Numerical studies with Minimally Doubled Fermions

Non-perturbative tuning of renormalisation coefficients

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Overview

1. Karsten-Wilczek Fermions: review of analytic results
2. Non-perturbative renormalisation: tuning of the anisotropy
3. Numerical results: chiral behaviour of the pseudoscalar ground state
4. Summary and Outlook

The Karsten-Wilczek action and its symmetries

[e.g. L.H. Karsten, Phys. Lett. B104 (1981) 315; F. Wilczek, Phys.Rev. Lett. 59 (1987) 2397]

The fermionic part of the **Karsten-Wilczek action** reads

$$\begin{aligned} S_\alpha^{KW} &= \sum_x \sum_\mu \tfrac{1}{2a}\left(\overline{\psi}_x \gamma_\mu U_\mu(x)\psi_{x+\hat{\mu}} - \overline{\psi}_{x+\hat{\mu}}\gamma_\mu U_\mu^\dagger(x-\hat{\mu})\psi_x\right) \\ &- \sum_{\mu\neq\alpha} i\tfrac{\zeta}{2a}\left(\overline{\psi}_x \gamma_\alpha U_\mu(x)\psi_{x+\hat{\mu}} + \overline{\psi}_{x+\hat{\mu}}\gamma_\alpha U_\mu^\dagger(x-\hat{\mu})\psi_x\right) + \left(\overline{\psi}_x\left(\frac{i}{a}3\zeta\gamma_\alpha + m_0\right)\psi_x\right). \end{aligned} \quad (1)$$

- The two **zero modes** of the free, massless KW action, $k_\alpha = \{0, \frac{\pi}{a}\}$, $k_\mu = 0\ \forall\ \mu \neq \alpha$, are identified with **two degenerate flavours** of quarks in the naïve continuum limit.
- The same naïve CL requires mixing of tastes ψ_1, ψ_2:
  [e.g. M.Pernici, Phys. Lett. B346 (1995) 99; B. Tiburzi, Phys.Rev. D82 (2010) 034511]

$$\psi_1(k) \equiv \begin{cases} \psi(k), & -\frac{\pi}{2a} < k_\alpha \leq \frac{\pi}{2a} \\ 0, & \text{else} \end{cases}, \quad \psi_2(l) \equiv \begin{cases} \mathcal{R}T_\pi^\alpha e^{i\delta}\psi(l), & \frac{\pi}{2a} < l_\alpha + \frac{\pi}{a} \leq \frac{3\pi}{2a} \\ 0, & \text{else} \end{cases}, \quad (2)$$

  with **rotation matrix** $\mathcal{R} = i\gamma_\alpha\gamma_5 = \mathcal{R}^\dagger = \mathcal{R}^{-1}$, **momentum shift** $T_\pi^\alpha f(k_\alpha) \equiv f(k_\alpha + \frac{\pi}{a})T_\pi^\alpha$ and unconstrained phase shift δ. The **Pauli matrix** τ_1 is represented $(\delta = 0)$ by $\tau_1 = \mathcal{R}T_\pi^\alpha$.
- Pending on commutativity with $\tau_1 = \mathcal{R}T_\pi^\alpha$, operators are isospin singlet or isospin non-singlet (τ_3). KW-term $\propto \zeta$ is **non-singlet**, **anisotropic** and **violates** T**-parity**.
  [e.g. P.F. Bedaque, M.I. Buchoff, B.C. Tiburzi and A. Walker-Loud, Phys. Rev. D78 (2008) 449]
- Conserved point-split vector and axial-vector currents are obtained using WTI.
  [e.g. S. Capitani, J. Weber, H. Wittig, Phys.Lett. B681 (2009) 105-112]

Counterterm structure and anisotropy

- Interactions cause operator mixing $\Rightarrow$ counterterms needed to **restore isotropy** to CL.
  [e.g. M.Pernici, Phys. Lett. B346 (1995) 99; S. Capitani, M. Creutz, J. Weber, H. Wittig, JHEP 1009 (2010) 027]

$$c \times \overline{\psi}_x \frac{i}{a} \gamma_\alpha \psi(x), \tag{3}$$

$$d \times \frac{1}{2a} \left(\overline{\psi}_x \gamma_\alpha U_\alpha(x) \psi_{x+\hat{\alpha}} - \overline{\psi}_{x+\hat{\alpha}} \gamma_\alpha U_\alpha^\dagger(x-\hat{\alpha}) \psi_x \right), \tag{4}$$

$$d_P \times \frac{\beta}{N_c} \sum_\mu \Re \, Tr_c \, P_{\alpha\mu}(x). \tag{5}$$

