First results for SU(2) Yang-Mills with one adjoint Dirac Fermion

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The XXXI International Symposium on Lattice Field Theory
July 29 - August 03 2013
Mainz, Germany

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Overview

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Generalities

- We investigate $SU(2)$ with 1 Dirac flavour in the adjoint representation.
- This can be reexpressed as $SU(2)$ with 2 Majorana flavours in the adjoint representation.
- Why is this interesting?
  - Could this provide a route to lattice Supersymmetry?
  - Probe the end of the Conformal Window?
    - $SU(2)$ with 2 Dirac / 4 Majoranas is conformal (MWT).
    - $SU(2)$ with 1.5 Dirac / 3 Majoranas is predicted to be borderline.
    - $SU(2)$ with 1 Dirac / 2 Majoranas is predicted to be confining.
  - Technicolor (TC).
    - This cannot be an explanation of EWSB.
    - Too few goldstone bosons.
    - However it might have large anomalous dimension.

Dirac Decomposition: Why?

- We would like to investigate the spectrum in two separate ways:
  - Dealing with Majorana flavours.
  - Dealing with Dirac flavours.
- Hence, reexpress relevant quantities in terms of Majorana fields:
  - Spinors $\psi \longrightarrow (\psi_{\mathrm{M}+}, \psi_{\mathrm{M}-})$
  - Action $S_f^{\mathrm{Dirac}} \longrightarrow S_f^{\mathrm{Majorana}}$
  - Bilinears eg $\overline{\psi}\Gamma'(\Gamma)\psi \longleftrightarrow f(\overline{\psi}_{\mathrm{M}+}\Gamma\psi_{\mathrm{M}+}, \overline{\psi}_{\mathrm{M}+}\Gamma\psi_{\mathrm{M}-}, \overline{\psi}_{\mathrm{M}-}\Gamma\psi_{\mathrm{M}+}, \overline{\psi}_{\mathrm{M}-}\Gamma\psi_{\mathrm{M}-})$

Dirac Decomposition: Algebra

- We work in the chiral representation of the Dirac algebra:

$$\gamma^\mu = \begin{pmatrix} 0 & \overline{\sigma}_\mu \\ \sigma_\mu & 0 \end{pmatrix}$$

with

$$\sigma^\mu = (1_2, \vec{\sigma}), \quad \overline{\sigma}^\mu = (1_2, -\vec{\sigma}) \ .$$

- And γ_5:

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix} \ .$$

- Charge Conjugation:

$$C = \gamma_0\gamma_2\gamma_5 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix} , \qquad \tilde{C} = iC\gamma_5 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} \ .$$

Dirac Decomposition: Dirac $\rightarrow$ Majorana

- Decomposition of a Dirac Spinor ψ into two Majorana spinors.

  See also I. Montvay, Nucl.Phys. B466 (1996) 259-284, [hep-lat/9510042].

- A Dirac spinor is written as: $\psi = \psi_{\mathrm{M}+} + i\psi_{\mathrm{M}-}$.

- Hence:

$$\psi_{\mathrm{M}+} = \frac{\psi + \tilde{C}\overline{\psi}^{\mathrm{T}}}{2} \quad \text{and} \quad \psi_{\mathrm{M}-} = \frac{\psi - \tilde{C}\overline{\psi}^{\mathrm{T}}}{2i} \tag{1}$$

- Majorana Condition: $\psi_{\mathrm{M}} = \tilde{C}\overline{\psi}_{\mathrm{M}}^{\mathrm{T}}$ is satisfied.

- with:

$$\overline{\psi}_{\mathrm{M}+} = \frac{\overline{\psi} + \psi^{\mathrm{T}}\tilde{C}}{2} \quad \text{and} \quad \overline{\psi}_{\mathrm{M}-} = \frac{\psi^{\mathrm{T}}\tilde{C} - \overline{\psi}}{2i} \tag{2}$$

- Using (1) and (2) one can reexpress the action and the fermionic bilinears in terms of Majorana spinors.

Dirac Decomposition: Reexression of the action

- The fermionic action $S_f^{\text{Dirac}} = \overline{\psi}\partial\!\!\!/\psi + m\overline{\psi}\psi$ should be reexpressed in terms of Majorana Spinors.
- The mass term takes the form:

$$\overline{\psi}\psi = \overline{\psi}_{\text{M}+}\psi_{\text{M}+} + \overline{\psi}_{\text{M}-}\psi_{\text{M}-}$$

- The kinnetic term takes the form:

$$\overline{\psi}\partial\!\!\!/\psi = \overline{\psi}_{\text{M}+}\partial\!\!\!/\psi_{\text{M}+} + \overline{\psi}_{\text{M}-}\partial\!\!\!/\psi_{\text{M}-}$$

- The fermionic action can take the form:

$$S_f^{\text{Majorana}} = \overline{\psi}_{\text{M}+}\partial\!\!\!/\psi_{\text{M}+} + \overline{\psi}_{\text{M}-}\partial\!\!\!/\psi_{\text{M}-} + m\left(\overline{\psi}_{\text{M}+}\psi_{\text{M}+} + \overline{\psi}_{\text{M}-}\psi_{\text{M}-}\right)$$

- The massless fermionic action has an SU(2) global chiral symmetry.
- In the presence of a nonzero condensate it breaks to SO(2)=U(1) (Baryon Number B in Dirac basis).

$$\longrightarrow \overline{\psi}\gamma_5\psi \text{ has } B = -1+1 = 0$$

Lattice Formulation: Lattice discretization

- Lattice Action:

$$S = S_G + S_f$$

- Gauge Action:

$$S_G = \beta \sum_p \mathrm{Tr}\left[1 - U(p)\right]$$

- Fermionic Action:

$$S_f^{\mathrm{Dirac}} = \sum_{x,y} \overline{\psi}(x) D(x,y) \psi(y)$$

with massive Dirac operator:

$$D(x,y) = \delta_{x,y} - \frac{\kappa}{2}\left[\left(1-\gamma_\mu\right) U_\mu(x)\delta_{y,x+\mu} + \left(1+\gamma_\mu\right) U_\mu^\dagger(x-\mu)\delta_{y,x-\mu}\right]$$

and:

$$\beta = \frac{2(N=2)}{g^2} \qquad\qquad \kappa = \frac{1}{8+2am}$$

Lattice Formulation: Correlation Functions

- We calculate correlation functions:

$$\langle \mathscr{O} \rangle = \frac{\int D\overline{\psi} D\psi dU \mathscr{O} \mathrm{e}^{-S}}{\int D\overline{\psi} D\psi dU \mathrm{e}^{-S}}$$

  with $\mathscr{O}$ the operator (correlator).

- We use Monte-Carlo techniques to evaluate this integral.
- We produce Gauge Configurations using:
  - RHMC (HiRep).
- Masses can be calculated using as operators $\mathscr{O} = \mathrm{O}^\dagger(\vec{x}, t)\mathrm{O}(\vec{0}, 0)$.
- Correlation functions asymptotically behave as:

$$\lim_{t\to\infty} \left\langle \mathrm{O}^\dagger(\vec{x}, t)\mathrm{O}(\vec{0}, 0) \right\rangle = A\mathrm{e}^{-amn_t}$$

- Operators O encode the right quantum numbers.

Lattice Formulation: Fermionic Bilinears

- Express Majorana Bilinears in terms of Dirac Bilinears.
- Using $\overline{\psi}_{\mathrm{Mi}}\Gamma\psi_{\mathrm{Mj}} = O_{ij}(\Gamma)$ with $i, j \in \{+,-\}$ we obtain:

$$O_{\pm\mp}(\Gamma) = \begin{cases} \frac{1}{4i}\left(\psi^{\mathrm{T}}\tilde{C}\Gamma\psi - \overline{\psi}\Gamma\tilde{C}\overline{\psi}^{\mathrm{T}}\right) & \Gamma = 1, \gamma_5\gamma_\mu, \gamma_5 \\ \pm\frac{1}{2i}\overline{\psi}\Gamma\psi & \Gamma = \gamma_\mu, \gamma_0\gamma_5\gamma \end{cases} \tag{3}$$

$$O_{\pm\pm}(\Gamma) = \begin{cases} \frac{1}{4}\left(2\overline{\psi}\Gamma\psi \pm \psi^{\mathrm{T}}\tilde{C}\Gamma\psi \pm \overline{\psi}\Gamma\tilde{C}\overline{\psi}^{\mathrm{T}}\right) & \Gamma = 1, \gamma_5\gamma_\mu, \gamma_5 \\ 0 & \Gamma = \gamma_\mu, \gamma_0\gamma, \gamma_0\gamma_5\gamma\,, \end{cases}$$

- We extract masses from Dirac and Majorana bilinears...
- Dirac correlator an example:

$$\begin{aligned} &\left\langle \left(\psi^{\mathrm{T}}\tilde{C}\Gamma\psi - \overline{\psi}\Gamma\tilde{C}\overline{\psi}^{\mathrm{T}}\right)^{\dagger}(x)\left(\psi^{\mathrm{T}}\tilde{C}\Gamma\psi - \overline{\psi}\Gamma\tilde{C}\overline{\psi}^{\mathrm{T}}\right)(0)\right\rangle \\ &= -\mathrm{tr}\overline{\Gamma}\tilde{C}{D^{-1}}^{\mathrm{T}}(0;x)\tilde{C}\Gamma D^{-1}(0;x) + \mathrm{tr}\overline{\Gamma}\tilde{C}^{\mathrm{T}}{D^{-1}}^{\mathrm{T}}(0;x)\tilde{C}\Gamma D^{-1}(0;x) \\ &\quad -\mathrm{tr}\tilde{C}\overline{\Gamma}D^{-1}(x;0)\Gamma\tilde{C}{D^{-1}}^{\mathrm{T}}(x;0) + \mathrm{tr}(\tilde{C}\overline{\Gamma})^{\mathrm{T}}D^{-1}(x;0)\Gamma\tilde{C}{D^{-1}}^{\mathrm{T}}(x;0) \\ &= -\frac{1}{4}\mathrm{tr}\overline{\Gamma}D^{-1}(x;0)\Gamma D^{-1}(0;x) \end{aligned}$$

Lattice Formulation: Fermionic Bilinears

Dirac bilinears	Majorana bilinears	$U(1)^P$	correlators
$\bar{\psi}\gamma_0\gamma_5\psi$	$O_{++}(\gamma_0\gamma_5)+O_{--}(\gamma_0\gamma_5)$	0^-	singlet γ_5, $\gamma_0\gamma_5$
$\bar{\psi}\gamma_5\psi$	$O_{++}(\gamma_5)+O_{--}(\gamma_5)$		
$\psi^T C\gamma_5\psi$	$-i(O_{++}(1)-O_{--}(1)+2iO_{+-}(1))$	2^-	triplet 1
$\psi^\dagger C\gamma_5\psi^*$	$-i(O_{++}(1)-O_{--}(1)-2iO_{+-}(1))$	-2^-	
$\bar{\psi}\psi$	$O_{++}(1)+O_{--}(1)$	0^+	singlet 1, γ_0
$\bar{\psi}\gamma_0\psi$	$O_{+-}(\gamma_0)$		
$\psi^T C\psi$	$-i(O_{++}(\gamma_5)-O_{--}(\gamma_5)+2iO_{+-}(\gamma_5))$	2^+	triplet γ_5, $\gamma_0\gamma_5$
$\psi^T C\gamma_0\psi$	$-i(O_{++}(\gamma_5\gamma_0)-O_{--}(\gamma_5\gamma_0)+2iO_{+-}(\gamma_5\gamma_0))$		
$\psi^\dagger C\psi^*$	$-i(O_{++}(\gamma_5)-O_{--}(\gamma_5)-2iO_{+-}(\gamma_5))$	-2^+	
$\psi^\dagger C\gamma_0\psi^*$	$-i(O_{++}(\gamma_5\gamma_0)-O_{--}(\gamma_5\gamma_0)-2iO_{+-}(\gamma_5\gamma_0))$		
$\bar{\psi}\gamma_5\vec{\gamma}\psi$	$O_{++}(\gamma_5\vec{\gamma})+O_{--}(\gamma_5\vec{\gamma})$	0^+	singlet $\gamma_5\vec{\gamma}$, $\gamma_0\gamma_5\vec{\gamma}$
$\bar{\psi}\gamma_0\gamma_5\vec{\gamma}\psi$	$O_{+-}(\gamma_0\gamma_5\vec{\gamma})$		
$\bar{\psi}\gamma_0\vec{\gamma}\psi$	$O_{+-}(\gamma_0\vec{\gamma})$	0^-	singlet $\vec{\gamma}$, $\gamma_0\vec{\gamma}$
$\bar{\psi}\vec{\gamma}\psi$	$O_{+-}(\vec{\gamma})$		
$\psi^T C\vec{\gamma}\psi$	$-i(O_{++}(\gamma_5\vec{\gamma})-O_{--}(\gamma_5\vec{\gamma})+2iO_{+-}(\gamma_5\vec{\gamma}))$	2^-	triplet $\gamma_5\vec{\gamma}$
$\psi^\dagger C\vec{\gamma}\psi^*$	$-i(O_{++}(\gamma_5\vec{\gamma})-O_{--}(\gamma_5\vec{\gamma})-2iO_{+-}(\gamma_5\vec{\gamma}))$	-2^-	

Results: Observables

- We calculate the PCAC Mass:

$$m_{\text{PCAC}} = \frac{\sum_{\vec{x}} \langle \partial_0 A_0(x) P(0) \rangle}{2 \sum_x \langle P(x) P(0) \rangle}$$

with $A_0(x) = \overline{\Psi}_{M+}(x)\gamma_0\gamma_5\psi_{M-}(x)$ and $P(x) = \overline{\Psi}_{M+}(x)\gamma_5\psi_{M-}(x)$

- Meson Masses
- Glueball Masses

$\rightarrow$ Mass of 0^{++} glueball

- Gluino-Glue mass (spin $1/2$ Majorana fermion) G. Bergner *et al.* JHEP 09 (2012) 108, [arXiv:1206.2341]

Operator: $\mathrm{O} = \sum_{\mu\nu} \frac{1}{2} [\gamma_\mu, \gamma_\nu] \, \mathrm{Tr}[F^{\mu\nu}\psi_{\mathrm{M}}]$

