Crystalline Confinement

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Introduction

- Lattice gauge theories $\rightarrow$ fundamental contribution towards understanding of strongly correlated systems.
- Most non-perturbative computations done in Euclidean space with Wilson formulation.
- Ultra-cold atoms toolbox $\rightarrow$ quantum dynamics of gauge theories.
- Questions of real-time evolution and finite baryon density.
- Alternate formulation of gauge theories (Horn,1981; Orland, Rohrlich, 1990; Chandrasekharan, Wiese, 1997) and QCD with domain wall fermions (Brower, Chandrasekharan, Wiese, 1999) are particularly relevant.
- These realize **continuous** gauge symmetries using **discrete** quantum link variables, having finite dimensional Hilbert space $\rightarrow$ extension of Wilson formulation of gauge theories.
- Excellent candidate models to be implemented in cold-atom systems.
- Allows construction of very efficient algorithms to study static properties.

Hamiltonian $U(1)$ LGT: Wilson formulation

- U(1) gauge invariant Hamiltonian:

$$H = \frac{g^2}{2} \sum_{x,i} e_{x,i}^2 - \frac{1}{2g^2} \sum_{\square} (u_{\square} + u_{\square}^{\dagger})$$

- $u = \exp(i\varphi)$; $u^{\dagger} = \exp(-i\varphi)$; $e = -i\partial_{\varphi}$;
  $\Rightarrow$ are operators in the Hamiltonian formulation, operating in an **infinite** dimensional Hilbert space on a single link
- U(1) gauge transformations generated by Gauss Law:

$$\begin{aligned} G_x &= \sum_i (e_{x,i} - e_{x-\hat{i},i}); \quad [G_x, H] = 0 \\ V &= \prod_x \exp(i\alpha_x G_x); \quad u'_{xy} = V u_{xy} V^{\dagger} = \exp(i\alpha_x) u_{xy} \exp(-i\alpha_y) \end{aligned}$$

- Commutation relations realizing gauge invariance:

$$[e, u] = u, \quad [e, u^{\dagger}] = -u^{\dagger}$$

- $[u, u^{\dagger}] = 0$

Hamiltonian $U(1)$ LGT: Quantum Links

- U(1) gauge invariant Hamiltonian:

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{\square} (U_{\square} + U_{\square}^{\dagger})$$

- $U = S^1 + iS^2 = S^+; \;\; U^{\dagger} = S^1 - iS^2 = S^-; E = S^3$
  $\Rightarrow$ are operators in the Hamiltonian formulation, operating in a **finite** dimensional Hilbert space on a single link
- U(1) gauge transformations generated by Gauss Law:

$$G_x = \sum_i (E_{x,i} - E_{x-\hat{i},i}); \;\; [G_x, H] = 0$$

$$V = \prod_x \exp(i\alpha_x G_x); \;\; U'_{xy} = V U_{xy} V^{\dagger} = \exp(i\alpha_x) U_{xy} \exp(-i\alpha_y)$$

- Commutation relations realizing gauge invariance:

$$[E, U] = U, \;\; [E, U^{\dagger}] = -U^{\dagger}$$

- $[U, U^{\dagger}] = 2E$

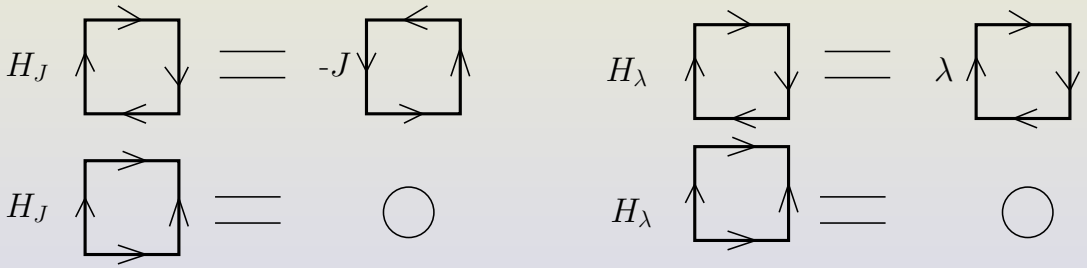
The (2+1)-d U(1) Quantum Link model

- Simplest Abelian pure gauge model: with spin $S = 1/2$
  $\rightarrow$ 2-dim Hilbert space per link

