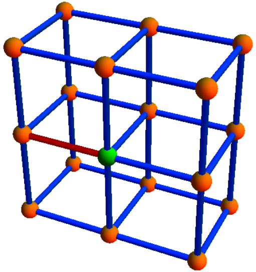
Clover fermions in the adjoint representation and simulations of supersymmetric Yang-Mills theory



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Supersymmetry

The project

DESY-Münster collaboration:
$\mathcal{N} = 1$ supersymmetric Yang-Mills theory on the lattice *(Investigation of non-perturbative aspects of a supersymmetric Yang-Mills theory)*
Members of the collaboration:

- G. Münster (WWU), I. Montvay (DESY)
- G. Bergner (Frankfurt)
- Phd students: U. D. Özugurel, S. Piemonte, D. Sandbrink (WWU)

Recent publication:

- *S. Musberg, G. Münster, S. Piemonte*: Perturbative calculation of the clover term for Wilson fermions in any representation of the gauge group SU(N), *JHEP 05 (2013) 143, arXiv:1304.5741 [hep-lat]*

Supersymmetry

Why do we study a supersymmetric theory?

Supersymmetry relates boson particles to fermion particles:

$$Q|\mathrm{boson}\rangle = |\mathrm{fermion}\rangle \qquad (1)$$

$$Q|\mathrm{fermion}\rangle = |\mathrm{boson}\rangle \qquad (2)$$

Exact supersymmetry *is not* realized in nature, but a lattice investigation of SUSY is motivated by:

1. It allows for checking the possibility of a spontaneous supersymmetry breaking in a non-perturbative way.
2. Orientifold equivalence between $N = 1$ QCD and $\mathcal{N} = 1$ SUSY
3. Tests of effective theory for low-energy dynamics

For these reasons we study $\mathcal{N} = 1$ SUSY with gauge group $SU(2)$

Supersymmetry

The action

The action for $\mathcal{N} = 1$ SUSY can be written in the continuum as:

$$S = \int d^4x \left\{ \frac{1}{4}(F^a_{\mu\nu} F^a_{\mu\nu}) + \frac{1}{2}\bar{\lambda}_a \gamma^\mu D^{ab}_\mu \lambda_b \right\} \tag{3}$$

where λ is a Majorana fermion in the adjoint representation:

$$\bar{\lambda}_a = \lambda_a^T C \tag{4}$$

$$D^{ab}_\mu \lambda_b = \partial_\mu \lambda_a + igA^c_\mu (T^A_c)^{ab} \lambda_b \tag{5}$$

The global supersymmetry interchanges boson gauge fields with fermion fields:

$$A_\mu \rightarrow A_\mu - 2i\bar{\lambda}\gamma_\mu \epsilon \tag{6}$$

$$\lambda^a \rightarrow \lambda^a - \sigma_{\mu\nu} F^a_{\mu\nu} \epsilon \tag{7}$$

Supersymmetry

The particle content

The spectrum of low-lying bound states of the theory can be organized in two supermultiplets:

1. A higher energy chiral supermultiplet:[1]
   - $a-\eta'$: $\bar{\lambda}_a\gamma_5\lambda_a$ a pseudoscalar meson 0^{-+}
   - $a-f_0$: $\bar{\lambda}_a\lambda_a$ a scalar meson 0^{++}
   - $g\tilde{g}$: $F^a_{\mu\nu}\sigma_{\mu\nu}\lambda_a$ the gluino-glue, a spin $1/2$ Majorana fermion
2. A lower energy chiral supermultiplet:[2]
   - gg: $F^{\mu\nu}F_{\mu\nu}$ a scalar glueball 0^{++}
   - gg: $\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ a pseudoscalar glueball 0^{-+}
   - $g\tilde{g}$: $F^a_{\mu\nu}\sigma_{\mu\nu}\lambda_a$ a lower gluino-glue state

If exact supersymmetry is realized, then the masses in the two supermultiplets must be degenerate.

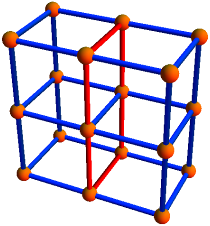
[1] G. Veneziano, S. Yankielowicz, Phys. Lett. B113 (1982) 231.