- Torelon Masses

$\rightarrow$ String Tension σ

- Wilson Loops

$\rightarrow$ String Tension σ

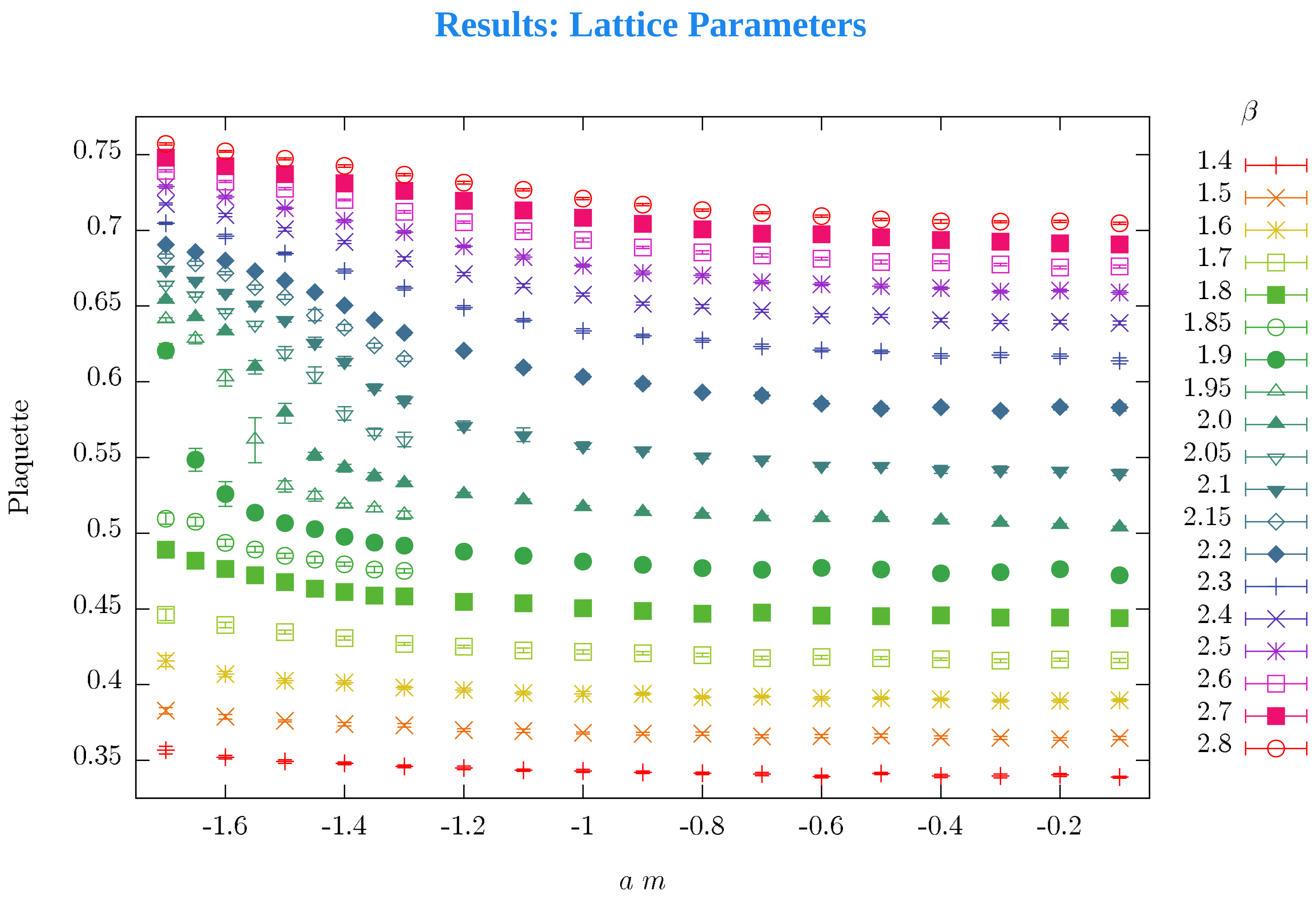
- Dirac Mode Number $\overline{\nu}(\Omega)$

$\rightarrow$ Mass Anomalous Dimension

RESULTS

Results: Lattice Parameters

- This theory has not been investigated on the Lattice before.
- What parameters (β and am) shall we use?
- We looked at the average plaquette on a 4^4 lattice for $1.4 \leq \beta \leq 2.8$ and $-1.7 \leq am \leq -0.1$



Results: Lattice Parameters
Plaquette
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
-1.6
-1.4
-1.2
-1
-0.8
-0.6
-0.4
-0.2
a m
β
1.4
1.5
1.6
1.7
1.8
1.85
1.9
1.95
2.0
2.05
2.1
2.15
2.2
2.3
2.4
2.5
2.6
2.7
2.8

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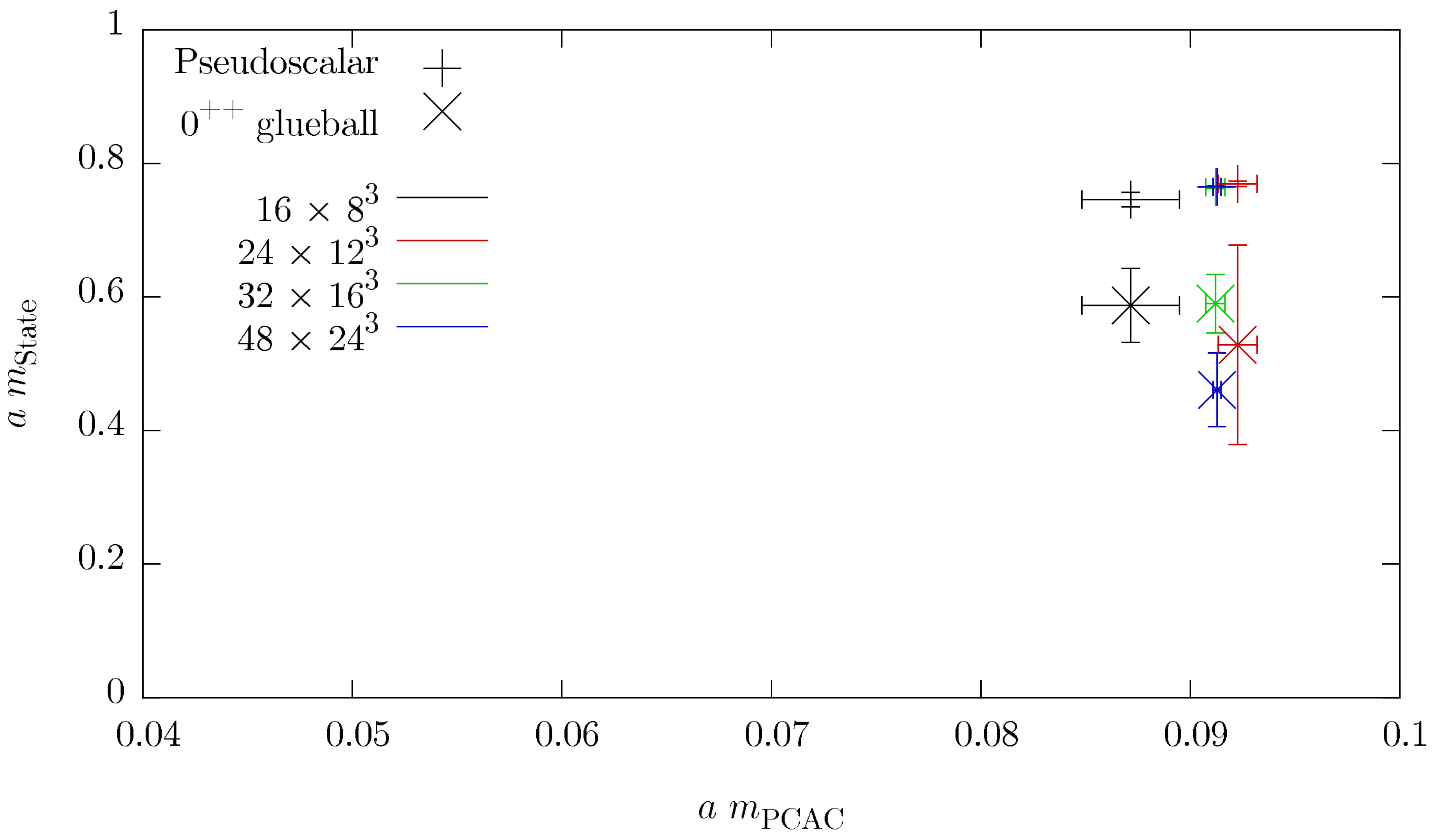
- What parameters (β and am) shall we use?
- We looked at the average plaquette on a 4^4 lattice for $1.4 \leq \beta \leq 2.8$ and $-1.7 \leq am \leq -0.1$
- Bulk phase transition at $\beta = 1.9$ and $am = -1.65$.
- Perform spectroscopy for $8^3 \times 16$, $12^3 \times 24$, $16^3 \times 32$ and $24^3 \times 48$ and $\beta = 2.05$ and $am \sim -1.5$
- We use the HiRep code.

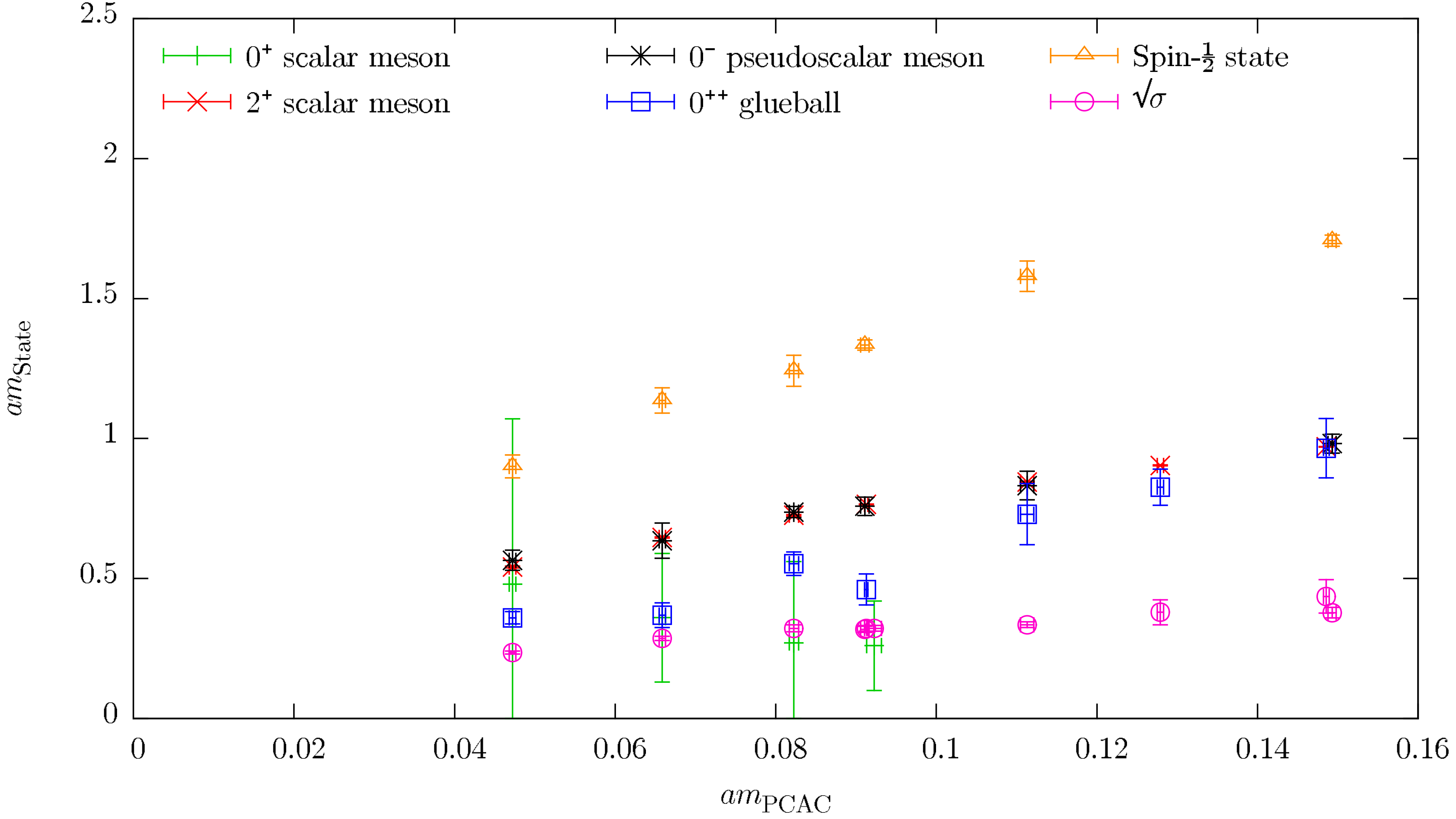
Results: Lattice Parameters

Lattice	V	$-am$	am_{PCAC}	N_{conf}
A1	16×8^3	1.475	0.1489(9)	2400
A2	16×8^3	1.500	0.1101(12)	2200
A3	16×8^3	1.510	0.0904(14)	2400
A4	16×8^3	1.510	0.0872(22)	4000
B1	24×12^3	1.475	0.1493(5)	2400
B2	24×12^3	1.500	0.1113(8)	2300
B3	24×12^3	1.510	0.09226(92)	4000
C1	32×16^3	1.475	0.1485(4)	2100
C2	32×16^3	1.490	0.1279(2)	2300
C3	32×16^3	1.510	0.09111(31)	2200
C4	32×16^3	1.510	0.09048(52)	2300
C5	32×16^3	1.514	0.08223(34)	2300
C6	32×16^3	1.519	0.06587(37)	2300
C7	32×16^3	1.523	0.04840(54)	2200
D1	48×24^3	1.510	0.09130(27)	1534
D2	48×24^3	1.523	0.04722(43)	2168

