Further studies of QCD with sextet quarks

D. K. Sinclair and J. B. Kogut

- Introduction
- Lattice QCD with colour-sextet quarks at finite temperature
- $N_f = 2$
- $N_f = 3$
- Summary and Conclusions

Introduction

In the standard Standard Model, the Higgs field is an elementary complex scalar, whose couplings to the $SU(2) \times U(1)$ electroweak gauge bosons $\gamma, W^{\pm}, Z$ are prescribed by gauge invariance. It has Yukawa couplings to the quarks and leptons, and a quartic self-coupling. Its quadratic self-coupling with dimensions of mass-squared is negative, so that it develops a vacuum expectation v which breaks $SU(2) \times U(1)$ spontaneously. The $W^{\pm}$ and Z gain masses by 'eating' the 3 Goldstone bosons. v gives masses to the fermions through the Yukawa couplings. The remaining (radial) component of the Higgs field is the so-called Higgs particle.

We consider the possibility that this simplest model of the Higgs sector of the Standard Model is merely an effective field theory, and that the Higgs fields are composite.

The simplest theories of this type are Technicolor theories. These are QCD-like gauge theories with massless (techni-)quarks, where the (techni-)pions play the role of the Higgs field giving masses to the Ws and Z.

Technicolor theories which are merely scaled-up QCD are not phenomenologically viable. It can be argued that Walking Technicolor theories, where the gauge group and fermion content are such that there is a range of length/mass scales where the running coupling evolves very slowly, might overcome these problems.

We are trying to identify gauge theories that walk. If we find such a theory, we then need to check if it is indeed phenomenologically viable.

An important aspect of an acceptable theory is that it must describe the light ($m_H \approx \frac{1}{2}v$) Higgs-like particle observed at the LHC.

Gauge theories with fermion content such that the theory is asymptotically free, but the 1- and 2-loop contributions to the β-function have opposite signs, are expected to be either conformal or walking.

Our candidate theory is (techni-)QCD with 2 massless (techni-)colour-sextet (techni-)quarks, which could be either walking or conformal. We contrast this theory with the 3-(techni-)flavour version, which is expected to be conformal.

QCD with 2 colour-sextet quarks has 3 Goldstone bosons – the correct number to give mass to the Ws and Z with none left over.

Because chiral symmetry breaking and confinement occur at very different scales, one expects (techni-)hadrons associated with both scales. Hence light hadrons at the confinement scale could well have masses $< f_{\pi(TC)} = v$. Thus we can get light Higgs-like particles other than the dilaton.

Other groups working on this model include DeGrand *et al.* and Fodor *et al.*.

We simulate lattice QCD with 2 sextet quarks at finite temperature, and measure the running of the couplings at the deconfinement and chiral transitions, as the lattice spacing is varied. On lattices with finite temporal extent $N_t a$ and spatial extent $N_s a$ with $N_s >> N_t$, the temperature is $T = 1/N_t a$. If our transitions are finite temperature transitions, they will remain at fixed temperatures as the lattice spacing a is varied. If we increase $N_t \longrightarrow \infty$ at either transition, $a \longrightarrow 0$. The bare (lattice) coupling g at the transition should approach zero as $N_t \longrightarrow \infty$ in the manner described by the (perturbative) β-function.

If on the other hand, the transition is a bulk transition, g will approach a non-zero limit as $N_t \longrightarrow \infty$. In this case the field theory is conformal.

Since the deconfinement transition occurs at a value of $\beta = 6/g^2$ that is too small to observe asymptotic freedom, for the N_ts we use, we concentrate our effort on the chiral transition β, β_χ.

We simulate the $N_f = 2$ theory on lattices with $N_t = 4, 6, 8, 12$ and hope to extend this to larger N_t.

Preliminary results indicate that $\beta_\chi(N_t = 12)$ is significantly larger than $\beta_\chi(N_t = 8)$, but by less than what the 2-loop β-function would predict.

We simulate the $N_f = 3$ theory on lattices with $N_t = 4, 6, 8$ and plan to extend this to $N_t = 12$.

Preliminary results indicate that $\beta_\chi(N_t = 8)$ is significantly greater than $\beta_\chi(N_t = 6)$ which would indicate that we are not yet at weak enough coupling.

QCD with colour-sextet staggered quarks at finite T

We use the simplest (Wilson) gauge action:

$$S_g = \beta \sum_{\Box} \left[1 - \frac{1}{3} \mathrm{Re}(\mathrm{Tr} UUUU) \right]. \quad (1)$$

Formally, the unimproved staggered quark action action is:

$$S_f = \sum_{sites} \left[\sum_{f=1}^{N_f/4} \psi_f^\dagger [\not{D} + m] \psi_f \right], \quad (2)$$

where where $\not{D} = \sum_\mu \eta_\mu D_\mu$ with

$$D_\mu \psi(x) = \frac{1}{2} [U_\mu^{(6)}(x) \psi(x + \hat{\mu}) - U_\mu^{(6)\dagger}(x - \hat{\mu}) \psi(x - \hat{\mu})]. \quad (3)$$

We use the RHMC algorithm to simulate values of $N_f/4$ which are not integers, in particular $N_f = 2, 3$.

$N_f = 2$

$N_t = 8$

Mainly on a $16^3 \times 8$ lattice.

Simulations performed on a $24^3 \times 8$ lattice at $\beta = 6.7$ and $\beta = 6.9$, both with $m = 0.0025$, indicate that finite lattice size errors are small for $N_s = 16$.

No new $16^3 \times 8$ results since Lattice 2012.

Simulations were performed at $m = 0.02$, $m = 0.01$, $m = 0.005$, and $m = 0.0025$.

In the neighbourhood of the chiral transition, $6.6 \leq \beta \leq 6.8$, we have performed runs of 50,000 length-1 trajectories at $m = 0.02$, $m = 0.01$, and $m = 0.005$ for each β and m. At $m = 0.0025$ we have performed runs of 100,000 trajectories for each β in this range.