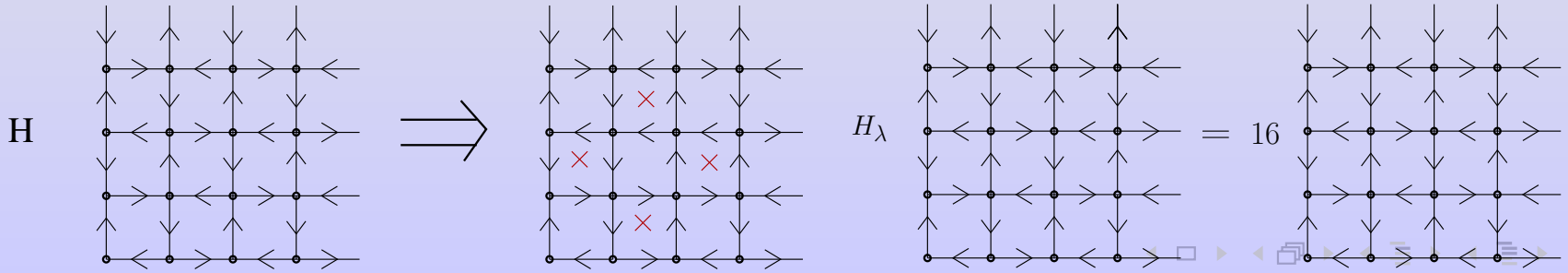
$$E|\uparrow\rangle = \frac{1}{2}|\uparrow\rangle; \quad E|\downarrow\rangle = -\frac{1}{2}|\downarrow\rangle; \quad U|\uparrow\rangle = 0; \quad U|\downarrow\rangle = |\uparrow\rangle; \quad U^\dagger|\uparrow\rangle = |\downarrow\rangle; \quad U^\dagger|\downarrow\rangle = 0$$

- E^2 contributes a constant for $S = 1/2$.

$$H = -J\sum_{\square}\left(U_{\square} + U_{\square}^\dagger\right) + \lambda\sum_{\square}\left(U_{\square} + U_{\square}^\dagger\right)^2$$



- Plaquettes are flipped only if they have flux in the right order; second term ($= H_\lambda$) counts the number of flippable plaquettes

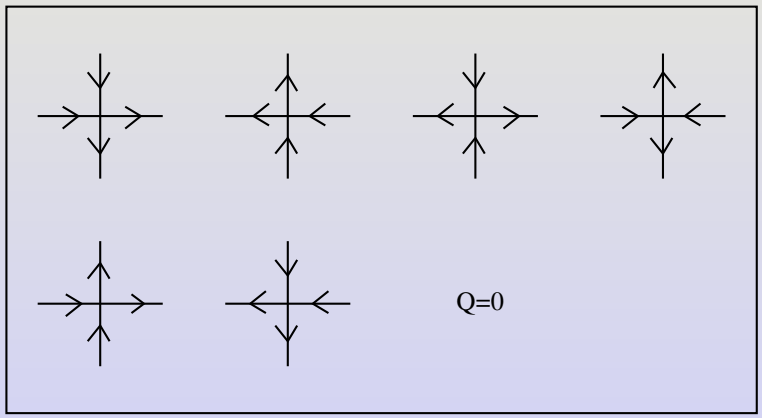


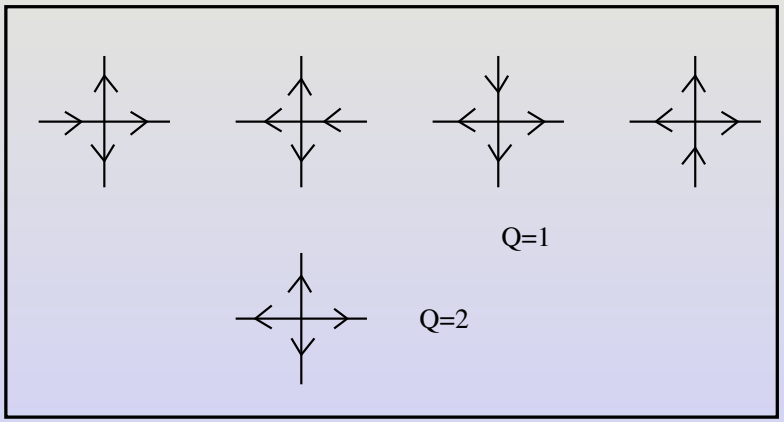
Gauss Law and Charge Sectors

To define the path integral $\mathcal{Z} = \mathrm{Tr}\,(\exp(-\beta H)\mathcal{P}_{\mathcal{G}})$,
the Gauss Law must be implemented :

