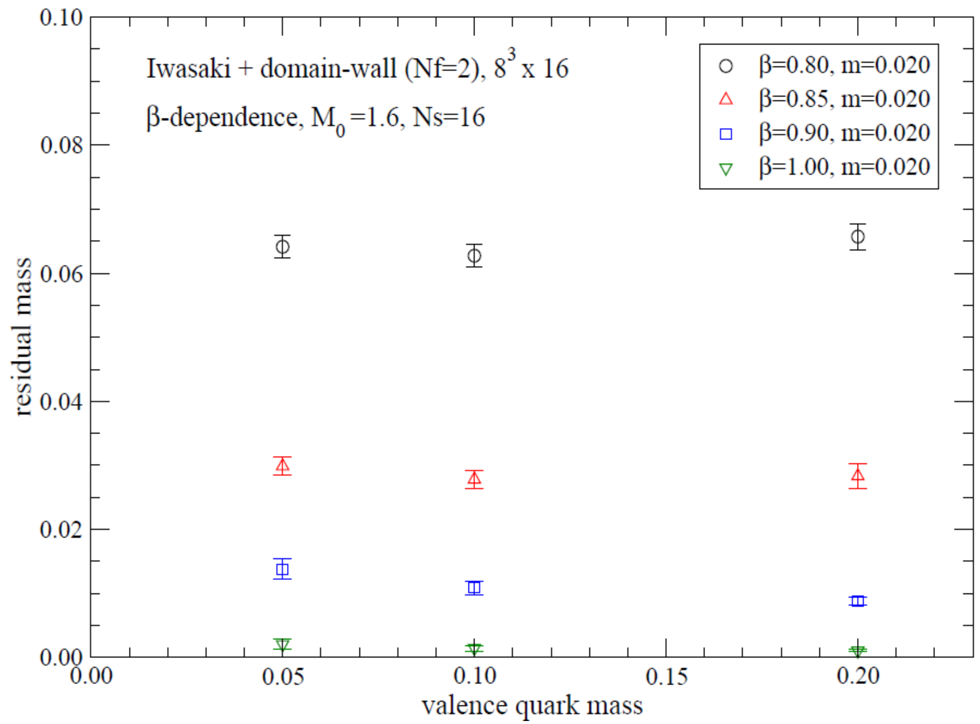
Meson spectrum and residual mass

- Valence quark mass dependence
- Local-local correlators, 20-40 configurations
- m_{PS}^2 is proportional to m_{val}