Since it is difficult if not impossible to extrapolate either the unsubtracted or subtracted chiral condensates to zero quark mass with sufficient reliability to determine the position of the chiral-symmetry-restoration phase transition accurately, we estimate the value of β_χ from the peaks in the (disconnected) chiral susceptibility:

$$\chi_{\bar\psi\psi} = \frac{V}{T}\left[\langle(\bar\psi\psi)^2\rangle - \langle\bar\psi\psi\rangle^2\right]$$

extrapolated to $m = 0$.

Since the positions of the peaks in these susceptibilities show little mass dependence, we take the position of the peak for $m = 0.0025$ as our estimate of β_χ. This gives $\beta_\chi = 6.69(1)$.

These susceptibilities are shown in figure 1.

For $m = 0.0025$ and $\beta = 6.7$ and $\beta = 6.9$, these susceptibilities are consistent with those measured on a $24^3 \times 8$ lattice.

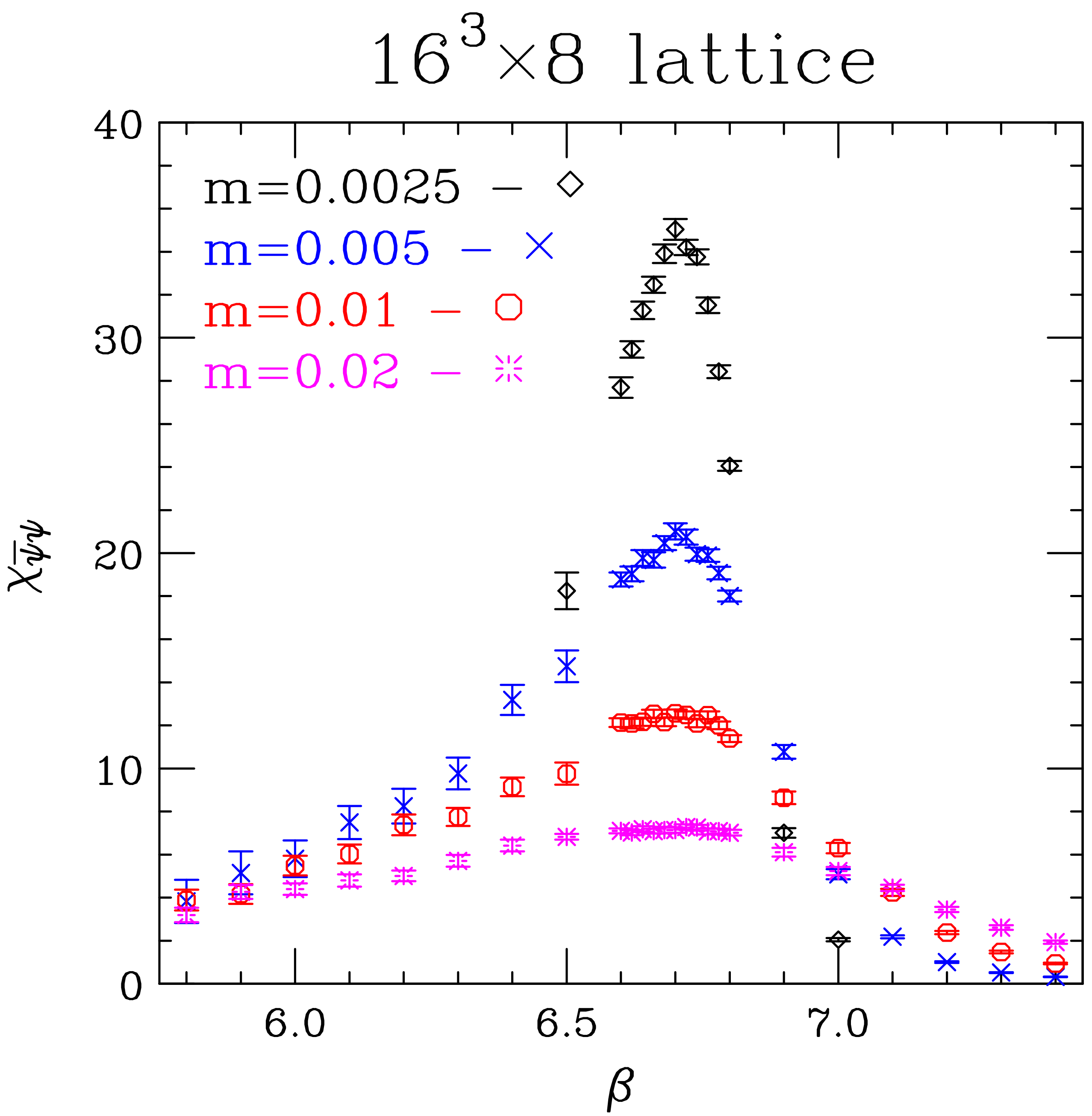

Figure 1: Chiral susceptibilities on a $16^3 \times 8$ lattice.

$N_t = 12$

We are now extending our simulations to $N_t = 12$ on $24^3 \times 12$ lattices, at quark masses $m = 0.01$, $m = 0.005$, and $m = 0.0025$.

In the neighbourhood of the chiral transition $6.6 \leq \beta \leq 6.9$ we perform simulations at βs spaced by $\delta\beta = 0.02$.

So far we have simulated 25,000 – 50,000 length-1 trajectories at each (β, m) in this range (with 2 exceptions).

Figure 2 shows the unsubtracted chiral condensates $\langle\bar{\psi}\psi\rangle$ measured in these simulations.

Note that, although these suggest that the condensate will vanish in the chiral limit for β sufficiently large, any attempt to extrapolate to $m = 0$ to estimate β_χ would be plagued with systematic uncertainties. Using subtracted condensates improves the situation, but not sufficiently to extract β_χ accurately.