- WTI currents are **conserved even before inclusion of counterterms**.
- The **self-energy** at 1-loop level has anisotropic contributions,

$$\Sigma = \Sigma_1 i \not{p} + \Sigma_2 m_0 + d_{1L}\, i(\gamma_\alpha\, p_\alpha) + c_{1L} \frac{i}{a}\, \gamma_\alpha. \tag{6}$$

- The 1-loop fermionic contribution of the **vacuum polarisation** has an anisotropic part,
  [e.g. S. Capitani, M. Creutz, J. Weber, H. Wittig, JHEP 1009 (2010) 027]

$$\left(p_\mu p_\nu (\delta_{\alpha\mu} + \delta_{\alpha\nu}) - \delta_{\mu\nu} (p^2 \delta_{\alpha\mu} \delta_{\alpha\nu} + p_\alpha^2) \right) \times d_{P,\,1L}. \tag{7}$$

- **Removing the anisotropies** (for $\zeta = +1$) requires tuning the counterterms like

$$c = c_{1L} = -29.5320\, C_F\, b, \;\; d = d_{1L} = -0.12554\, C_F\, b, \;\; d_P = -12.69766\, C_2\, b, \;\; b = \frac{g_0^2}{16\pi^2}. \tag{8}$$

Isospin structure of counterterms

- Only the renormalisation constant c **inherits isospin structure** and T-**parity violation**:

$$c_{1L}(-\zeta) = -c_{1L}(\zeta),\ d_{1L}(-\zeta) = +d_{1L}(\zeta),\ d_{P,\,1L}(-\zeta) = +d_{P,\,1L}(\zeta). \tag{9}$$

- **Boosted perturbation theory** predicts $b \to b/(U_0^4)$. Thus,

β	U_0^4	c_{1L}	c_{BPT}	d_{1L}	d_{BPT}	$d_{P,\,1L}$	$d_{P,\,BPT}$
6.0	0.594	−0.249	−0.420	−0.00106	−0.00179	−0.0893	−0.150
6.2	0.614	−0.241	−0.393	−0.00103	−0.00167	−0.0865	−0.141

Issues with simulations

- Fermion momentum and **taste cannot be fixed** on internal lines, e.g. n-p. propagators.
- **Separation** of the influence of different counterterms seems a priori **unclear**.

Whereas the KW action seems very similar to the Wilson action in its form, the choice of a feasible non-perturbative tuning strategy is subtle and the interpretation of hadronic content of correlation functions is intricate.

Non-perturbative renormalisation strategy: anisotropy of the transfer matrix

- Correlation functions depend on interpolating operators and on the transfer matrix.
- **Euclidean components of the transfer matrix** differ if the action is anisotropic.
- Unwise choice of interpolating operators can generate additional anisotropies.

Implementation of the strategy: parameter scans in the quenched approximation

- **Four parameters** in the quenched approximation: β, m_0, c, d.
- Perturbative results in the chiral limit $\Rightarrow$ comparison requires chiral extrapolation.

The third counterterm

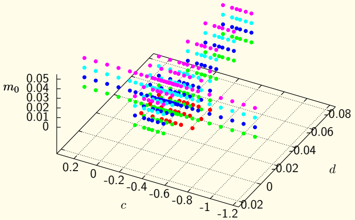
- Gauge field anisotropy due to feedback from **fermion loops**, needs dynamical fermions.
- Variation of d_P until the **anisotropy of plaquettes** vanishes tunes the 3rd parameter:

$$\sum_{\mu\neq\alpha} \Re Tr_c \left(P_{\alpha\mu}(x) - \sum_{\nu\neq\mu,\alpha} P_{\nu\mu}(x) \right) \stackrel{!}{=} 0. \tag{10}$$