Results: Finite Volume effects



Results: Spectrum



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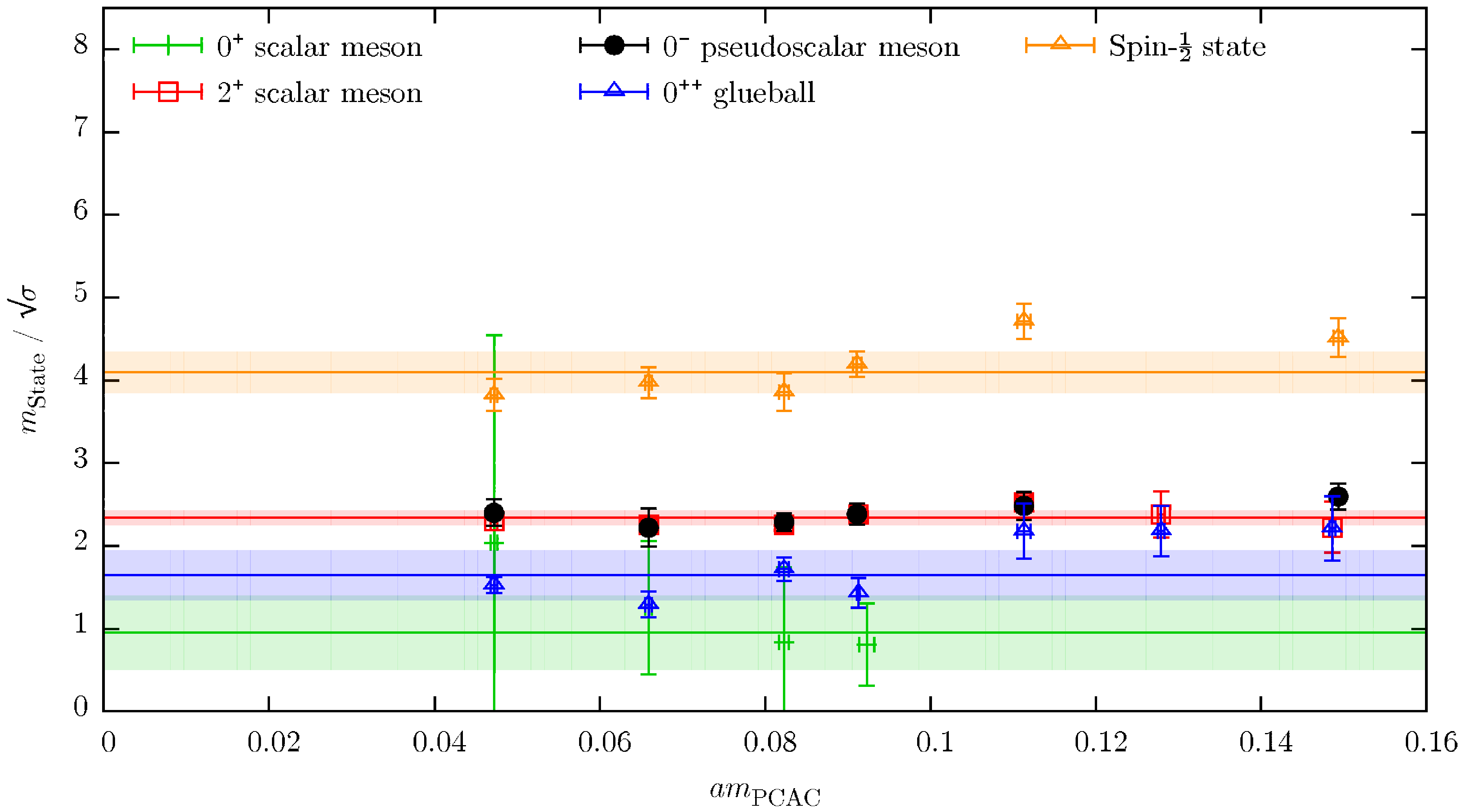

Results: Mass Anomalous Dimension

- The determination of the mass anomalous dimension γ_* is one of our goals.
- This is important for mass scaling.
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Let us vary $0.1 \leq \gamma_* \leq 2.0$

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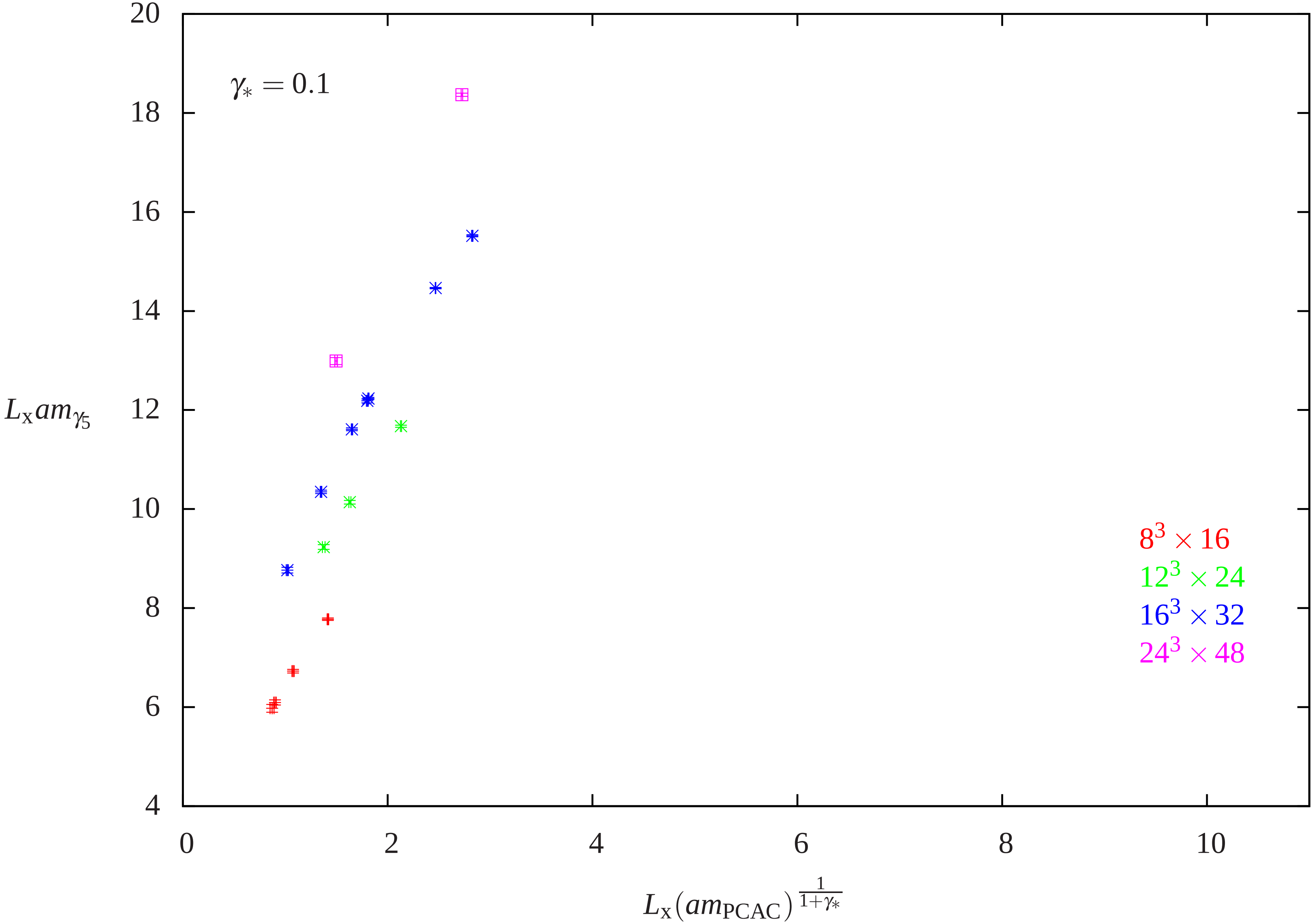

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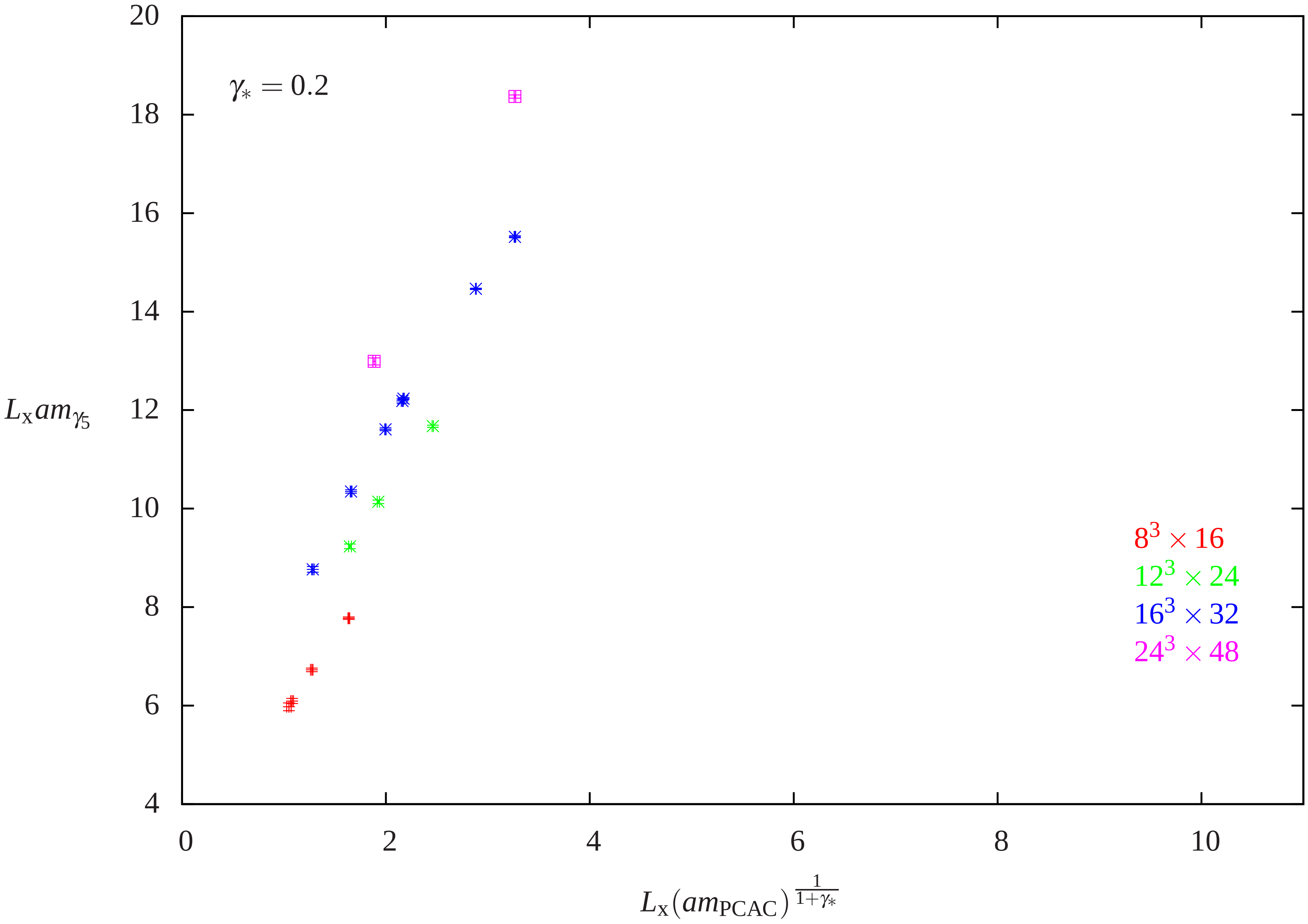

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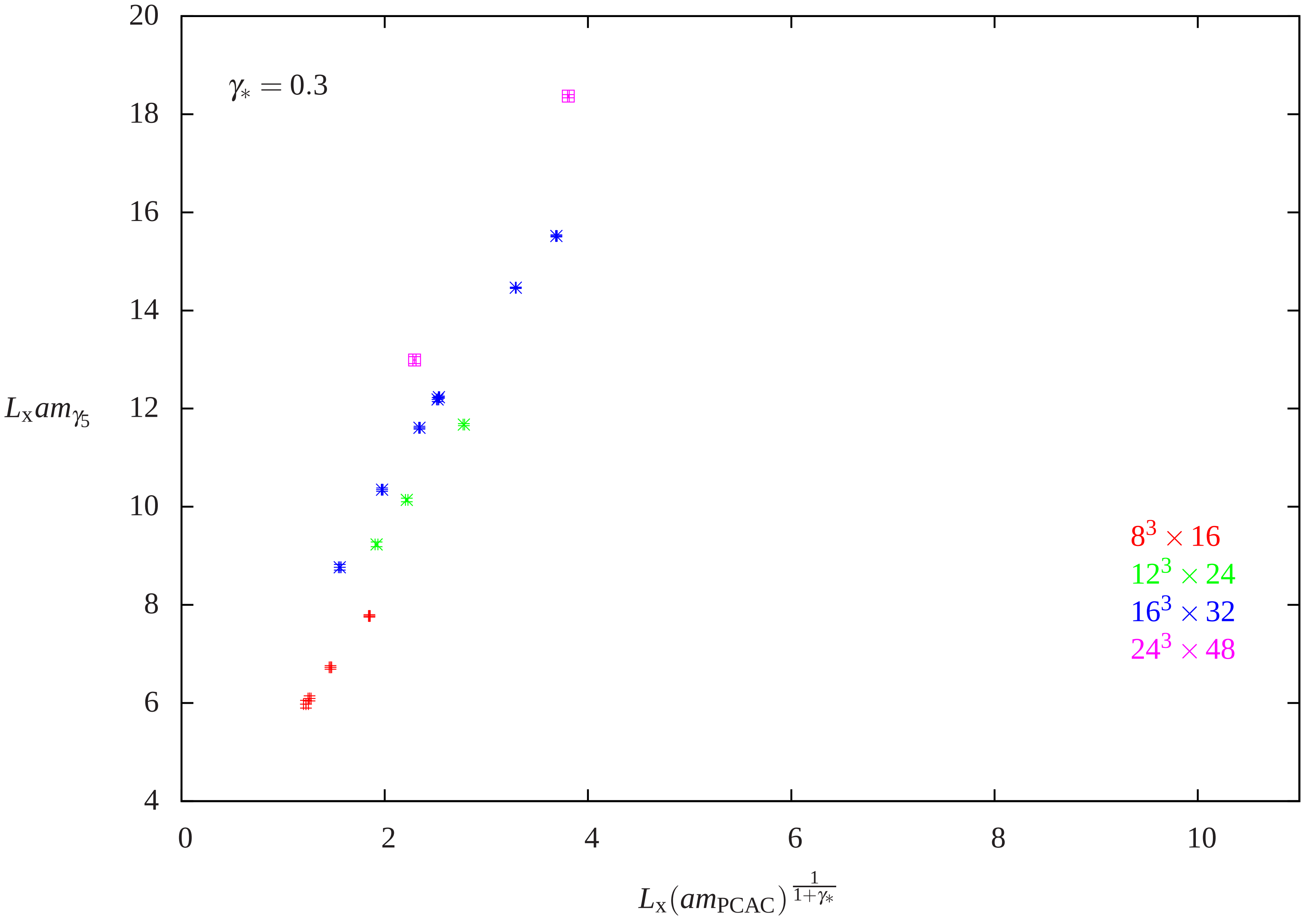

