LATTICE GAUGE THEORY WITH MPS

Mass spectrum of Schwinger model

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arXiv:1305.3765

MPQ
Max Planck Institut
für Quantenoptik

Mainz 30.7.2013

What can MPS/TNS say about LGT?

TNS

- TNS = Tensor Network States

Context: quantum many body systems

TNS

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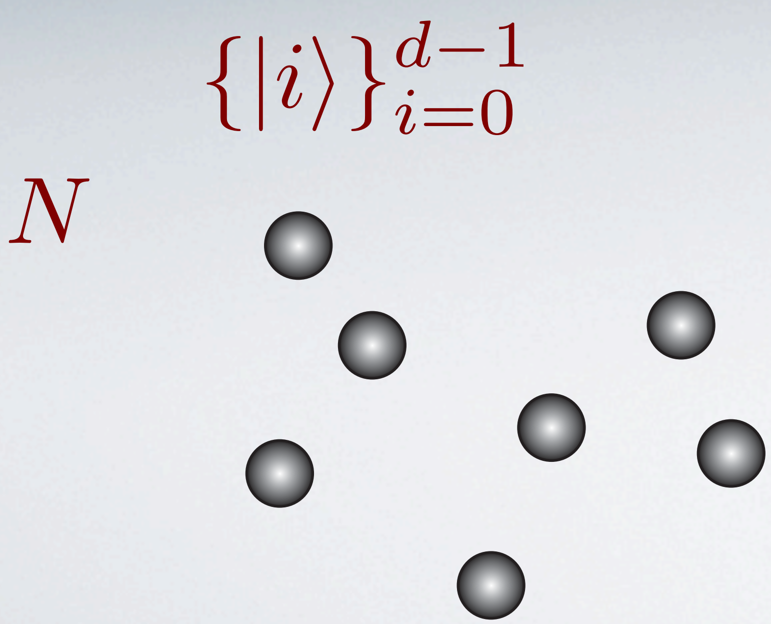
Context: quantum many body systems



TNS

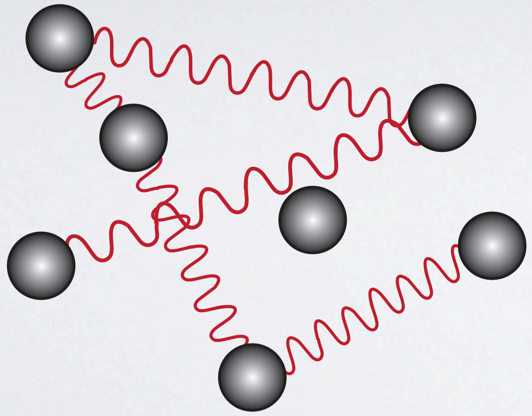
- TNS = Tensor Network States

Context: quantum many body systems

interacting with each other

$\{|i\rangle\}_{i=0}^{d-1}$

N



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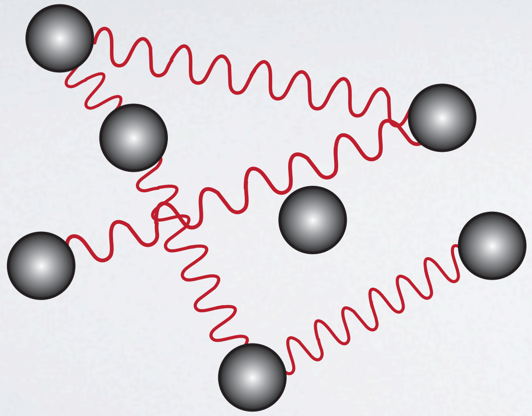

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Goal: describe equilibrium states

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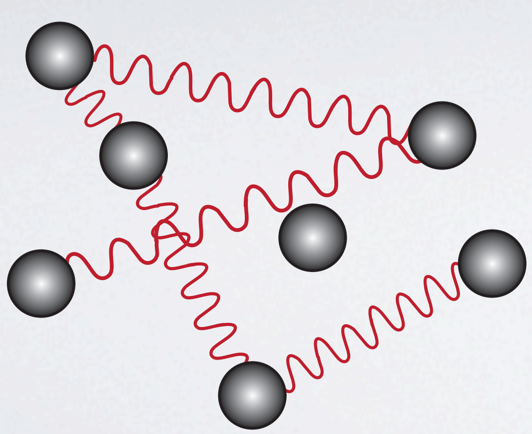

Goal: describe equilibrium states

ground, thermal states

TNS

- TNS = Tensor Network States

A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} c_{i_1 \ldots i_N} |i_1 \ldots i_N\rangle$$

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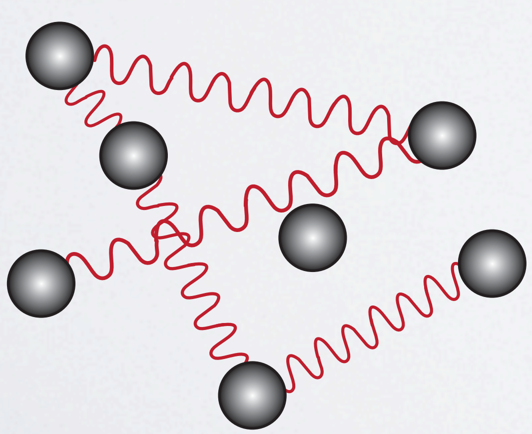

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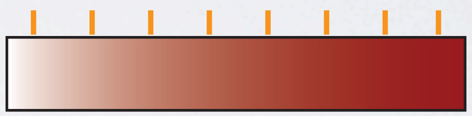
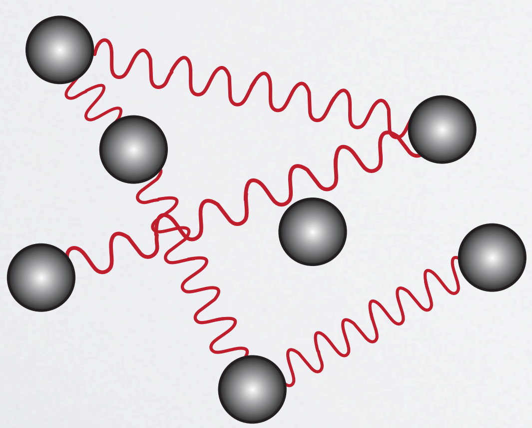


N-legged tensor

d^N

N



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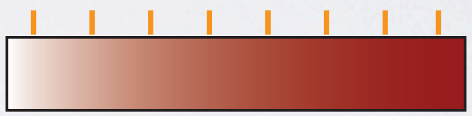

N-legged tensor

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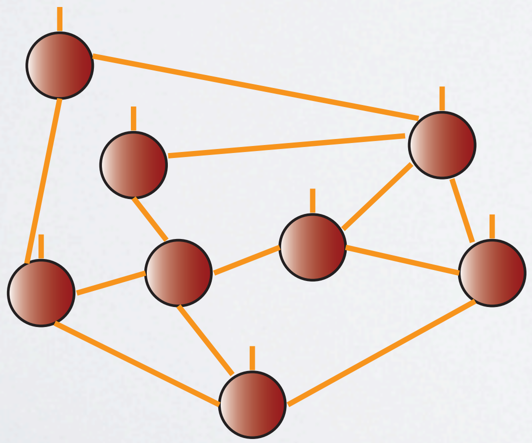
A TNS has only a polynomial number of parameters



$\mathrm{poly}(N)$

Non-perturbative for Hamiltonian systems

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Extremely successful for 1D systems (MPS)

TNS

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Promising improvements for higher dimensions

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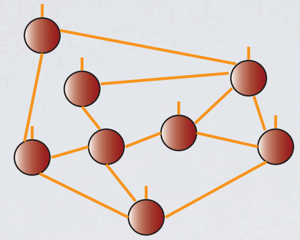
Promising improvements for higher dimensions

- ground states
- low-lying excitations
- thermal states
- time evolution



TNS

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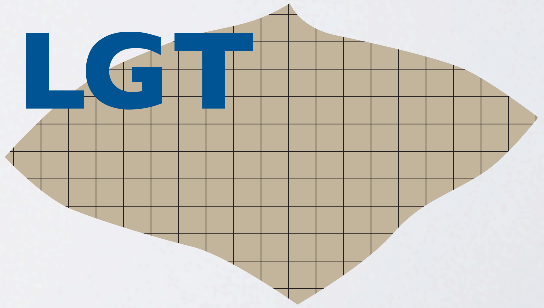
ground states
low-lying excitations
thermal states
time evolution

apply to



Precedents

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DMRG on Schwinger model

Byrnes et al. PRD 2002

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TN → extensions

time evolution, finite T

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MPS for LGT Z_2

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TNS for classical gauge models

Meurice et al. 2013

SCHWINGER MODEL AS TESTBENCH FOR TNS TECHNIQUES

SCHWINGER MODEL AS TESTBENCH FOR **MPS** TECHNIQUES

MPS

- MPS = Matrix Product States

$$|\Psi\rangle = \sum_{i_1 \ldots i_N} c_{i_1 \ldots i_N} |i_1 \ldots i_N\rangle$$

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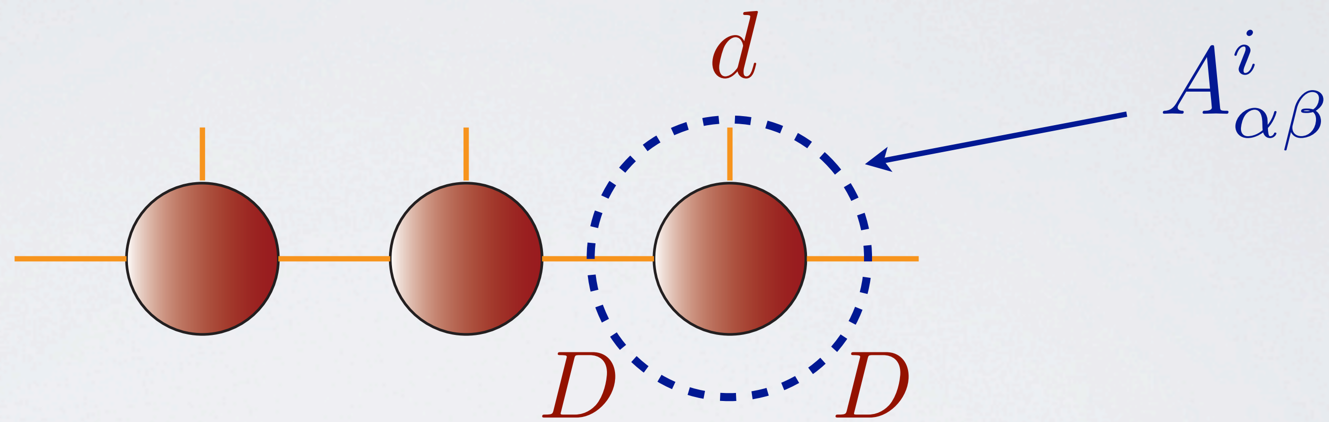

$$|\Psi\rangle = \sum_{i_1 \ldots i_N} \mathrm{tr}(A_1^{i_1} A_2^{i_2} \ldots A_N^{i_N}) |i_1 \ldots i_N\rangle$$

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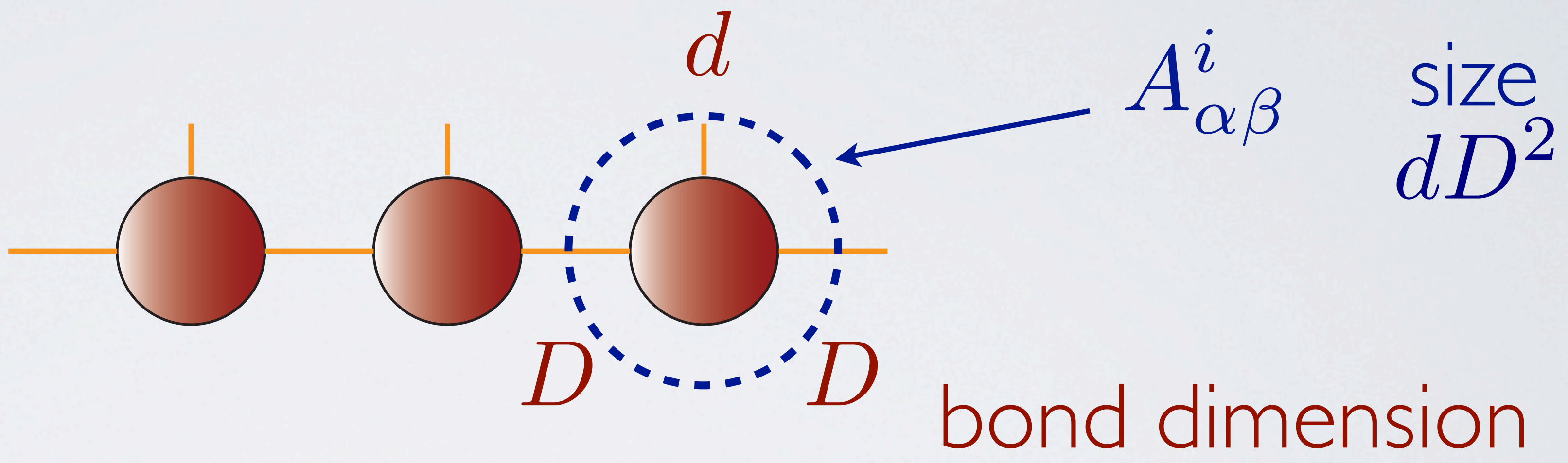

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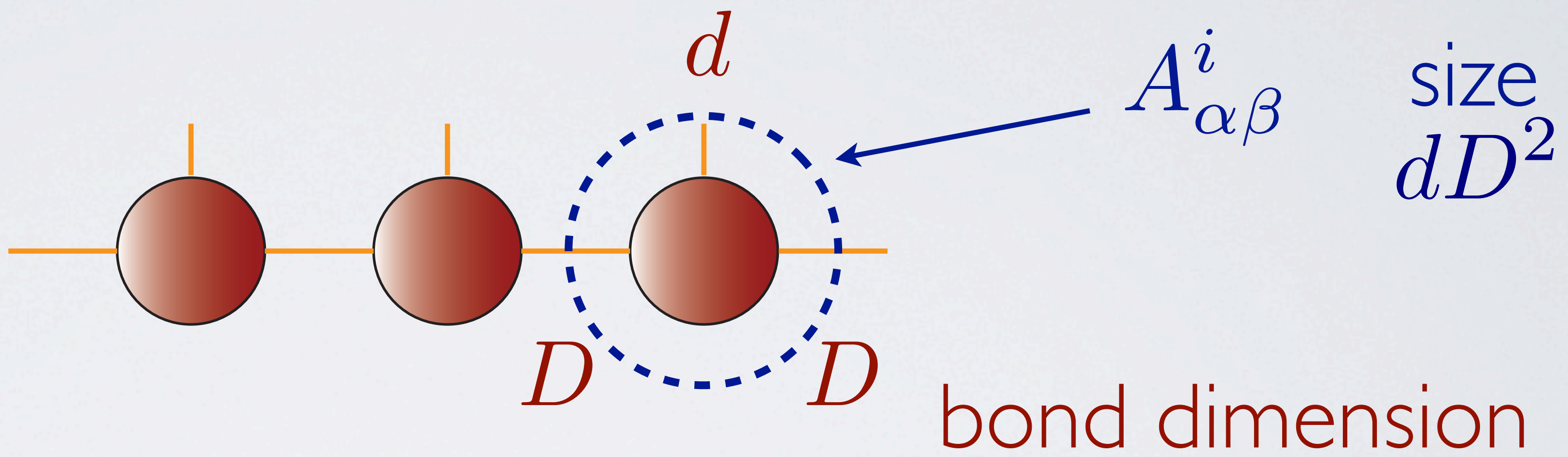

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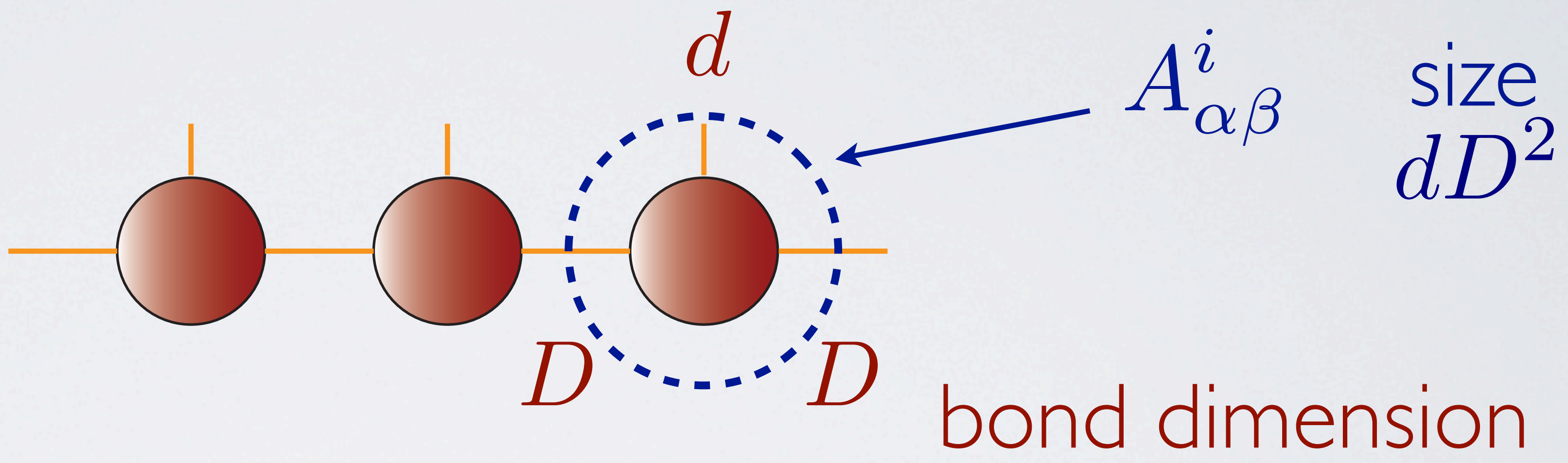

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$$|100\ldots\rangle + |010\ldots\rangle + |001\ldots\rangle + \ldots$$

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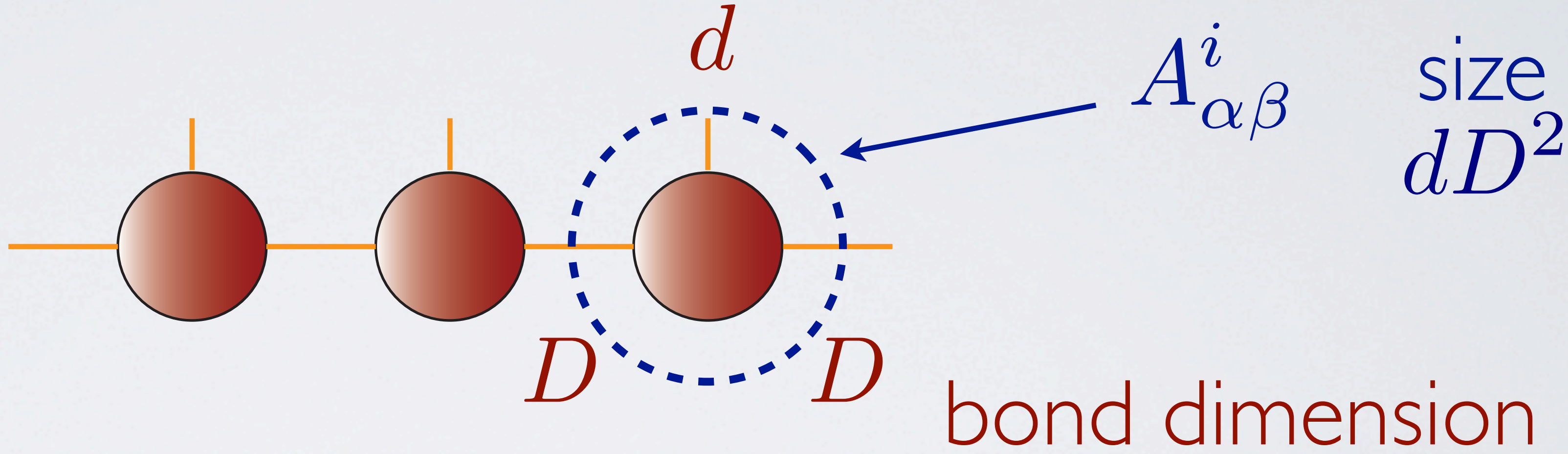

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$$A^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad A^1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad D = 2$$

$$|100\dots\rangle + |010\dots\rangle + |001\dots\rangle + \dots$$

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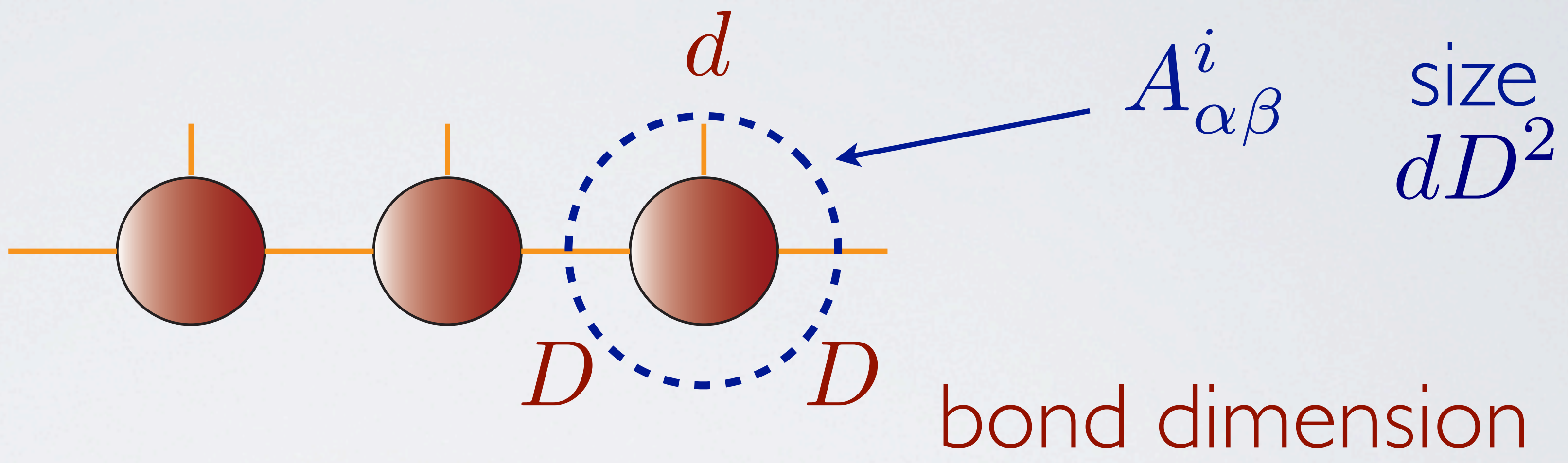