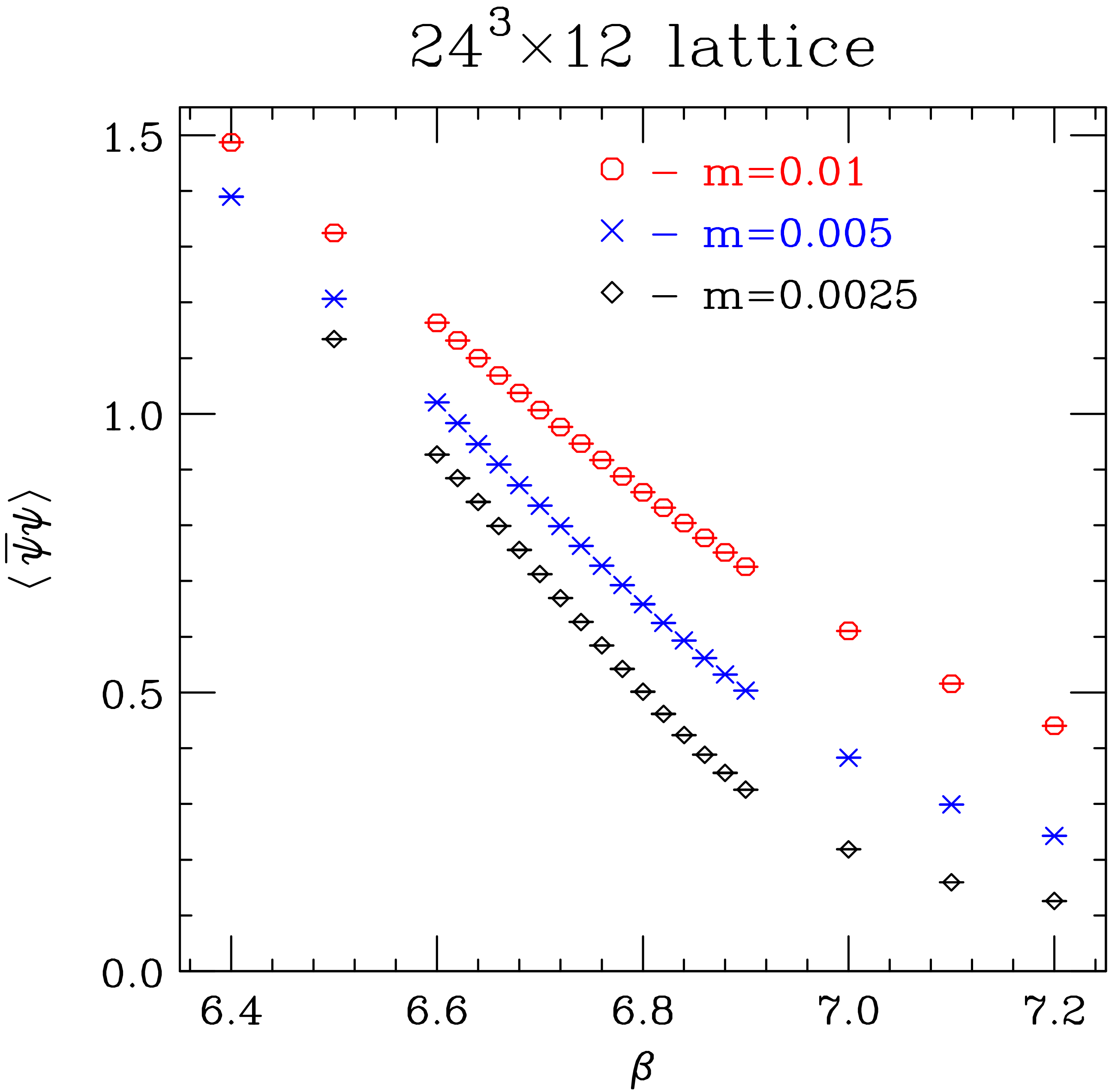

Figure 2: Chiral condensates on a $24^3 \times 12$ lattice.