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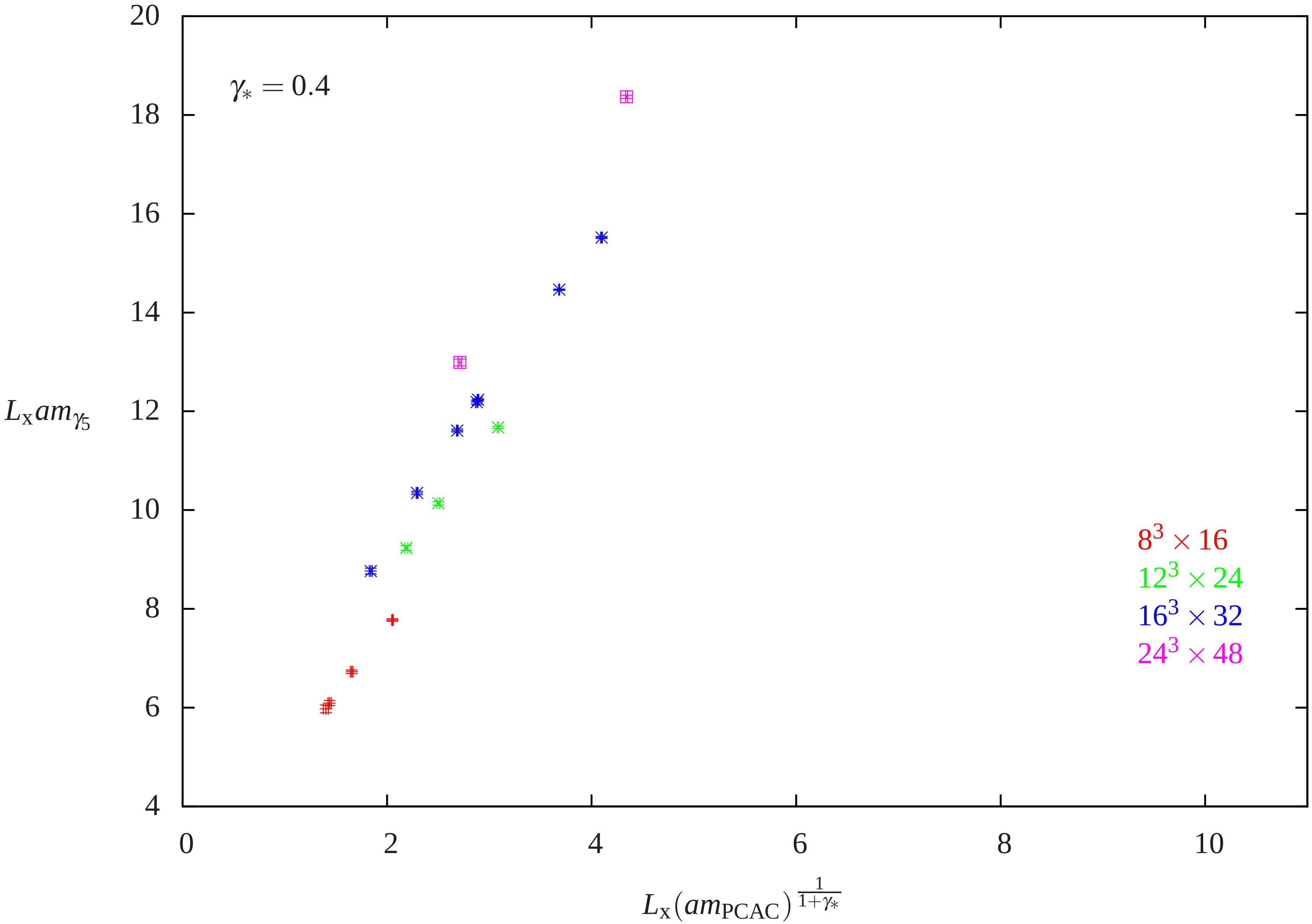

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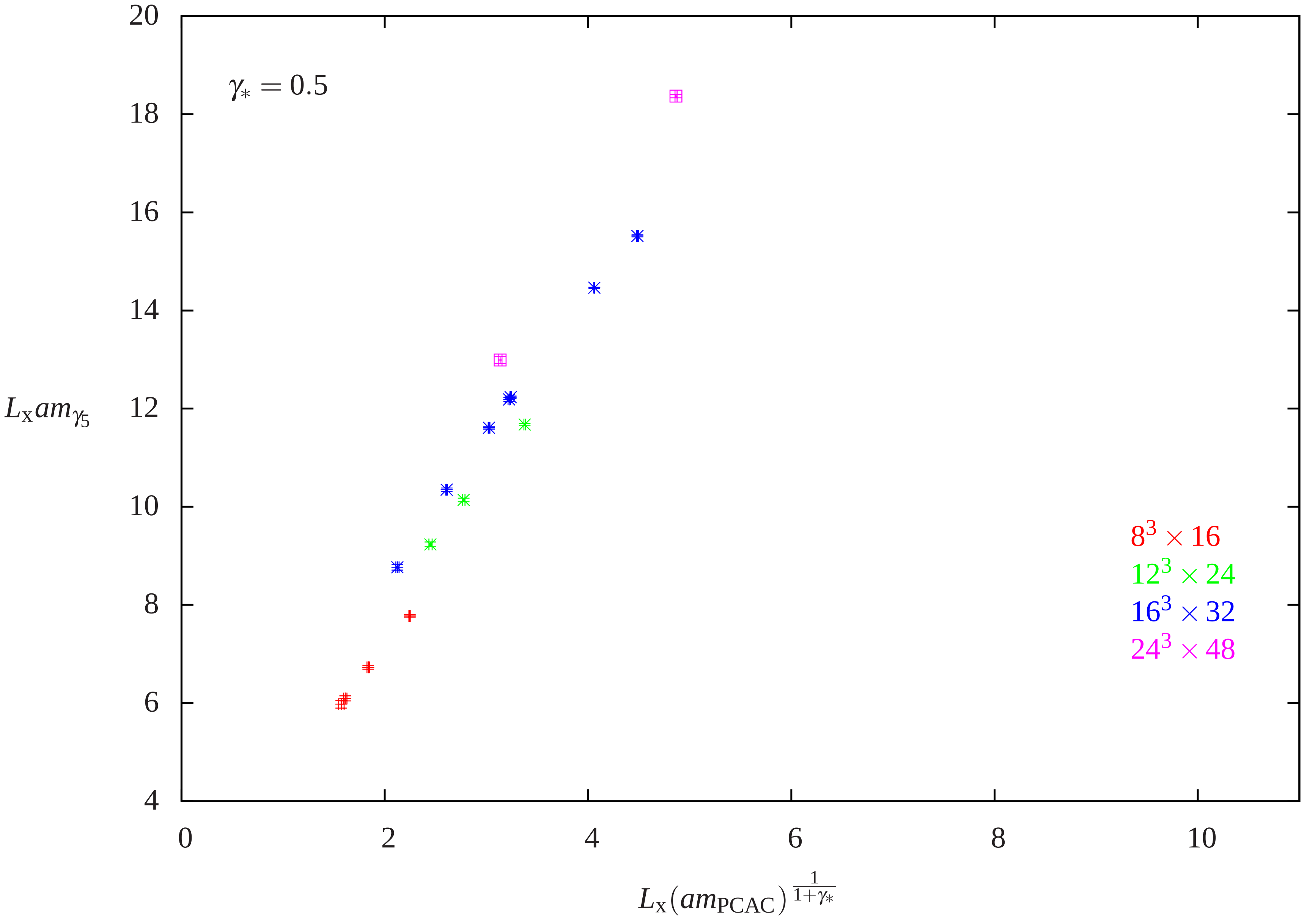

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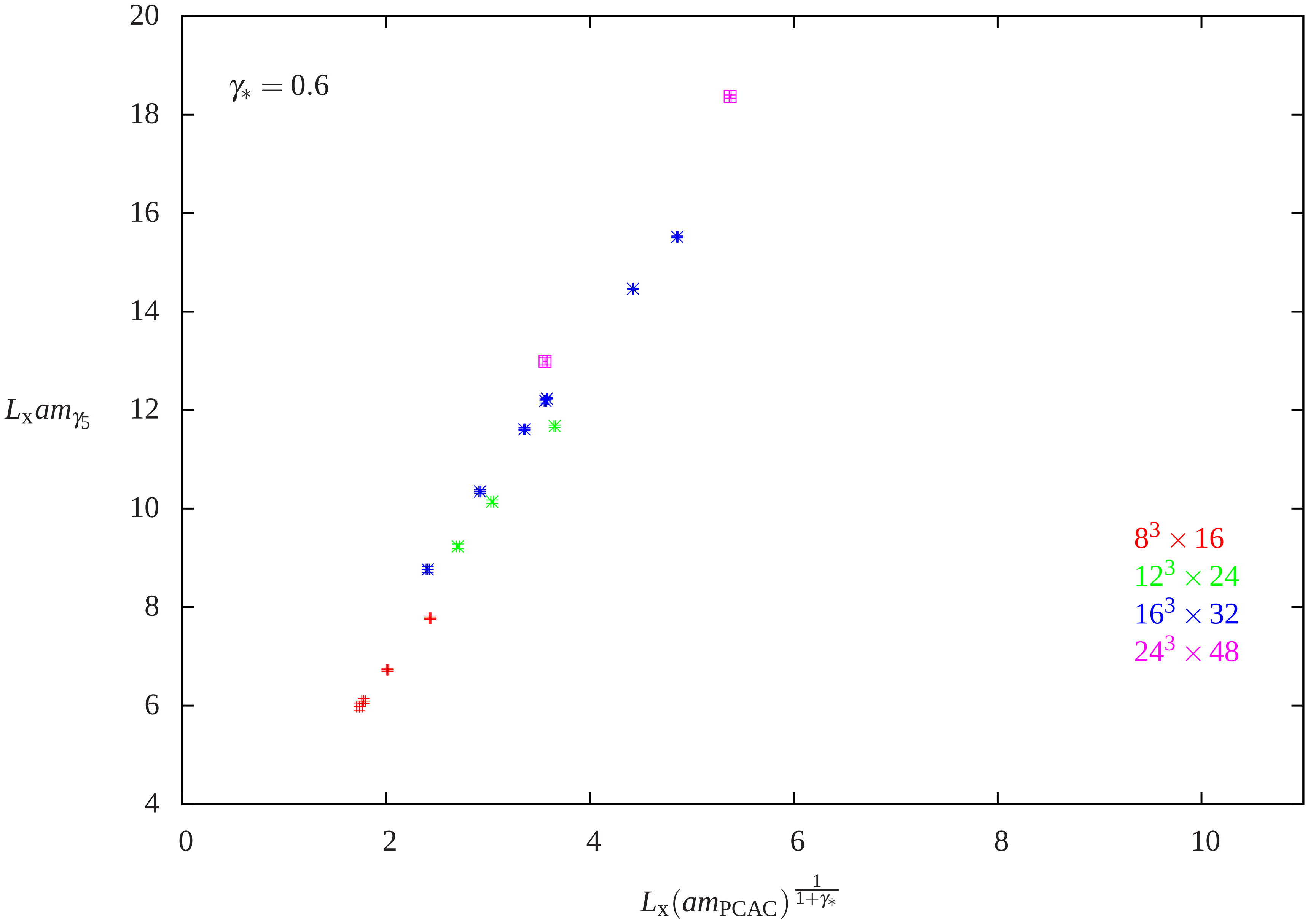

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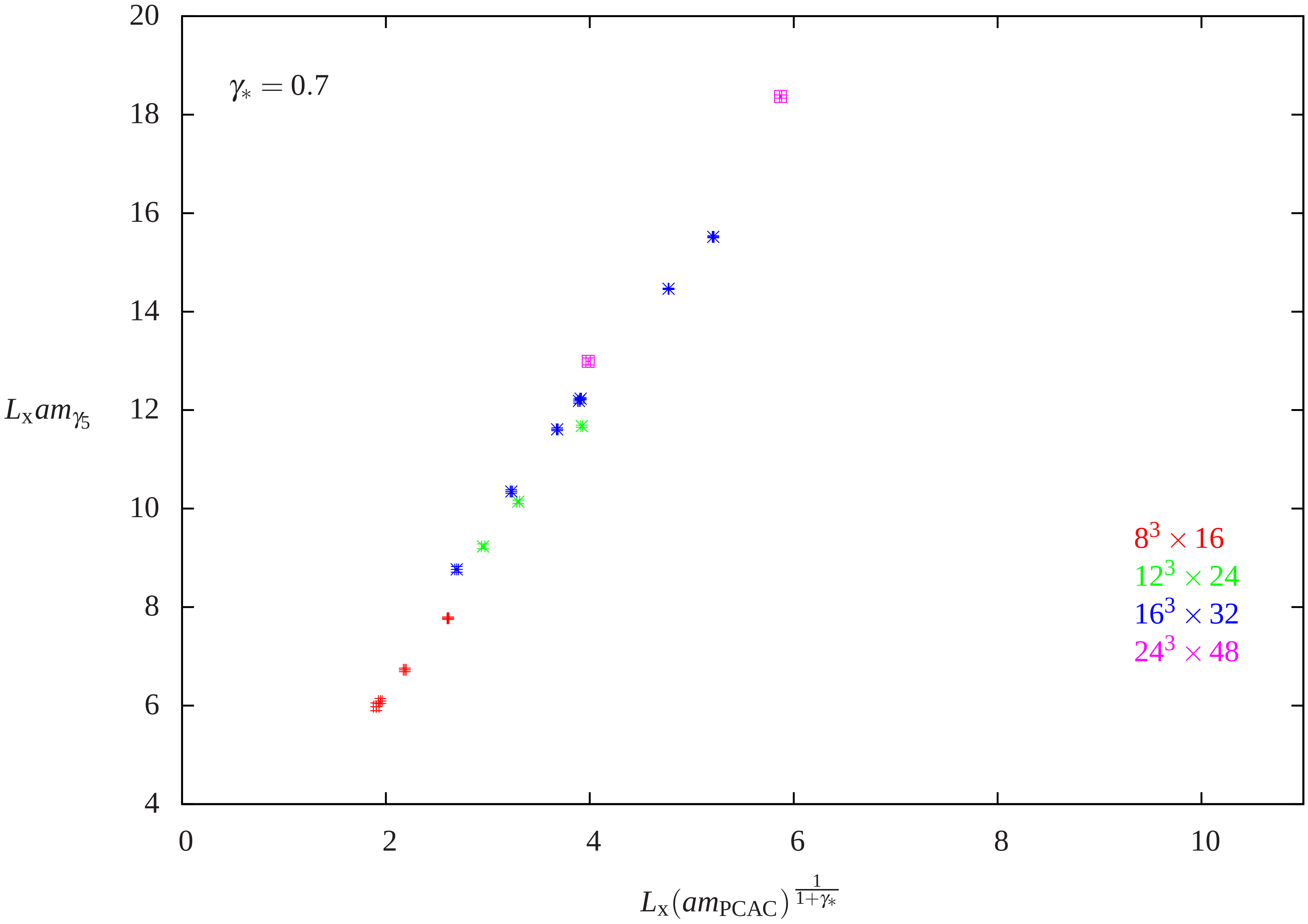