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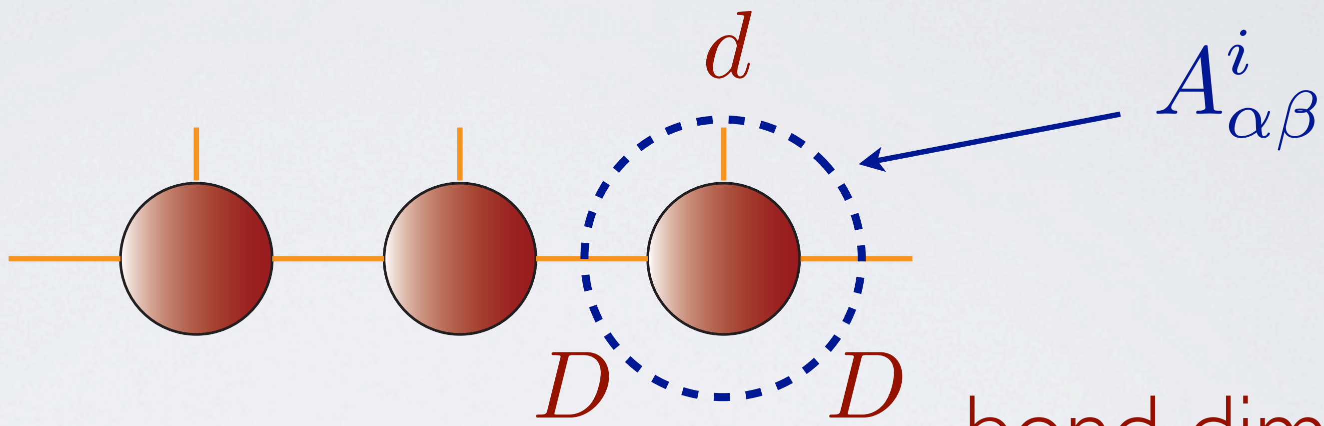

size dD^2

bond dimension

$$|\Psi\rangle = \sum_{i_1 \ldots i_N} \mathrm{tr}(A_1^{i_1} A_2^{i_2} \ldots A_N^{i_N}) |i_1 \ldots i_N\rangle$$

very efficient algorithms for:

ground states → also excitations

time evolution

mixed states (thermal)

SCHWINGER MODEL

discrete Hamiltonian (staggered) formulation

Kogut, Susskind ’75

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discrete Hamiltonian (staggered) formulation

Kogut, Susskind '75

Jordan-Wigner → spin model

$$H = \frac{1}{g^2a^2}\sum_n \left(\sigma_n^+ e^{i\theta_n}\sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n}\sigma_n^-\right)$$
$$+\frac{m}{ag^2}\sum_n \left(1+(-1)^n\sigma_n^3\right) + \sum_n L_n^2$$

Gauss Law

$$L_n - L_{n-1} = \frac{1}{2}[\sigma_n^3 + (-1)^n]$$

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$$|\ldots s_e\,\ell\;s_o\,\ell\;s_e\,\ell\;s_o\ldots\rangle$$

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Gauss Law fixes *photon* content

$$L_n = \ell_0 + \frac{1}{2}\sum_{k\le n}\sigma_n^3 + \ldots$$

eliminate gauge dof

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SCHWINGER MODEL

basis for MPS

$$|\ell_0 \ldots s_e\ s_o\ s_e\ s_o \ldots\rangle$$

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COMPUTING THE SPECTRUM WITH MPS

arXiv:1305.3765

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Scan parameters

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m/g

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m/g mass gaps and GS energy density in the continuum $ga \to 0$

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ga $1/(ga)^2 \in [5, 600]$

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m/g — mass gaps and GS energy density in the continuum $ga \to 0$

ga — $1/(ga)^2 \in [5,\, 600]$

N — $N \propto 1/(ga)$ (up to ~850)

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m/g	mass gaps and GS energy density in the continuum $ga \to 0$
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convergence ↑

D — $D \in [20,\ 120]$

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m/g — mass gaps and GS energy density in the continuum $ga \to 0$

ga — $1/(ga)^2 \in [5,\ 600]$

finite-size ↑

N — $N \propto 1/(ga)$ (up to ~850)

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$ga \to 0$

ga — $1/(ga)^2 \in [5,\ 600]$

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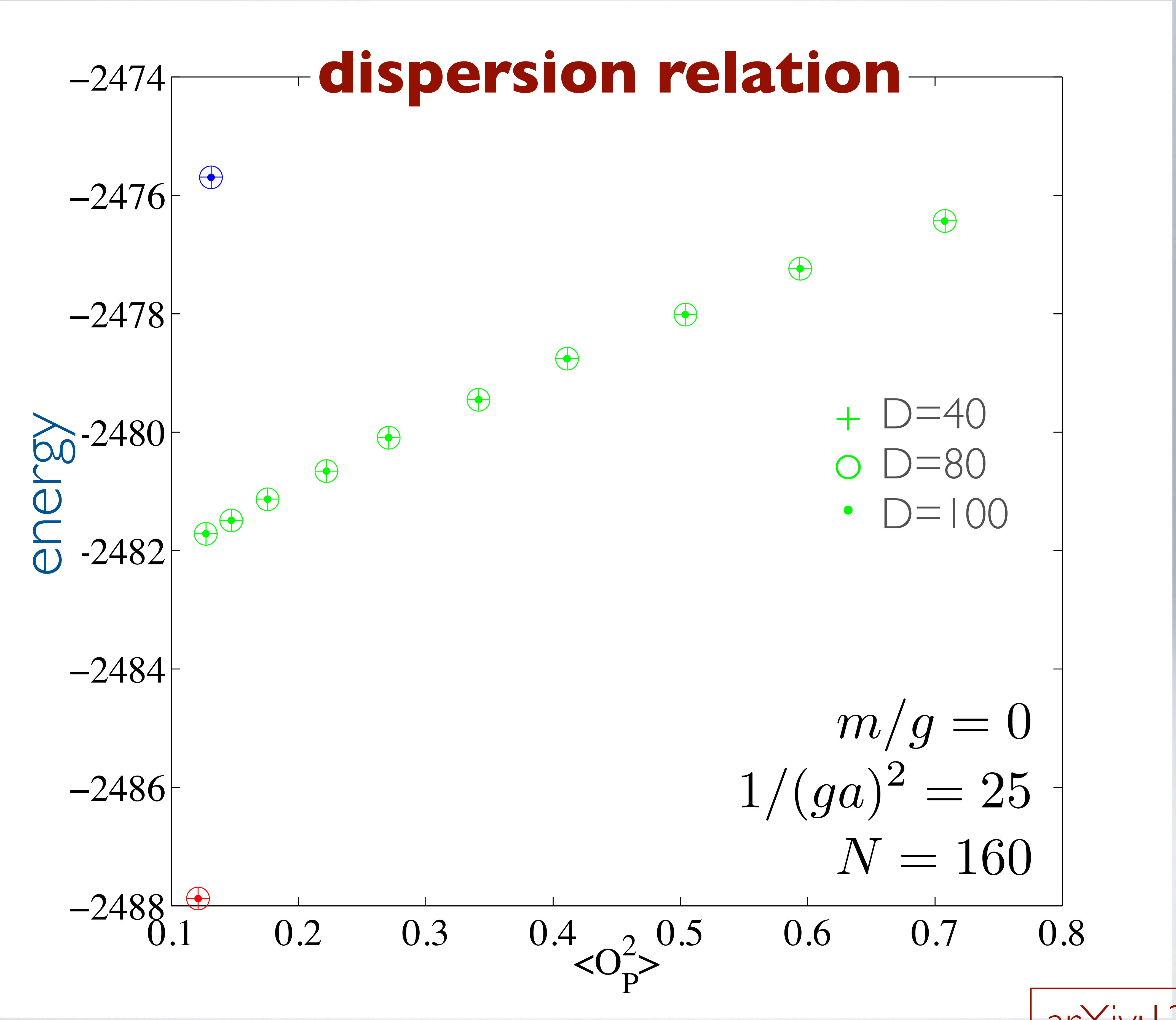
RESULTS



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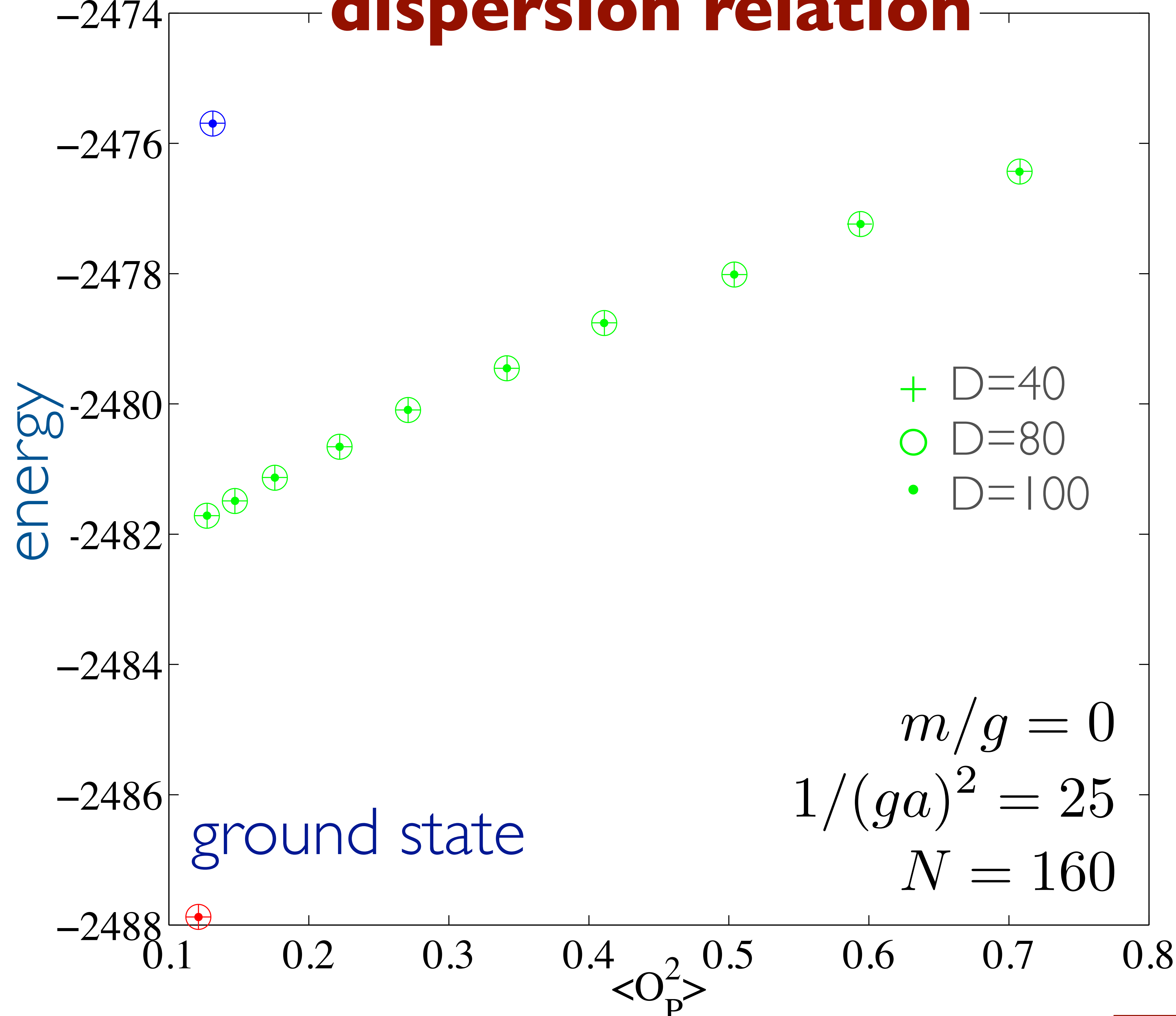
dispersion relation



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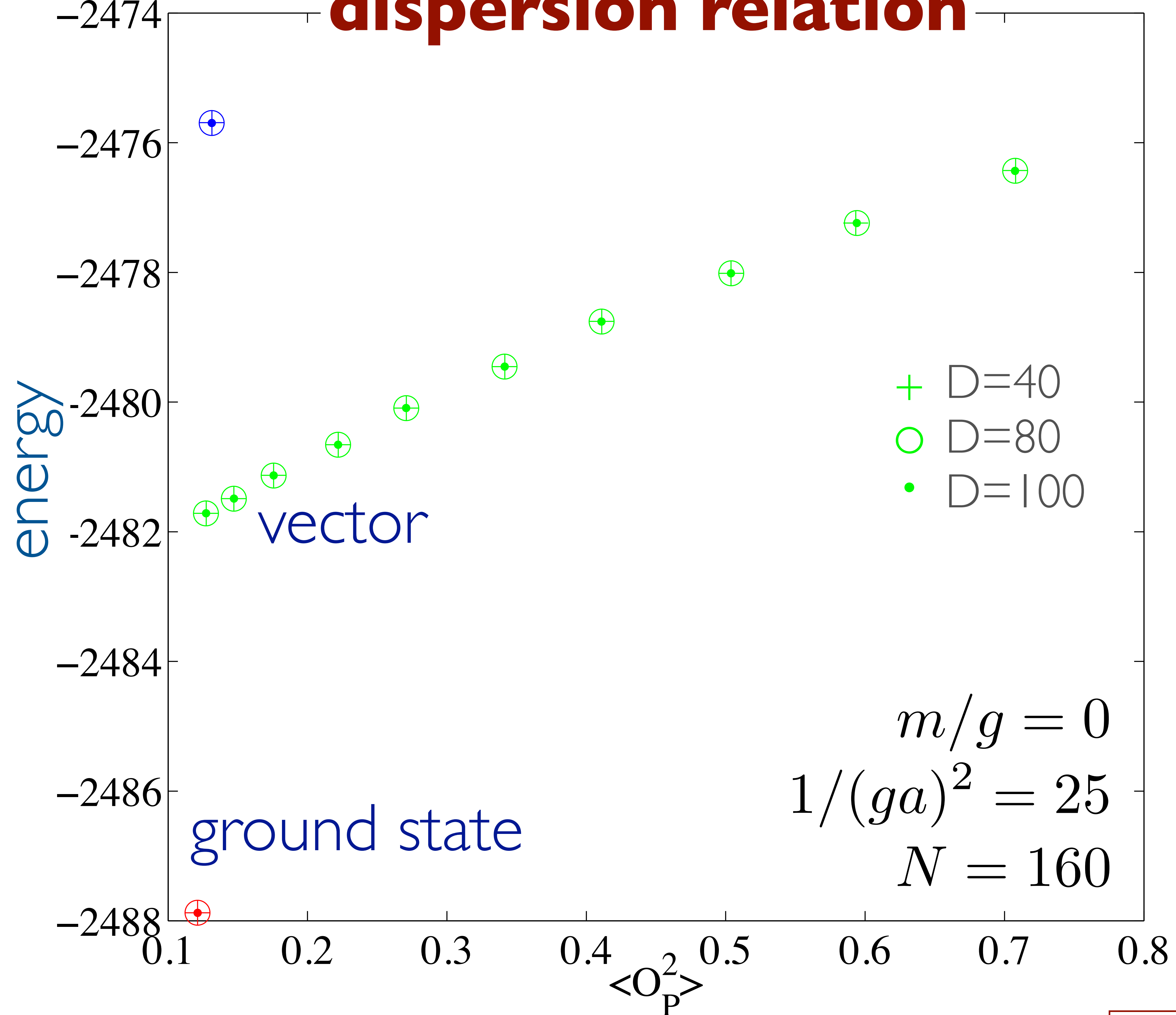

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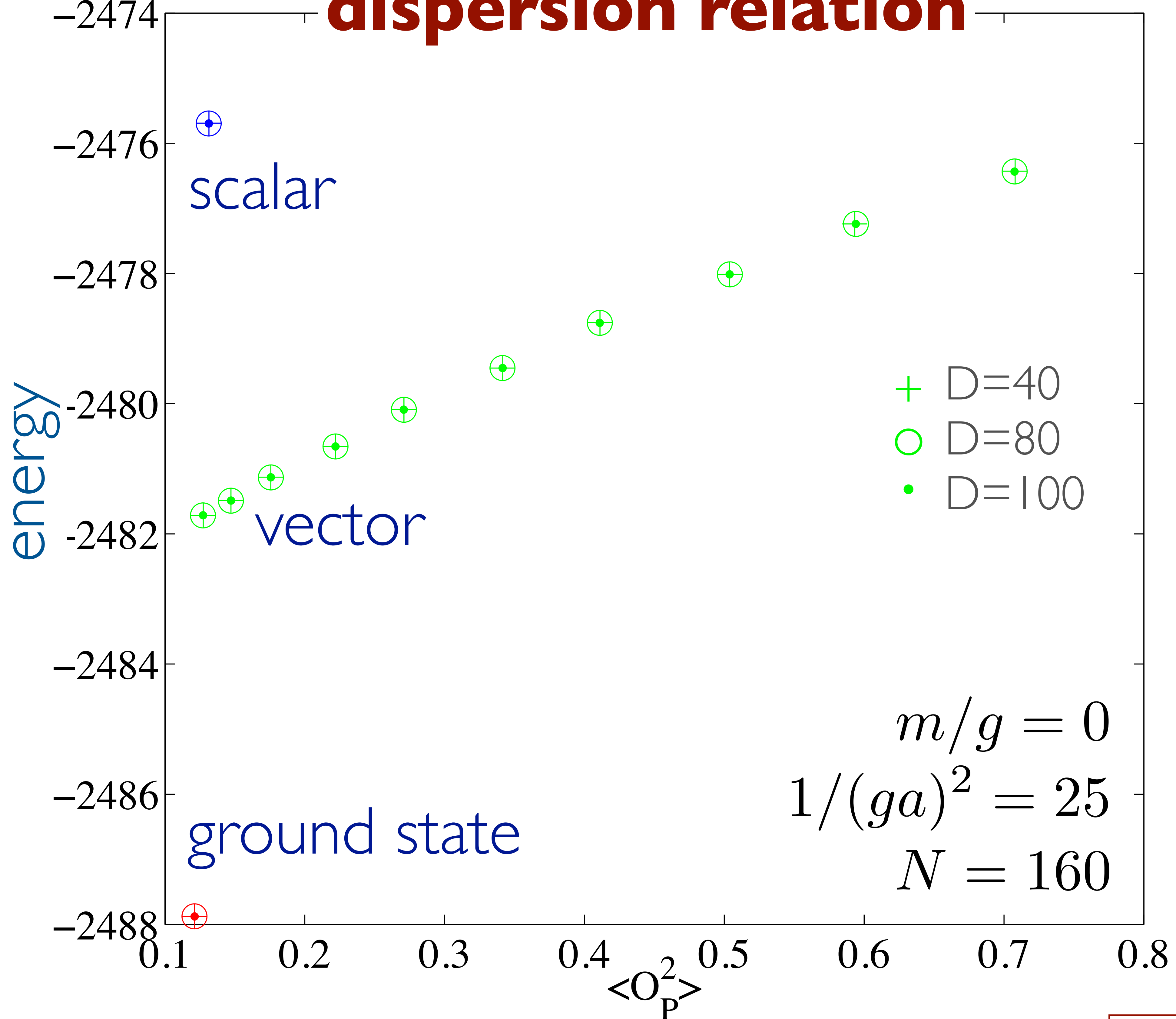