$$\sum_i \left(E_{x,i} - E_{x-\hat{i},i}\right) = Q_x$$

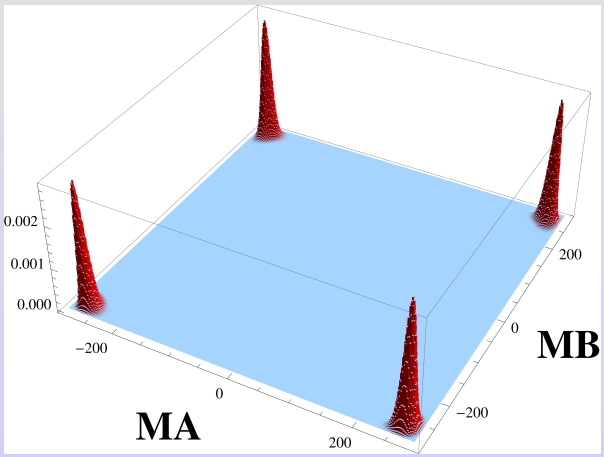
There is zero charge everywhere (charge-0 sector) unless external static charges are placed at vertices.

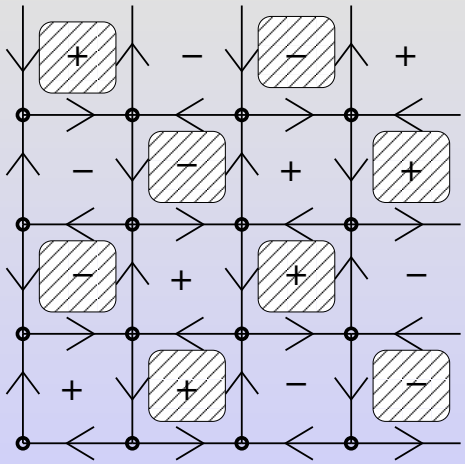


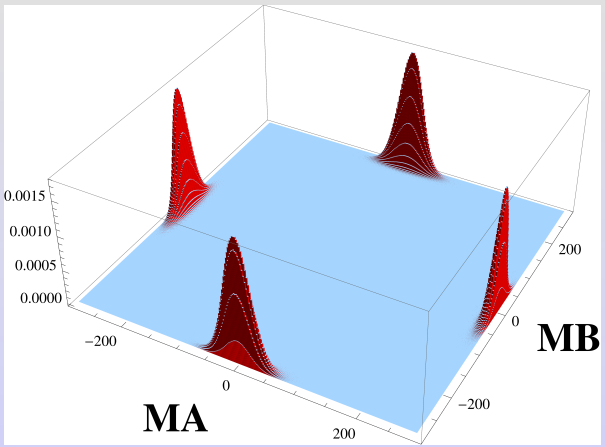


Symmetry breaking and phase transitions

- Discrete: Rotation by $\pi/2$, Reflection, **Charge Conjugation (C)**, **Translation($T = (T_x, T_y)$)**
- Continuous: $U(1)$ center symmetries in x- and y-directions
- Symmetry breaking patterns can be deduced very well from exact diagonalizations (next talk by Philippe Widmer).
- 2-component order parameter (M_A, M_B) to analyze the symmetry breaking patterns