• Residual mass: decreases as β increases

- Consistent with PS meson mass, $m_{res}(\beta=0.90) \sim 0.015$



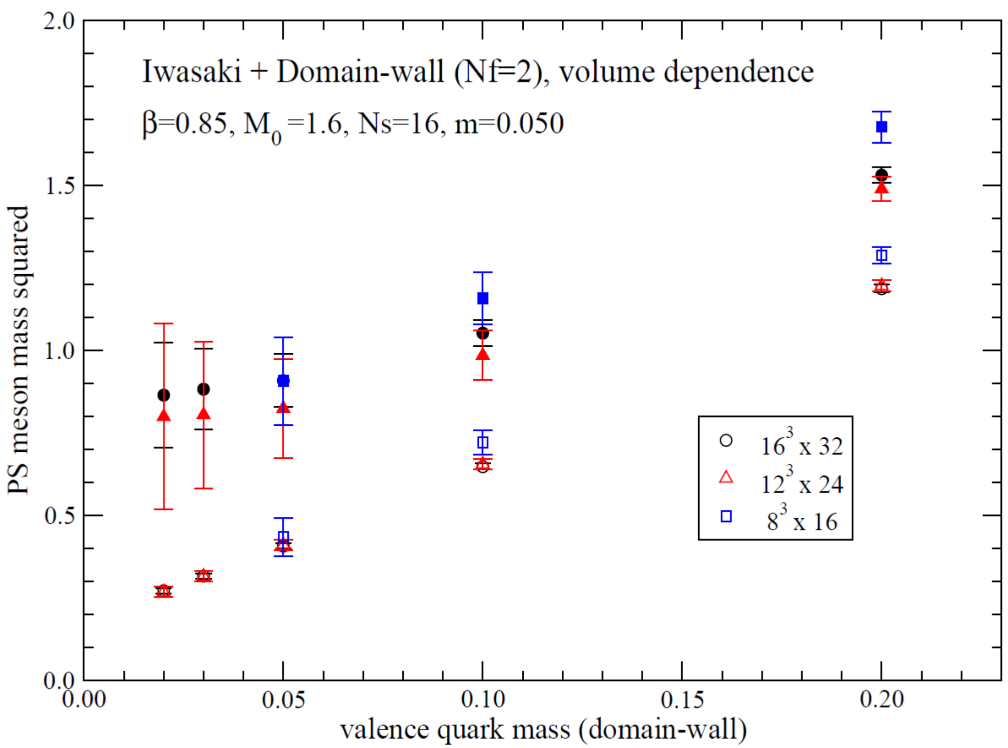


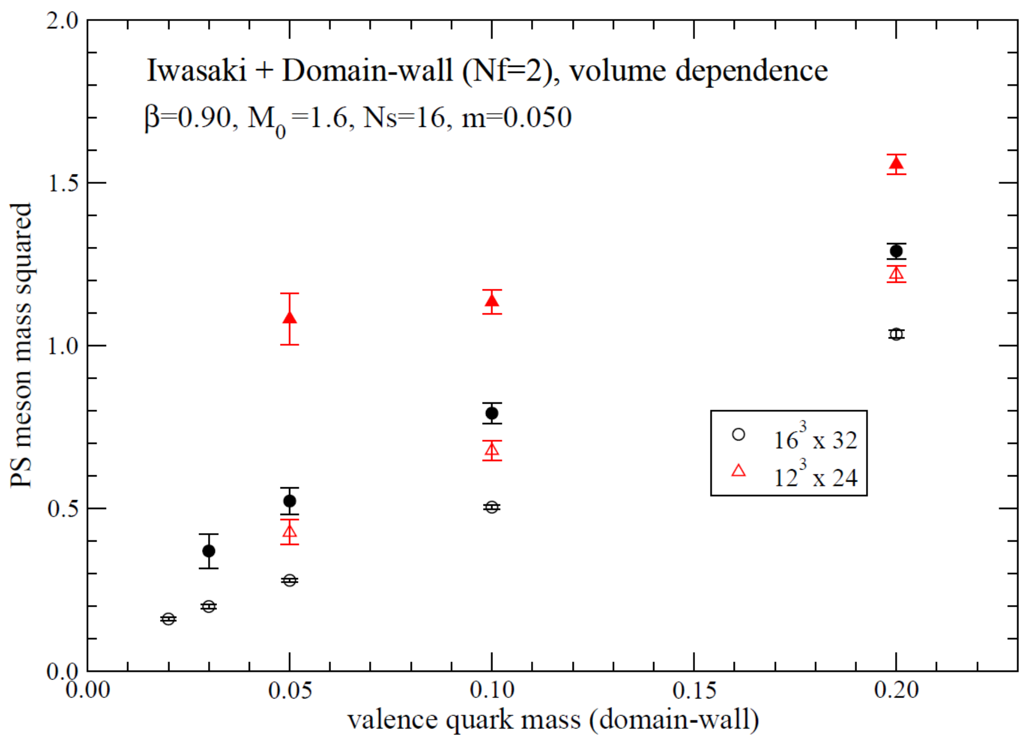
Finite size effect

- Comparison of 8^3x16, 12^3x24, 16^3x32 at the same beta and m
- At β=0.85, finite size effect small on 12^3x24 and 16^3x32

  $Lm_{PS}(m=0)$ ~ 5.4 (L=12), 3.6 (L=8)
- At β=0.90, rather large finite size effect: 12^3x24 underway