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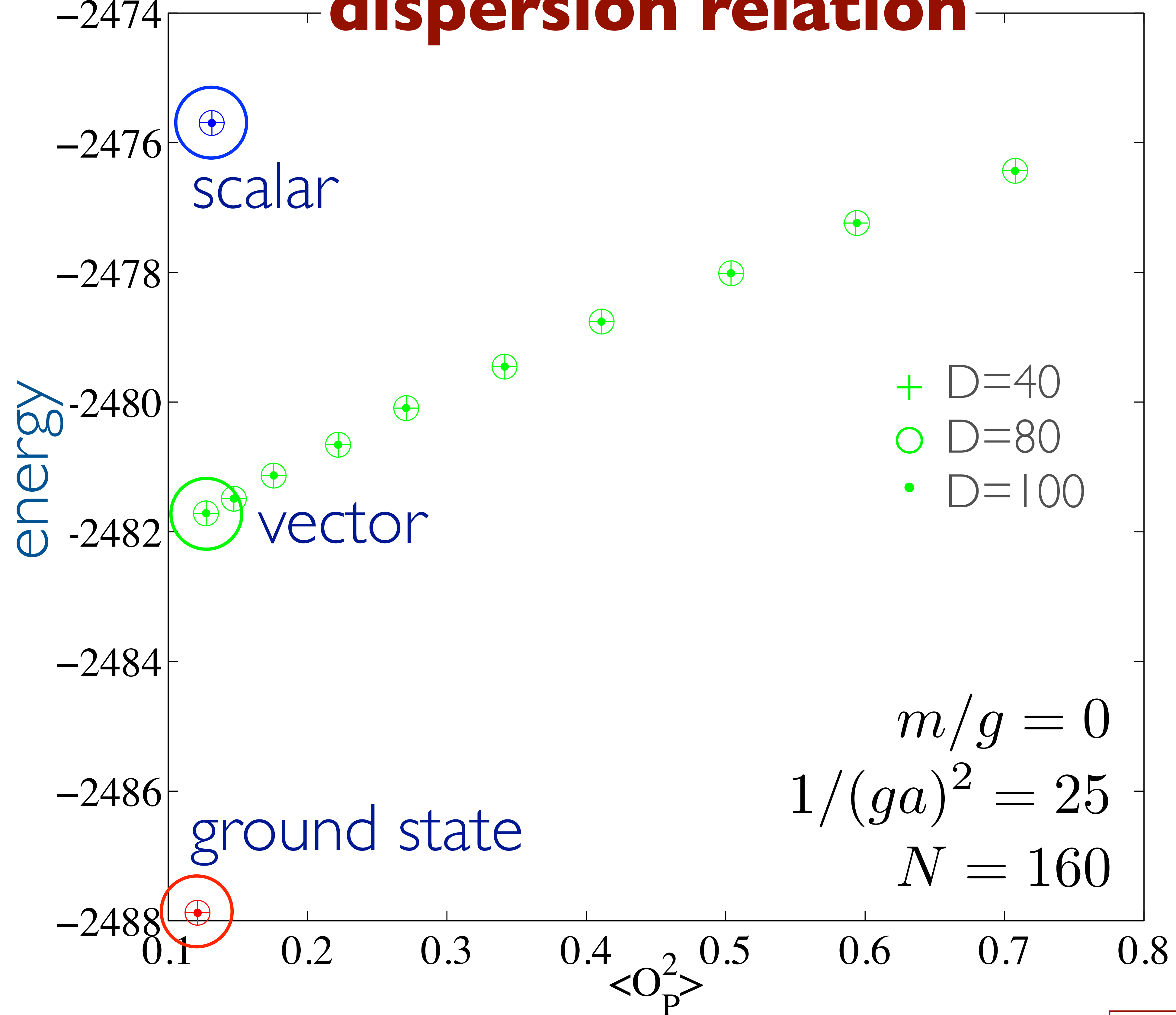

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① **truncation error**

$$m/g = 0 \quad 1/(ga)^2 = 100$$
$$N = 300$$

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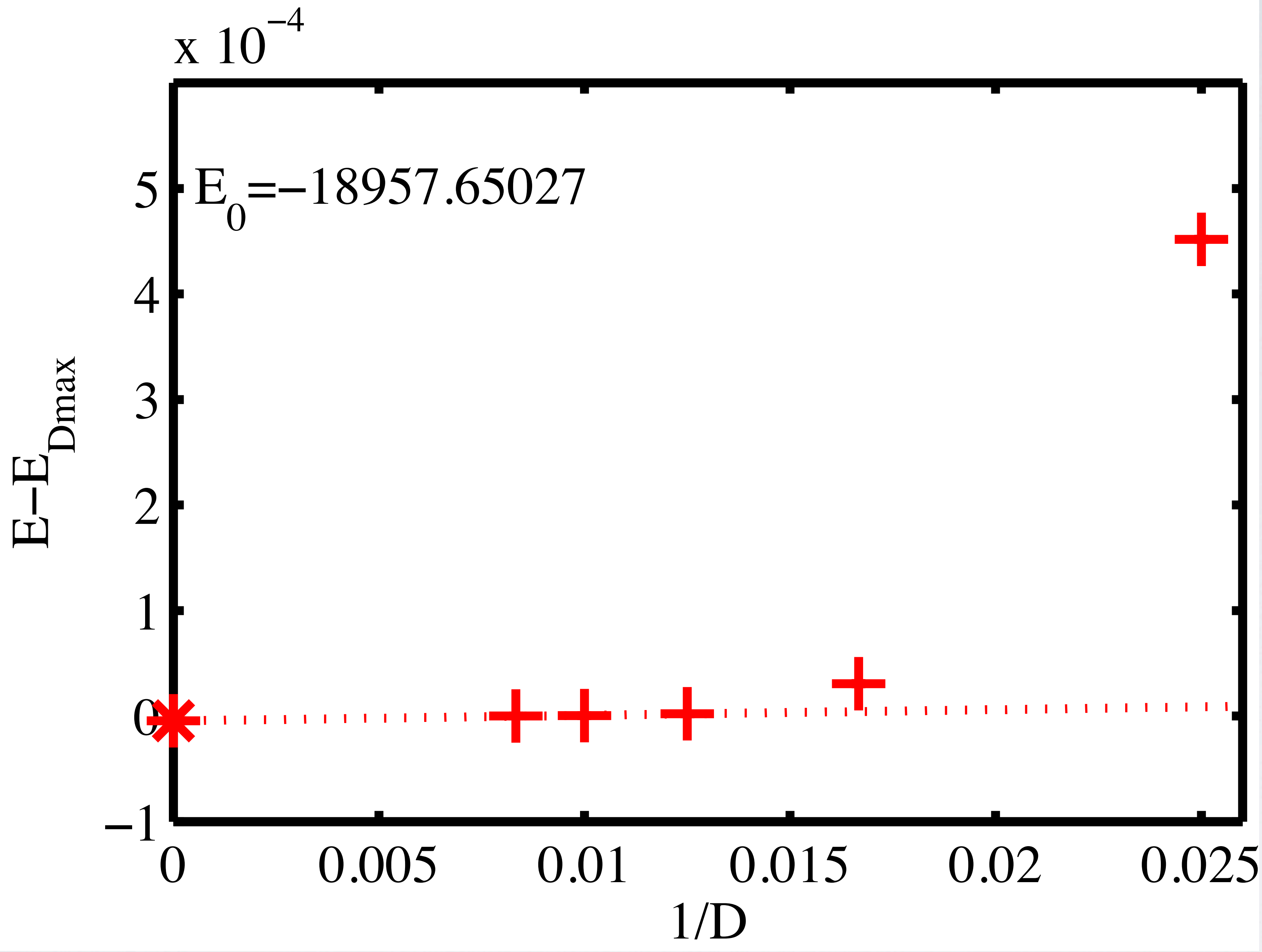

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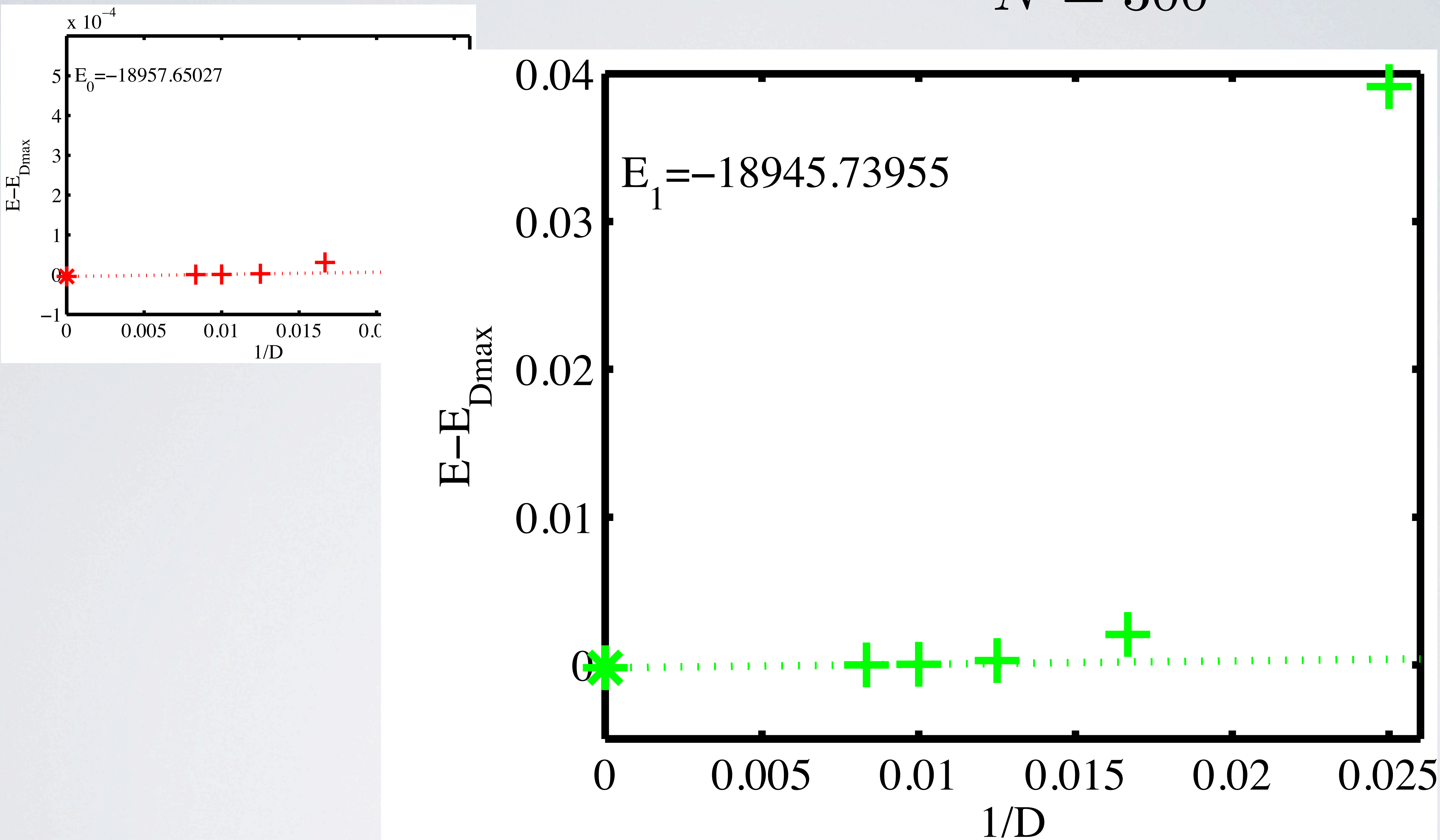

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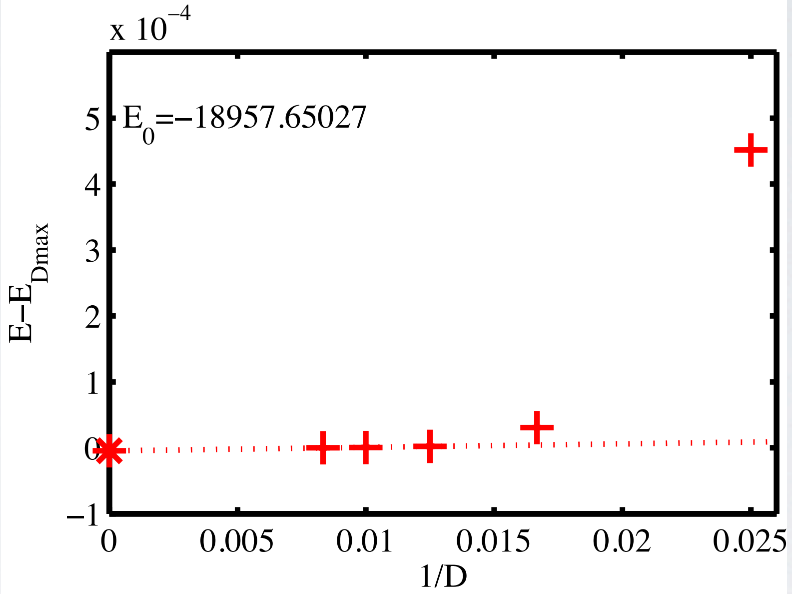

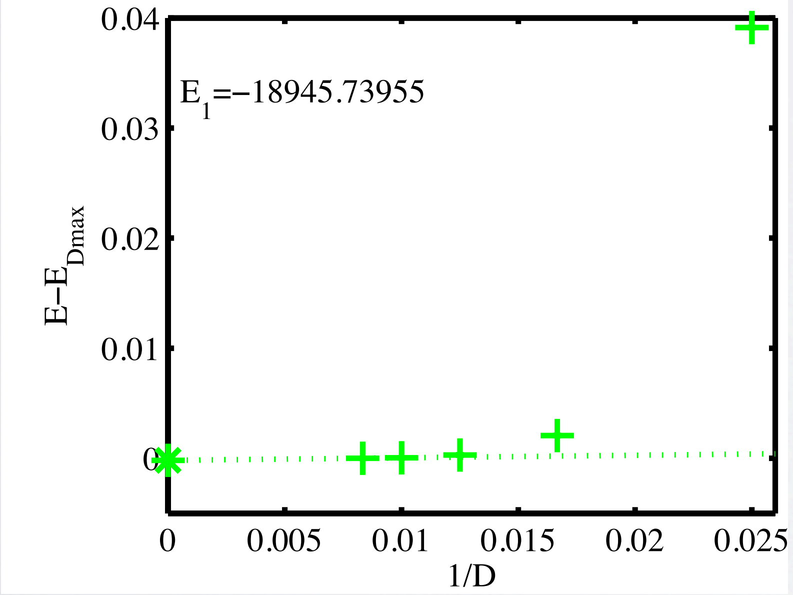

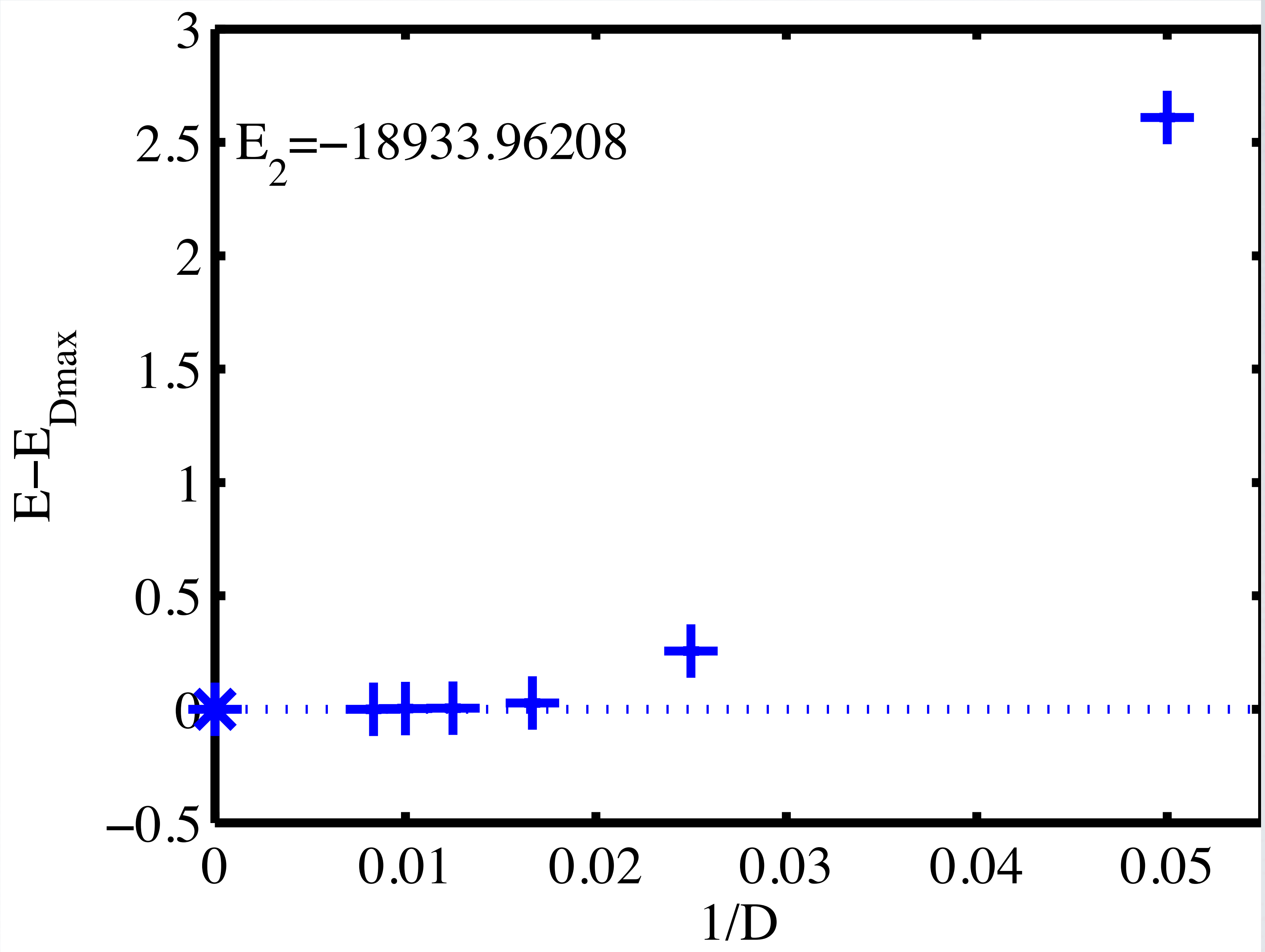

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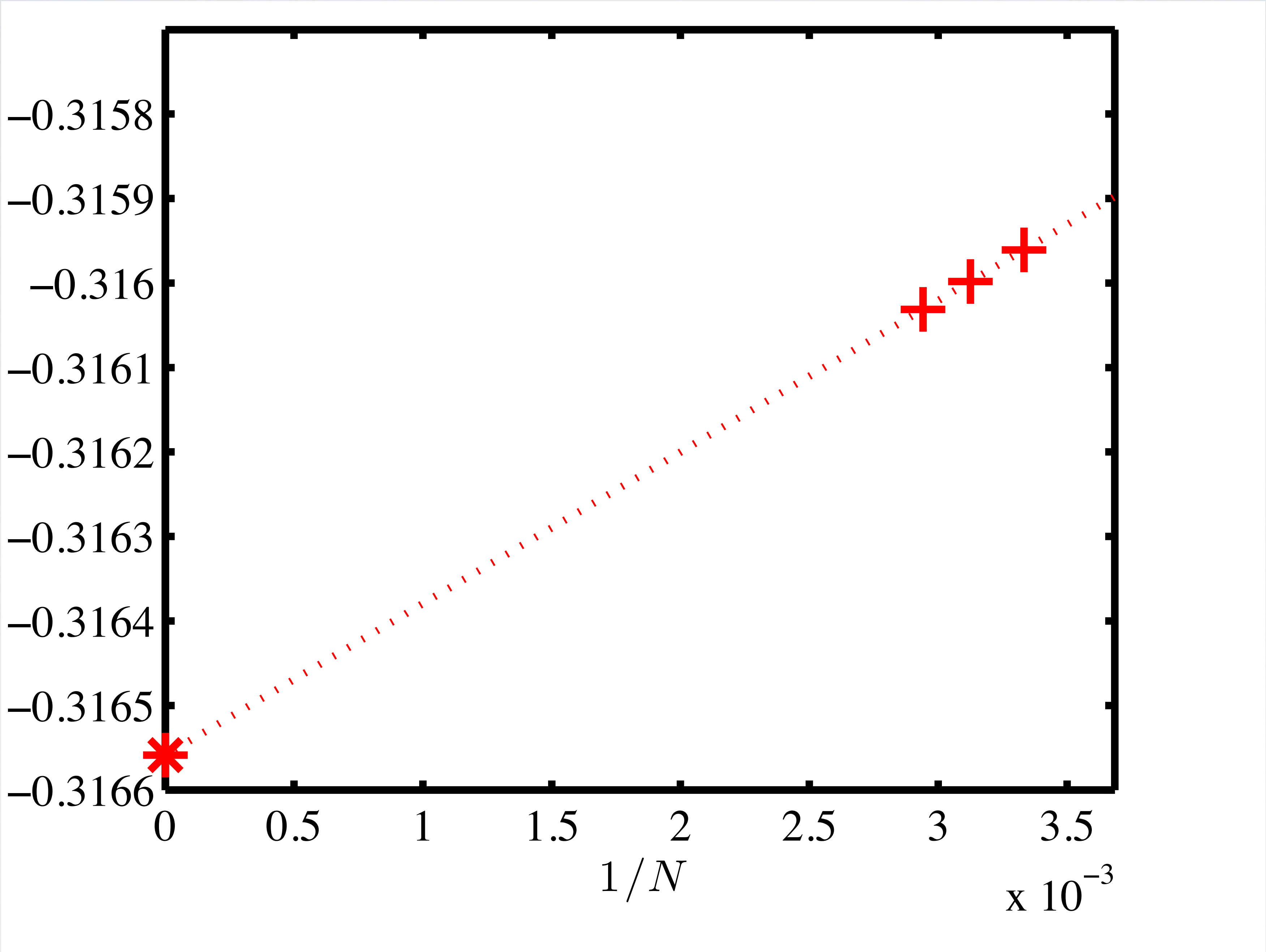

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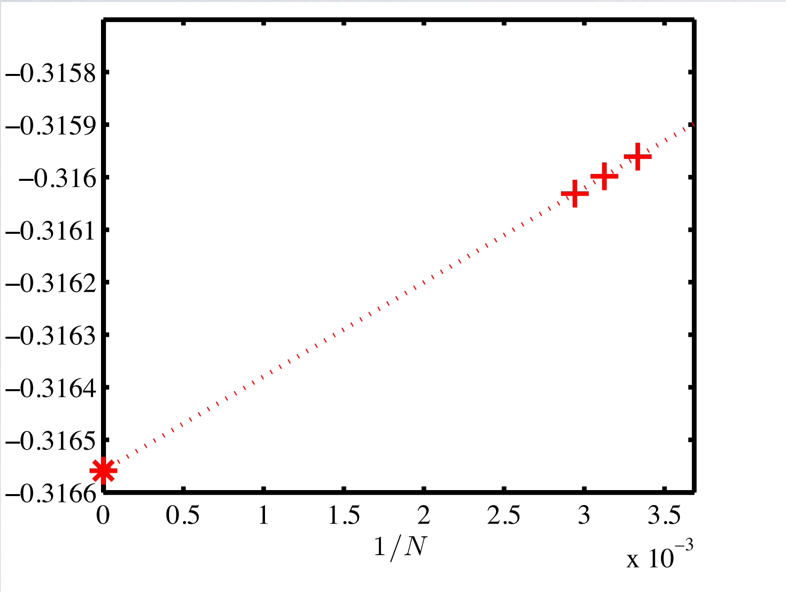