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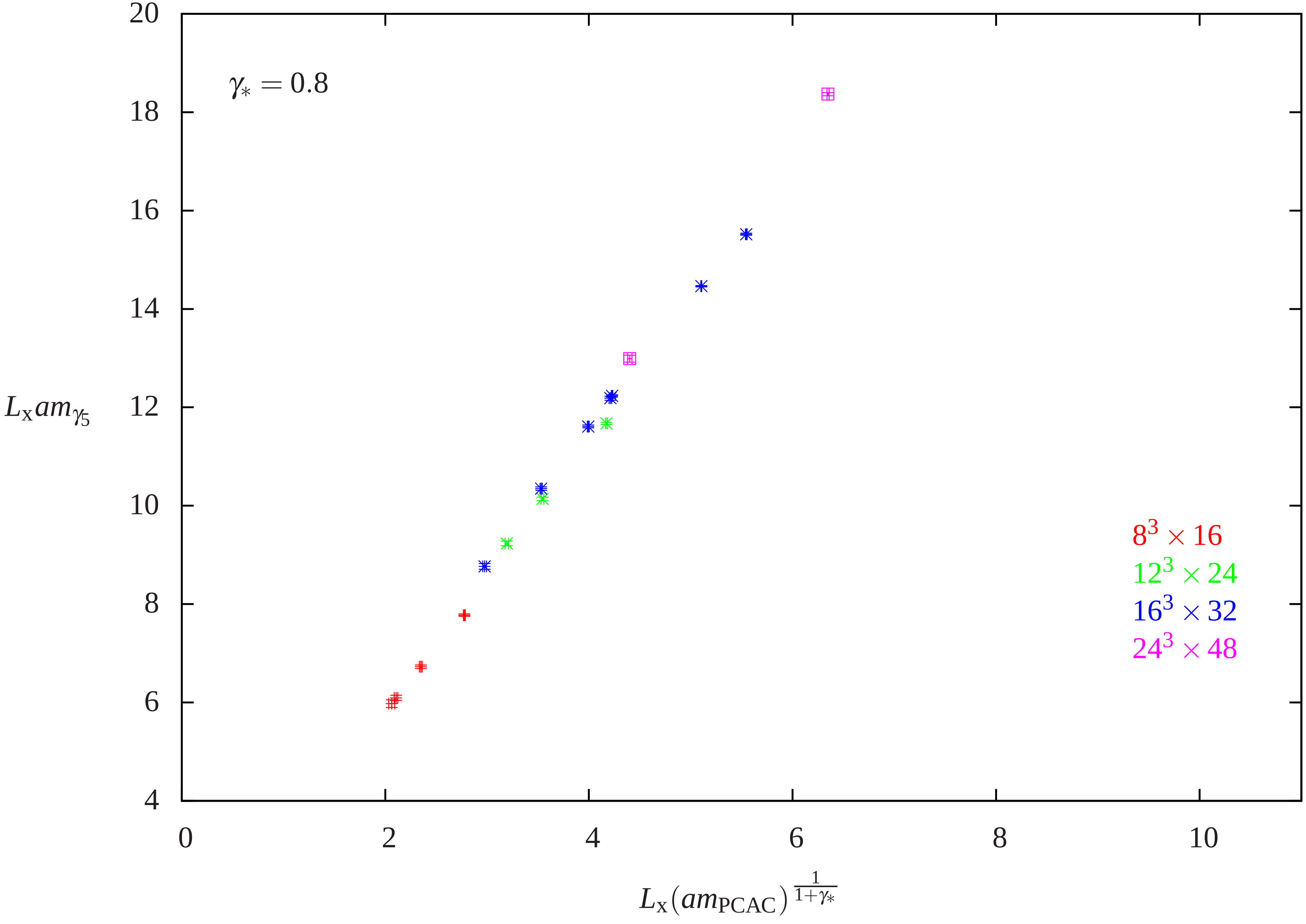

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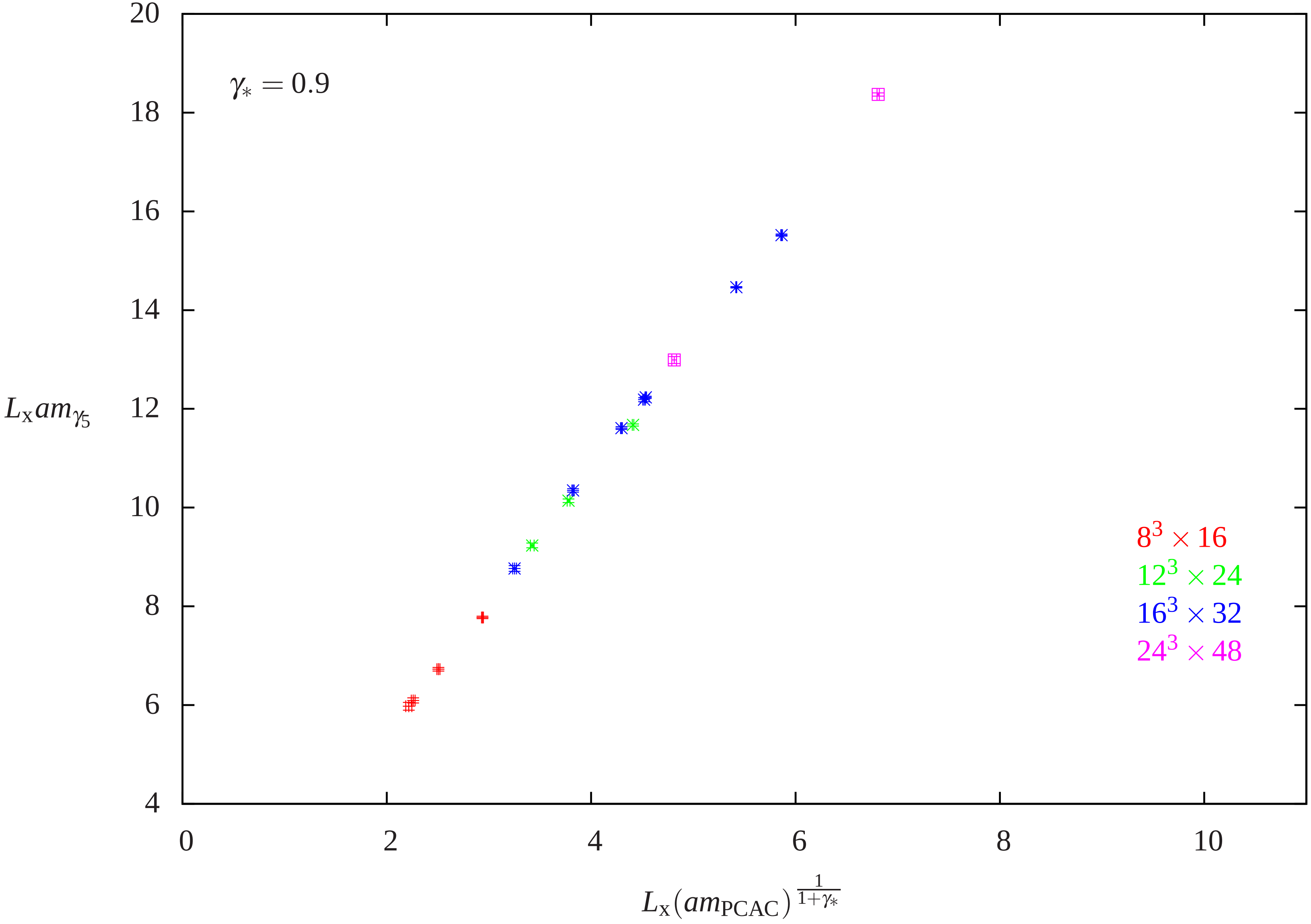

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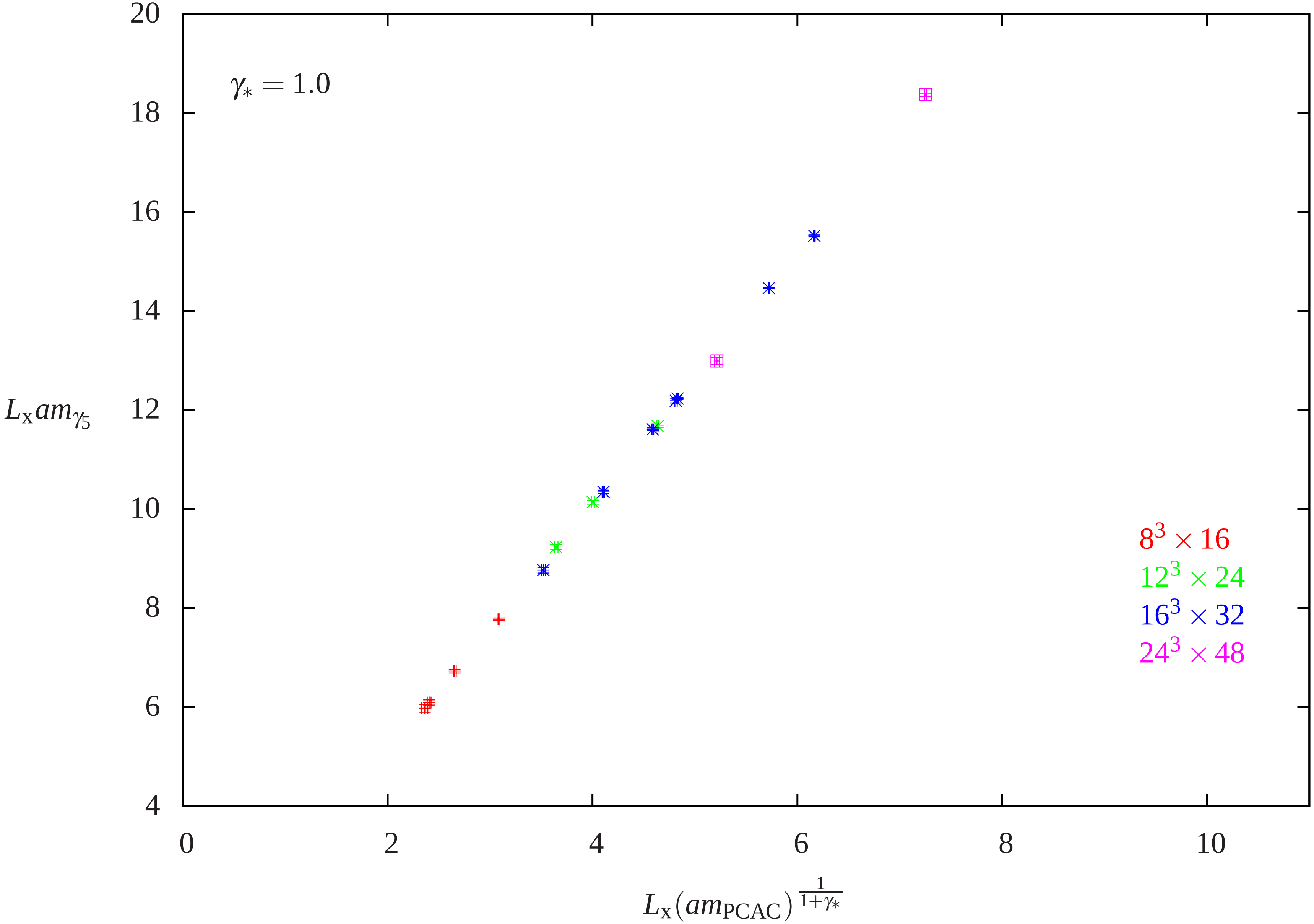

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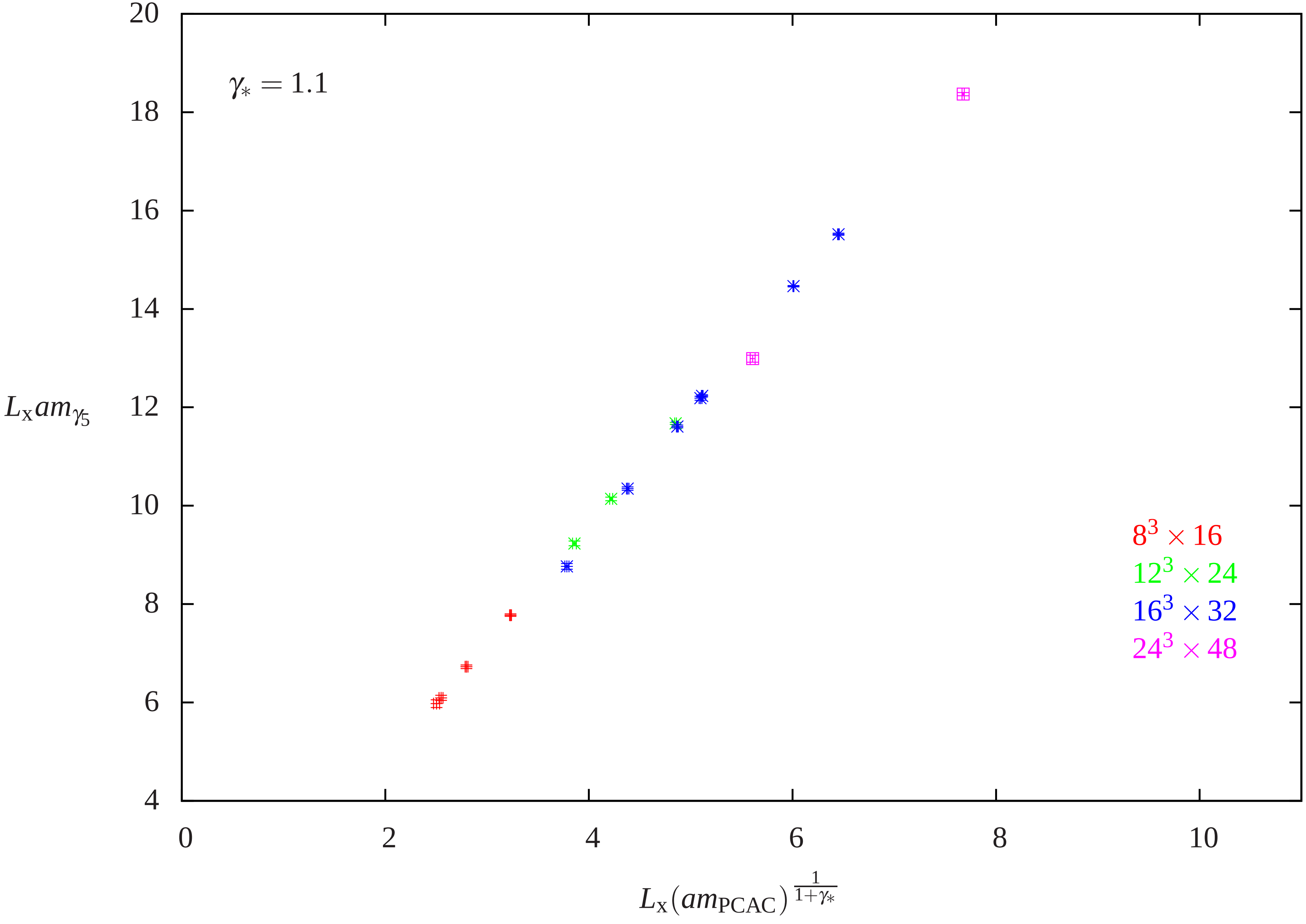