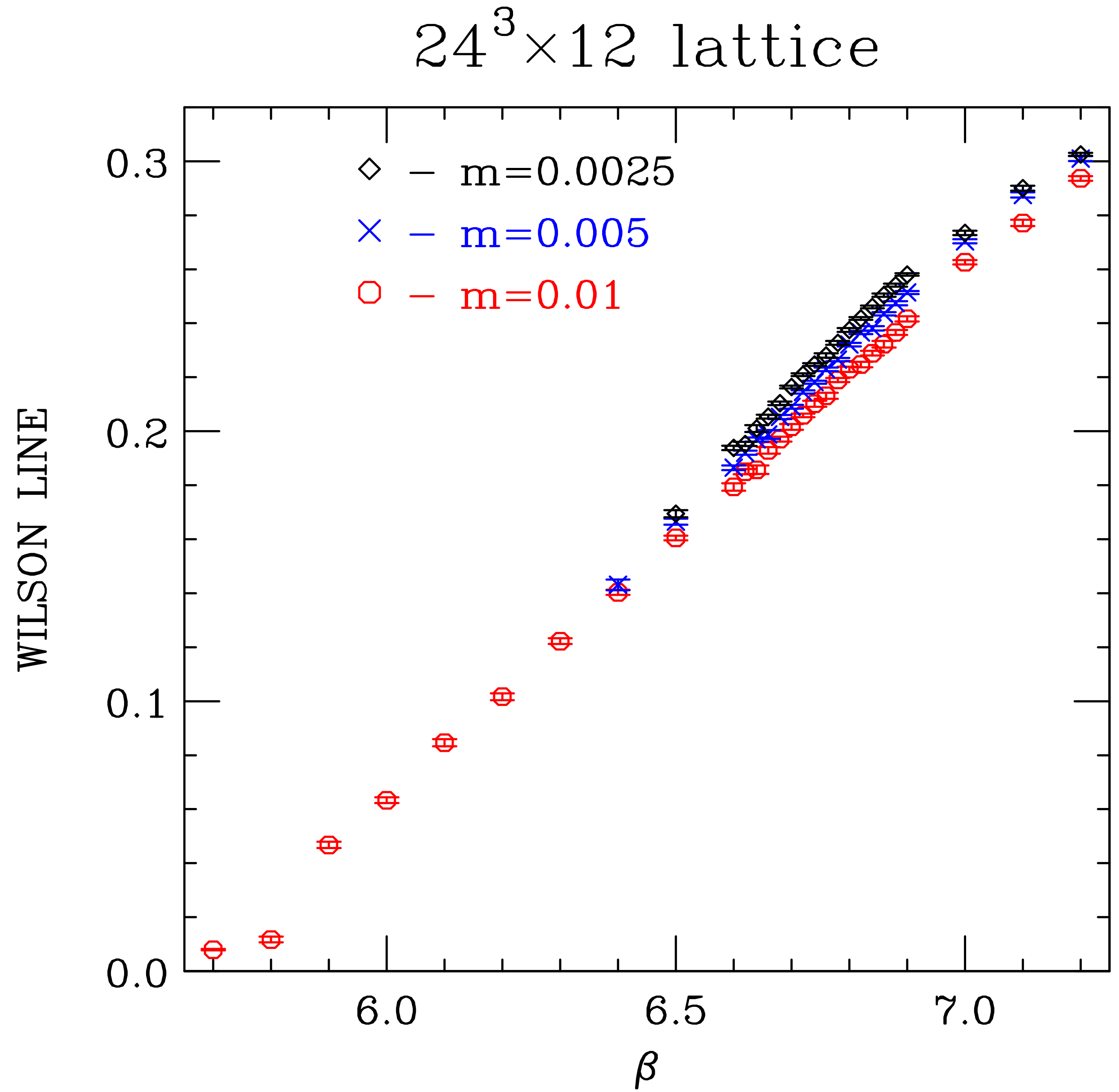

Figure 3: Wilson lines (Polyakov Loops) on a $24^3 \times 12$ lattice.

To determine the position β_χ of the chiral transition with sufficient accuracy, we examine the peak in the (disconnected) chiral susceptibility as a function of mass.

These susceptibilities are shown in figure 4.

Both the $m = 0.005$ and $m = 0.0025$ susceptibilities show peaks. While the 'data' is consistent with there being little mass dependence of the position of the peaks, it is not yet compelling. More statistics is needed.

Our best estimate of β_χ from the current data is $\beta_\chi = 6.78(2)$.

This implies that:

$$\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) = 0.09(2)$$

compared with the 2-loop perturbative prediction

$$\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) \approx 0.12$$

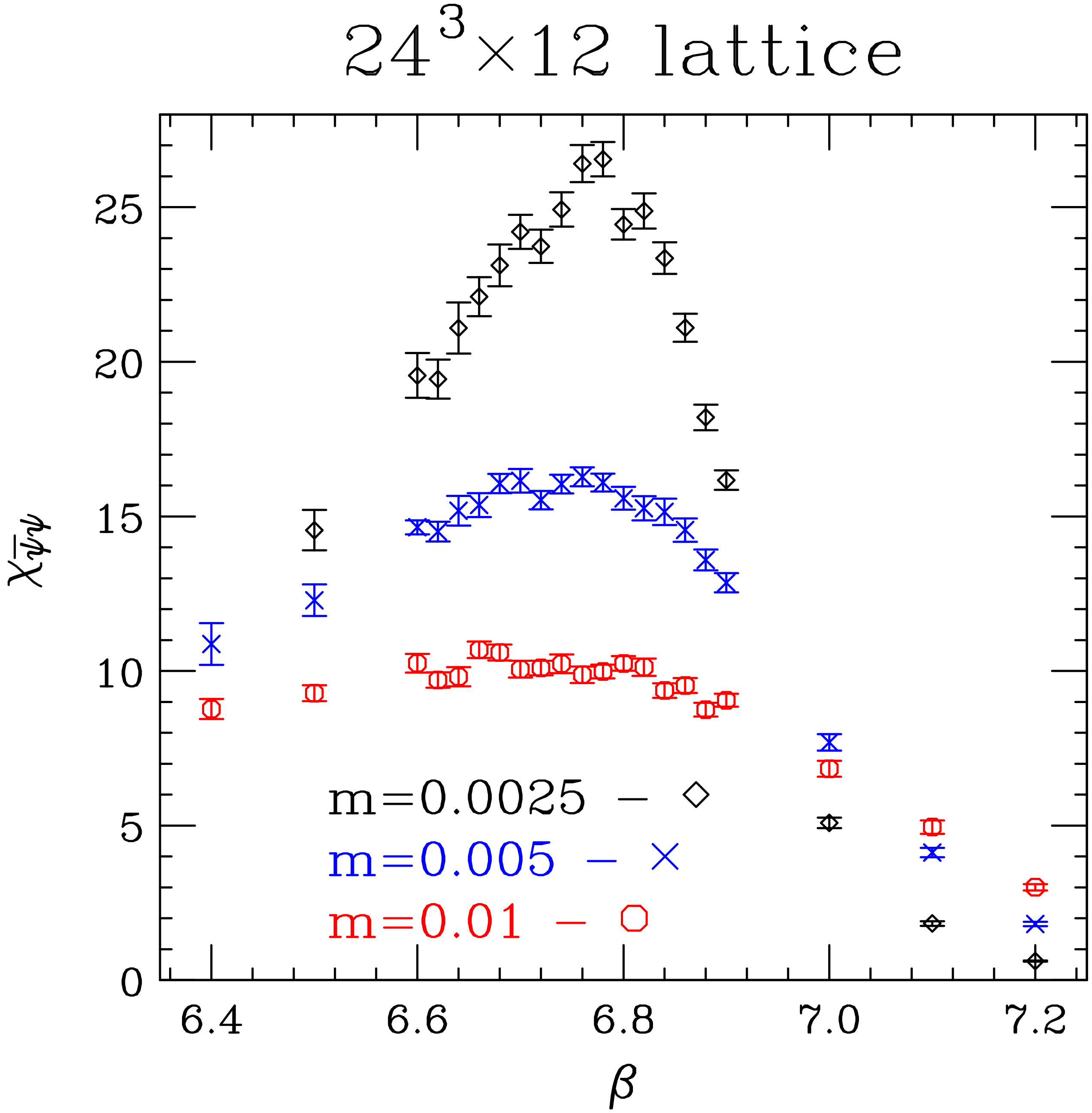

Figure 4: Chiral susceptibilities on a $24^3 \times 12$ lattice.

$24^3 \times N_t$

We simulate this 2-flavour theory on $24^3 \times N_t$ lattices with $N_t \le 24$ at fixed β, to search for evidence of a transition back to the chirally broken phase as N_t is increased.