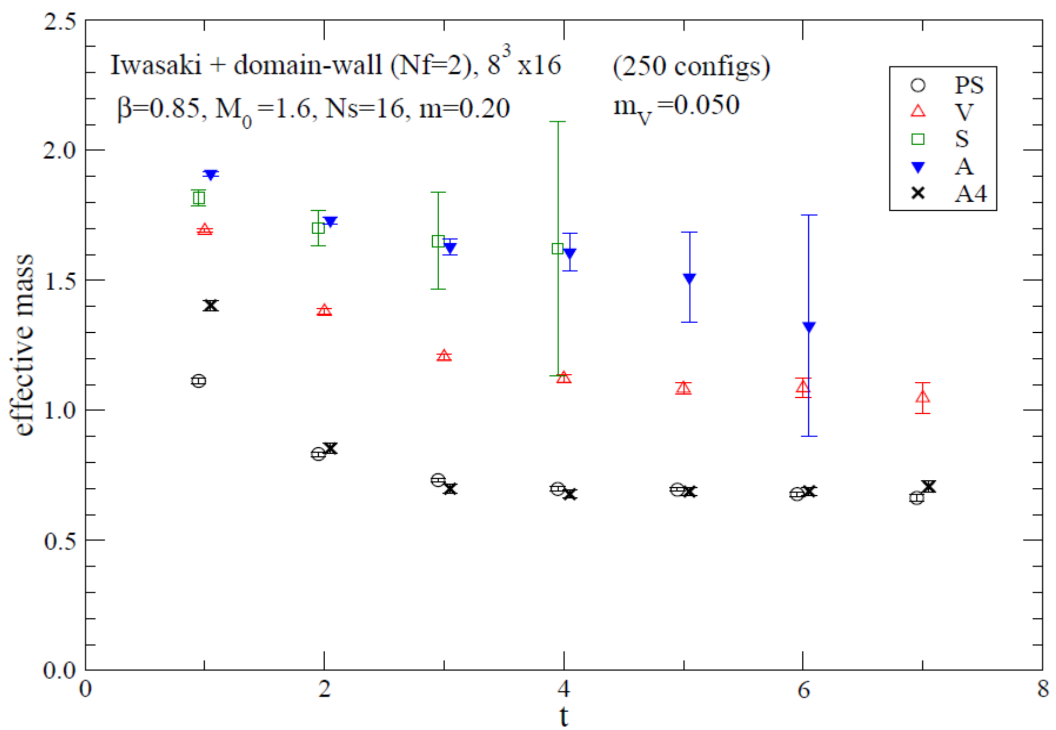
  $Lm_{PS}(m=0)$ ~ 5.8 (L=16), 3.6 (L=12)





Domain-wall analysis summary

- Spectroscopy
  - Spectrum and residual mass are consistent with standard behaviors with broken chiral symmetry
  - Ready to extend to large Nf/finite temperature
- Analysis In progress:
  - Other meson/diquark channels (need larger statistics)
  - PCAC mass, decay constant
  - Engenvalues



Valence overlap analysis

- Overlap operator

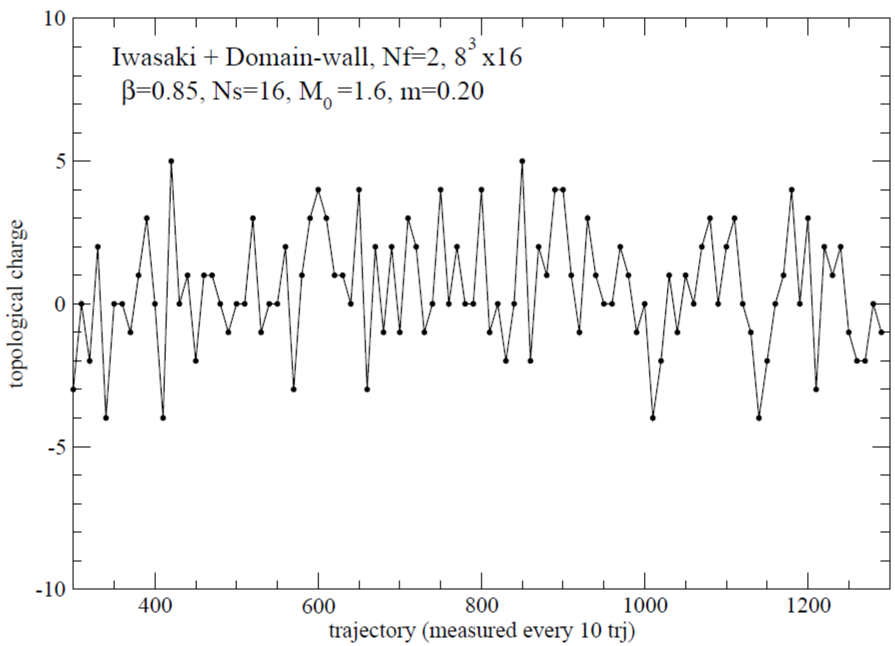
$$D = \frac{1}{Ra}\left[1 + \gamma_5 \,\mathrm{sign}(H_W(-m_0))\right]$$