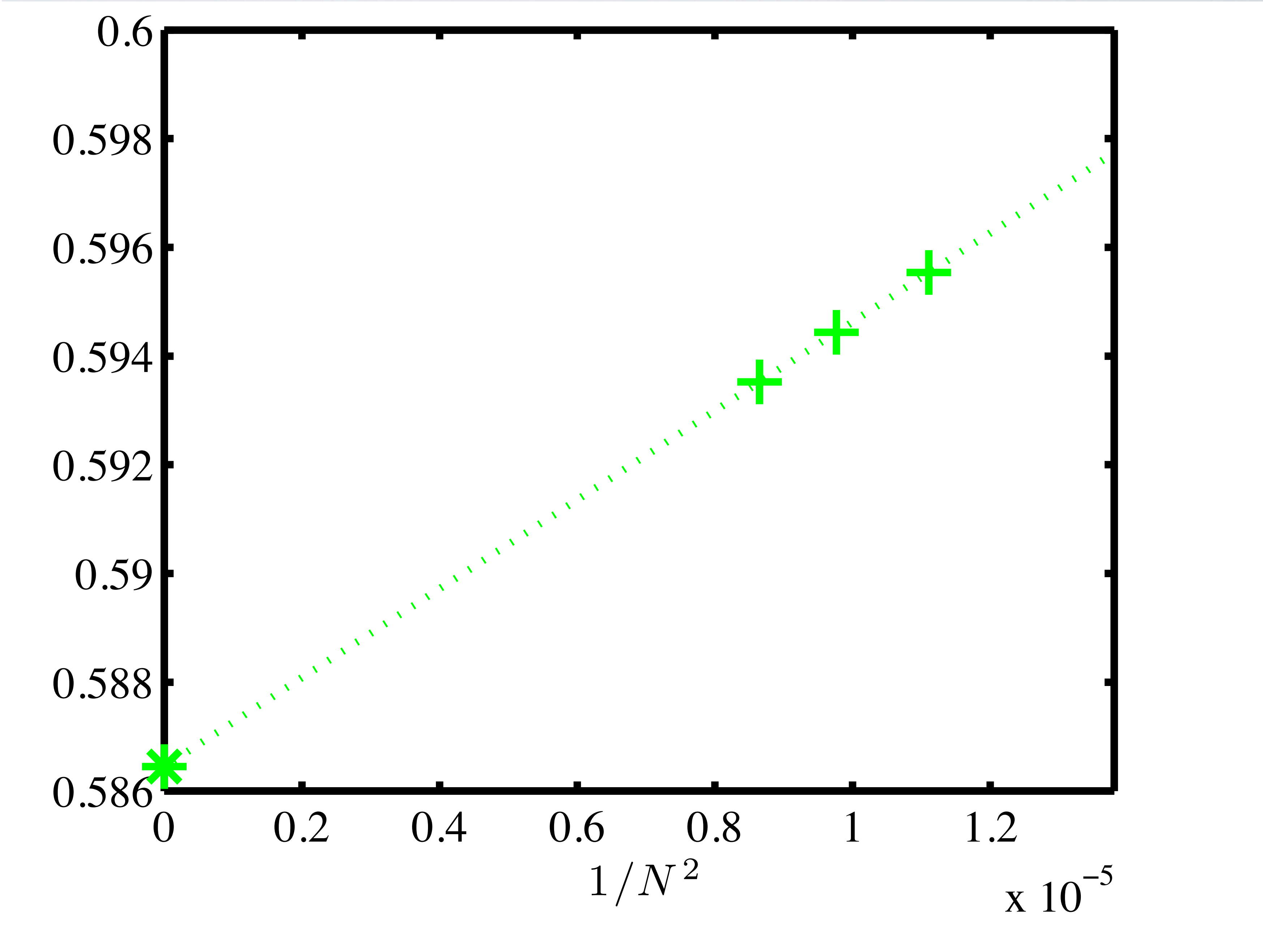

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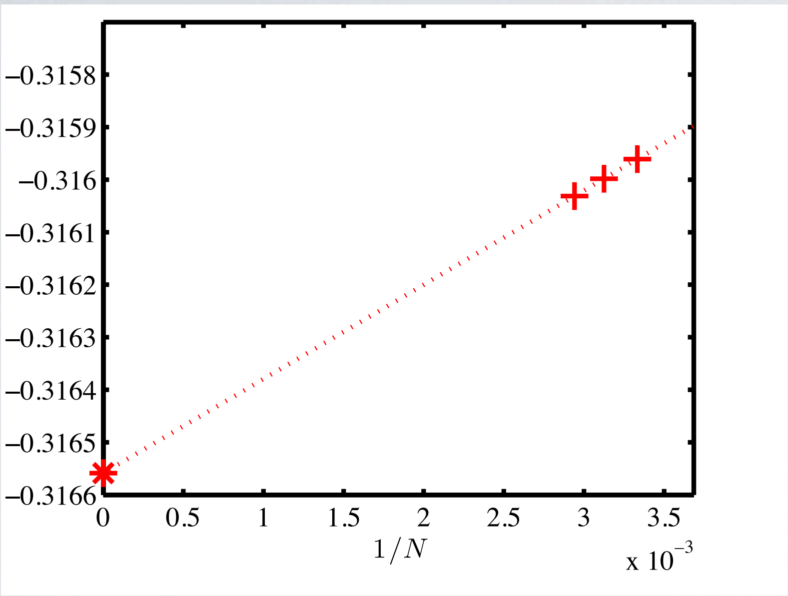
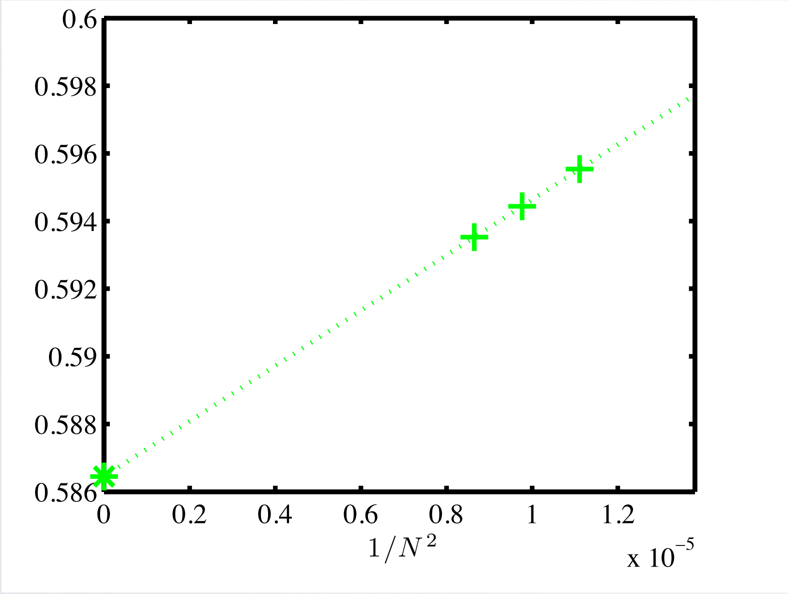
2 finite-size scaling $m/g = 0 \quad 1/(ga)^2 = 100$





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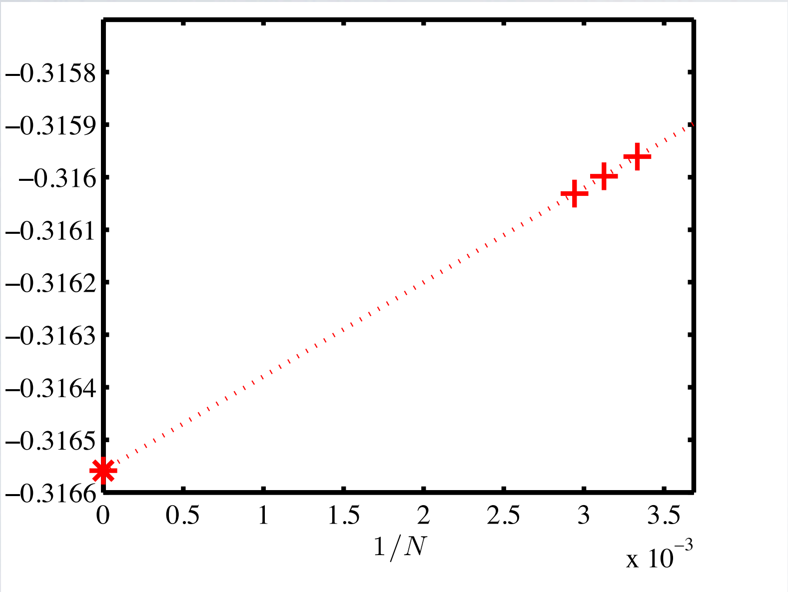
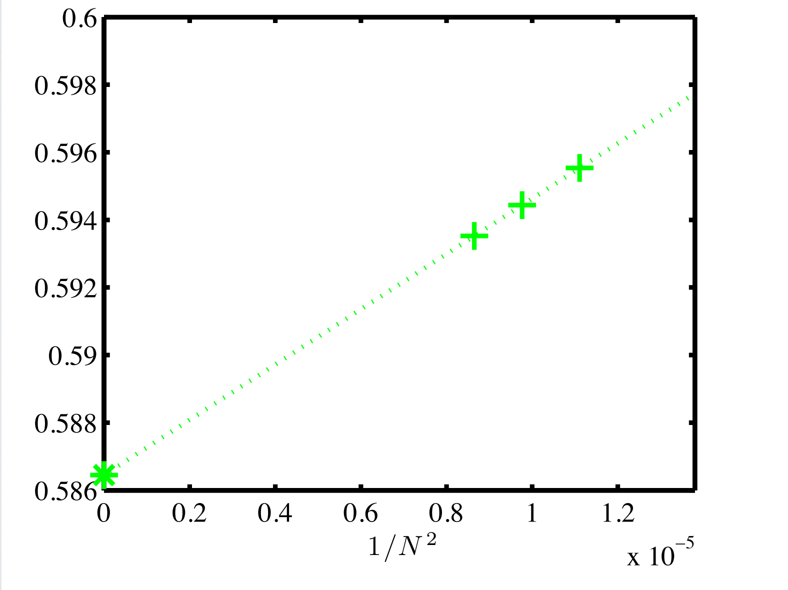
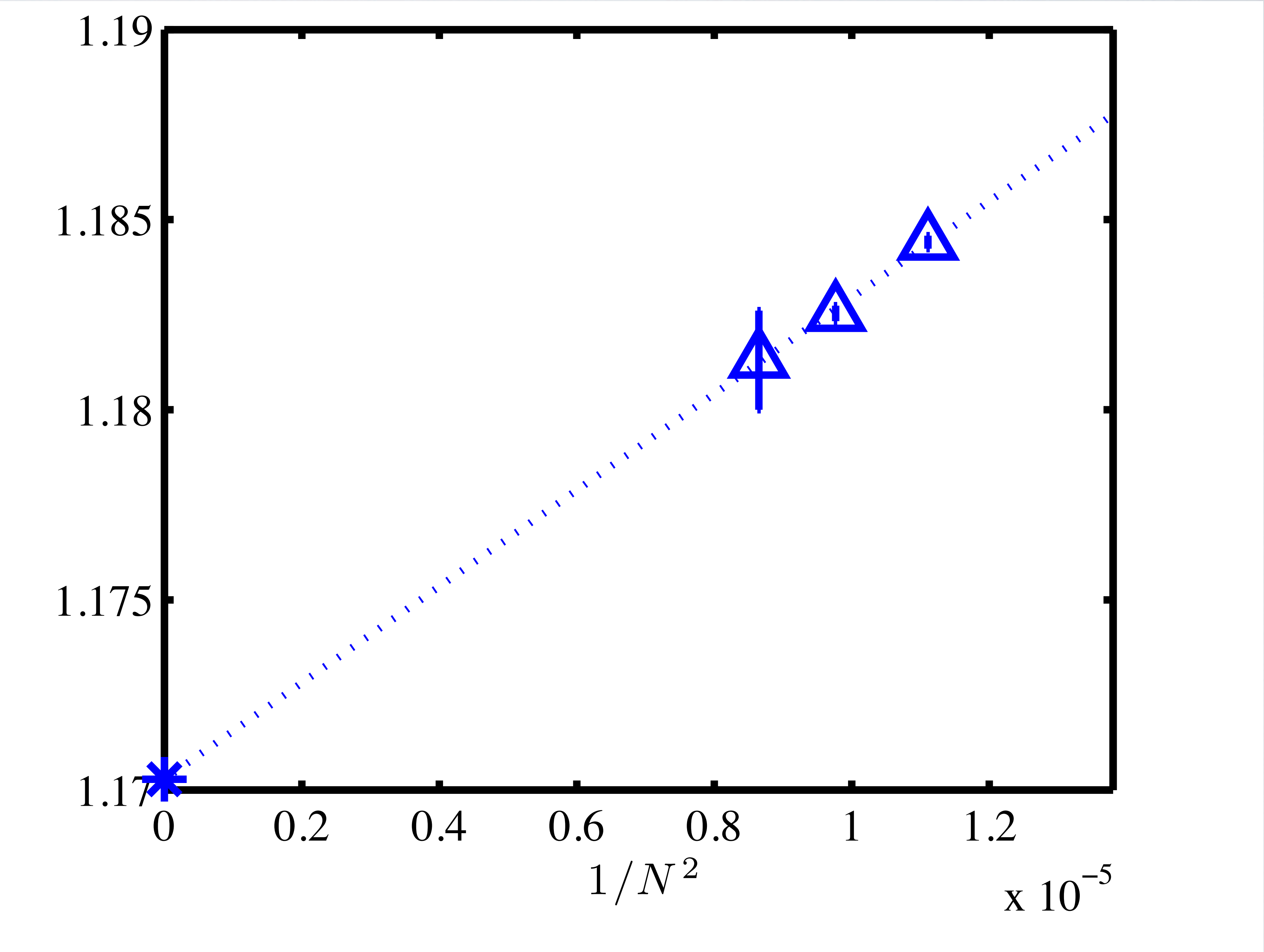
(2) finite-size scaling $m/g = 0 \quad 1/(ga)^2 = 100$







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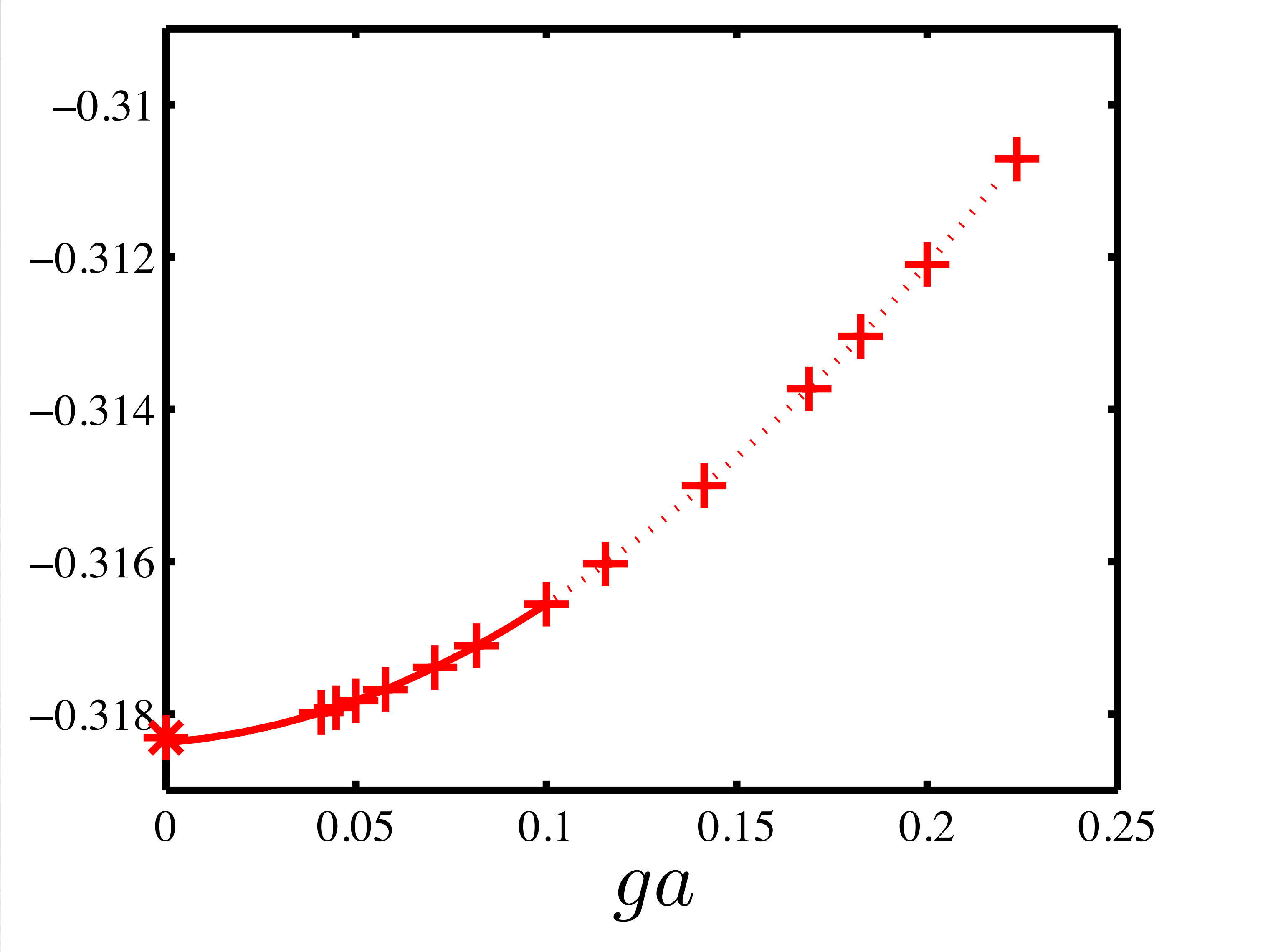
3 continuum limit $m/g = 0$

arXiv:1305.3765

3 **continuum limit** $m/g = 0$



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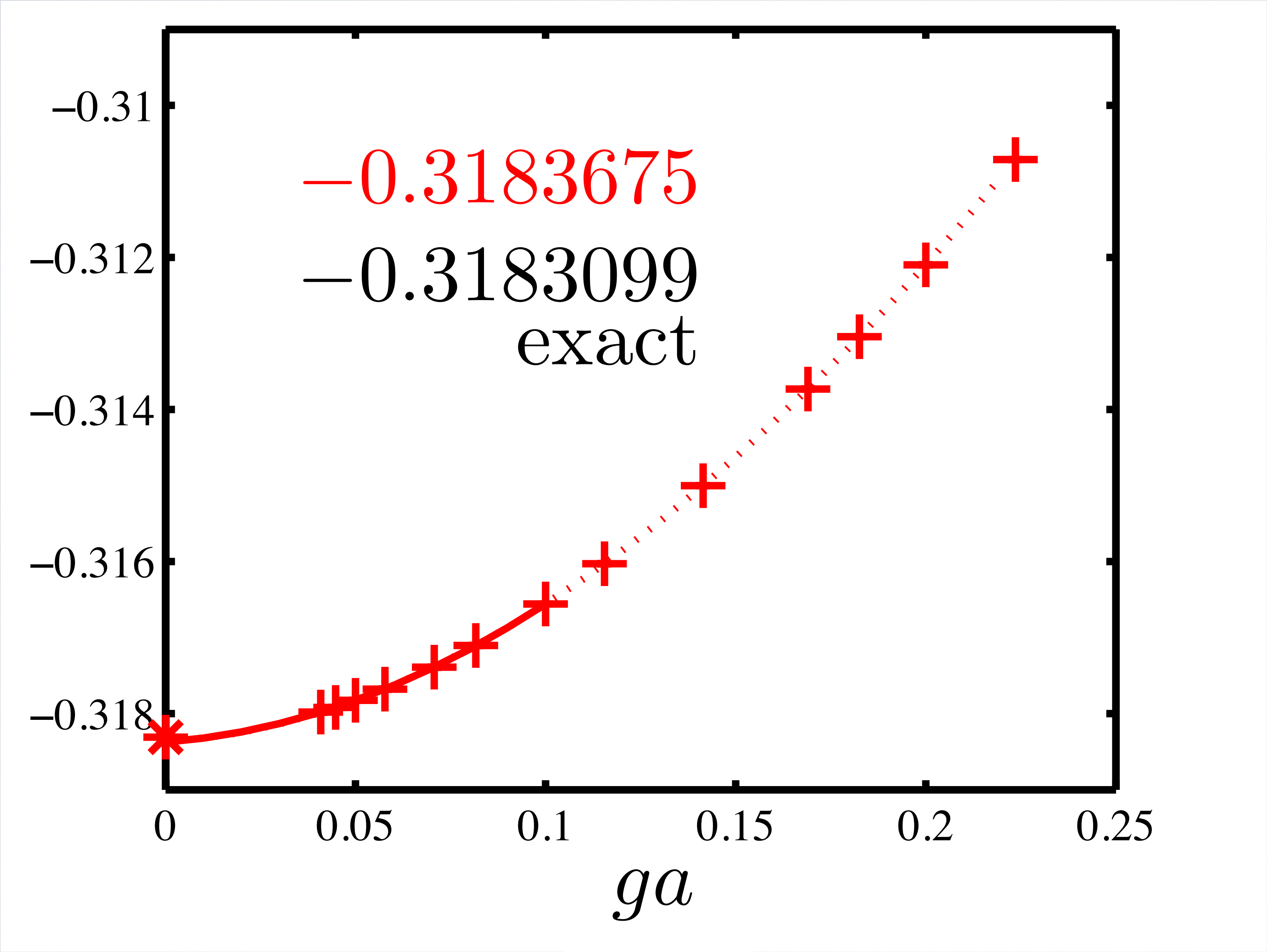