Parameter space scans

1. **Maximally symmetric lattice** to avoid accidental anisotropic effects due to euclidean lattice geometry
2. **Sufficient size** to avoid finite volume effects
3. Scan of **relevant parameter** c with heavy quark for **fixed choice of marginal parameter** $d(= 0.0)$
4. **Lowering of quark mass** in order to check persistence of pattern towards chiral limit
5. Scan of **marginal parameter** d in r.o.i. for c

Parameter space scan for $\beta = 6.0$ and 32^4

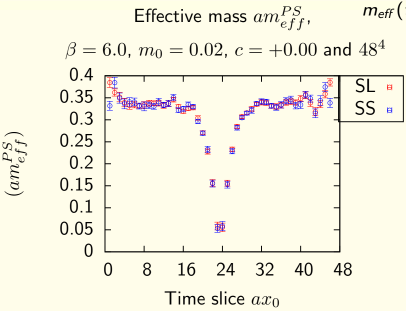


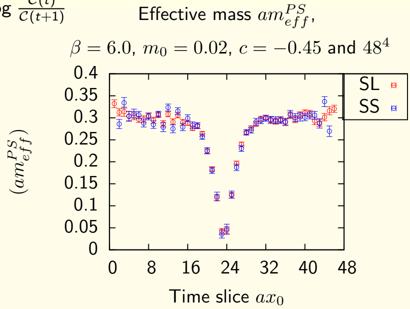
Analysis of correlation functions – non-trivial issues

1. T **parity violated** – forwards and backwards masses and spectral weights unequal?
2. Anisotropic action – **different spectral weights** for different euclidean components?
3. Arbitrariness of the **Wilczek parameter** ζ – how do choices of ζ affect the spectrum?

Effective mass plots – x_0 direction

$$m_{eff}(t) = \log \frac{\mathcal{C}(t)}{\mathcal{C}(t+1)}$$





1. **Broken T-parity could not be disentangled** from fluctuations of effective masses.
2. Sink-smeared x_0-correlator yields results compatible with local sink.

Effective mass plots – x_3 direction

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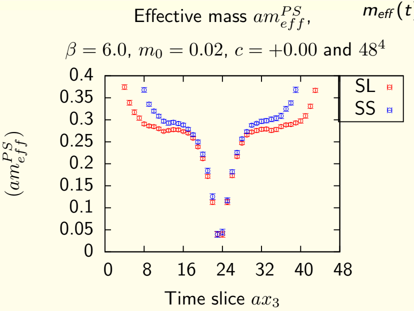
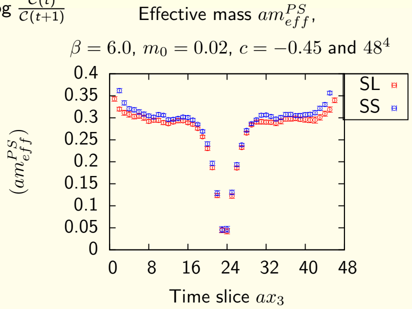


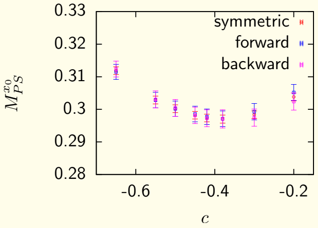


1. **Broken T-parity could not be disentangled** from fluctuations of effective masses.
2. Sink-smeared x_0-correlator yields results compatible with local sink.
3. **Sink-smeared x_3-correlator** not compatible with local sink – due to **lattice artifacts**?
4. Ground state plateau of x_3-correlator is very short before tuning.

Violation of T **parity** – obstacle or tuning criterion?

Forward, backward and symmetric $M_{PS}^{x_0}$,

$\beta = 6.0$, $am_0 = 0.02$, $d = 0$ and 32^4



Assumption: **mass difference between forward and backward** modes as tuning criterion

1. Separate exponentials for **forward and backward propagating** states

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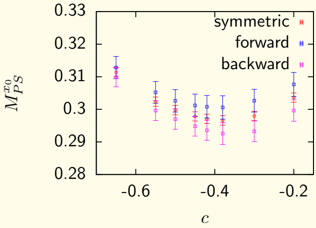