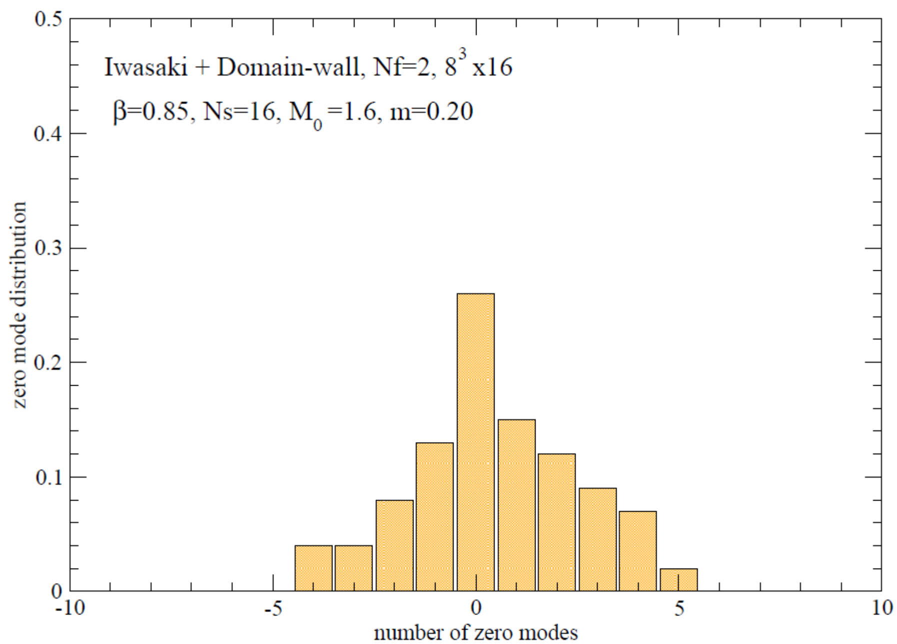
(Neuberger, 1998)

  - H_W : hermitian Wilson-Dirac operator
  - Zolotarev approx to sign function
  - Low-mode subtraction for H_W
- Eigenvalues/vectors
  - zero mode destribution
  - Low-mode distribution/density
  - Comparison with random matrix theory
  - Now most results are on 8^3x16
- Hadron spectroscopy is in progress

Overlap eigenvalues

- Eigenvalues of massless overlap fermion operator
  - Implicitly restarted Lanczos algorithm
- Topological charge distribution: number of zero modes
  - Autocorrelation: on 8^3x16, topology well frequently changes
  - Need to prepare to long autocorrelation on larger lattices