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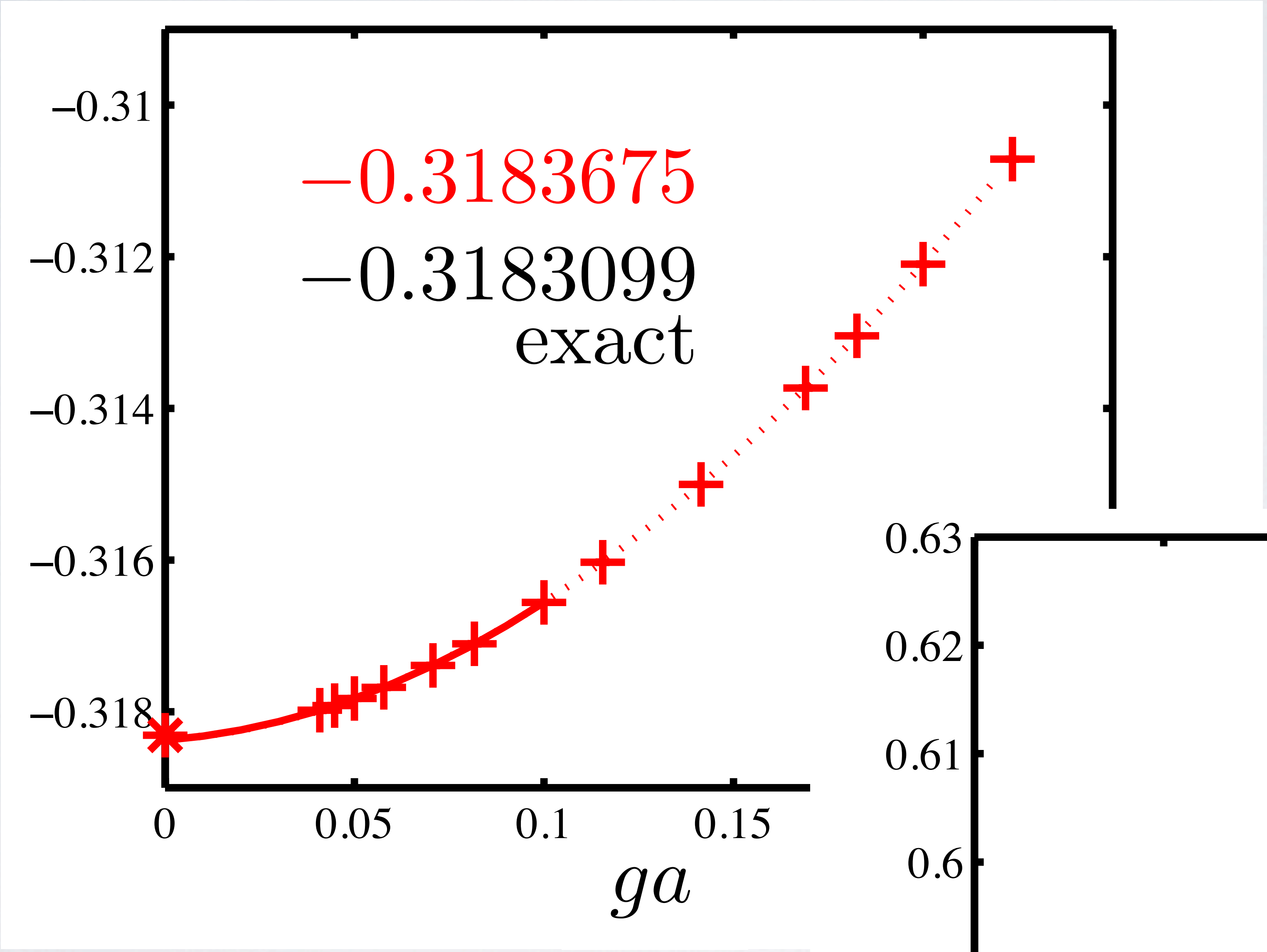

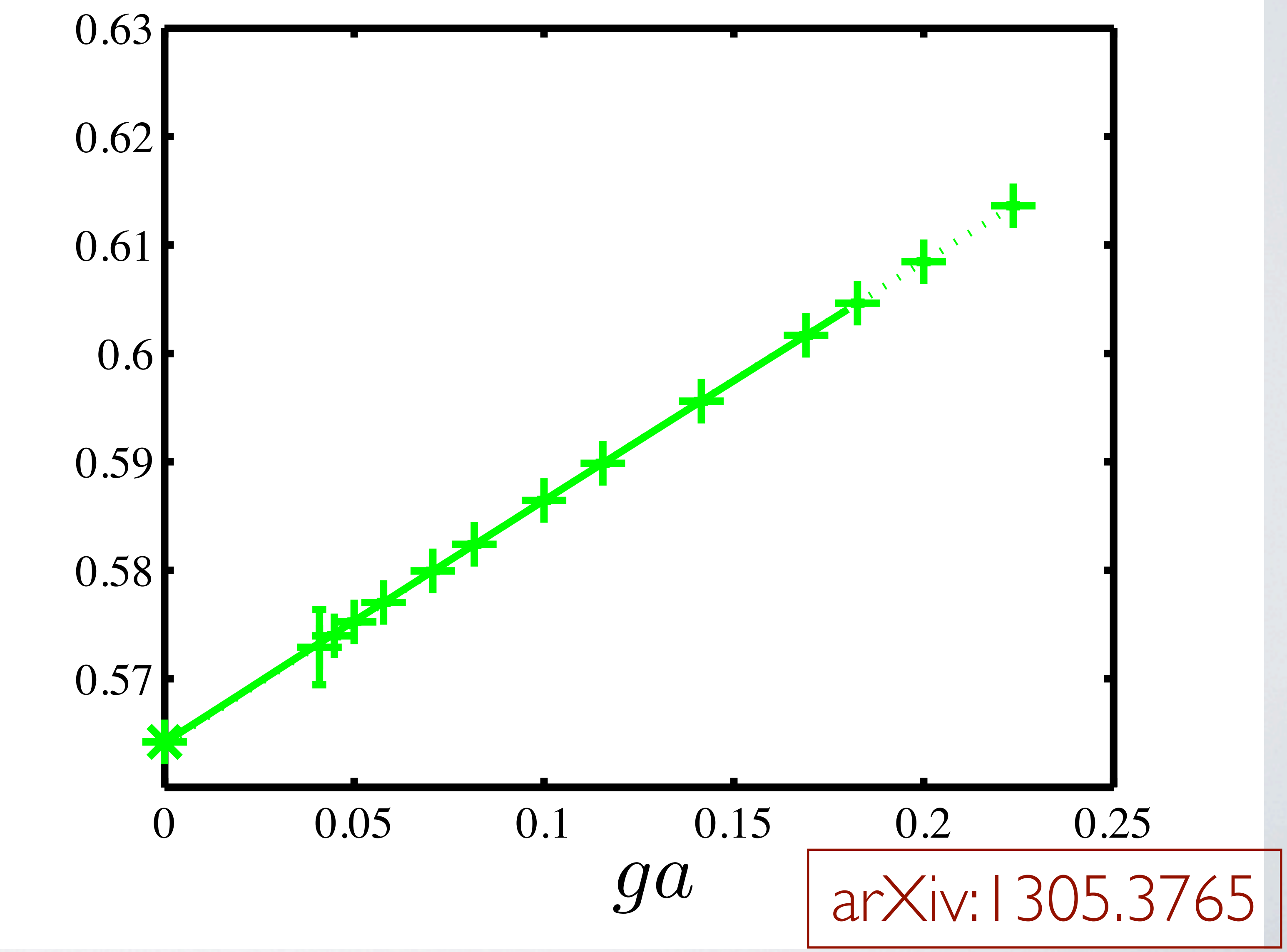

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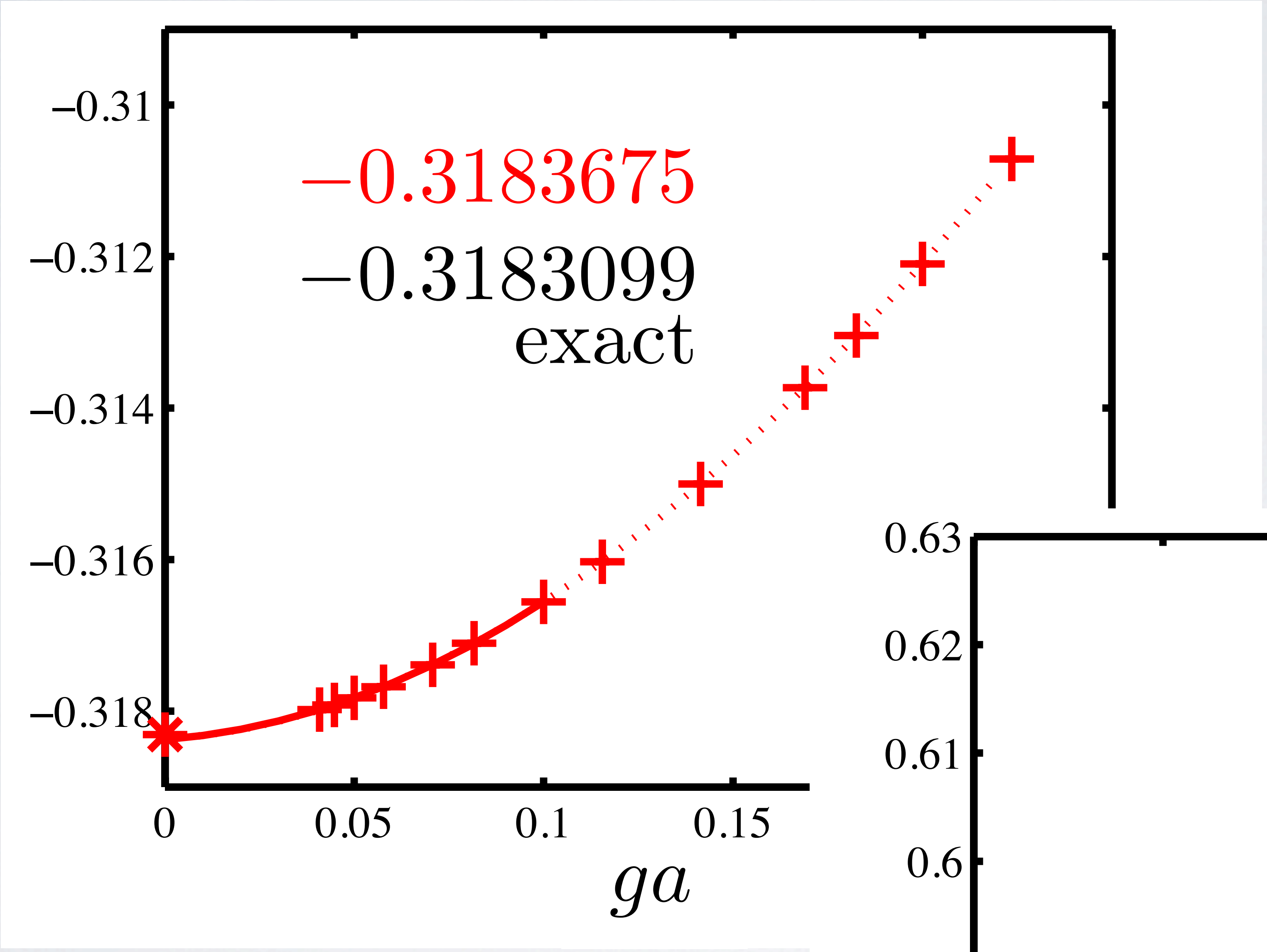
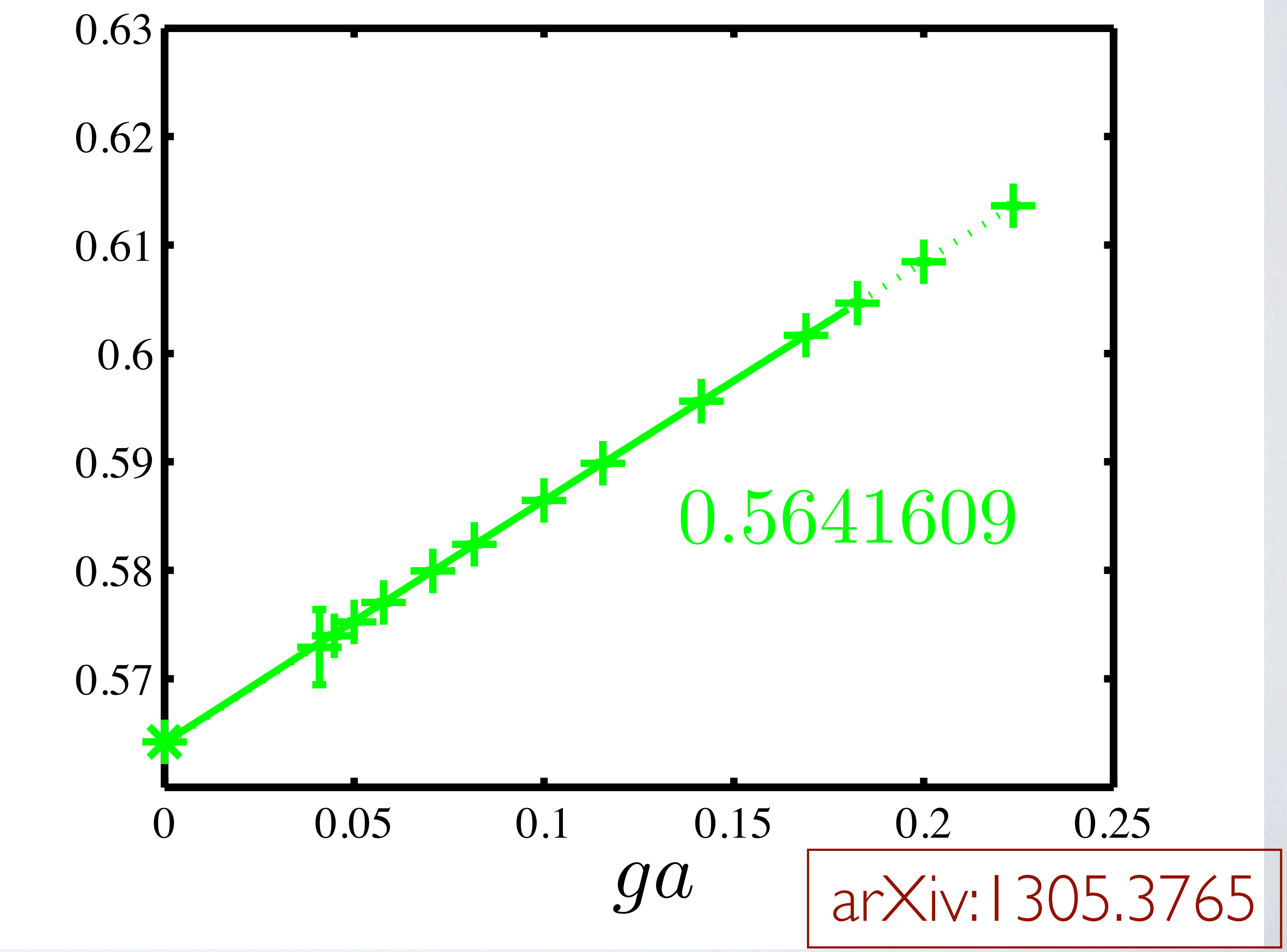
③ continuum limit $m/g = 0$





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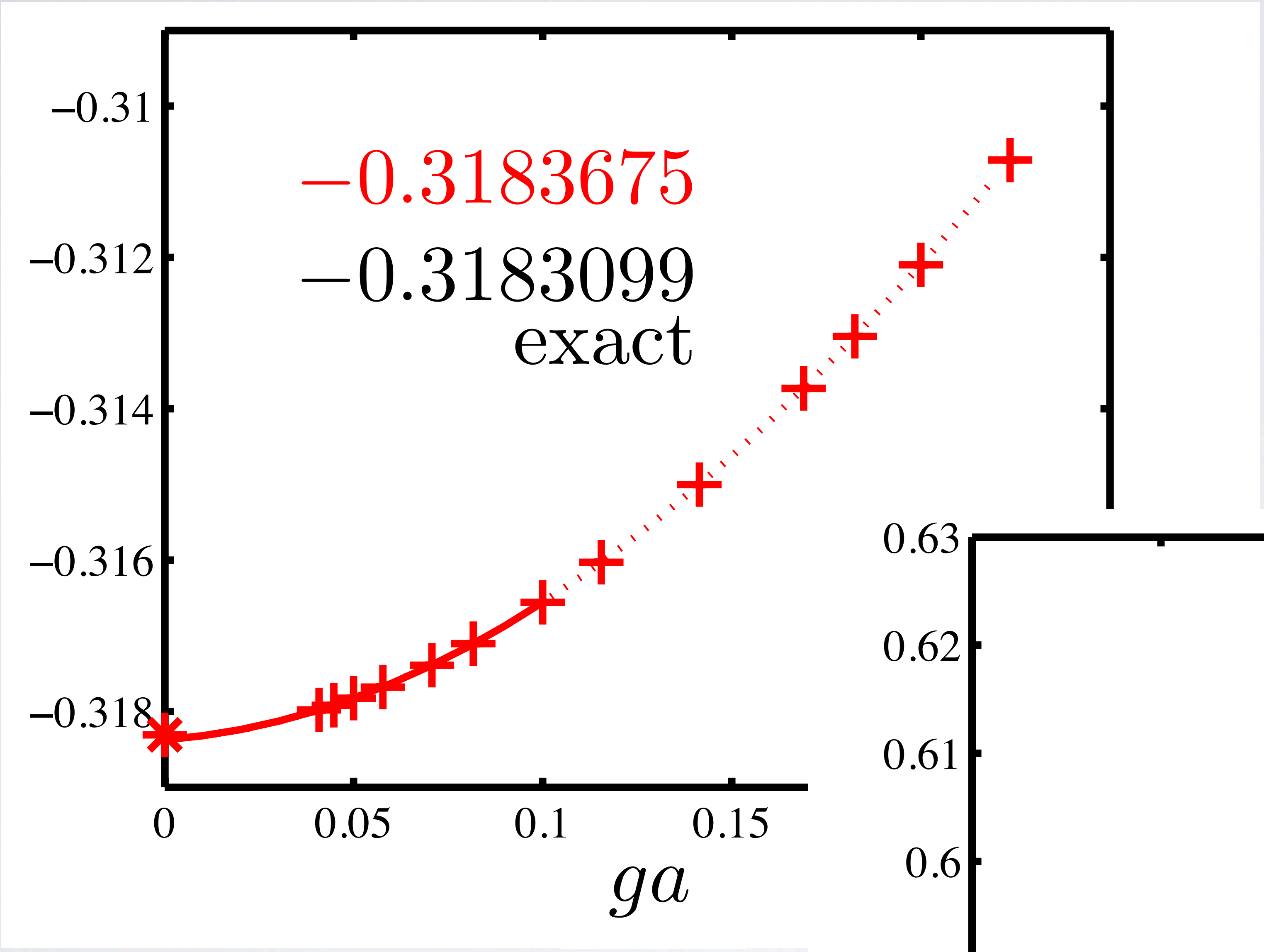