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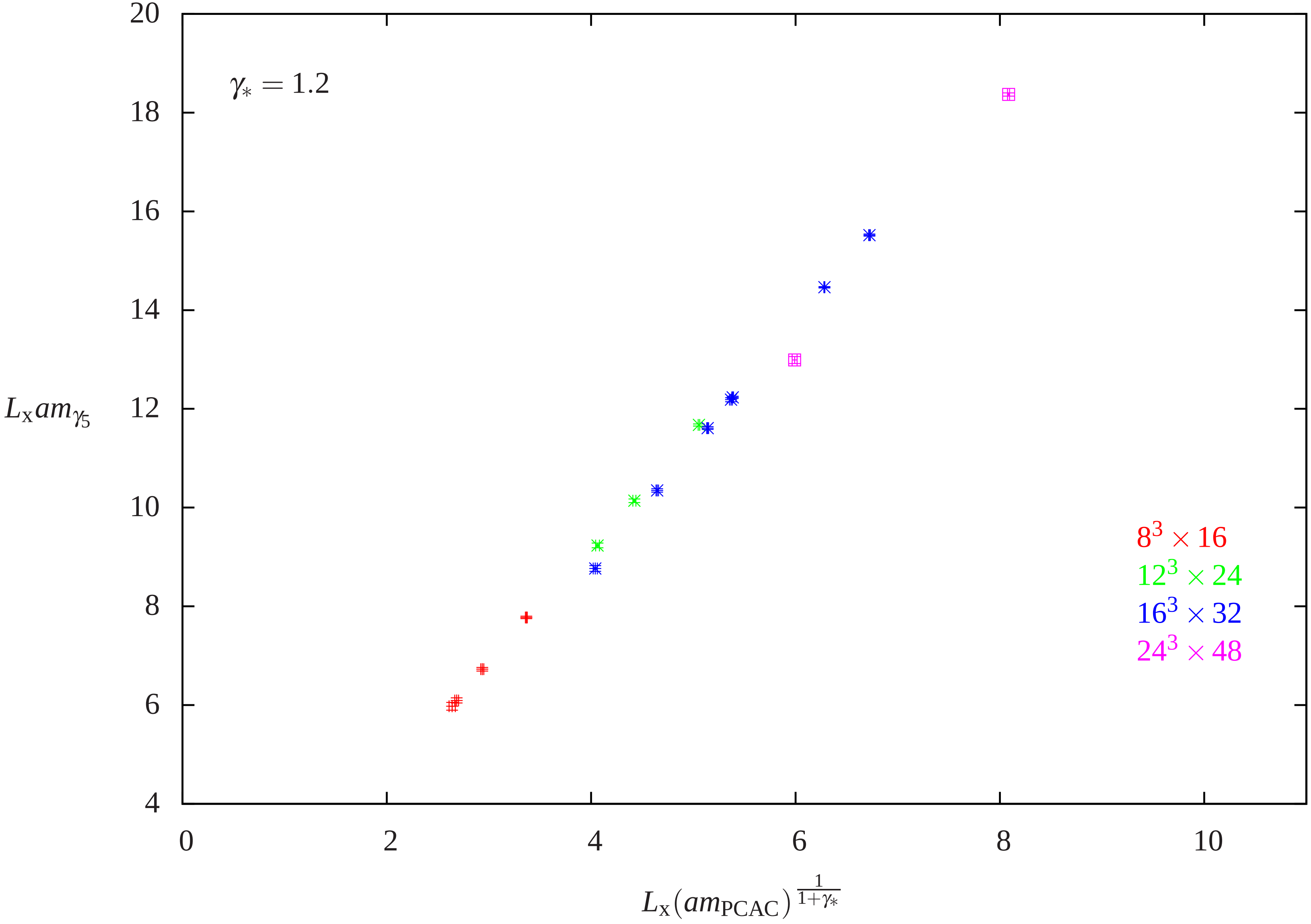

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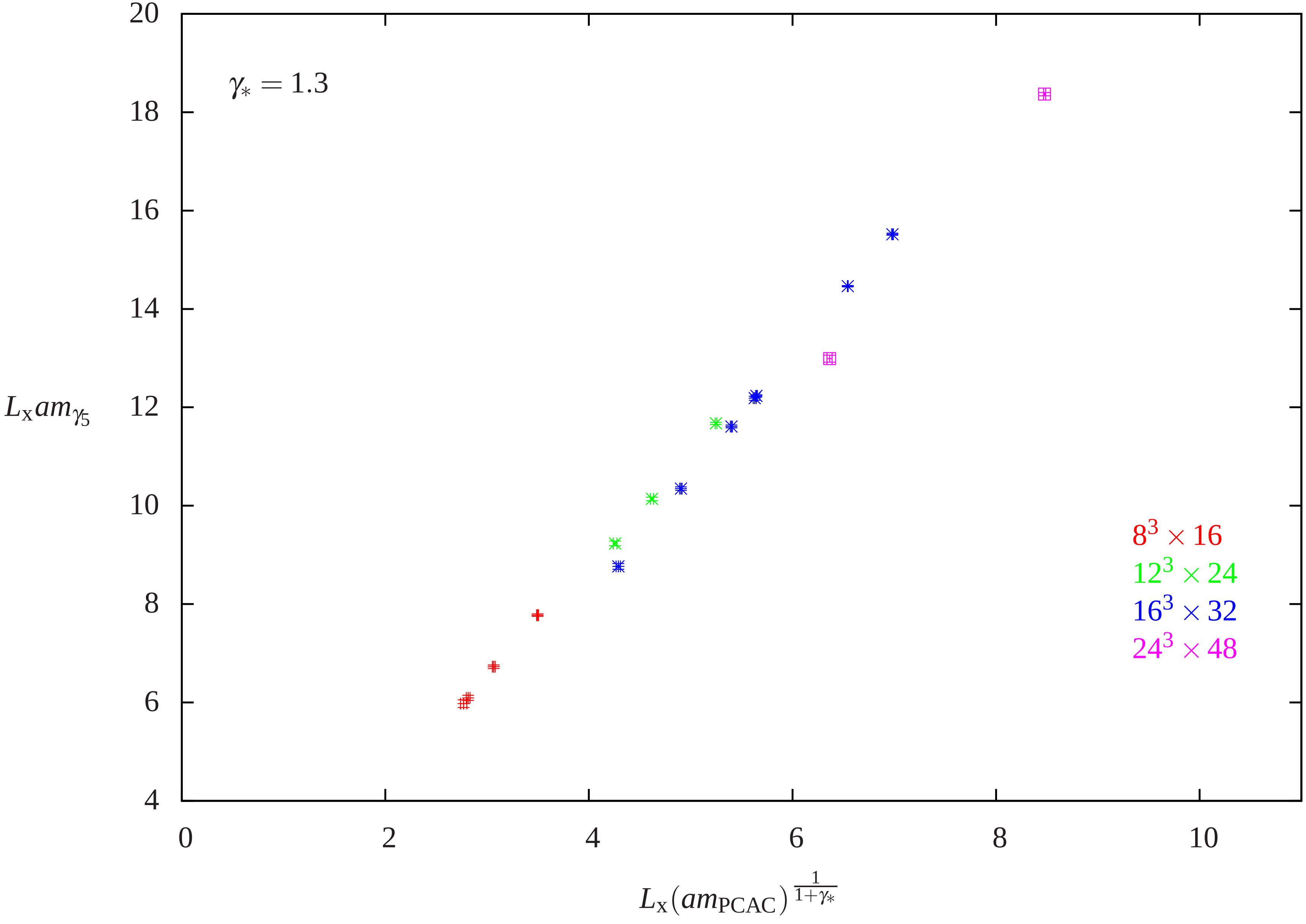

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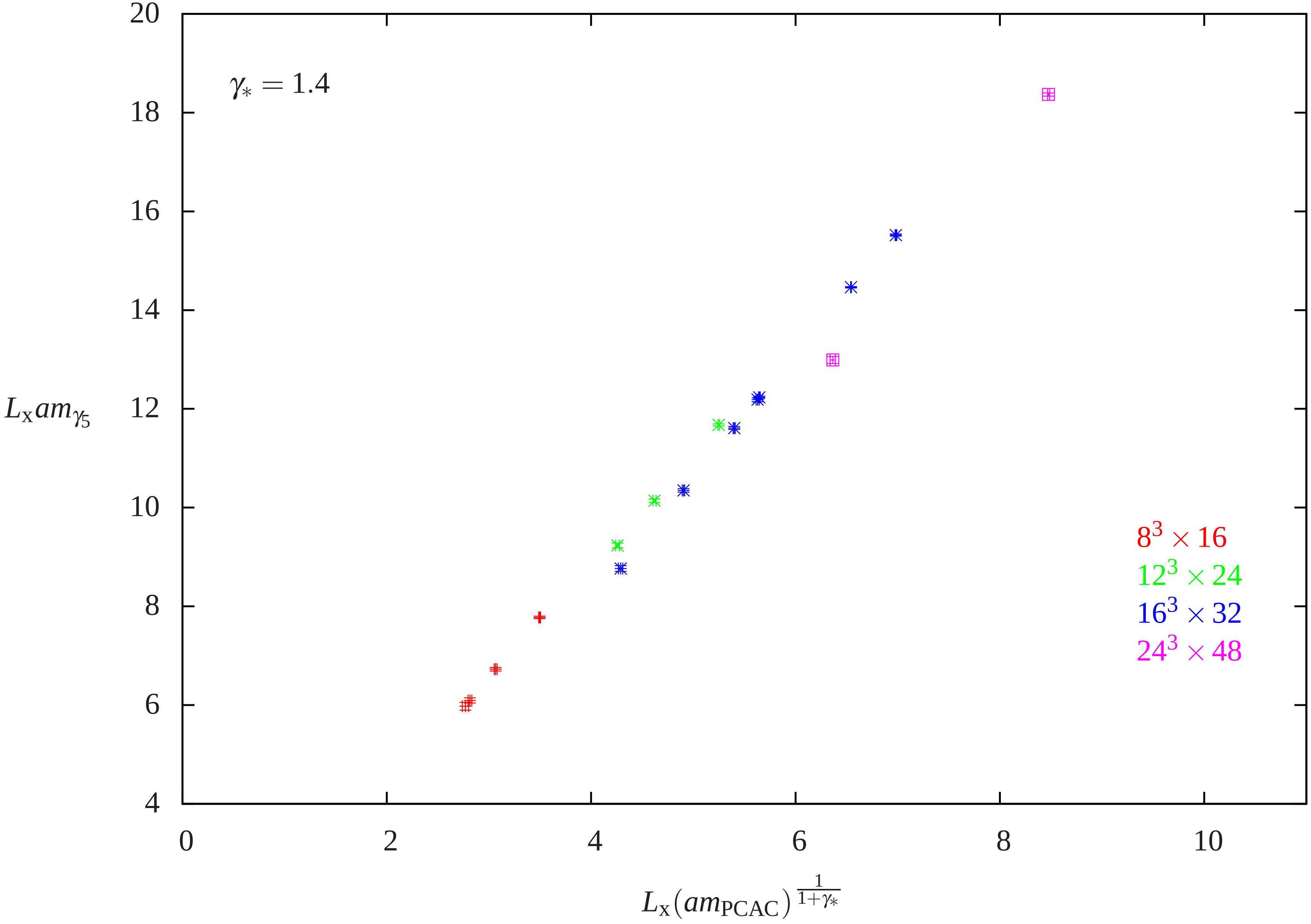

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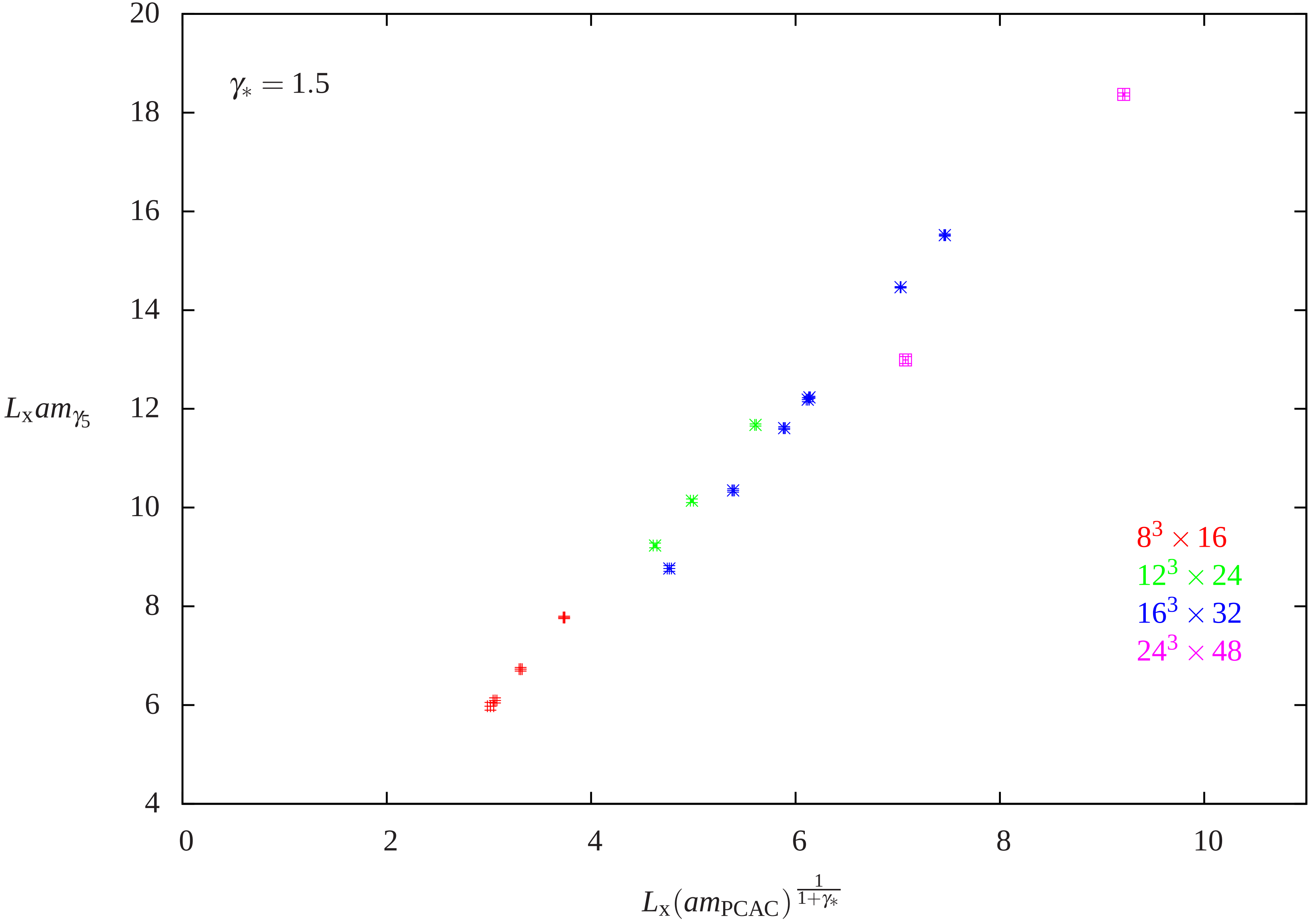