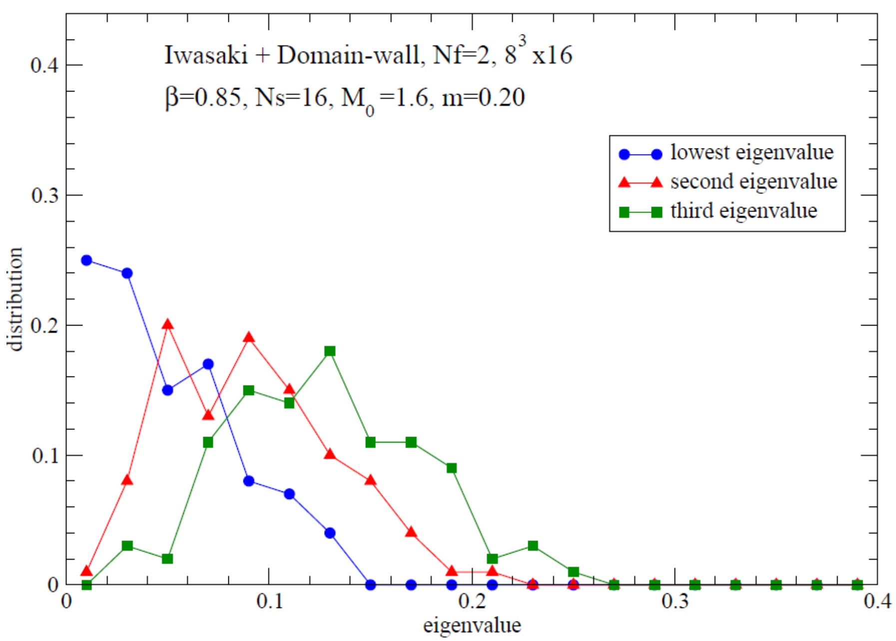


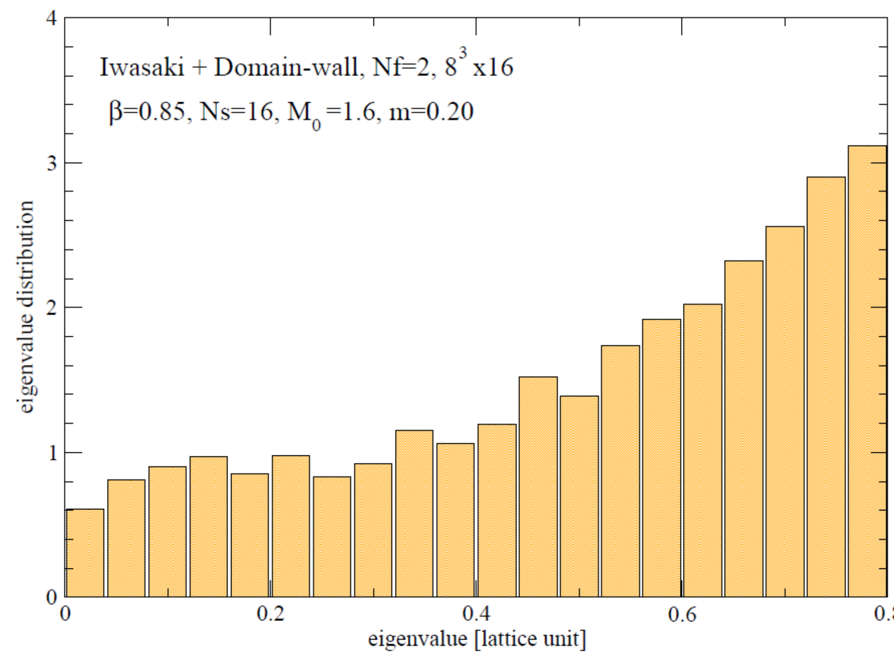


Overlap eigenvalues

- Eigenvalue distribution
  - Distribution of the lowest, 2nd, 3rd lowest modes except zero modes

