Emergence of a pseudo-Goldstone Boson in a (2+1)-d U(1) pure gauge theory

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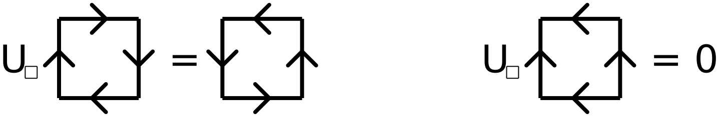
Introduction

- Conventional Wilson formulation of lattice gauge theory uses continuous gauge fields
- This formulation cannot be used to directly answer questions about real-time dynamics or the physics at finite baryon density
- Quantum Link Models (QLM) are an alternative formulation of gauge theories which uses discrete degrees of freedom to realize continuous gauge transformations [Horn, 1981], [Orland, Rohrlich, 1990], [Chandrasekharan, Wiese 1997]
- This allows, in principle, to realize these models on optical lattices or in ion-traps with atoms/molecules/ions
- QLMs also offer a possibility for improved algorithms
- Already the simplest case of an Abelian gauge symmetry offers interesting physics

The (2+1)d U(1) Quantum Link Model (QLM)

$$H = -J \sum_{\square} \left[U_{\square} + U_{\square}^{\dagger} - \lambda \left(U_{\square} + U_{\square}^{\dagger} \right)^2 \right]$$

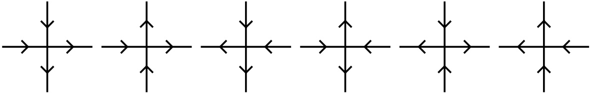
- where $U_{\square} := U_{x,i} U_{x+\hat{i},j} U^{\dagger}_{x+\hat{j},i} U^{\dagger}_{x,j}$
- Instead of $u_{x,i} = \exp(i\varphi_{x,i}) \in U(1)$ we use quantum links
- The links become spin raising operators, $U_{x,i} = S^{+}_{x,i}$
- Spin-$\frac{1}{2}$ represenation, i.e. 2d Hilbert space per link
- $U_{\square}$ and $U_{\square}^{\dagger}$ flip plaquettes with a closed flux loop



- The λ-term counts the number of flippable plaquettes

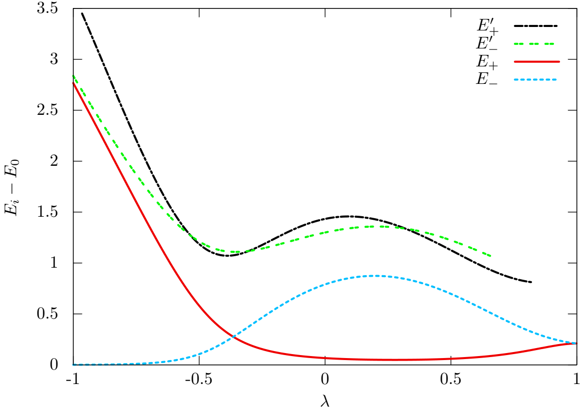
Gauge invariance and the Gauss law

- $[H, G_x] = 0$, where $G_x = \sum_i \left(E_{x,i} - E_{x-\hat{i},i} \right)$ are the generators of infinitesimal gauge transformations
- The Gauss law restricts the system to gauge invariant states, $G_x|\psi\rangle = 0$
- This leads to the following set of allowed configurations at a site x



- 6 instead of $2^4 = 16$ states per site
  $\Rightarrow$ Exact diagonalization

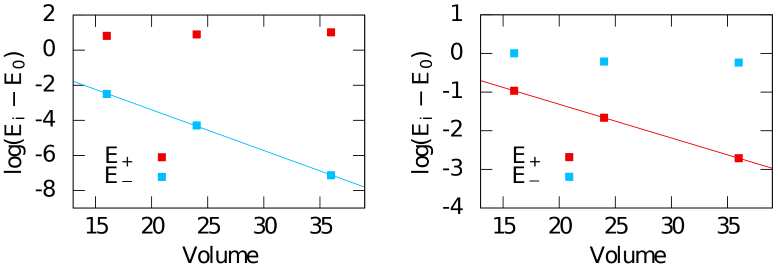
Exact diagonalization results - Energy gaps



E_- and E'_- have $C = -$ and $p = (\pi, \pi)$
E_+ and E'_+ have $C = +$ and $p = (\pi, \pi)$