We choose $\beta = 6.9$. If the evolution of β_χ is governed by the 2-loop β-function, $\beta_\chi \approx 6.9$ for $N_t = 18$. Hence we should see evidence for the chiral transition as N_t is increased.

At present, we run on $24^3 \times 8$, $24^3 \times 10$, $24^3 \times 12$, $24^3 \times 18$ and 24^4 lattices at $m = 0.005$, $m = 0.0025$ and $m = 0.00125$. We are just starting runs on $24^3 \times 20$ and $24^3 \times 22$ lattices.

Figure 5 shows the unsubtracted and subtracted chiral condensates as functions of N_t for these runs.

We follow Fodor *et al.*, defining a subtracted chiral condensate by:

$$\langle\bar{\psi}\psi\rangle_{sub} = \langle\bar{\psi}\psi\rangle - \left(m_V \frac{\partial}{\partial m_V}\langle\bar{\psi}\psi\rangle\right)_{m_V=m}$$

where m_V is the valence quark mass. We should try other schemes.

Figure 6 shows the chiral susceptibilities for these runs.

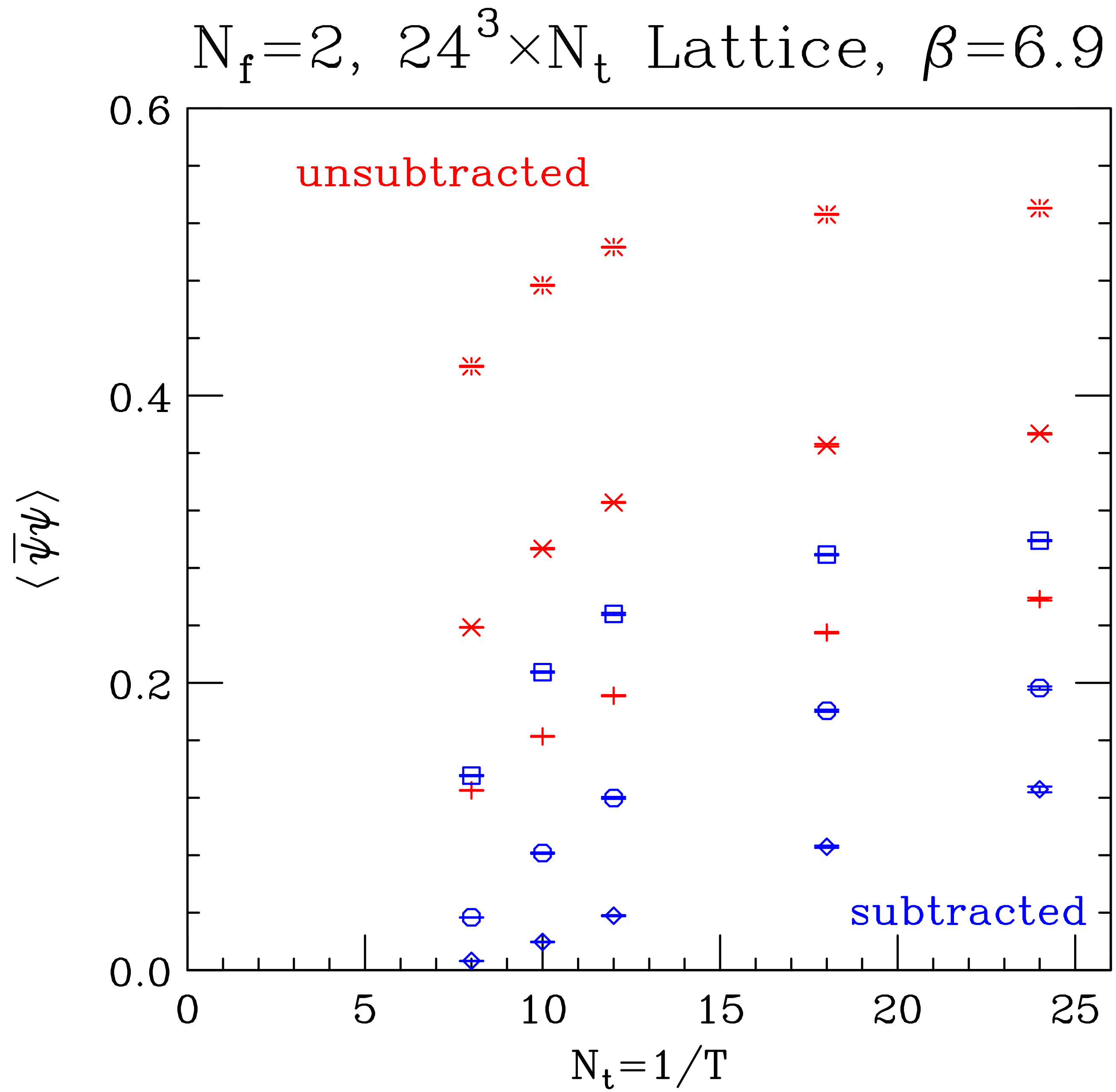

Figure 5: Chiral condensates on $24^3 \times N_t$ lattices at $\beta = 6.9$. From top to bottom, the masses are $m = 0.005$, $m = 0.0025$ and $m = 0.00125$.