[2] G. R. Farrar, G. Gabadadze, M. Schwetz, Phys. Rev. D58 (1998) 015009 [arXiv:hep-th/9711166]

The lattice

Wilson loops and Dirac-Wilson operator

For performing Monte Carlo simulations, we introduce a finite lattice spacing a, but it breaks explicitly SUSY:



$$\{Q_\alpha, Q_\beta\} = (\gamma^\mu C)_{\alpha\beta} P_\mu$$

The propagation of a gluino in the space-time is related to the inverse of the Dirac-Wilson operator D_W:

$$S_f = \frac{1}{2}\bar{\lambda}(D_W[V_\mu] + m)\lambda$$

The mass $m \neq 0$ is a renormalization parameter of the theory. The link V_μ in D_W are in the adjoint representation:

$$V_\mu(x)_{ab} = 2\mathrm{Tr}(U_\mu^\dagger(x) T_a^F U_\mu(x) T_b^F)$$

The lattice

Questions

Is supersymmetry restored in the continuum limit?

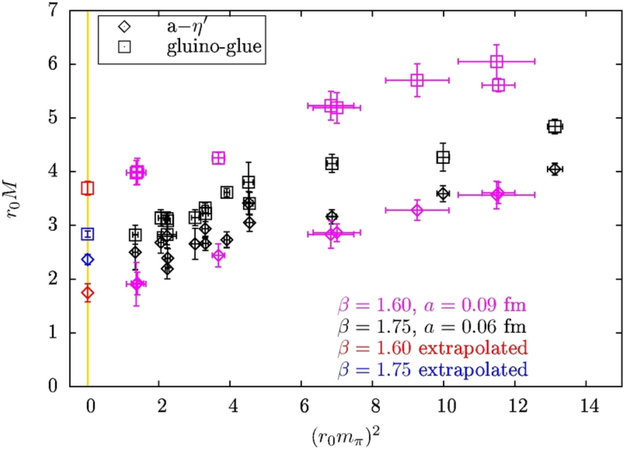
Fine tuning of (g, m) to the critical point in order to recover both Lorentz symmetry and zero gluino mass needed by SUSY.

Possible tests for checking a restored SUSY:

- Study the SUSY Ward identities - done
- Check the degeneracy in the supermultiplets - in progress
- Test $\langle \mathcal{H} \rangle$ as order parameter - future

Lattice SUSY

Physical particles - The supersymmetric limit[3]



[3] G. Bergner, T. Berheide, I. Montvay, G. Münster, U. D. Özugurel, D. Sandbrink: JHEP 09 (2012) 108, arXiv:1206.2341 [hep-lat]

Lattice SUSY

Source of Supersimmetry breaking

What are the possible sources of SUSY breaking in our simulations?

1. Finite volume - under control
2. Boundary conditions - to be tested
3. Finite lattice spacing - to be further reduced

The Wilson fermion action has an discretization error proportional to $O(a)$:

$$\lim_{a\to 0} \frac{m_{a-\eta'}(a)}{m_{a-f_0}(a)} = \frac{m^c_{a-\eta'}}{m^c_{a-f_0}}(1 + O(a)) \tag{8}$$

in contrast to the pure gauge theory with Wilson action, where the error scales as $O(a^2)$.

Symanzik program

The clover term

The Symanzik program reduces the discretization error $O(a)$ order by order in perturbation theory adding irrelevant operators to the Lagrangian:

$$\mathcal{L} = \mathcal{L}_0 - a\frac{c_{SW}}{4}O_{CL} \quad (9)$$

$$O_{CL} = \bar{\lambda}\sigma_{\mu\nu}F^{\mu\nu}\lambda \quad (10)$$

The Sheikholeslami-Wohlert coefficient:

$$c_{SW} = c^0_{SW} + c^1_{SW}g^2 + \ldots \quad (11)$$

can be tuned similarly to the QCD action, requiring no $O(a)$ errors for on-shell quantities (like the gluino-gluino scattering cross section).[4]

[4] S. Aoki and Y. Kuramashi: Phys. Rev. D 68 (2003) 094019 [hep-lat/0306015]

Symanzik program

The clover term

The gluino-gluino-gluon vertex operator has the generic form:

$$\begin{aligned}\Lambda(p,p')^{a}_{\mu;cd} &= g\left(i\gamma_\mu A + g\frac{a}{2}(p+p')_\mu(B-c_{SW}) + \right.\\ &\quad \left. + O(p^2,p'^2) + O(a^2)\right)(T^a_R)_{cd}\end{aligned}$$

At tree level $A = 1 + O(g^2)$ and $B = 1 + O(g^2)$, the terms proportional to a vanish if the clover term is set one:

$$c^0_{SW} = 1$$

independent from the representation of the fermions and from their number of degrees of freedom (Dirac/Majorana).[5]

[5] This is the unique choice that ensures the cancellation of the infrared divergences from the gauge theory.