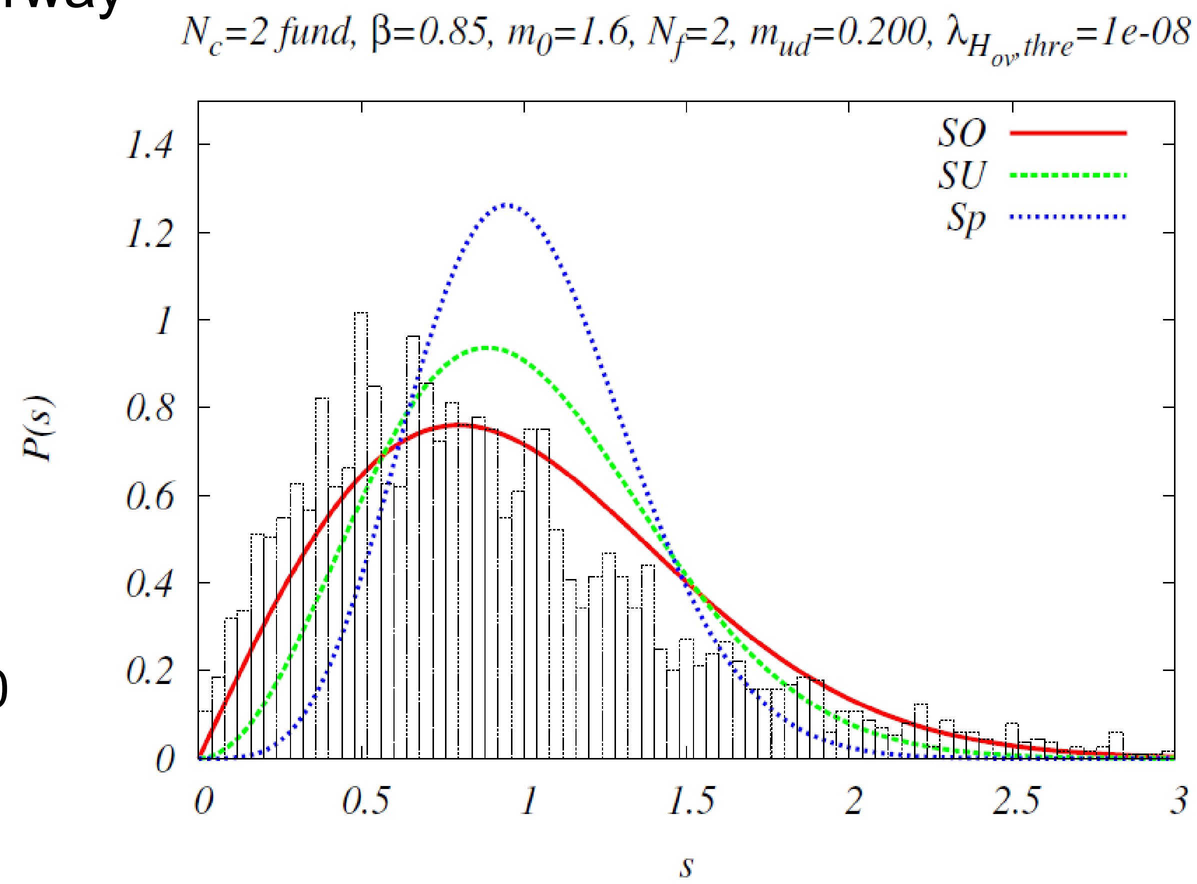
    -- May be correlated with meson correlator for small masses
  - Eigenvalue density: condednsate through Banks-Casher relation





Random matrix theory

- Level spacing statistics
  → Comparison with Random matrix theory
  - Roughly consistent with SO ensemble
  - Detailed analysis is underway



8^3x16, beta=0.85, m=0.20
500 configs (every 5 trj.)

Low-lying eigenmodes

- How domainwall fermion approaches overlap ?
  - As residual mass decreases
  - Low-lying eigenmodes: expected to become similar
- Correlation bwtween eigenmodes of overlap and 4D domainwall operators

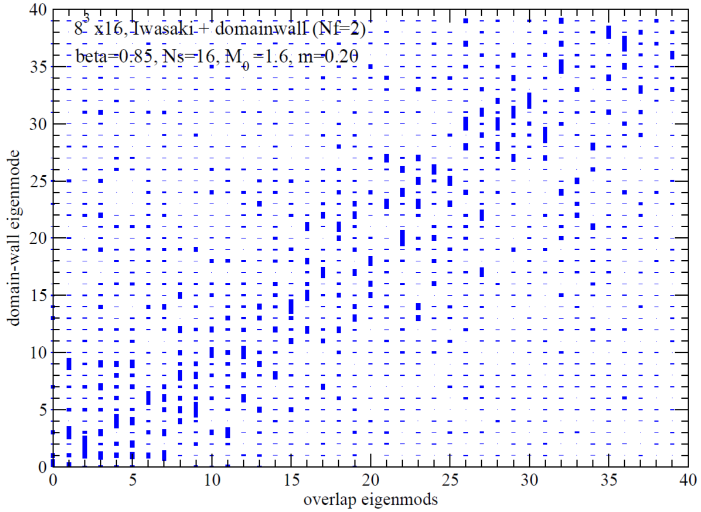
$$D^{(4)}(m) = \{P^{-1}[D_{DW}(1)]^{-1}D_{DW}(m)P\}_{11}$$