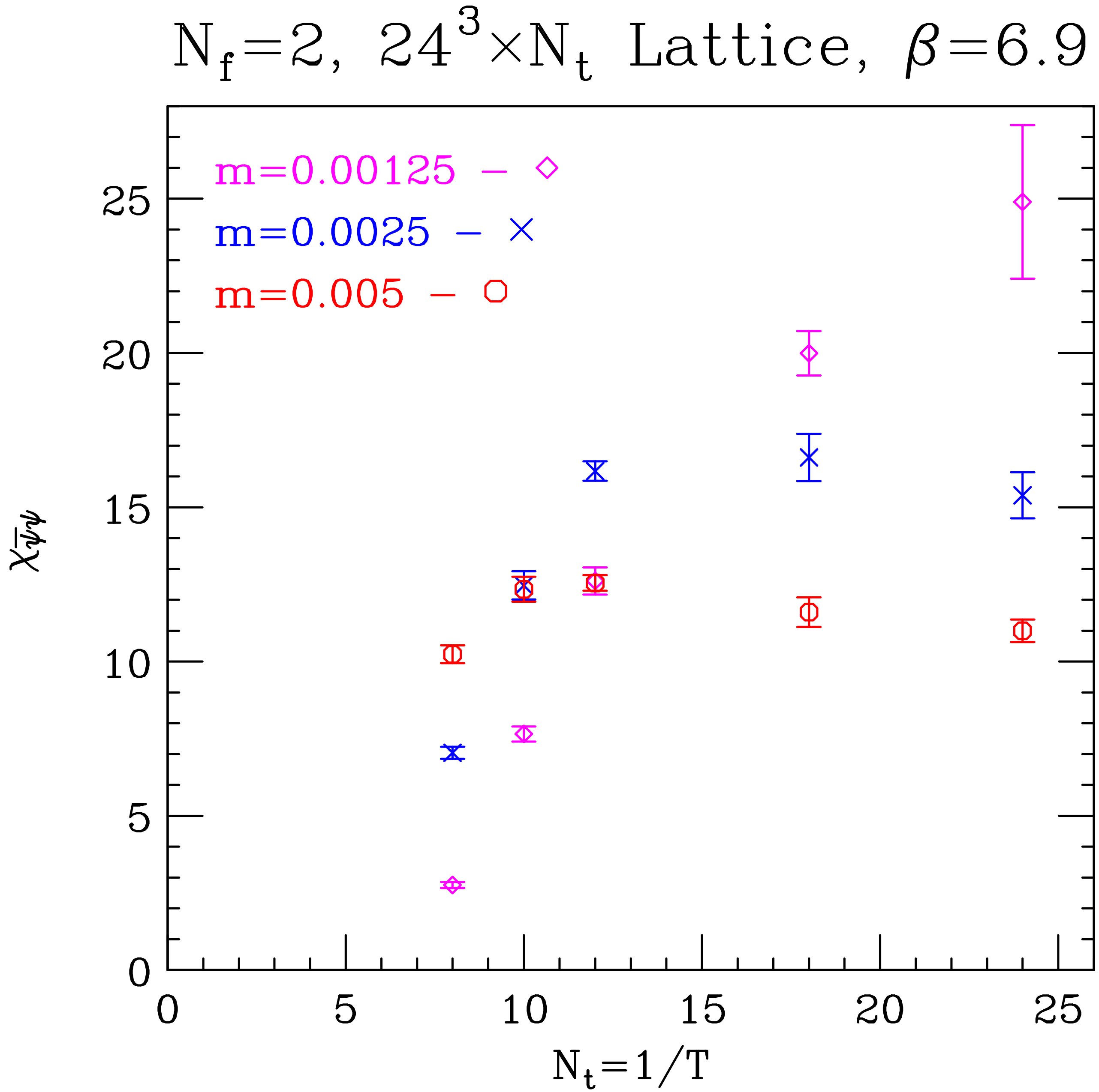

Figure 6: Chiral susceptibilities on $24^3 \times N_t$ lattices at $\beta = 6.9$.

$N_f = 3$

$N_t = 8$

We simulate lattice QCD with 3 light quark flavours on $16^3 \times 8$ lattices.

We run at $m = 0.01$ and $m = 0.005$.

For $6.3 \leq \beta \leq 6.5$ (close to the chiral transition) and $m = 0.005$, we perform runs of 50,000 – 100,000 trajectories at intervals of 0.02 in β.

Figure 7 shows the chiral susceptibilities for these runs.

From this we estimate that $\beta_\chi = 6.38(2)$, somewhat larger than that for $N_t = 6$.