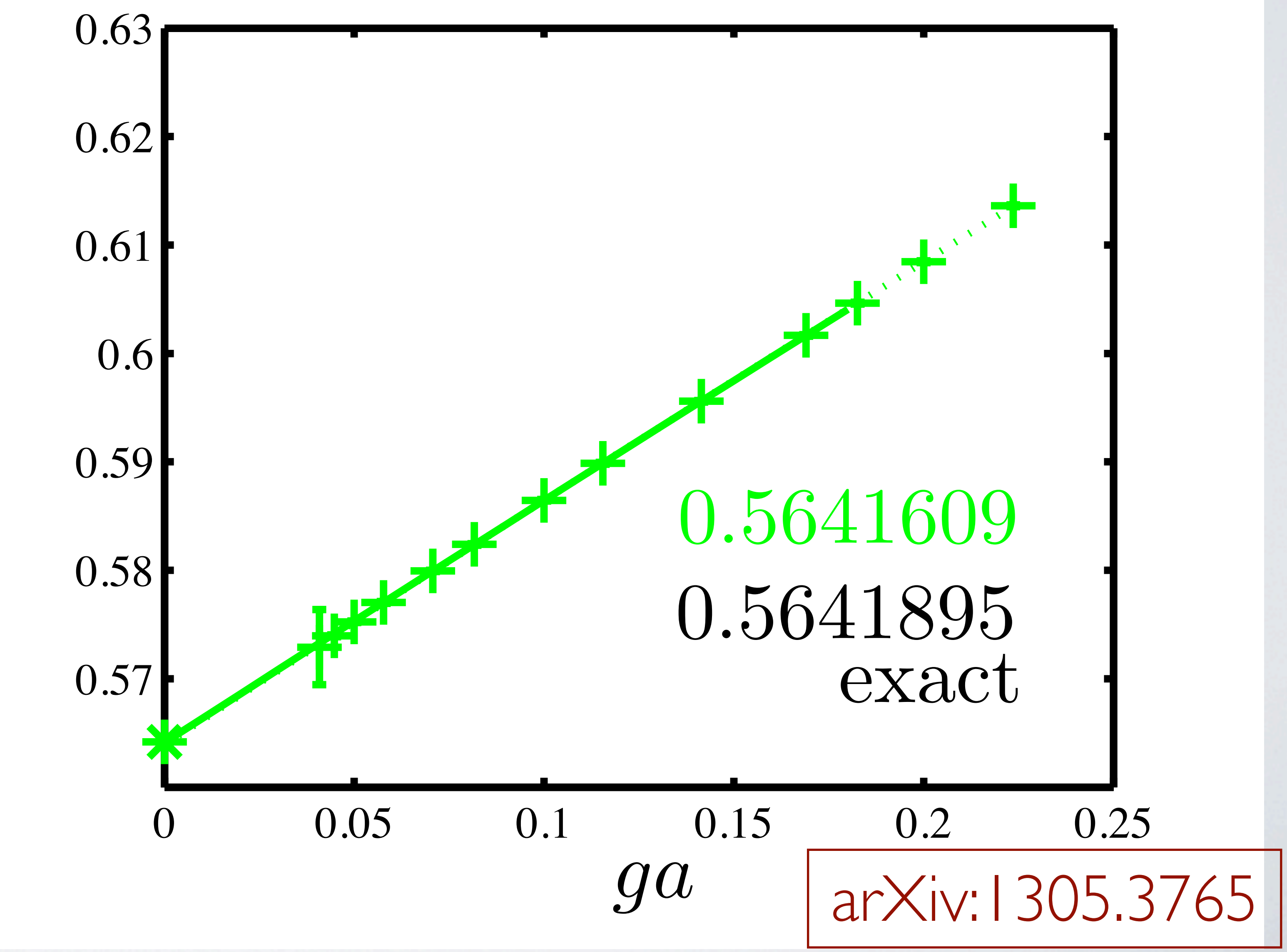

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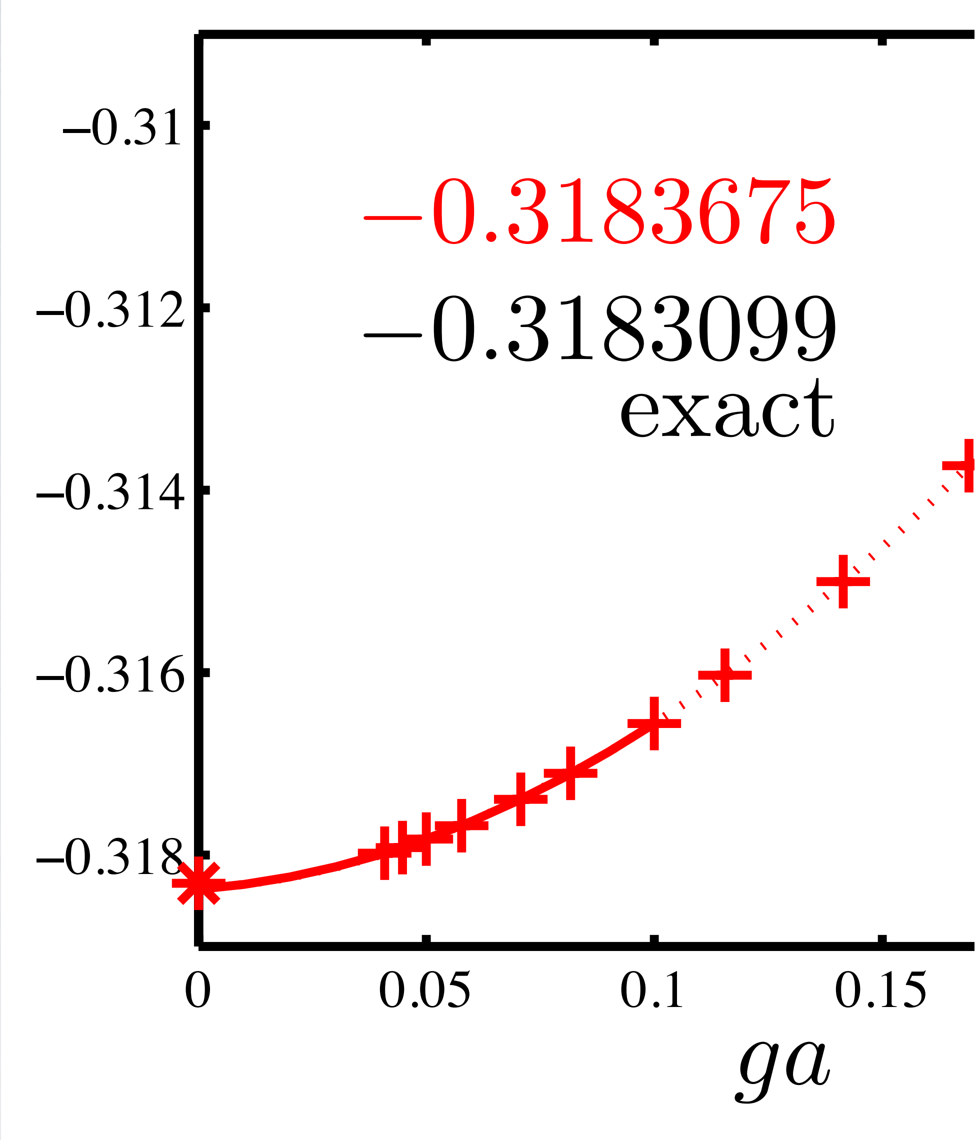
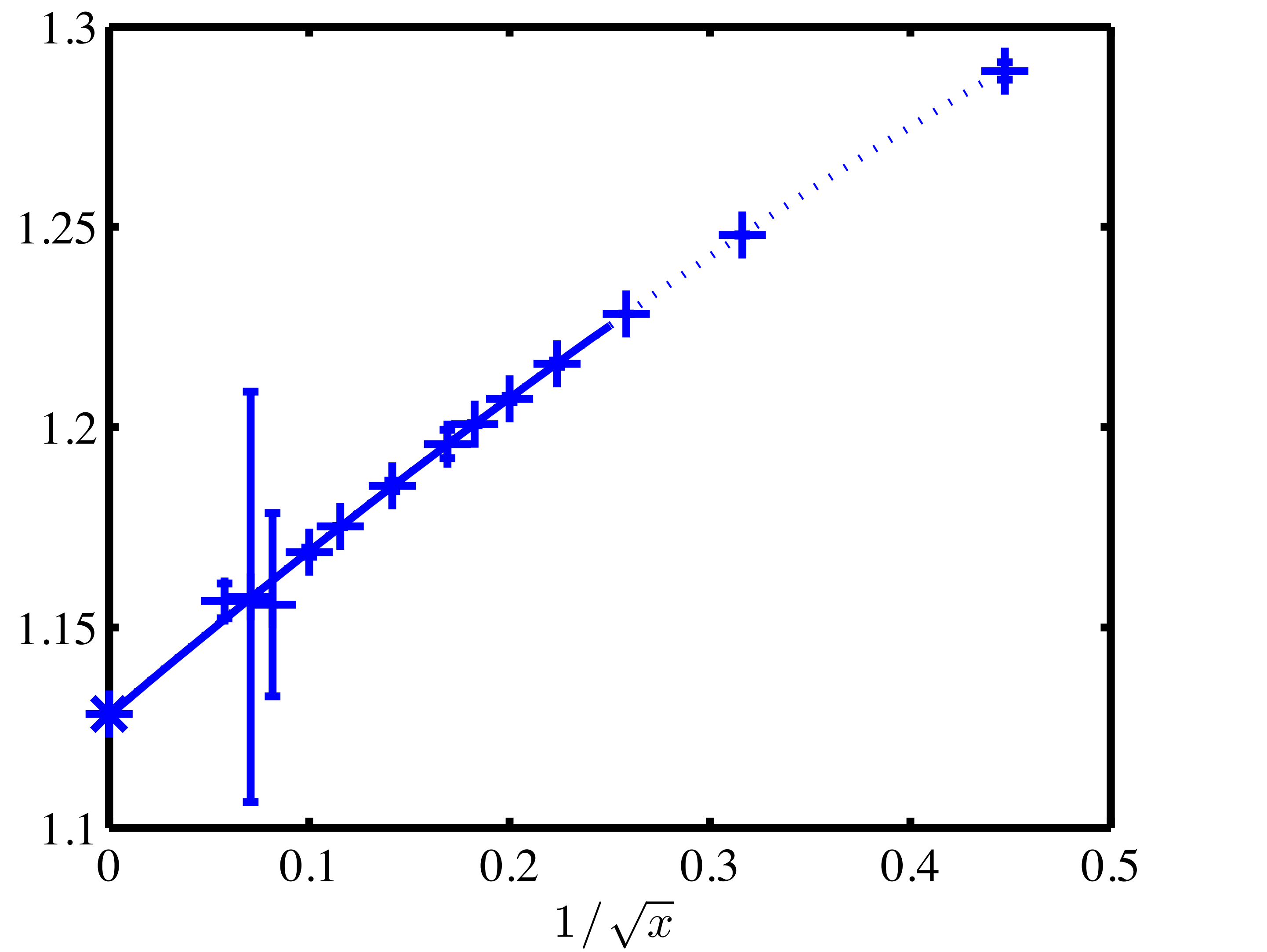
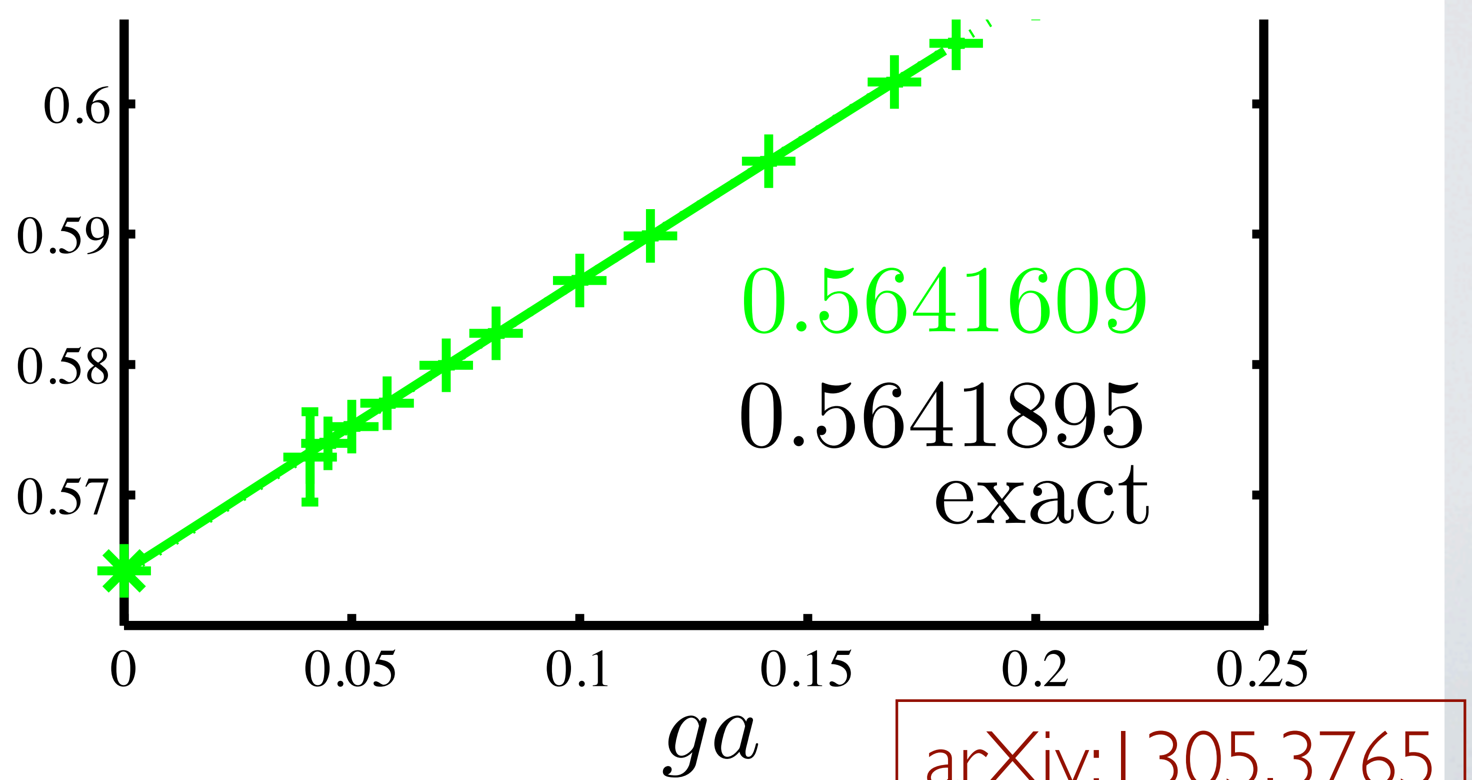
3 continuum lin