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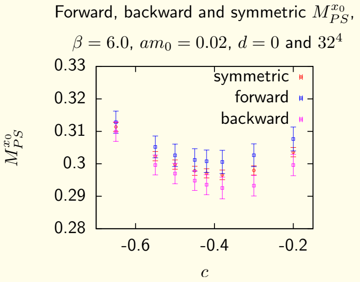
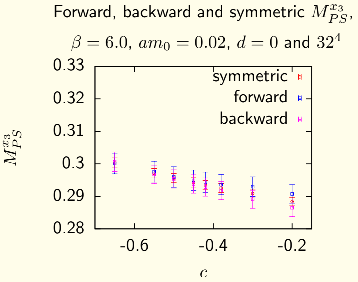
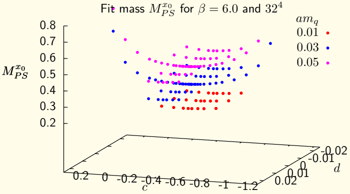


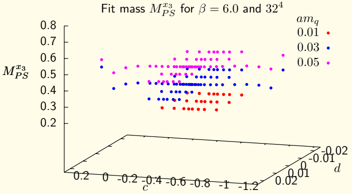


Assumption: **mass difference between forward and backward** modes as tuning criterion

1. Separate exponentials for **forward and backward propagating** states
2. Discrepancy depends strongly on fit range – **contribution to systematic error**
3. Discrepancy in $M_{PS}^{x_3}$ as well – **unrelated** to **violated** T**-parity** – but large error source

Masses from fits to 2-point correlators





1. Measurement of **PS 2-pt. correlator** in x_0 and x_3 directions **without sink smearing**
2. Study of correlators $\mathcal{C}_{PS}^{x_\mu}(t)$ with **cosh**, **two separate exponentials** with **like** or **unlike masses**

$$C_{PS}(t) \equiv A_f e^{-m_f t} + A_b e^{-m_b(T-t)} \tag{11}$$

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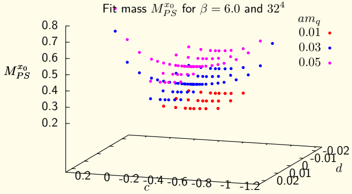

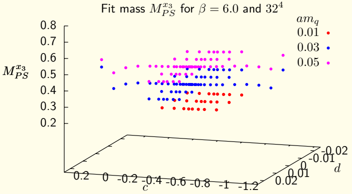

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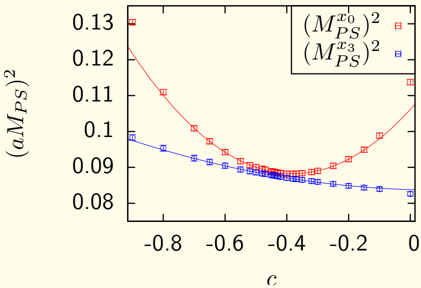
3. **Rapid degradation** of χ^2/dof for $\mathcal{C}_{PS}^{x_3}$ outside of r.o.i. – **extrapolation region limited**
4. Tuning criterion: **mass difference between** pseudoscalar correlators in **different directions**

$$\Delta(M_{PS}^2) = (M_{PS}^{x_0})^2 - (M_{PS}^{x_3})^2 \tag{12}$$

Minimisation of the anisotropy in M_{PS}

Extrapolation of $(M_{PS})^2$,

$\beta = 6.0$, $am_0 = 0.02$, $d = 0$ and 32^4



Minimisation of fit mass anisotropy,

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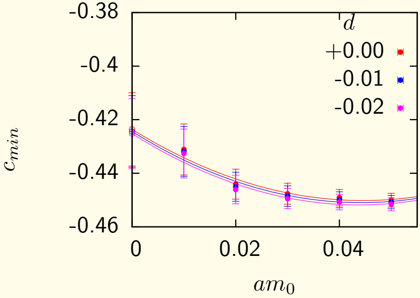
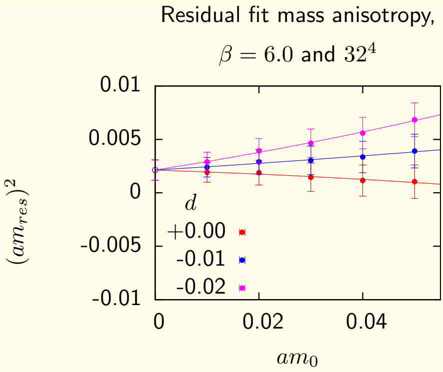