Symanzik program

The clover term

At one loop, in the Feynman diagrams traces and sums of the group generators will introduce a representation dependent contribution to B:

$$\begin{aligned} B &= 1 + g^2(0.16764(3)C_R + 0.01503(3)N) \\ C_R\delta_{bc} &= \sum_a (T_a^R T_a^R)_{bc} \end{aligned}$$

Therefore the clover term has to be fixed to:

$$c_{SW}^1 = 0.16764(3)C_R + 0.01503(3)N \tag{12}$$

independently from the number of degrees of freedom of the fermion (Dirac/Majorana).

Symanzik program

The clover term

The perturbative result is in agreement with the non-perturbative determination of the clover term for $SU(2)$ adjoint model:[6]

$$c_{SW}(g) = \frac{1 + 0.032653g^2 - 0.002844g^4}{1 - 0.314153g^2}$$

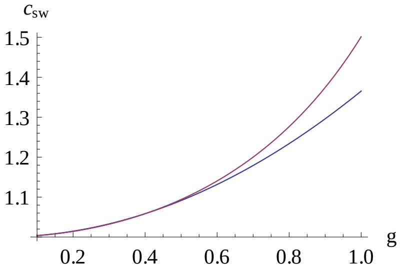


Figure: Comparison of the perturbative (blue line) vs non-perturbative (purple line) estimation of c_{sw}

[6] T. Karavirta, K. Tuominen, A. Mykkanen, J. Rantaharju and K. Rummukainen: PoS(Lattice 2010)064 [arXiv:1011.1781]

Symanzik program

Does the clover term improve the results?

The clover term at tree-level is already able to reduce the off-axis differences in the correlators:

$$\rho = \frac{C_\pi(\vec{x}_1)}{C_\pi(\vec{x}_2)} \underset{|\vec{x}_1|=|\vec{x}_2|}{\overset{a\to 0}{=}} 1$$

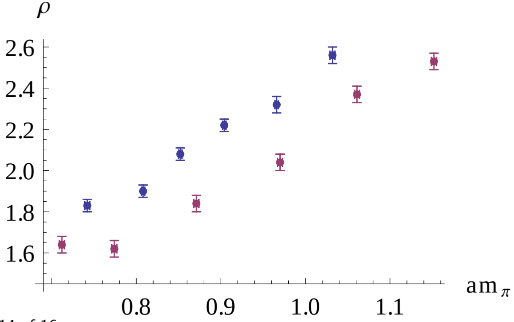


Figure: Comparison of the ratio of the correlators for different pion masses with $\vec{x}_2 = \{3, 0, 0, 4\}$ and $\vec{x}_1 = \{0, 0, 0, 5\}$, on a lattice 14^4 - $\beta = 1.75$

Blue: $c_{sw} = 0$ Purple: $c_{sw} = 1$

Symanzik program

Does the clover term improve the results?

The efficiency in the real Monte Carlo simulations of the improvements should be compared to the computational cost of the new terms in the action:

1. The HMC requires to compute the derivative of $dS_C/dU_\mu \rightarrow$ mostly a coding problem
2. The HMC and the mass spectroscopy requires to compute the inverse of the clover improved operator $\rightarrow$?

On parallel machine, the clover term has an additional cost of $20-30\%$, due to locality nearest neighborhood communications are not needed.

Conclusions

Clover fermions in the adjoint representation and simulations of supersymmetric Yang-Mills theory

The clover improvement of adjoint model or supersymmetry:

- are now calculated to one loop level using standard perturbation theory
- can be tested checking for the restored rotational symmetry
- have real advantage for reducing the ratio cost over errors in Monte Carlo simulations

Future works:

- Computation of the spectrum with clover improvements
- Possible procedure for non-perturbative improvements for SUSY?