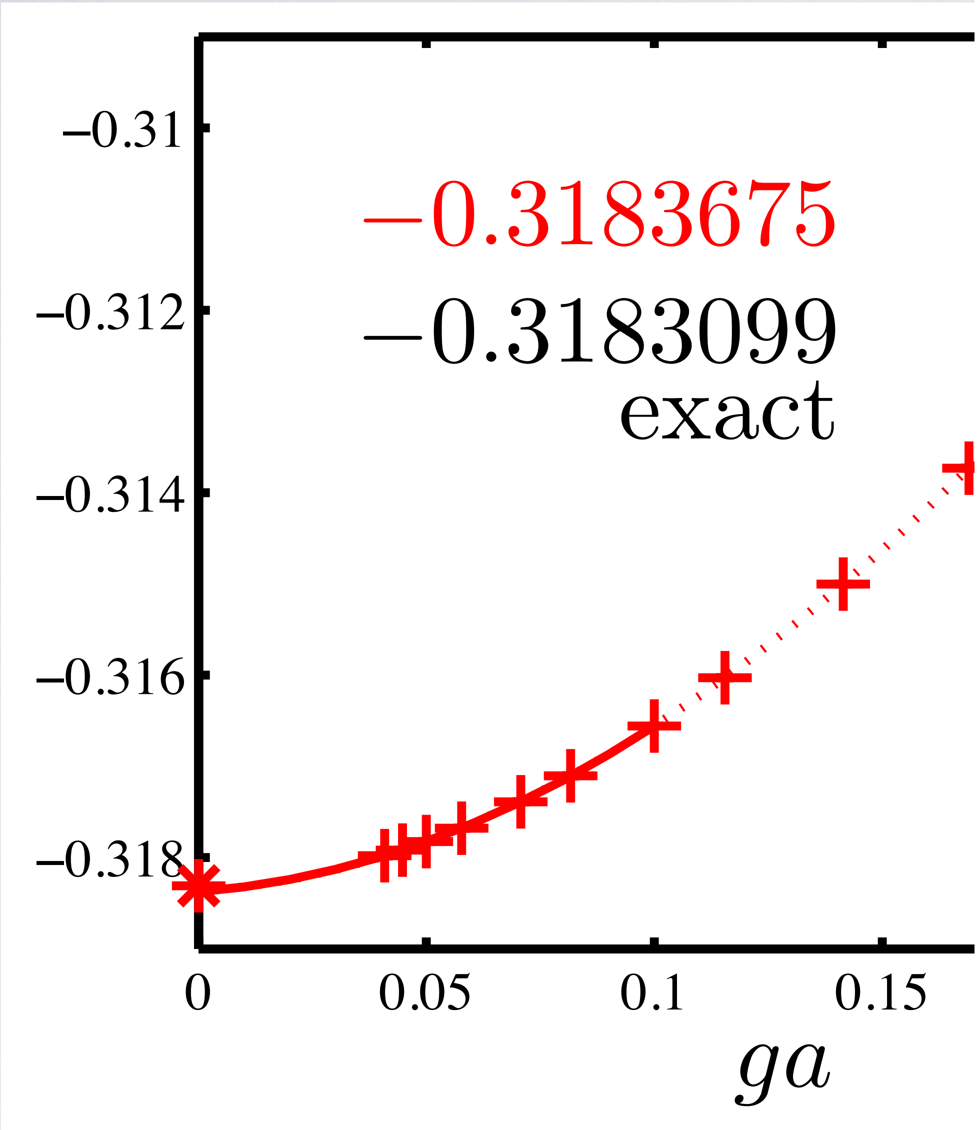


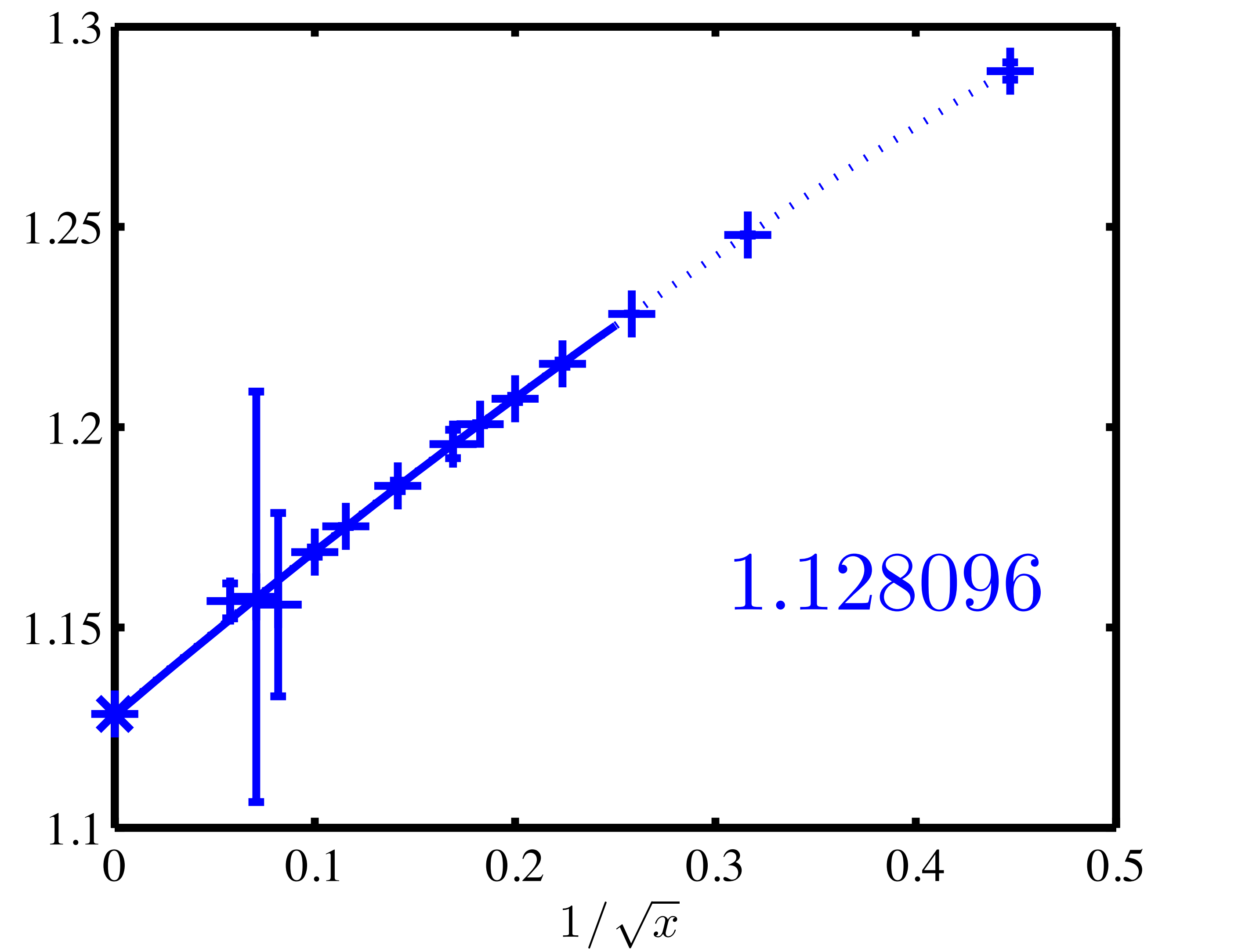


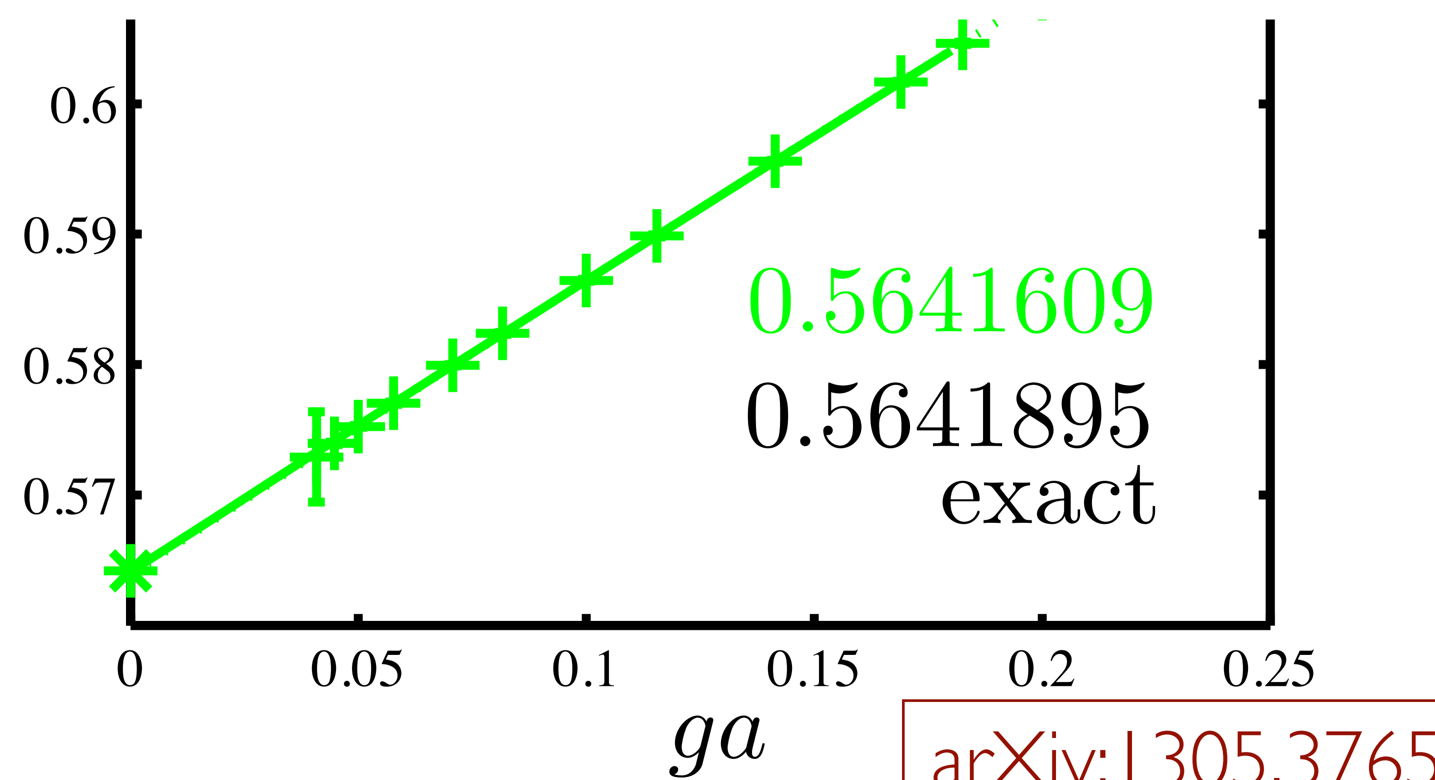


arXiv:1305.3765

3 continuum lim







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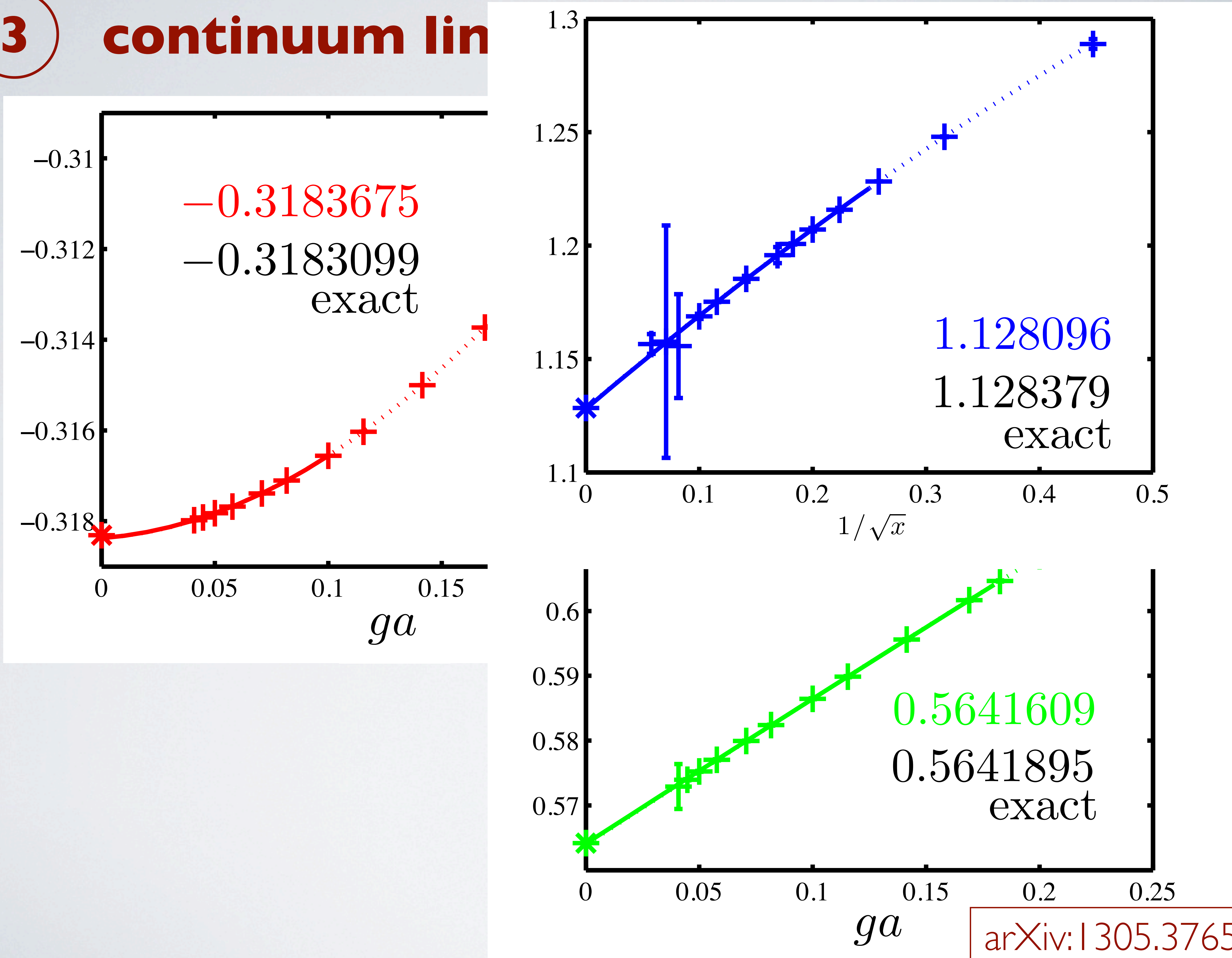

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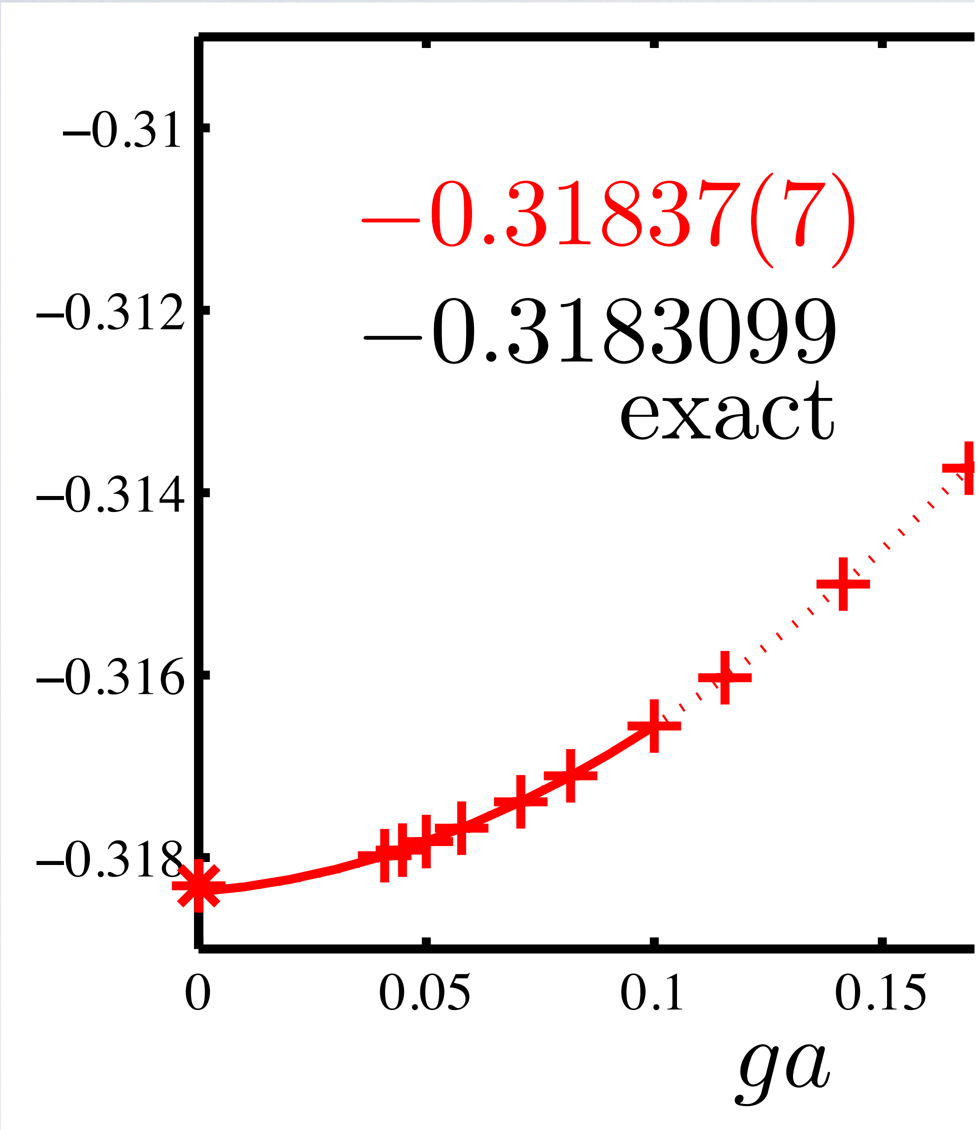

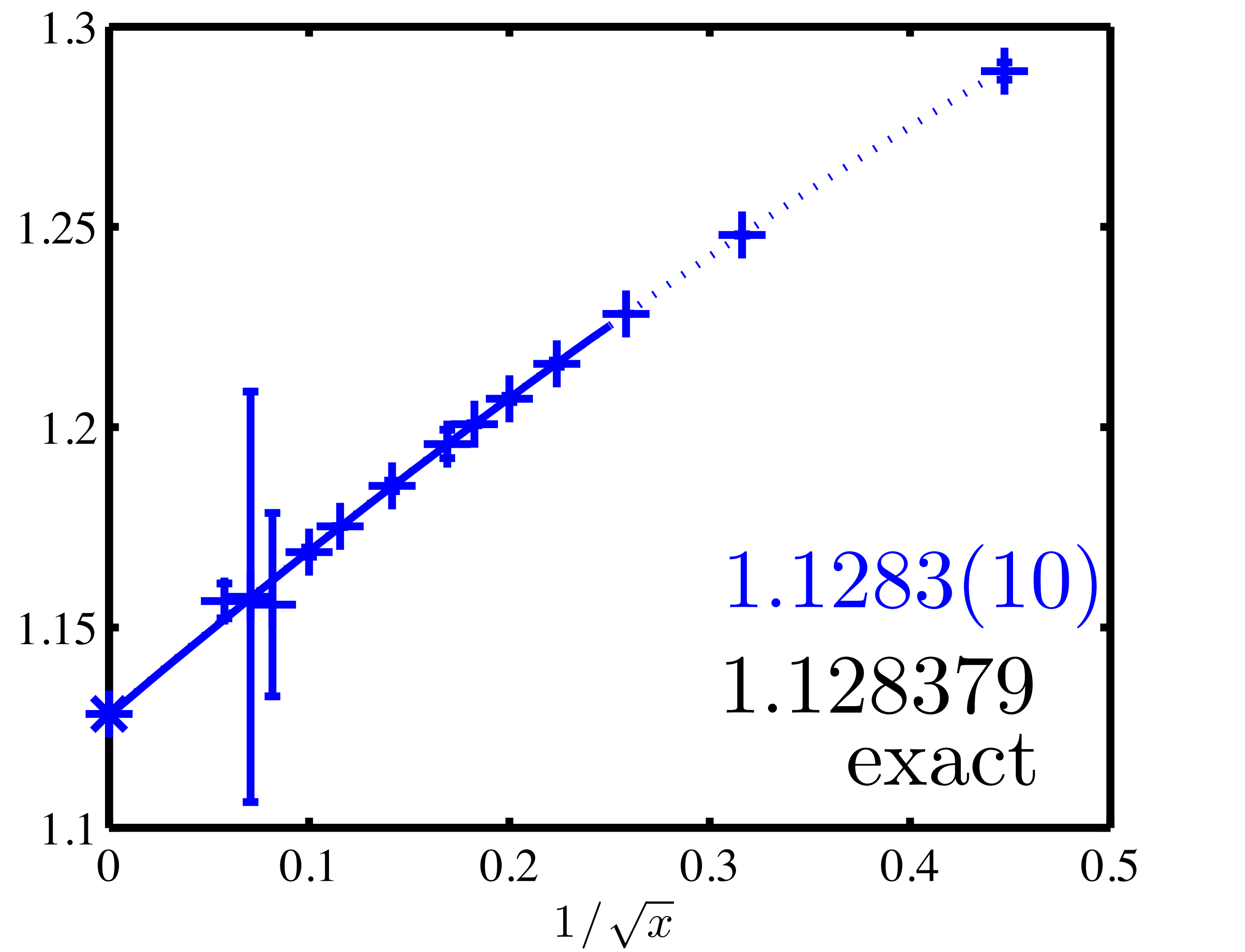

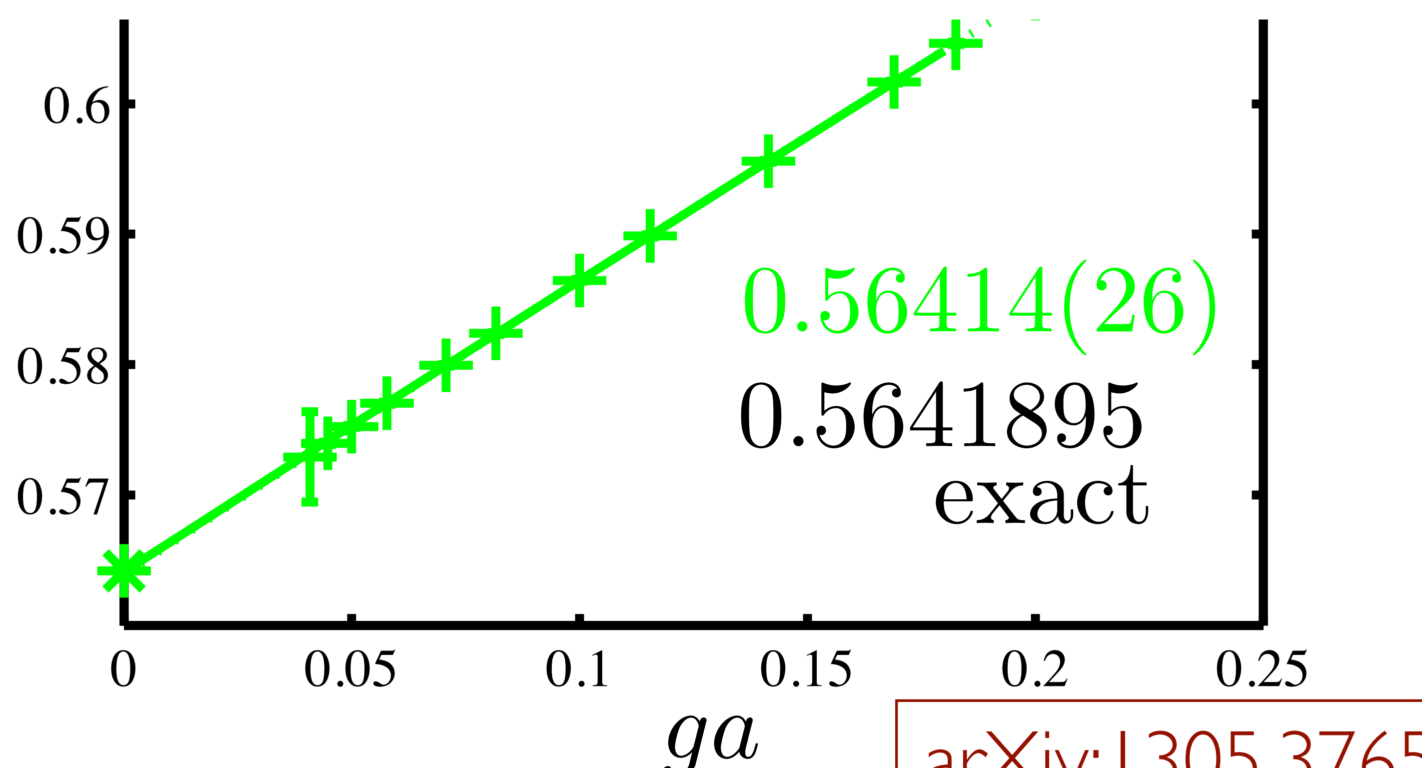

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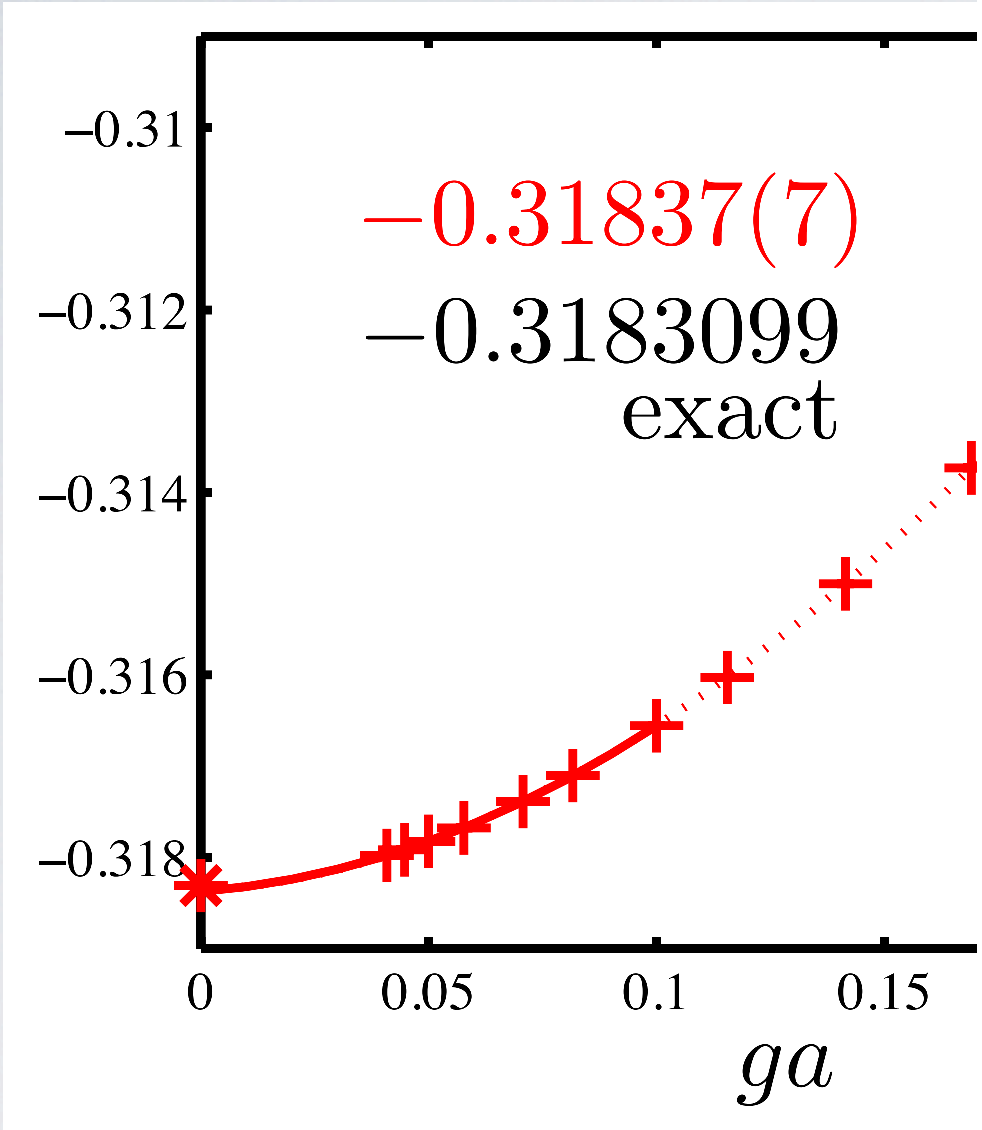