1. **Quadratic extrapolation** of $(M_{PS})^2$ for different quark masses and marginal parameters d
2. **Analytic determination** of c_{min} with minimal $\Delta(M_{PS}^2) = (M_{PS}^{x_0})^2 - (M_{PS}^{x_3})^2$ without further fit
3. **Chiral extrapolation** of c_{min} – dependence on d overshadowed by statistical errors

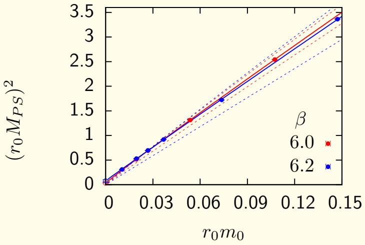
Incomplete tuning – residual mass difference $\Delta(M_{PS}^2)(c_{min})$



1. The mass difference is not reduced to zero. Residual anisotropies persist.
2. The dependence on d disappears in the chiral limit.

Chiral limit of the PS ground state – application of the tuned Karsten-Wilczek action

Chiral limit of M_{PS}^2, 48×24^3



1. **Sufficient tuning of parameters** is assumed: $c_{6.0} = -0.450$, $c_{6.2} = -0.400$, $d = -0.001$

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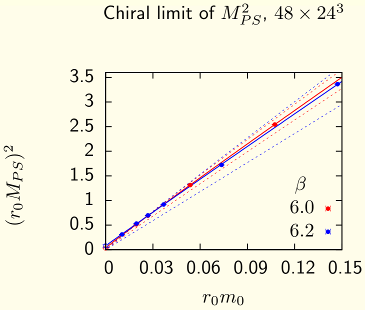

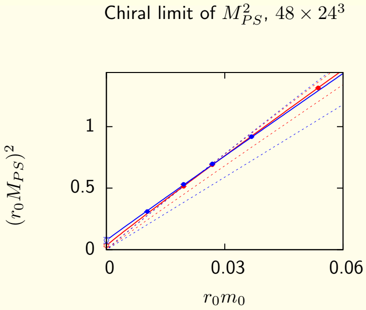

1. **Sufficient tuning of parameters** is assumed: $c_{6.0} = -0.450$, $c_{6.2} = -0.400$, $d = -0.001$
2. **Light masses** ($M_{PS} \leq 300 MeV$) might suffer **finite volume effects** ($M_{PS} L \sim 1.8 - 2.4$)
3. Chiral extrapolation includes **residual PS mass** term A and **quenched chiral logarithms**:

$$(r_0\, M_{PS})^2 = r_0^2\, A + (r_0\, B_0)(r_0\, m_0)\left((1-\delta) - \delta \log(m_0/r_0)\right) \tag{13}$$

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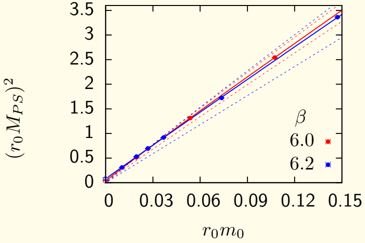
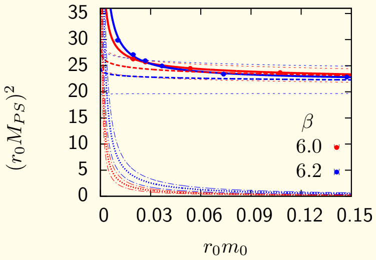


Chiral limit of M_{PS}^2/m_0, 48×24^3



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4. Possible evidence for quenched chiral logarithms – results still inconclusive

Summary

1. **First simulations** with minimally doubled fermions in the quenched approximation
2. **Non-perturbative tuning** prescription for c via **anisotropy of PS ground state mass**
3. Results largely **insensitive to parameter** d
4. **No statistically significant** evidence for T-**parity violation** in PS ground state

β	c_{BPT}	c_{NPT}	d_{BPT}
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6. Quenched chiral logarithms not unambiguously resolved

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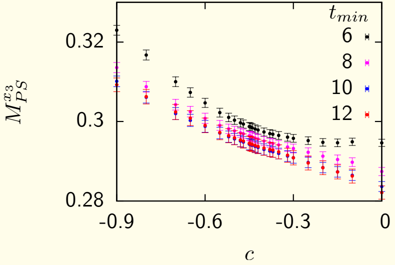
Outlook

1. Improve study of **chiral behaviour** – larger volumes, different lattice spacings
2. Study of **other observables** (vector mesons, baryons) and **improvement of tuning condition**
3. Study **quark-disconnected contribution** to PS correlator and **lattice chiral perturbation theory** in order to pin down charged and neutral pions
4. Implement **dynamical minimally doubled fermions**

Systematic errors of the approach: **finite size effects**?