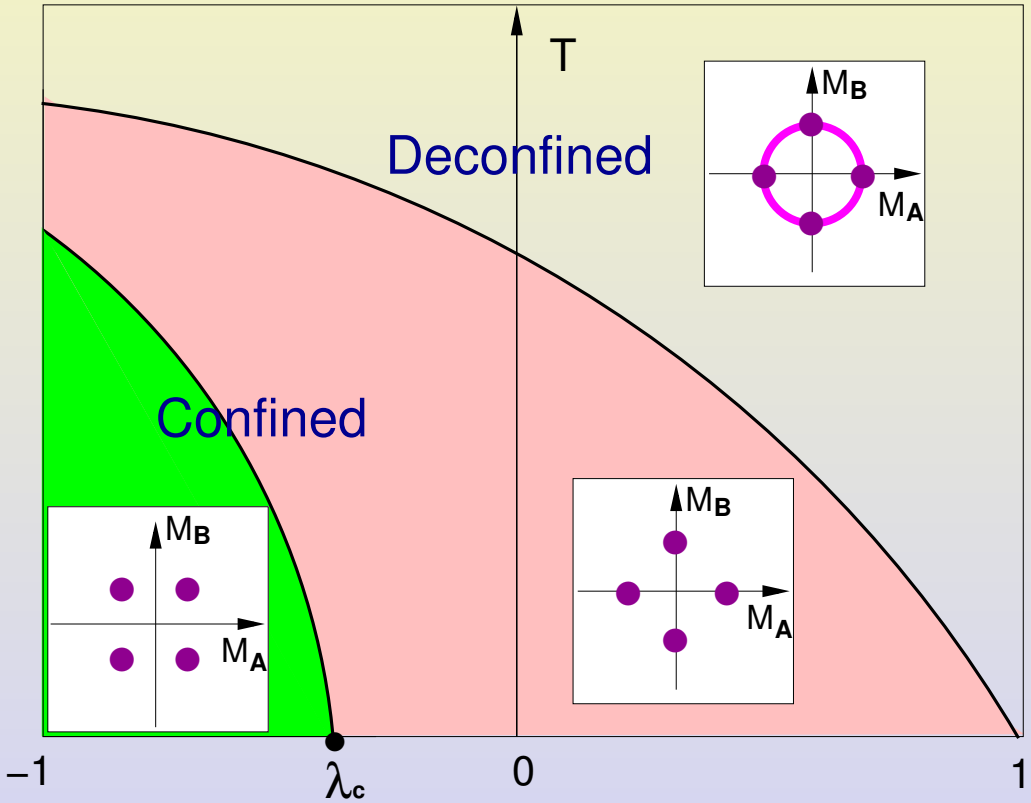


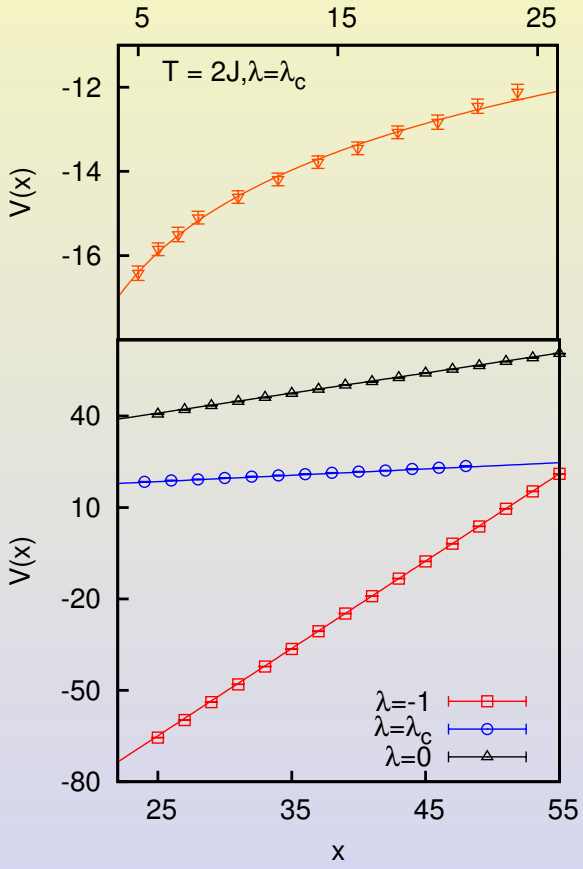




Phase diagram

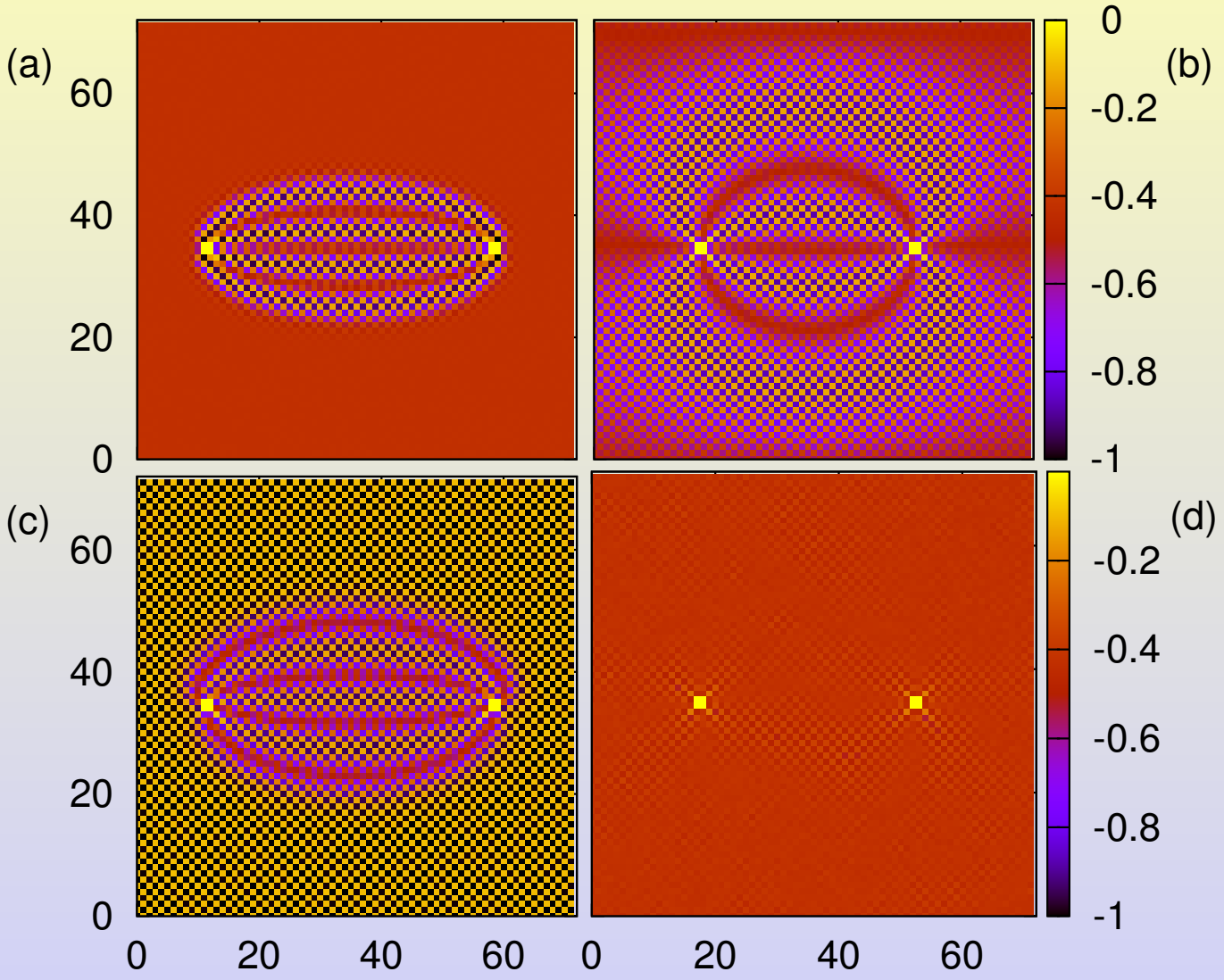
Explored with exact diagonalization and a newly developed cluster algorithm using dualization techniques.





An *approximate global SO(2) symmetry is emergent* at λ_c. A description in terms of a low-energy *effective theory suggests a weak 1st order transition*. Next talk by P. Widmer provides more details