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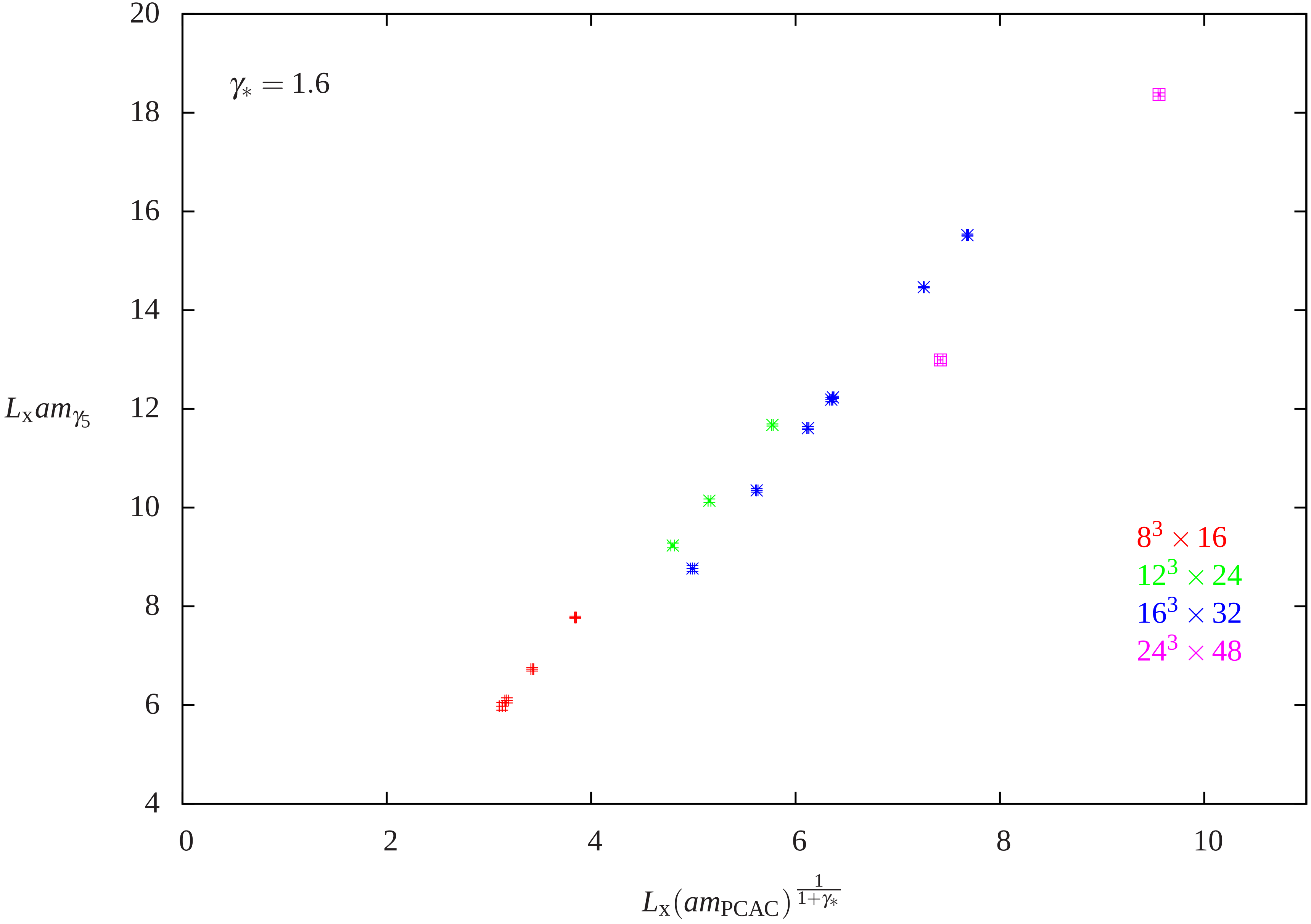

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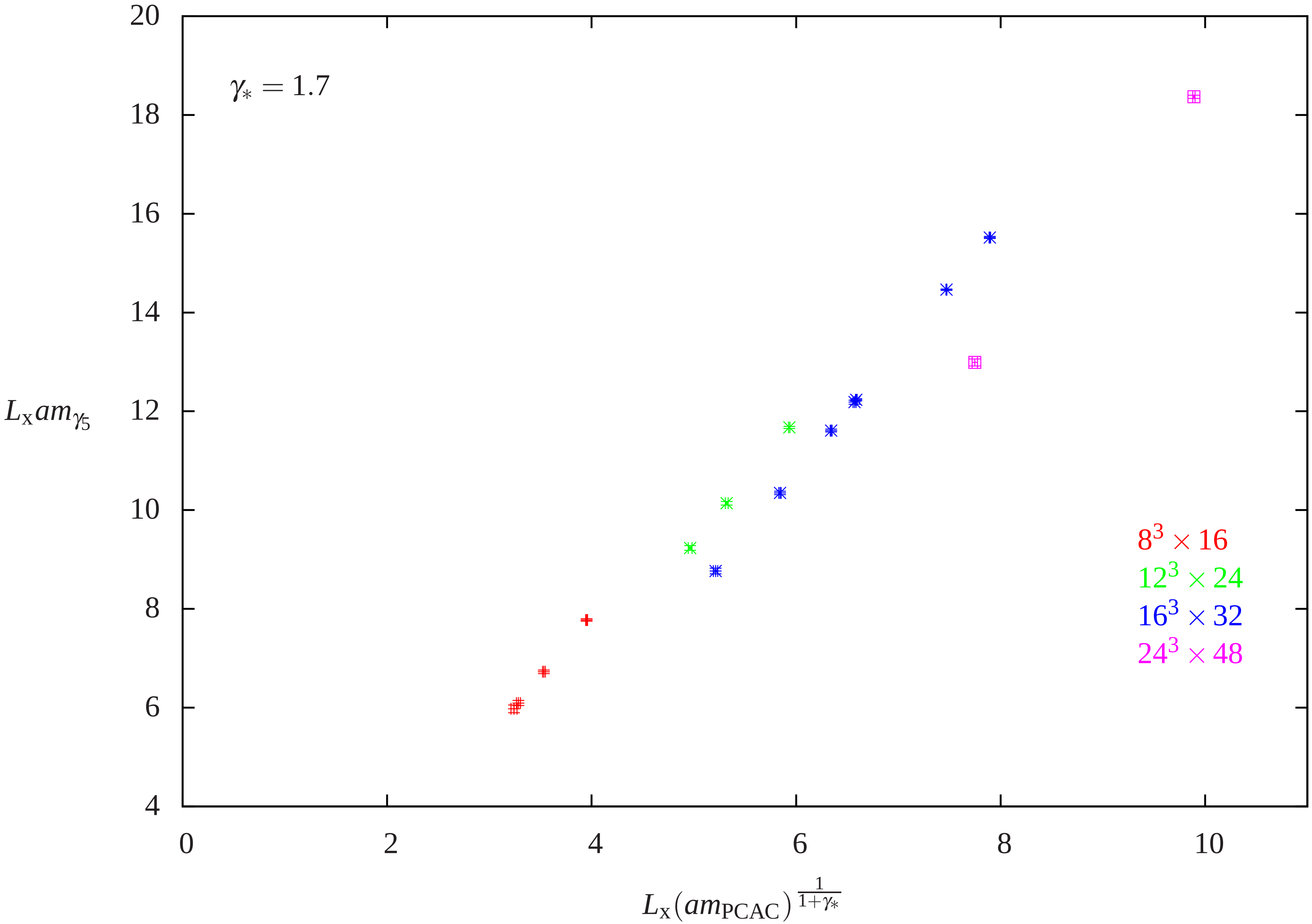

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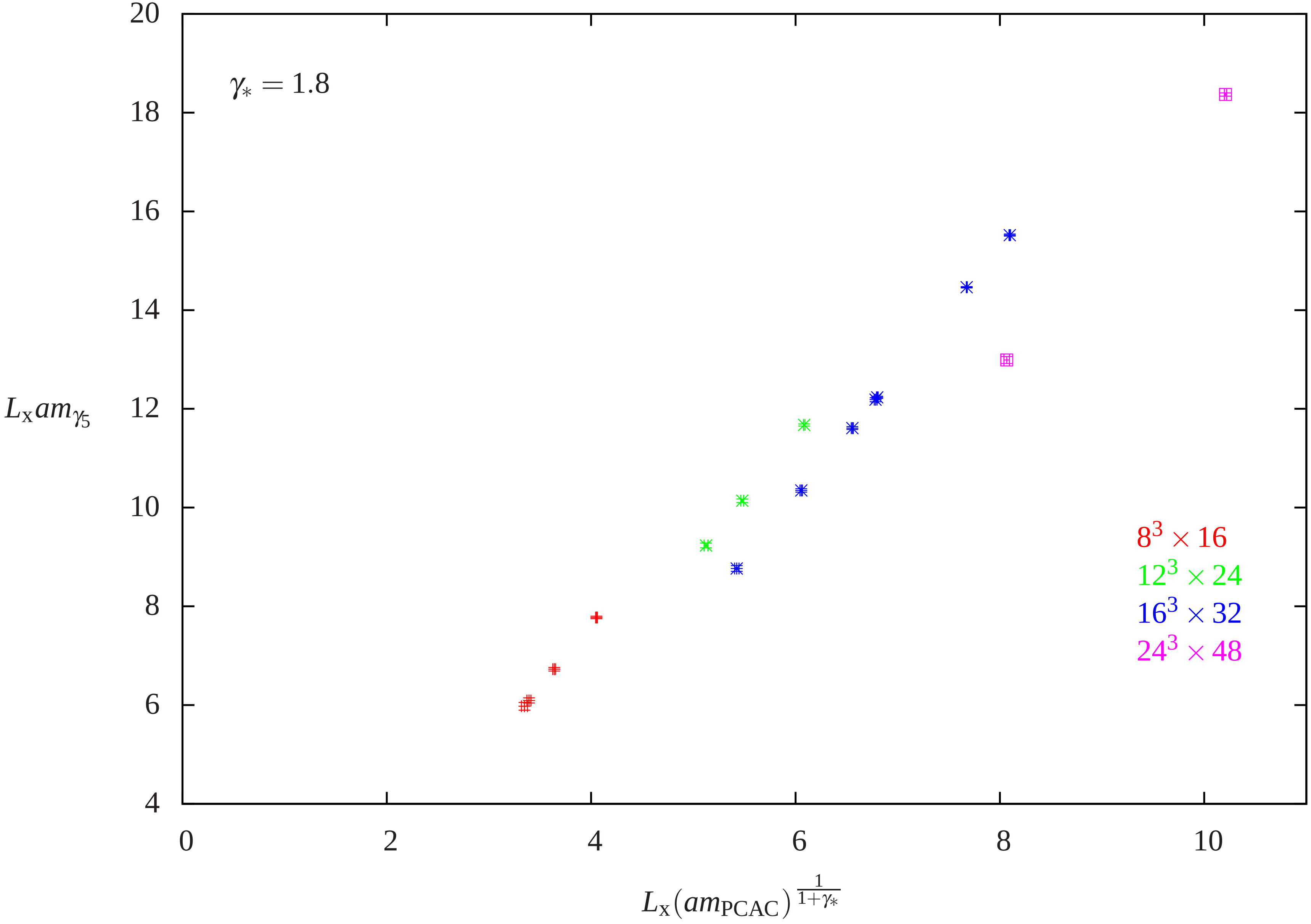

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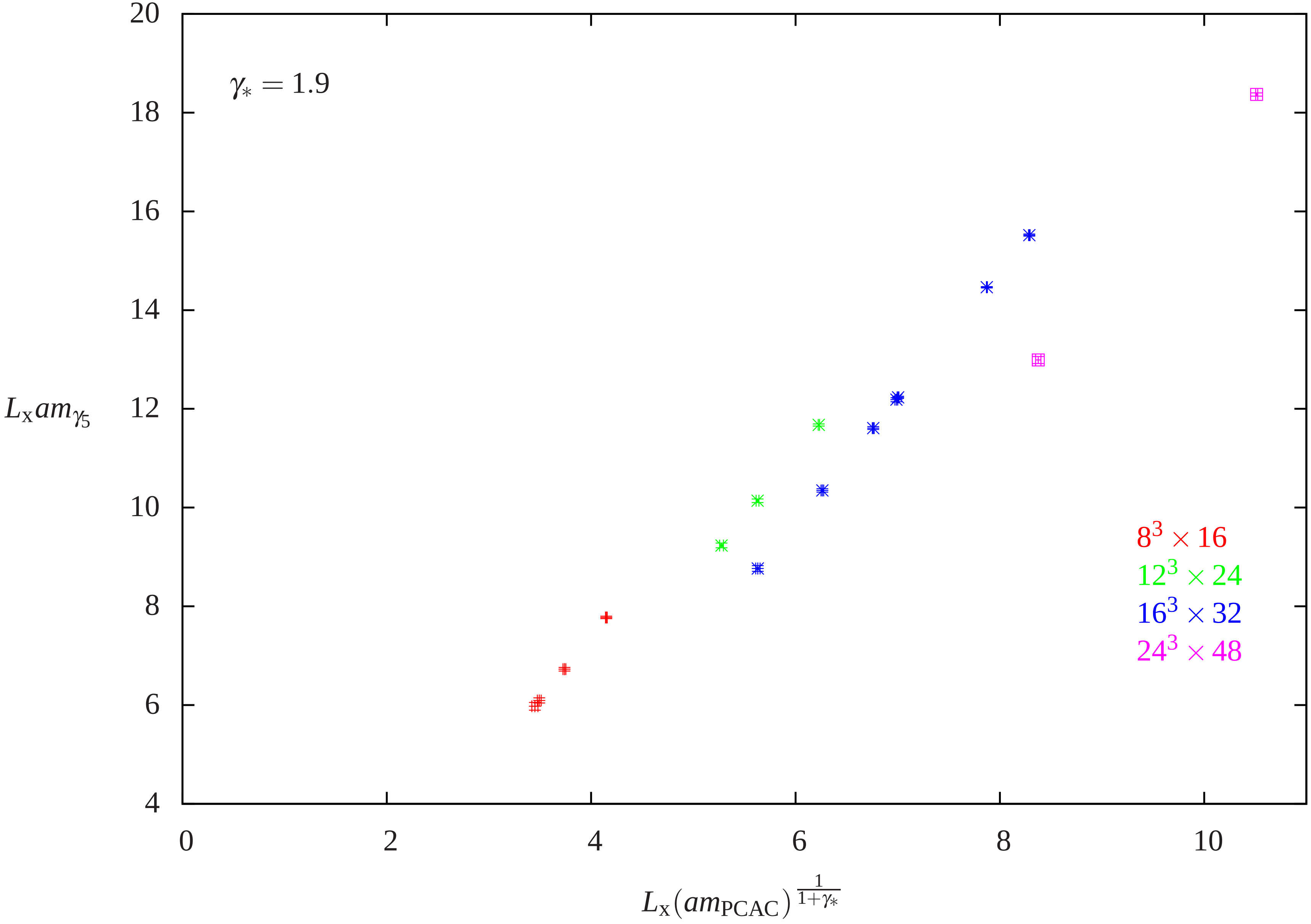