Exponential energy gaps

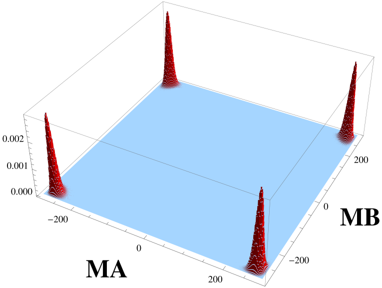
- Spontaneous symmetry breaking: $E_1 - E_0 = A\exp\left(-\sigma L_x L_y\right)$
- Energy gaps at $\lambda = -1$ (left) and $\lambda = 0$ (right):



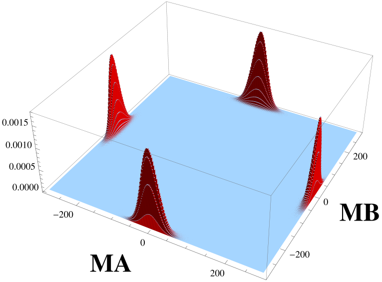
$\Rightarrow C, T$ (left) and T (right) spontaneously broken

Probability distributions $p(M_A, M_B)$ at $T = 0$

Define order parameters M_A and M_B associated with the even and odd dual sublattices A and B to distinguish different symmetry breaking patterns



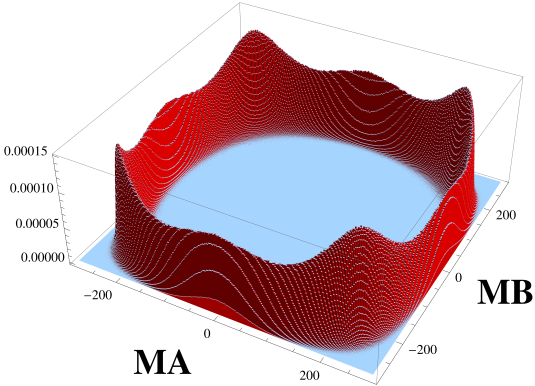
$\lambda = -1$



$\lambda = 0$

$$^{T}M_A = -M_B \quad ; \quad ^{T}M_B = M_A \quad ; \quad ^{C}M_A = M_A \quad ; \quad ^{C}M_B = -M_B$$

Probability distributions $p(M_A, M_B)$ at $T = 0$



$\lambda = \lambda_c$

Effective theory (ET)

- Near λ_c, exact diagonalization shows (approximate) finite-volume rotor spectrum $E_m = \frac{m^2 c^2}{2\rho L_1 L_2}, m$ even
- Emergent $SO(2)$ symmetry that is spontaneously broken
- Formulate effective theory in terms of unit-vector field $\vec{e}(x) = (\cos\varphi(x), \sin\varphi(x))$ representing direction of (M_A, M_B)
- (M_A, M_B) indistinguishable from $(-M_A, -M_B)$
  $\Rightarrow \mathbb{R}P(1)$ symmetry instead of $SO(2)$
- Therefore only states invariant against sign changes of $\vec{e}(x)$ belong to the physical Hilbert space

Effective theory

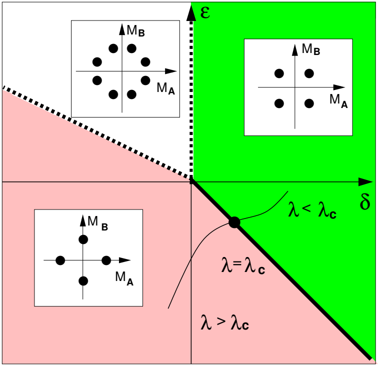
- Effective Euclidean action (using $\partial_3 = \partial_{ct}$):

$$S[\varphi] = \int d^3x \frac{1}{c} \left[\frac{\rho}{2} \partial_\mu \varphi \partial_\mu \varphi + \delta \cos^2(2\varphi) + \varepsilon \cos^4(2\varphi) \right]$$

- $\delta + \epsilon$ measures deviation from the phase transition
- δ breaks emergent symmetry from $SO(2)$ to $\mathbb{Z}(4)$
- This leads to small Goldstone boson mass $Mc = 2\sqrt{2|\delta|/\rho}$
- The ϵ-term is needed in order to avoid a vanishing string tension at λ_c (which we see neither in exact diagonalizations nor in simulations)