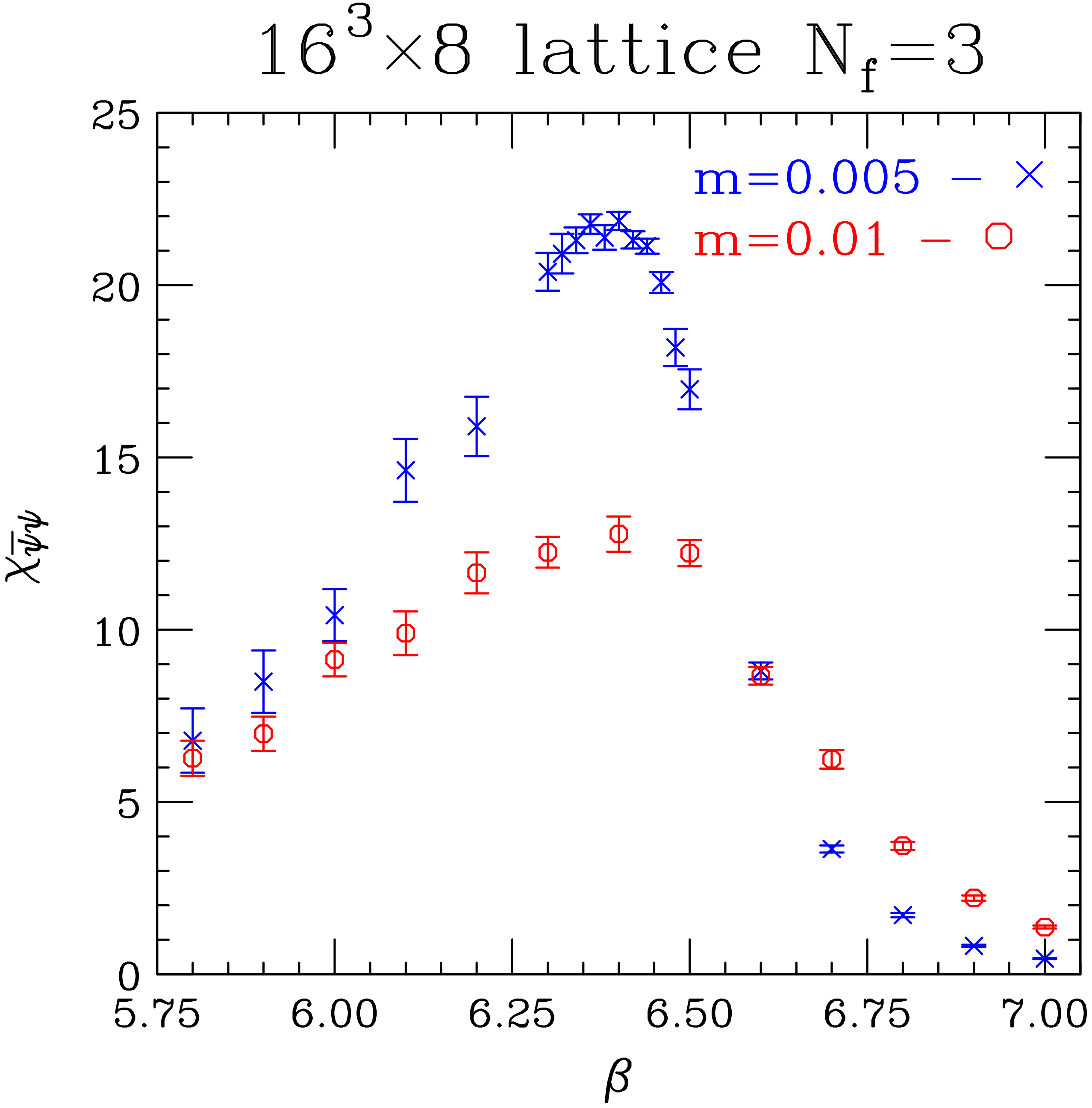

Figure 7: Chiral susceptibilities for $N_f = 3$ on a $16^3 \times 8$ lattice.

Discussions and Conclusions

N_t	β_d	β_χ
4	5.40(1)	6.3(1)
6	5.54(1)	6.6(1)
8	5.65(1)	6.69(1)
12	5.75(5)	6.78(2)

Table 1: $N_f = 2$ deconfinement and chiral transitions for $N_t = 4, 6, 8, 12$.

- We are simulating lattice QCD with 2 light colour-sextet quarks at finite temperature, to test whether it has an infrared fixed-point and is thus conformal, or if it is QCD-like, but with a slowly-evolving running coupling constant, i.e. if it 'walks' and is thus a Walking-Technicolor candidate.
- We have extended our simulations to $N_t = 12$. As seen in the table above, the chiral phase transition moves to larger β as N_t is increased. However, $\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) = 0.09(2)$ compared with ≈ 0.12 predicted by the 2-loop perturbative β-function.

- More statistics is needed to ratify this result. We should also test if this is a finite size effect. It is possible that the 3-loop term in the β-function is large in this lattice regularization. Larger N_ts might be needed to clarify this issue. We should also extend our $N_t = 6$ simulations to obtain a better estimate of the position of its chiral phase transition.

- Note that the chiral susceptibility is very sensitive to long 'time'-constant modes describing the system's evolution. These often have small amplitudes, so that they are not evident in other observables.

- Our simulations of the 3-flavour theory at $N_t = 8$ indicate that the increase in β, $\beta_\chi(N_t = 8) - \beta_\chi(N_t = 6)$ is appreciably larger than the ≈ 0.0025 predicted by 2-loop perturbation theory. Nor can we see any evidence that $\beta_\chi(N_t)$ is approaching a non-zero constant as $N_t \to \infty$ as expected, since the this theory is expected to be conformal. We will therefore need to perform simulations at $N_t = 12$.

- To study the spectrum of the 2-flavour theory, we need to restrict ourselves to the region $\beta < \beta_d$. For this β to lie in the weak-coupling domain, we will require rather large lattices, since our experience with the chiral transition tells us that the crossover from strong- to weak-coupling occurs somewhere in the regime $\beta =$ 6.3–6.6 while $\beta_d(N_t = 12) \approx 5.75$.
- For zero temperature physics, f_π is the relevant scale, since we know that $f_\pi = v \approx 246$ GeV.
- Another promising Walking Technicolor candidate, which we intend to study is $SU(2)$ gauge theory with 3 adjoint Majorana/Weyl quarks.

These simulations were performed on Hopper and Carver at NERSC, Kraken at NICS, and Fusion at LCRC, Argonne.