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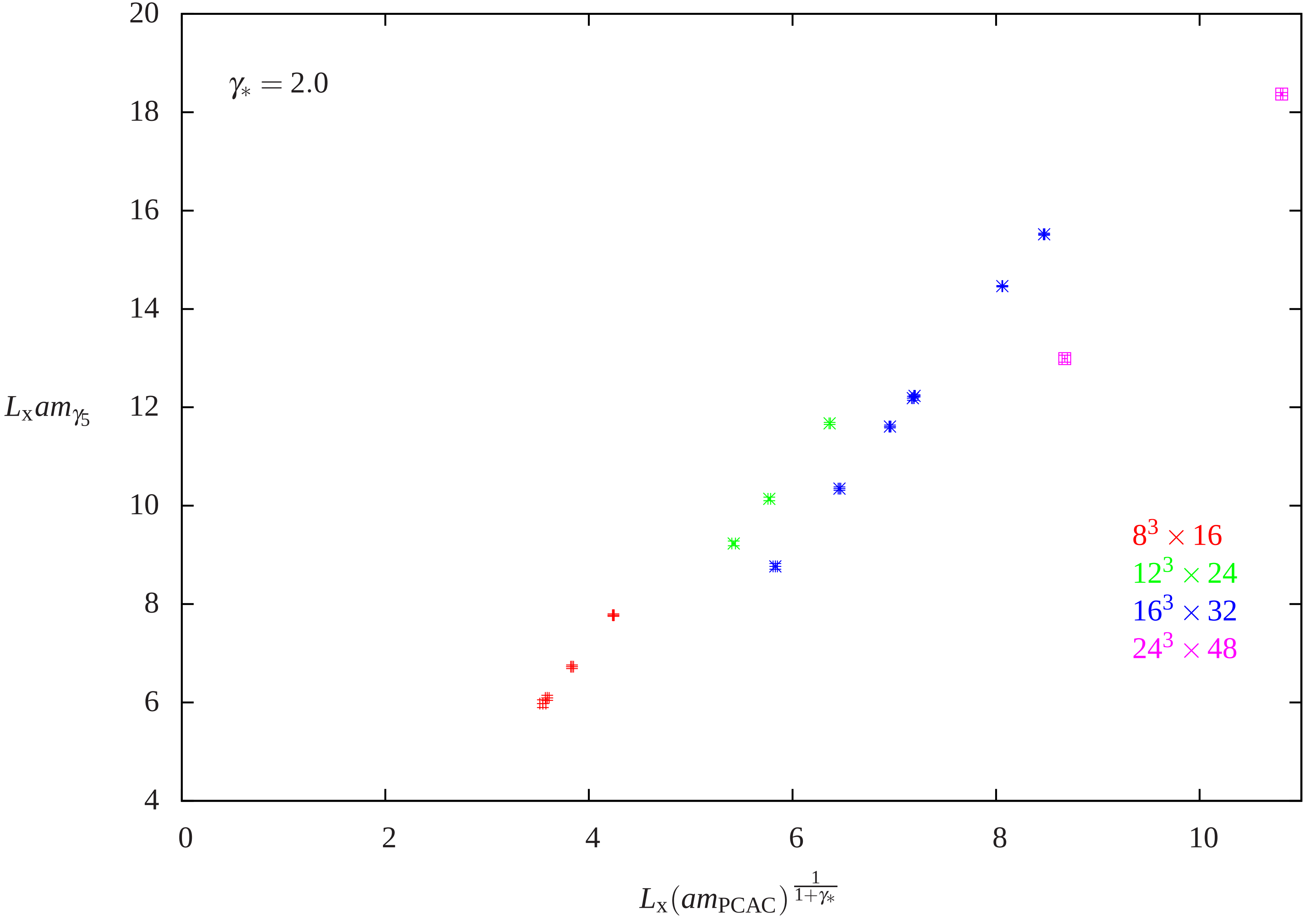

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  - The Dirac Mode Number per unit volume $\overline{\nu}(\Omega)$:

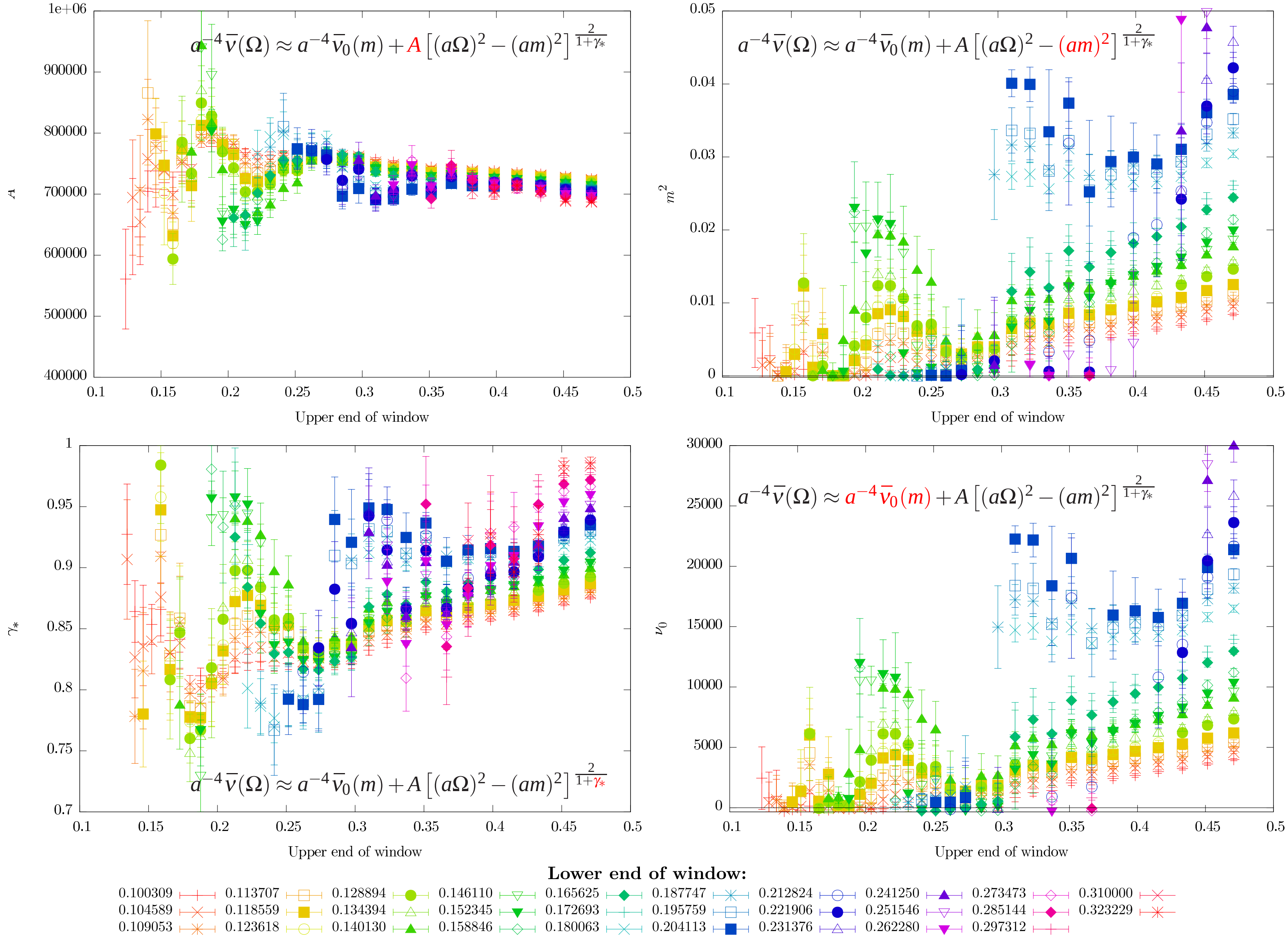
Fit:

$$a^{-4}\overline{\nu}(\Omega) \approx a^{-4}\overline{\nu}_0(m) + A\left[(a\Omega)^2 - (am)^2\right]^{\frac{2}{1+\gamma_*}} .$$

according to the details provided in A. Patella, PhysRevD.86:025006,2012 [arXiv:1204.4432].

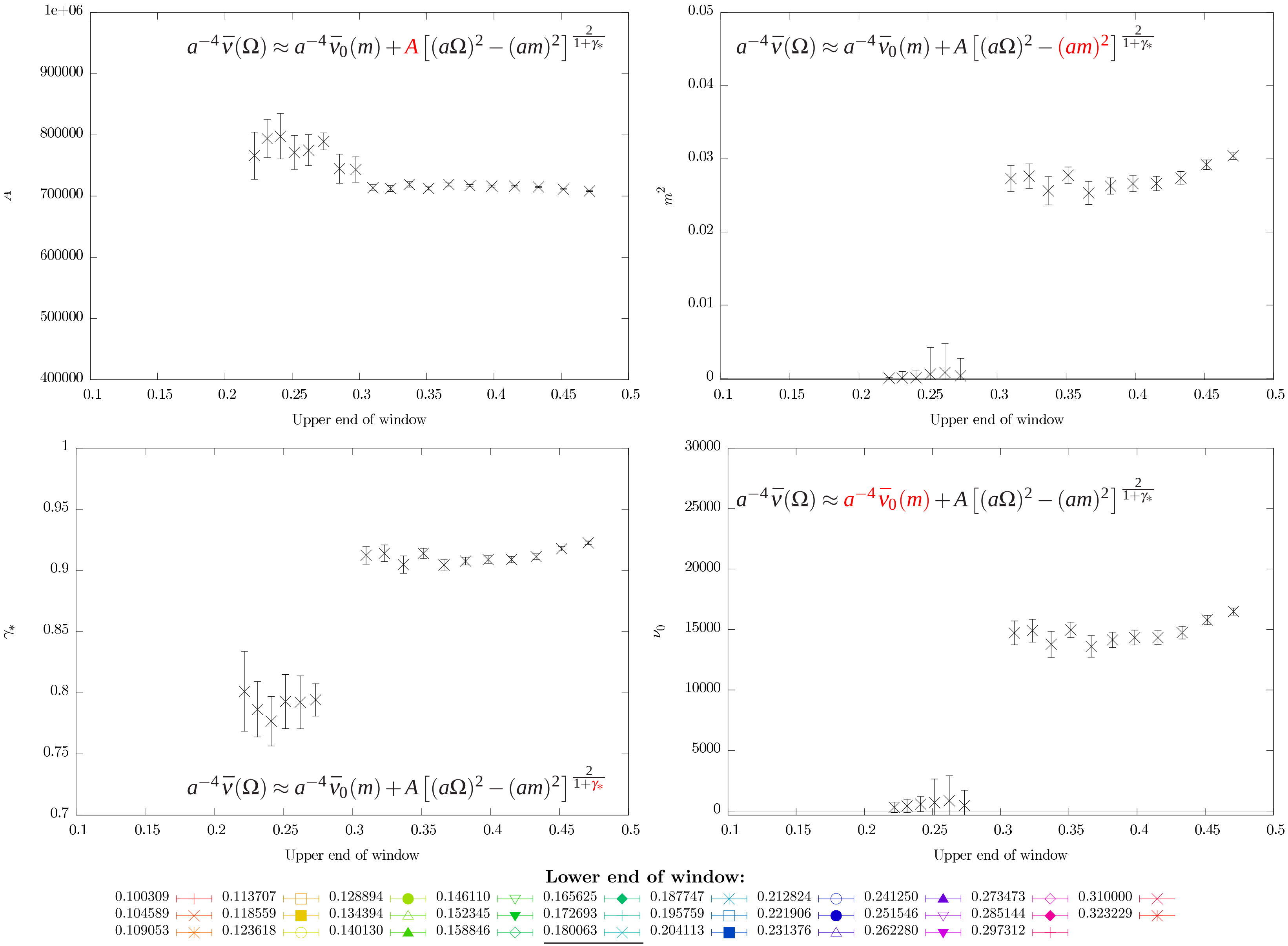
Results: Mass Anomalous Dimension

All lower ends



Results: Mass Anomalous Dimension

Lower end at 0.180063



Results: Mass Anomalous Dimension

- The determination of the mass anomalous dimension γ_* is one of our goals.
- This is important for mass scaling.
- For phenomenologically viable TC model a large anomalous dimension is required $\gamma_* \simeq 1$.
- Determination of the mass anomalous dimension.
  - Simple inspection:

$$L_x a m_{\gamma_5} \propto L_x a m_{\text{PCAC}}^{\frac{1}{1+\gamma_*}}$$

Let us vary $0.1 \leq \gamma_* \leq 2.0$

$$\rightarrow \gamma_* \sim 0.9 - 1.1$$

  - The Dirac Mode Number per unit volume $\overline{\nu}(\Omega)$:

Fit:

$$a^{-4}\overline{\nu}(\Omega) \approx a^{-4}\overline{\nu}_0(m) + A\left[(a\Omega)^2 - (am)^2\right]^{\frac{2}{1+\gamma_*}} .$$

according to the details provided in A. Patella, PhysRevD.86:025006,2012 [arXiv:1204.4432].

$$\rightarrow \gamma_* \sim 0.9 - 0.95$$

Discussion and Conclusions

- We have carried out the first look at $SU(2)$ with one Dirac adjoint fermion.
- We have performed an initial exploratory study of the phase structure in the $\beta - m$ plane.
- Quantitative studies at a variety of lattice sizes.
  - Mesons spectrum.
  - Gluino-Glue spectrum.
  - Glueball spectrum.
  - Torelon masses (String Tension).
  - Static Potential (String Tension).
  - Mass anomalous dimension.
- Results suggest that the theory is IR-Conformal or near-conformal.
- With large anomalous dimension.

THANK YOU!!!

Results: Potential