Fit range dependence $M_{PS}^{x_3}$,

$\beta = 6.0$, $am_0 = 0.02$, $d = 0$ and 32^4



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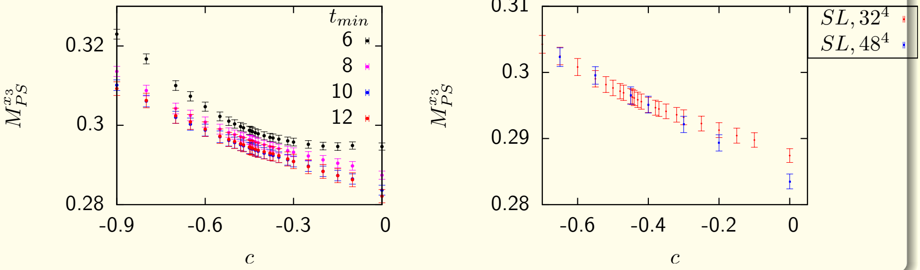
Volume dependence of $M_{PS}^{x_3}$, $\beta = 6.0$, $am_0 = 0.02$, $d = 0$



1. $M_{PS}^{x_3}$ has strong sensitivity on fit range – indicates **finite size effects** ?
2. Finite size effects **less prominent in proximity of** c_{min}

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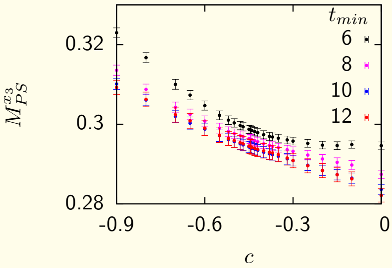
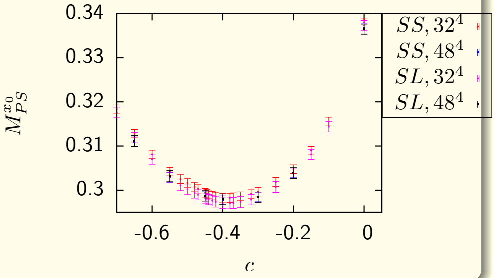


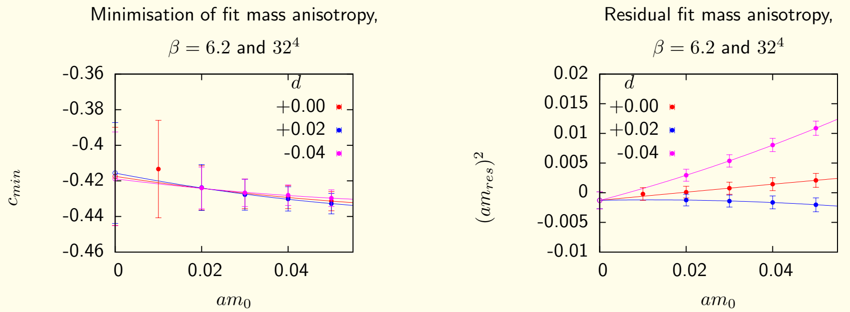
Volume dependence of $M_{PS}^{x_0}$,

$\beta = 6.0$, $am_0 = 0.02$, $d = 0$



1. $M_{PS}^{x_3}$ has strong sensitivity on fit range – indicates **finite size effects** ?
2. Finite size effects **less prominent in proximity of** c_{min}
3. Volume dependence of $M_{PS}^{x_0}$ less than statistical errors

Continuum limit – renormalisation on finer lattices



1. Investigation of finer lattice with $\beta = 6.2$ – compatible with perturbative prediction.
2. Plateaus in $M_{PS}^{\chi 3}$ difficult to obtain – source of systematic errors due to finite size effects.
3. Dependence of residual mass $(am_{res})^2$ on d disappears in the chiral limit.