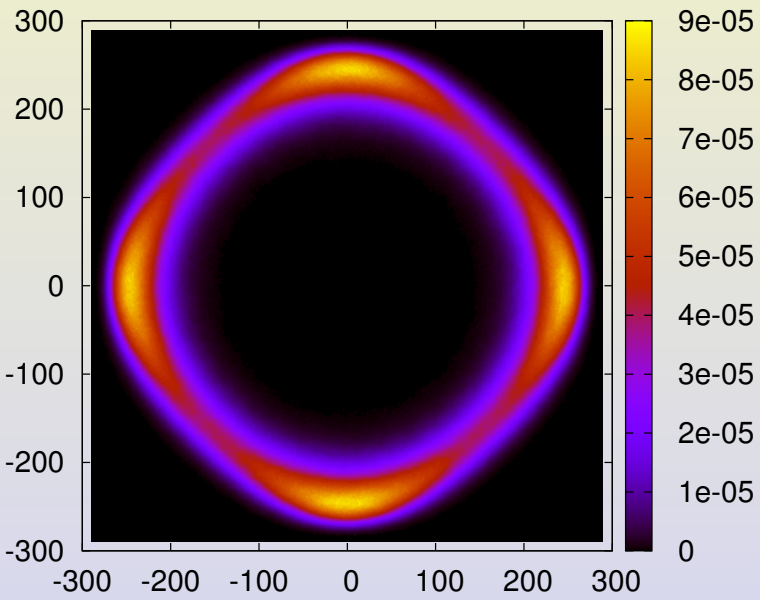
Crystalline confinement

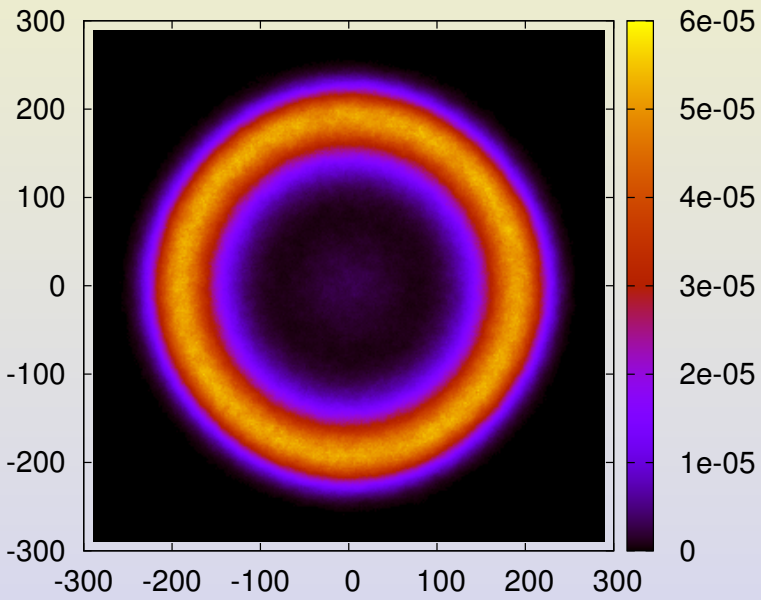


Energy density $\langle H_J \rangle$ of two charges $Q = \pm 2$ placed along the axis on $L = 72$ lattice

Deconfined Crystal?

Universality arguments predict the finite temperature transition to be of BKT type. Systematic investigation underway; hints of a high-temperature phase with broken T symmetry, which gets smoothly restored with increasing temperature.





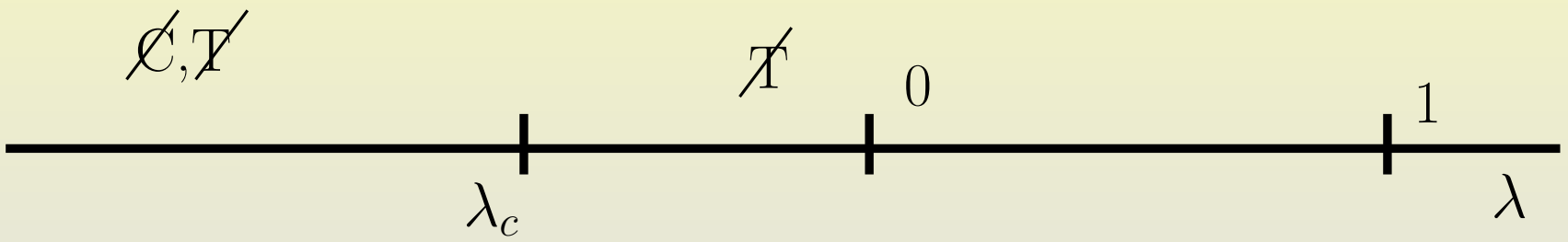
Order parameter contour plots (M_A, M_B) for L=24; $\lambda = 0$;