Appendix

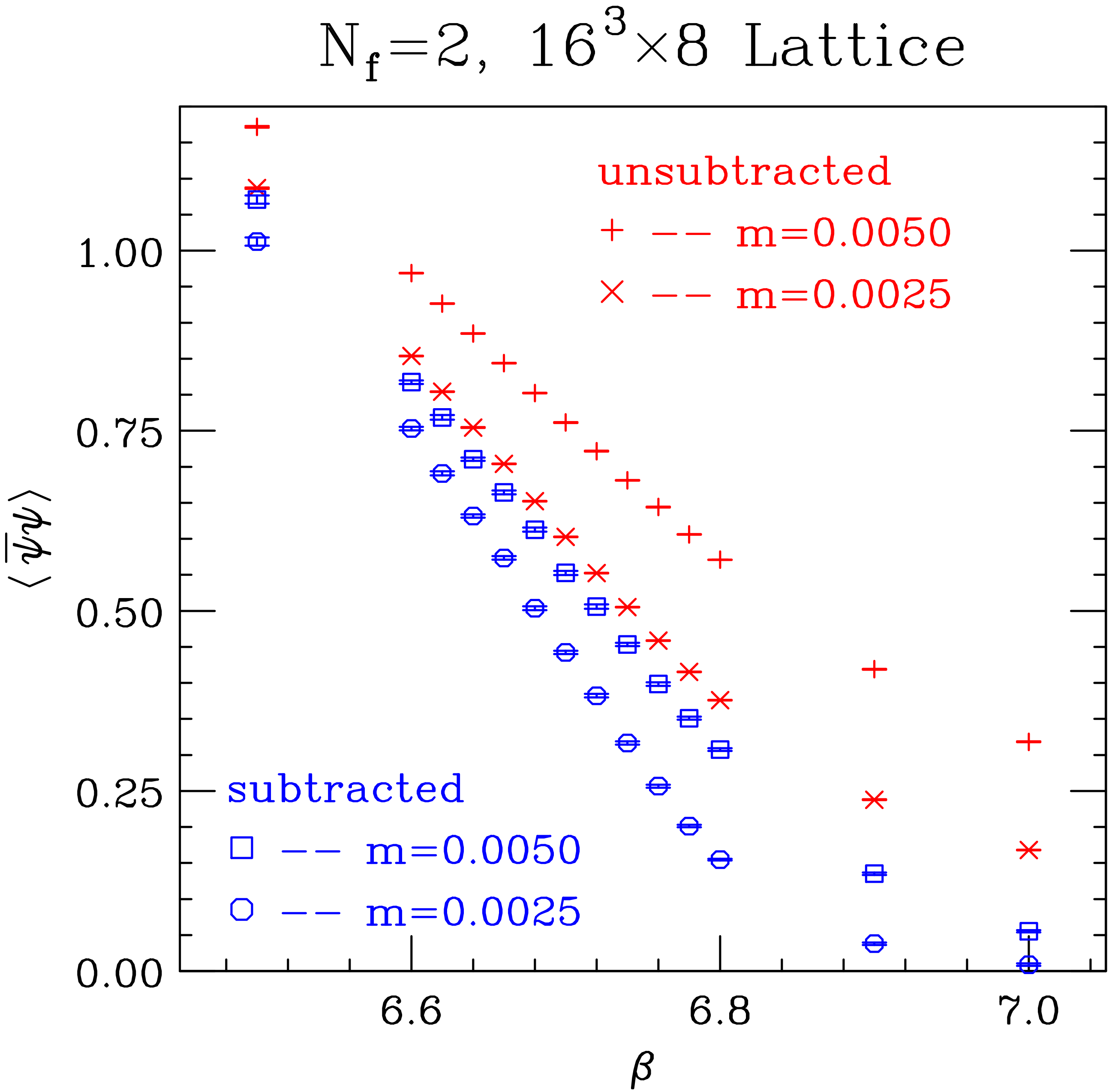

Figure 8: Unsubtracted and subtracted chiral condensates on a $16^3 \times 8$ lattice. $N_f = 2$.

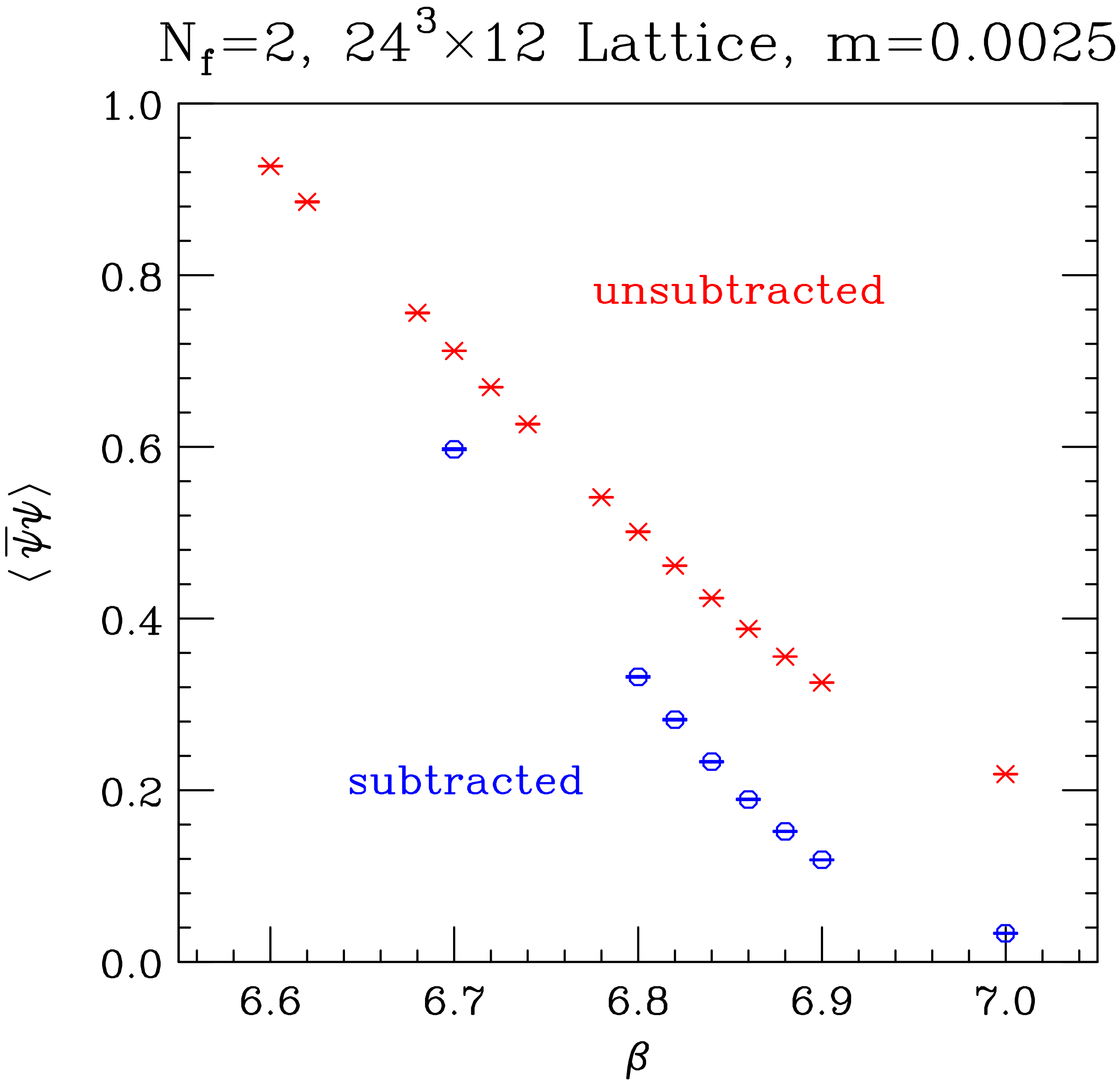

Figure 9: Unsubtracted and subtracted chiral condensates on a $24^3 \times 12$ lattice at $m = 0.0025$. $N_f = 2$.