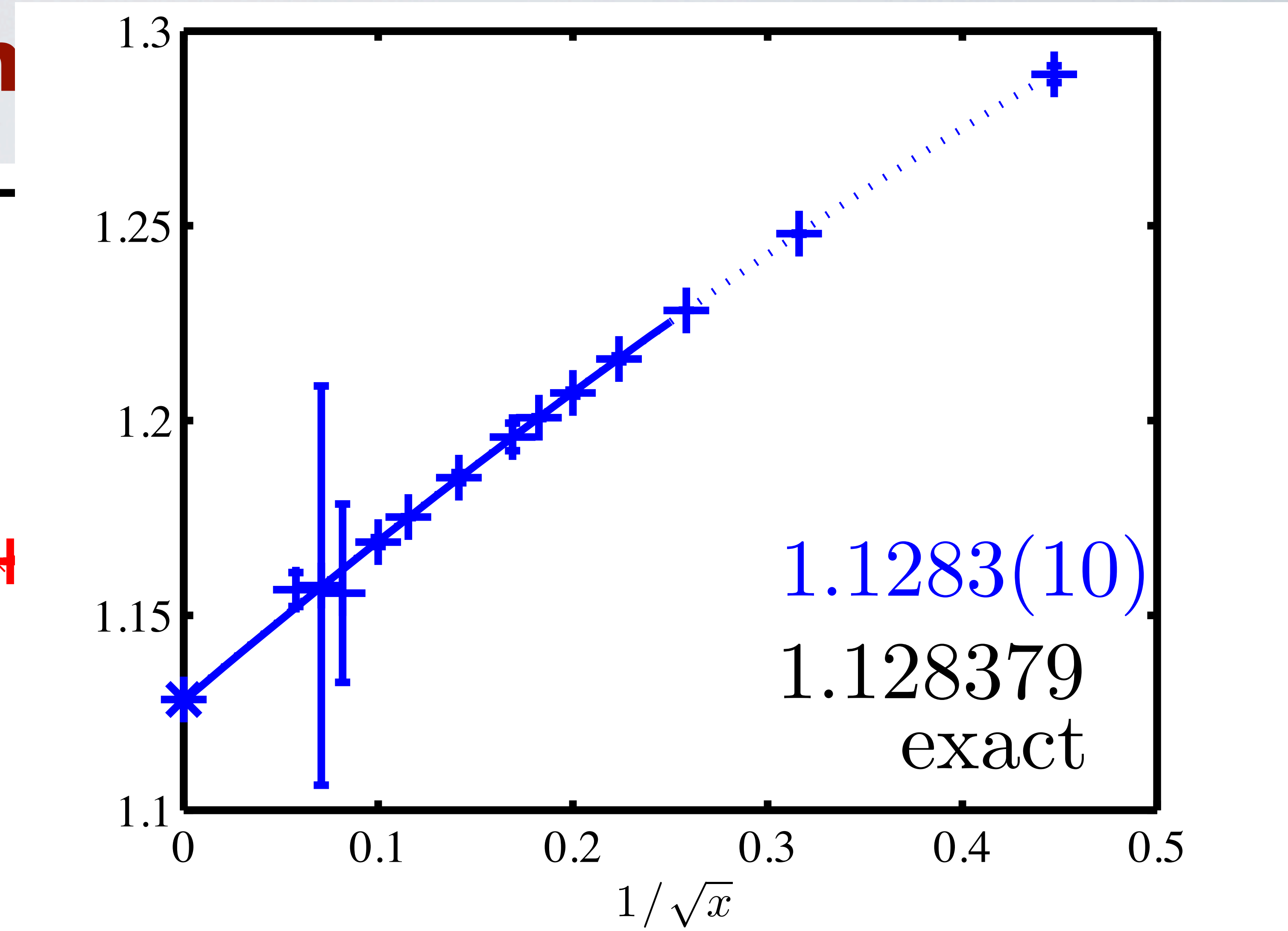

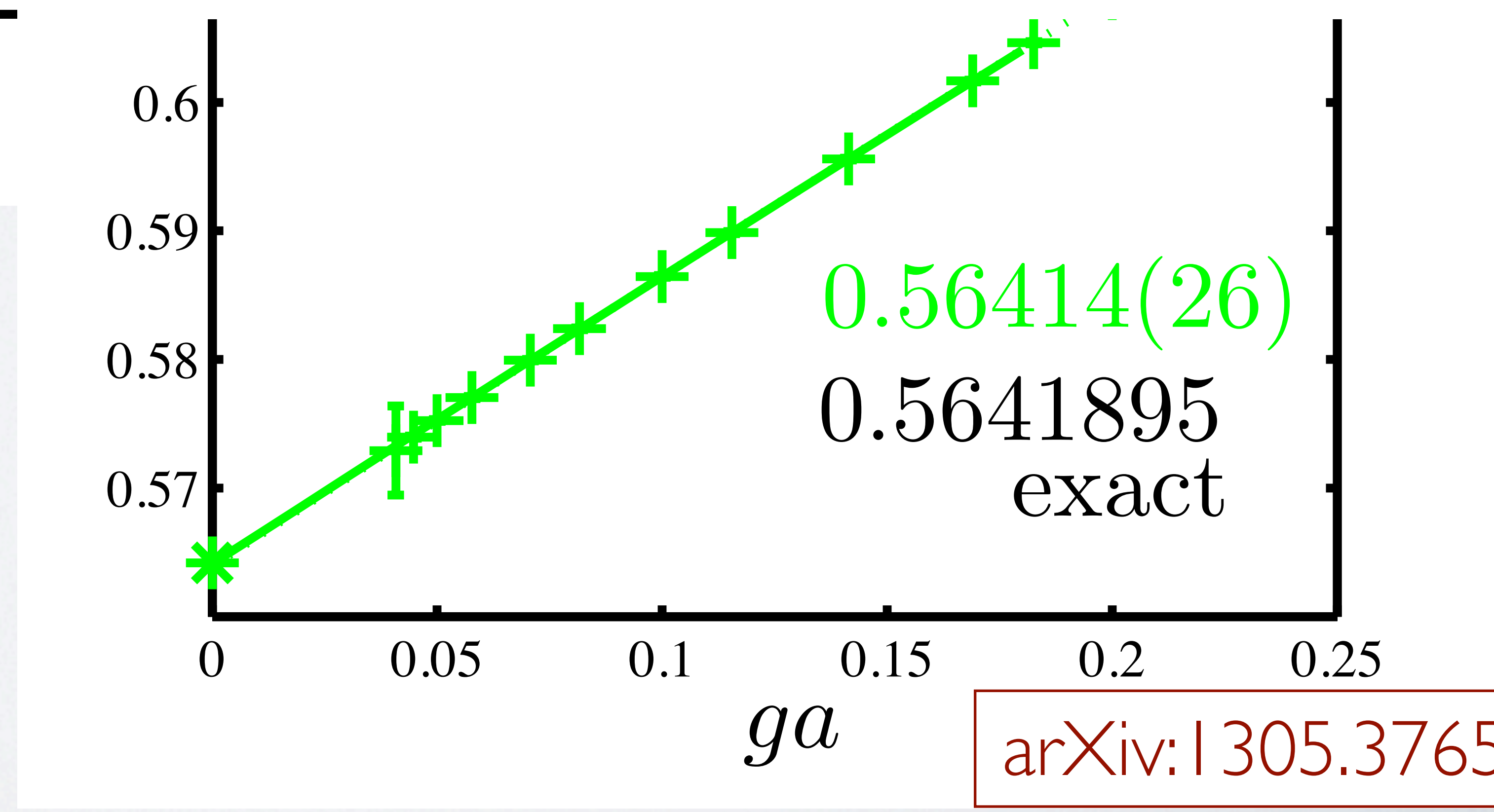

good agreement with exact values

arXiv:1305.3765

same game for massive case

1. **truncation error**
2. **finite-size scaling**
3. **continuum limit**

arXiv:1305.3765

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m/g
0.125
0.25
0.5

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same game for massive case

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m/g	DMRG
0.125	0.53950(7)
0.25	0.51918(5)
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m/g	DMRG	MPS with OBC
0.125	0.53950(7)	0.53946(20)
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m/g	DMRG	MPS with OBC	SCE
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0.25	0.51918(5)	0.51915(14)	1.24(3)
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m/g	DMRG	MPS with OBC	SCE	MPS with OBC
0.125	0.53950(7)	0.53946(20)	1.22(2)	1.221(2)
0.25	0.51918(5)	0.51915(14)	1.24(3)	1.239(6)
0.5	0.48747(2)	0.48748(6)	1.20(3)	1.231(5)

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comparable or better precision than available numerics

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MPS STATES → OBSERVABLES

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compute the chiral condensate in the GS

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in the spin language

$$\frac{\sqrt{x}}{L}\sum_n (-1)^n \frac{1+\sigma_n^3}{2}$$

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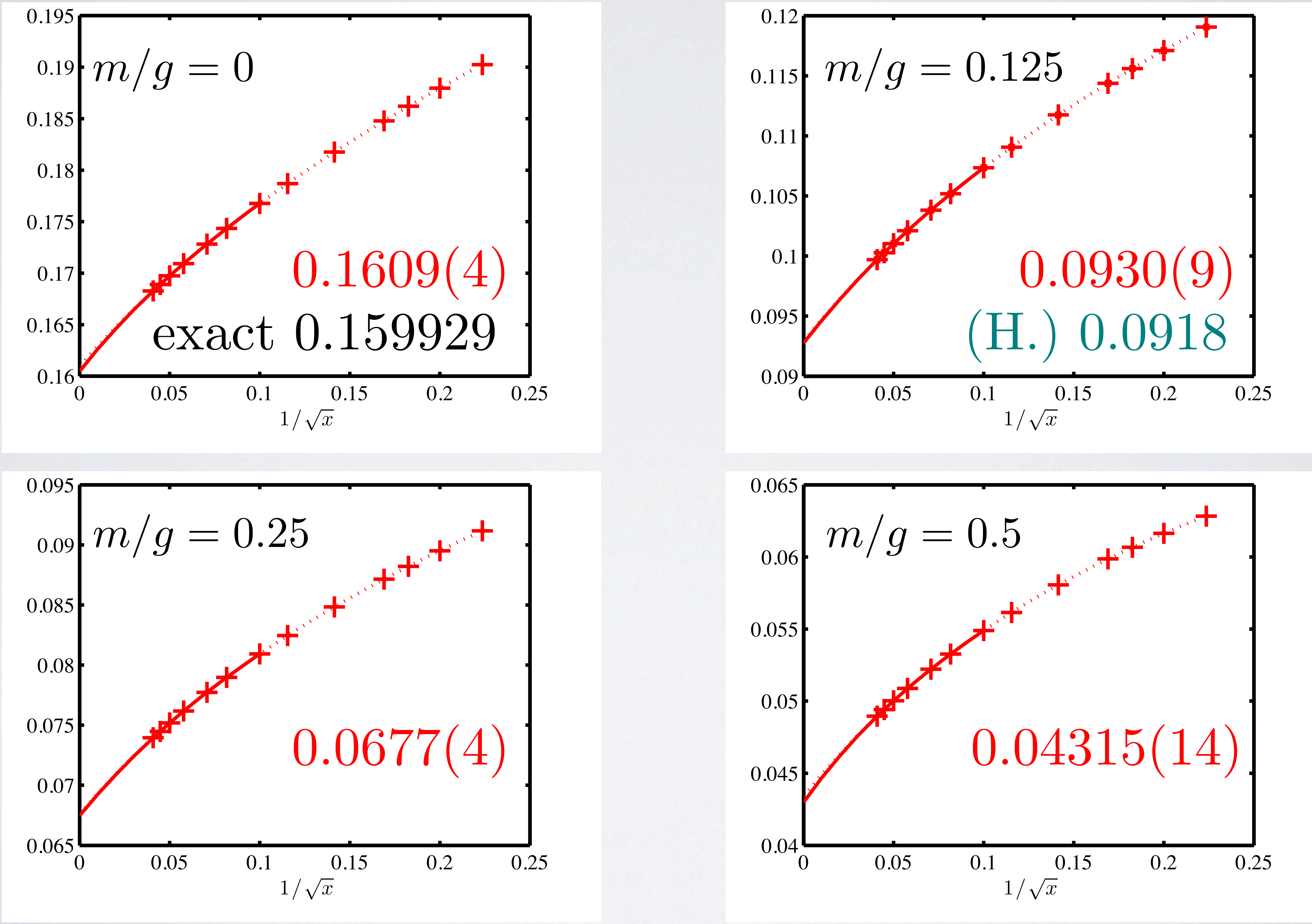
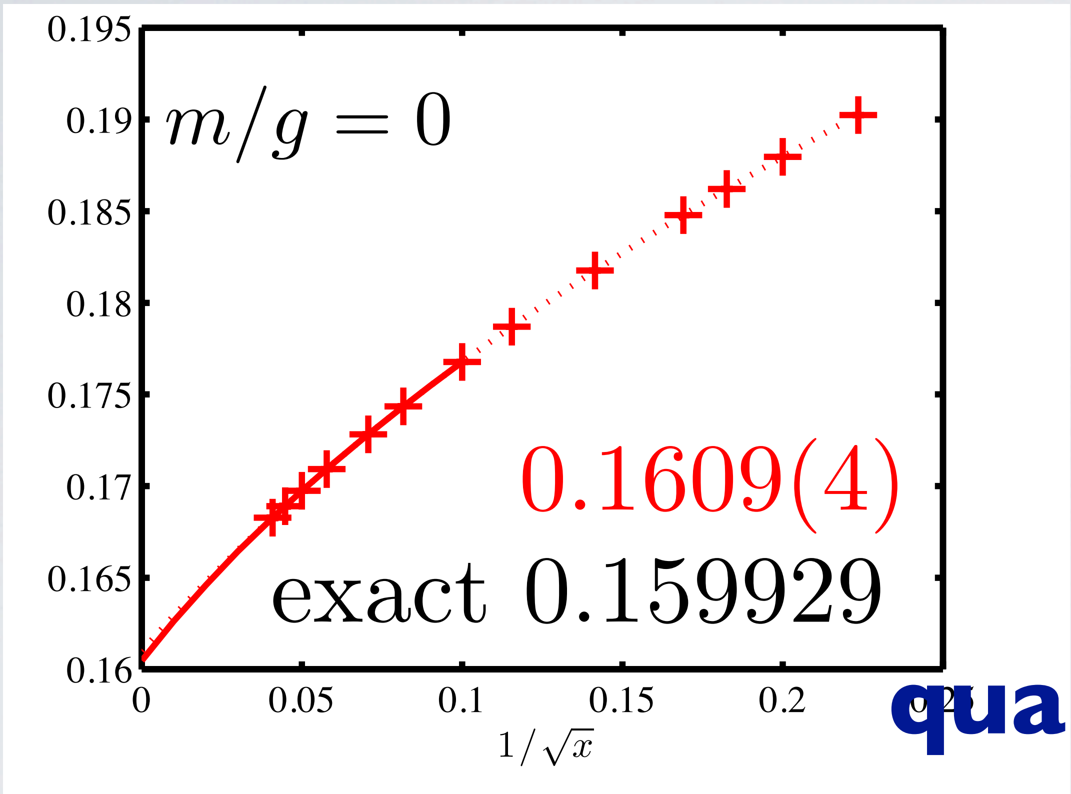
divergent ➞ substracted condensate from exact solution of non-interacting case

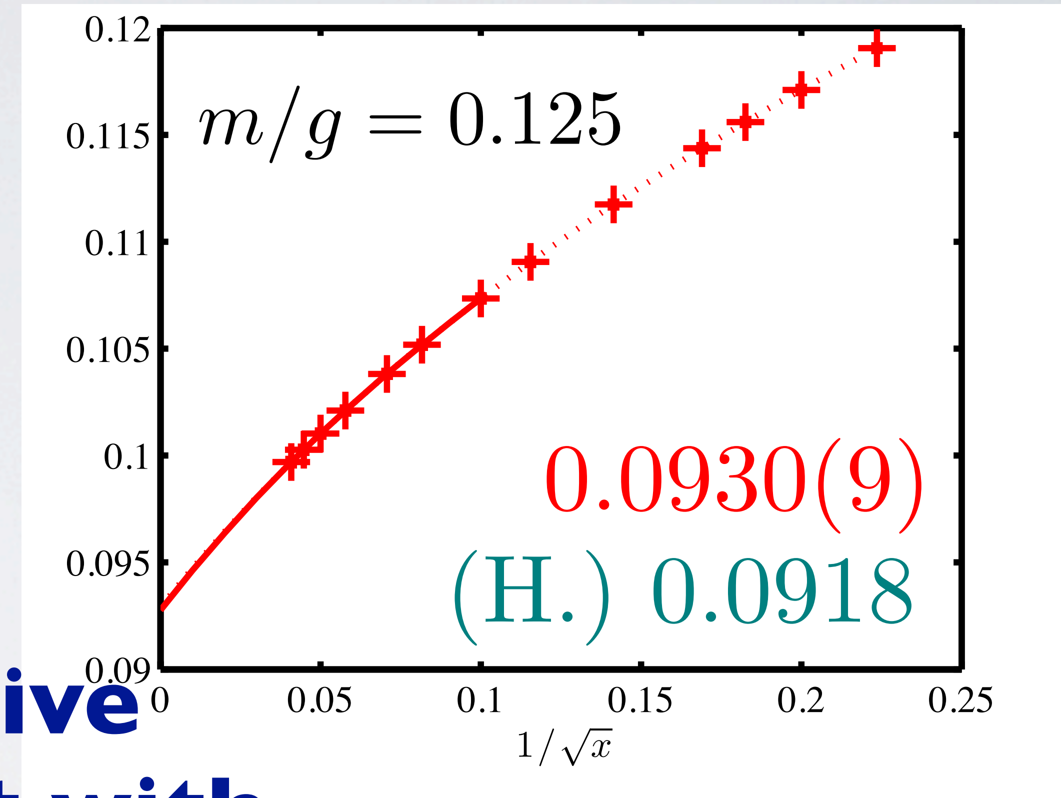
CHIRAL CONDENSATE



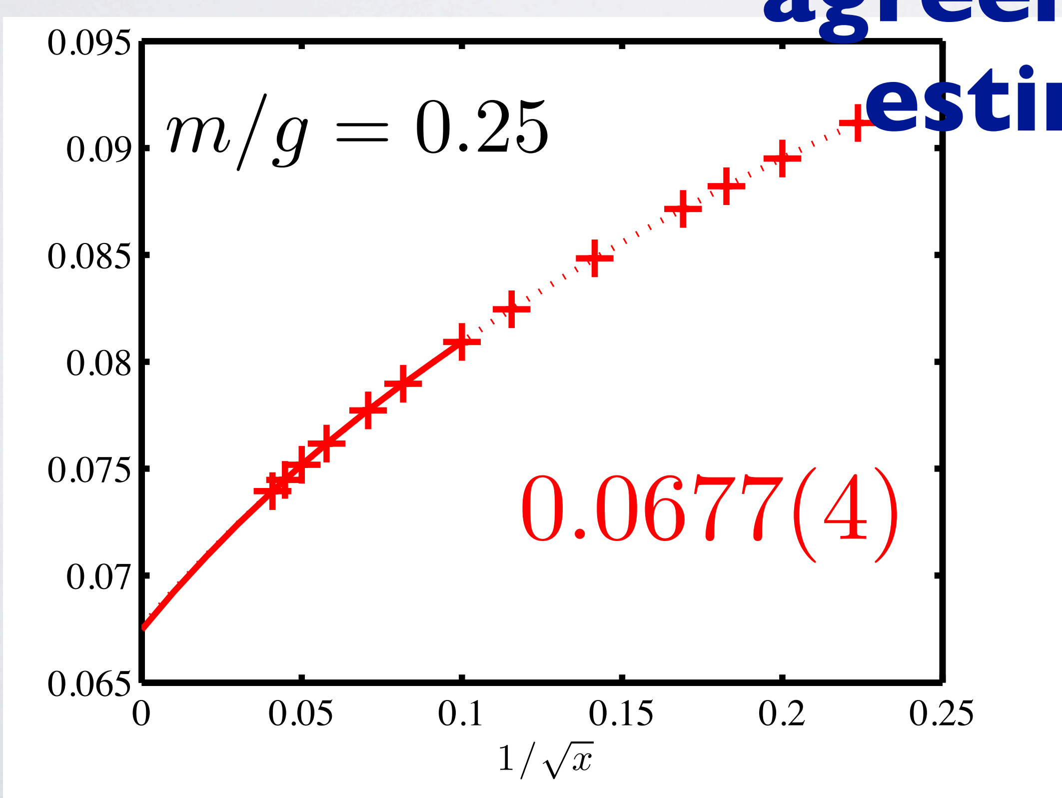
preliminary results

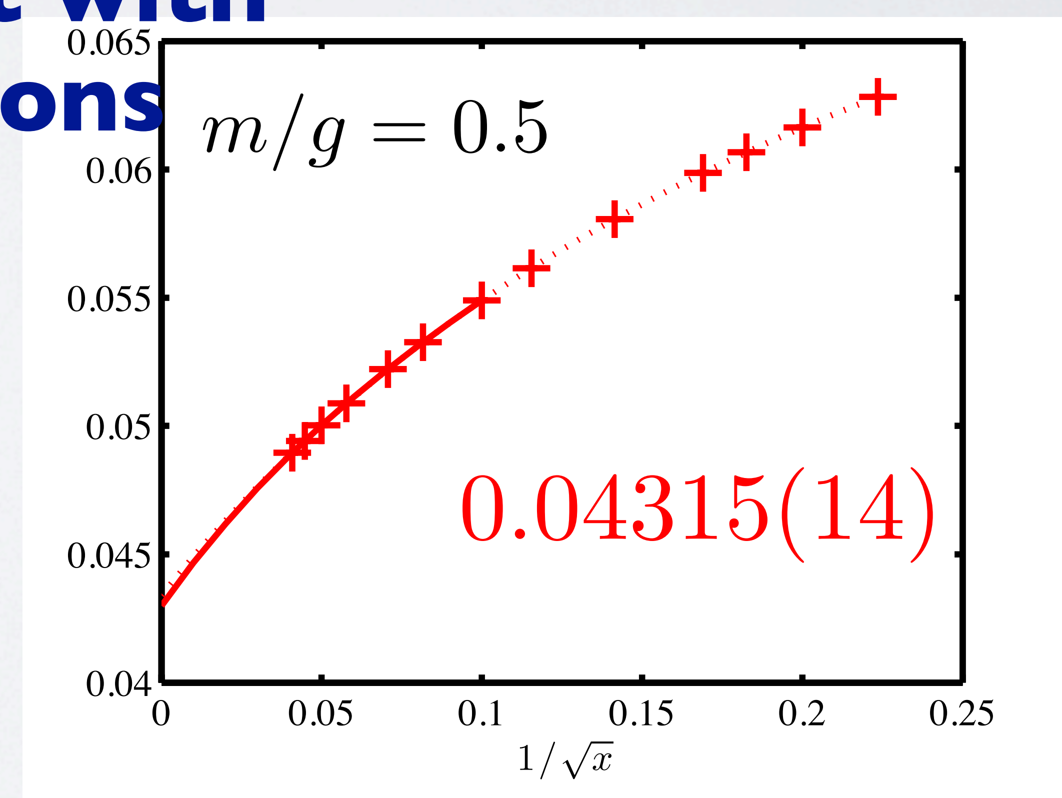
CHIRAL CONDENSATE





qualitative agreement with estimations





TO CONCLUDE

m/g	DMRG	MPS with OBC	SCE	MPS with OBC
0	0.5641859	0.56414(26)	1.128379	1.1283
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also condensate, other observables

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THANK YOU!

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