Phase diagram in the δ-ϵ-plane

- From mean field theory applied to the effective theory:

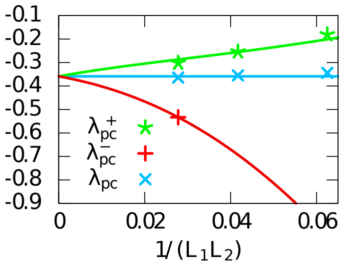


- Solid line is 1st order, dotted lines are 2nd order

Global fit of exact diagonalization data

- We perturbatively analyzed the effective theory spectrum
- A global fit of exact diagonalization data with these predictions for the energy eigenvalues yields

$$\lambda_c = -0.359(5)$$
$$\delta_c = -\epsilon_c = 0.01(1)\, J/a^2$$
$$\rho = 0.45(3)\, J$$
$$c = 1.5(1)\, Ja$$



- These values together with phase diagram from ET imply a weak first order phase transition

Conclusions

- Exact diagonalization on relatively small lattices already provides very useful information about the phase structure in the ultimate large volume regime
- We observed an emergent SO(2) symmetry with an associated pseudo-Goldstone boson
- More work with the cluster algorithm is under way to get more precise calculations of the ET parameters
- Quantum Link Models together with quantum simulations might be a (long-term) solution to currently unsolvable problems in Lattice QCD
- We are working together with the group of P. Zoller in Innsbruck on schemes for quantum simulating this model (to be published shortly).

Definition of M_A and M_B

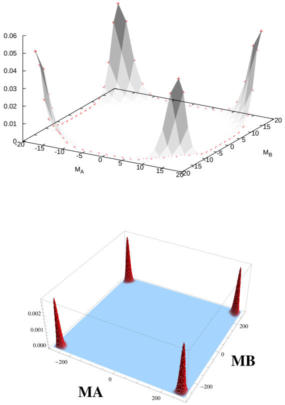
$$h^A_{\widetilde{x}} = 0, 1 \quad ; \quad h^B_{\widetilde{x}} = \pm\frac{1}{2} \quad ; \quad \widetilde{x} = (x_1 + \frac{1}{2}, x_2 + \frac{1}{2})$$

$$E_{x,x+\hat{i}} = [h^X_{\widetilde{x}} - h^{X'}_{\widetilde{x}+\hat{i}-\hat{1}-\hat{2}}] \text{ mod } 2 = \pm\frac{1}{2}, \quad X, X' \in \{A, B\}$$

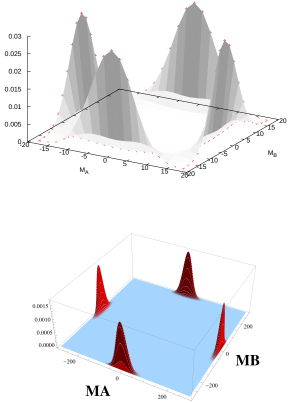
$$M_A = \sum_{\widetilde{x}\in A} (-1)^{(\widetilde{x}_1 - \widetilde{x}_2)/2}\, h^A_{\widetilde{x}}$$

$$M_B = \sum_{\widetilde{x}\in B} (-1)^{(\widetilde{x}_1 - \widetilde{x}_2 + 1)/2}\, h^B_{\widetilde{x}}$$

M_A, M_B from exact diagonalization



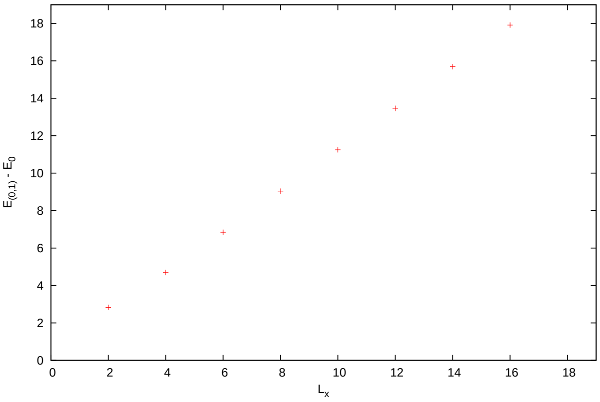
$\lambda = -1$



$\lambda = 0$

Confinement

Energy of a string wrapping around the lattice ($\lambda = 0$):



Sketch of the conjectured phase diagram