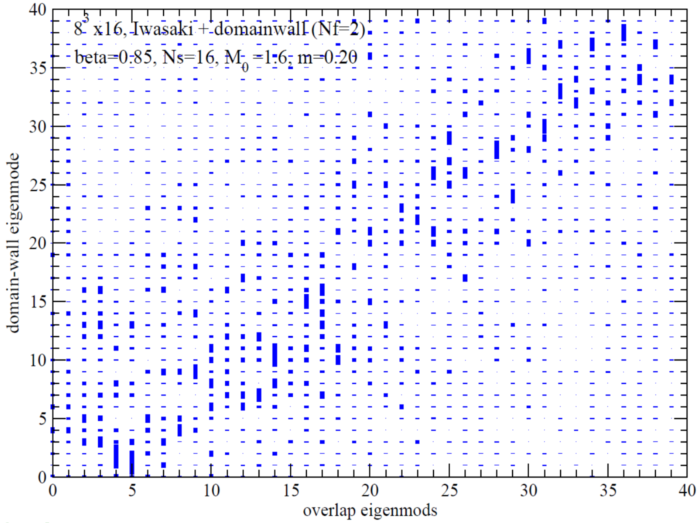
Kikukawa-Noguchi (99), Borici (99)

  - Approximation to overlap operator
- Numerical result: on 8^3x16
  - 40 low-lying eigenmodes measured on each config.
  - Results in next page: correlation on each config.
  - As residual mass decreases, correlation increases

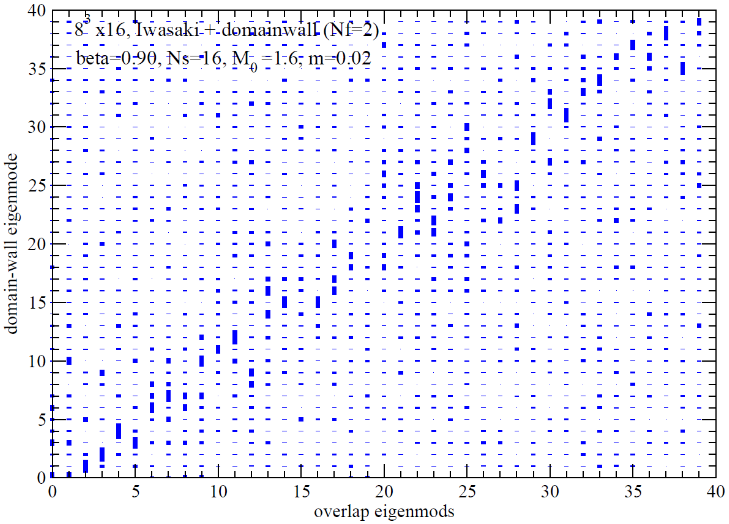
Low-lying eigenmodes

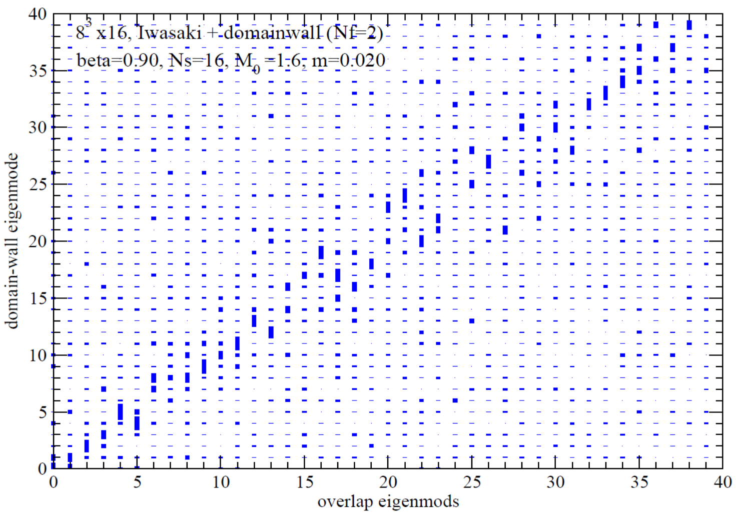
- 8^3x16, Beta=0.85, m=0.20: m_{res} ~ 0.03





- 8^3x16, Beta=0.90, m=0.02: m_{res} ~ 0.015





Summary/outlook

- Summary
  - SU(2) gauge theory is explored with chirally symmetric fermions
  - Iwasaki + Nf=2 dynamical domainwall fermions successfully works
  - Domainwall/overlap observables: complementary analyses

- Outlook
  - Large Nf, finite temperature/density
  - Improved signals needed → low-mode averaging/all-to-all prop.
  - Residual mass: better to dcrease → optimal domainwall ?
  - Other gauge group, fermion repr. : adjoint fermions underway
  - Dynamical overlap (fixed topology) in epsilon-regime