(left) βJ=1.4 and (right) βJ=0.8

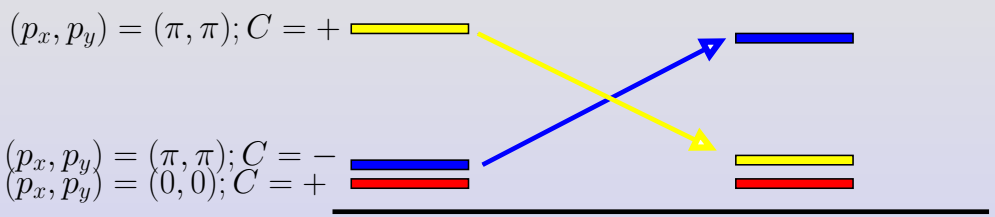
Conclusion

- The (2+1)-d QLM has interesting non-trivial physics, including exotic phases not seen before in gauge theories; and emergence of an approximate global symmetry
- More details about this will be discussed in the next talk.
- The first efficient cluster algorithm for a quantum link model allows us to investigate the phase structure in great detail!
- These models are also related to quantum dimer models, which are extensively studied in condensed matter physics with reference to high-T_c phenomena $\rightarrow$ applicability of new algorithms.
- Broader applicability of the dualization techniques! Playground for ideas for extending the construction to other theories.
- The possibility of "observing" this physics in engineered cold-atom systems makes the study worthwhile.

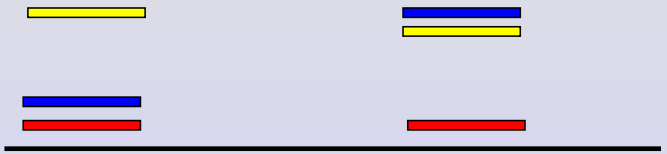
Backup 1: Level crossing



Non-trivial quantum phase transition:

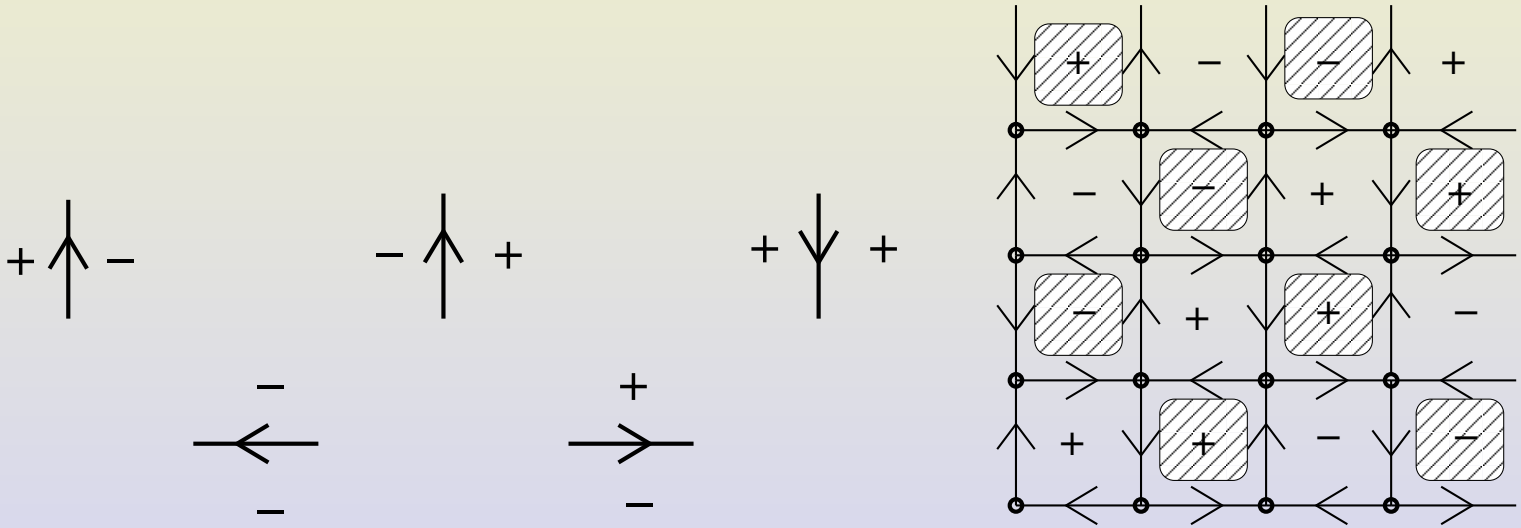


Uninteresting scenario:



Backup 2: Dualization and cluster algorithm

- Using duality, the $\mathcal{Z}$ can be rewritten as a height model with a 6-height variable interaction. The height variables are dual to the plaquette variables of the U(1) gauge theory
- A highly efficient cluster algorithm can be constructed for the resulting height model



- Dualization naturally suggests a 2-component order parameter to analyze the phase transition: (M_A, M_B) measured on the even (unshaded) and odd (shaded) sublattice.