Towards Continuum Limit for the QCD Critical Point

Saumen Datta, Rajiv V. Gavai and Sourendu Gupta
T. I. F. R., Mumbai, India

Non-zero Temperature and Density : Session 5 A

31st International Symposium on Lattice Field Theory

Mainz, Germany

Introduction

- QCD Critical Point in the temperature (T)-baryon density (μ_B) plane is a major focal theme of many experiments, such as the beam energy scan at RHIC or the upcoming FAIR (Germany) and NICA (Russia).

- Many models suggest its existence (e.g.,Asakawa-Yazaki, NPA 1989) but the best and most reliable way to locate it, if it exists, maybe Lattice.

Introduction

- QCD Critical Point in the temperature (T)-baryon density (μ_B) plane is a major focal theme of many experiments, such as the beam energy scan at RHIC or the upcoming FAIR (Germany) and NICA (Russia).

- Many models suggest its existence (e.g.,Asakawa-Yazaki, NPA 1989) but the best and most reliable way to locate it, if it exists, maybe Lattice.

- Beset with strong challenges : Its investigation needs

  — 2 light and 1 moderately heavy flavour with good control on $U_A(1)$-anomaly.

  — method(s) to handle the complex Fermion Determinant for nonzero μ_B.

Introduction

- QCD Critical Point in the temperature (T)-baryon density (μ_B) plane is a major focal theme of many experiments, such as the beam energy scan at RHIC or the upcoming FAIR (Germany) and NICA (Russia).

- Many models suggest its existence (e.g.,Asakawa-Yazaki, NPA 1989) but the best and most reliable way to locate it, if it exists, maybe Lattice.

- Beset with strong challenges : Its investigation needs

  — 2 light and 1 moderately heavy flavour with good control on $U_A(1)$-anomaly.

  — method(s) to handle the complex Fermion Determinant for nonzero μ_B.

- Mostly staggered quarks used in the present-day simulations. Broken flavour and spin symmetry on lattice: $2 + 1$ flavours ? Anomaly ??

Introduction

- QCD Critical Point in the temperature (T)-baryon density (μ_B) plane is a major focal theme of many experiments, such as the beam energy scan at RHIC or the upcoming FAIR (Germany) and NICA (Russia).

- Many models suggest its existence (e.g.,Asakawa-Yazaki, NPA 1989) but the best and most reliable way to locate it, if it exists, maybe Lattice.

- Beset with strong challenges : Its investigation needs

  — 2 light and 1 moderately heavy flavour with good control on $U_A(1)$-anomaly.

  — method(s) to handle the complex Fermion Determinant for nonzero μ_B.

- Mostly staggered quarks used in the present-day simulations. Broken flavour and spin symmetry on lattice: $2 + 1$ flavours ? Anomaly ??
  — Overlap/DWF possible now (Gavai-Sharma, PLB 2012).

Lattice Approaches for $\mu \neq 0$

Several approaches proposed in the past two decades or more : None as satisfactory as the usual $T \neq 0$ simulations.

Lattice Approaches for $\mu \neq 0$

Several approaches proposed in the past two decades or more : None as satisfactory as the usual $T \neq 0$ simulations.

- A partial list :
  - Two parameter Re-weighting (Z. Fodor & S. Katz, JHEP 0203 (2002) 014).
  - Imaginary Chemical Potential (Ph. de Frocrand & O. Philipsen, NP B642 (2002) 290; M.-P. Lombardo & M. D'Elia PR D67 (2003) 014505).
  - Taylor Expansion (C. Allton et al., PR D68 (2003) 014507; R.V. Gavai and S. Gupta, PR D68 (2003) 034506).
  - Canonical Ensemble (K. -F. Liu, IJMP B16 (2002) 2017, S. Kratochvila and P. de Forcrand, Pos LAT2005 (2006) 167.)
  - Complex Langevin (G. Aarts and I. O. Stamatescu, arXiv:0809.5227 and its references for earlier work).

Lattice Approaches for $\mu \neq 0$

Several approaches proposed in the past two decades or more : None as satisfactory as the usual $T \neq 0$ simulations.

- A partial list :
  - Two parameter Re-weighting (Z. Fodor & S. Katz, JHEP 0203 (2002) 014).
  - Imaginary Chemical Potential (Ph. de Frocrand & O. Philipsen, NP B642 (2002) 290; M.-P. Lombardo & M. D'Elia PR D67 (2003) 014505).
  - Taylor Expansion (C. Allton et al., PR D68 (2003) 014507; R.V. Gavai and S. Gupta, PR D68 (2003) 034506).
  - Canonical Ensemble (K. -F. Liu, IJMP B16 (2002) 2017, S. Kratochvila and P. de Forcrand, Pos LAT2005 (2006) 167.)
  - Complex Langevin (G. Aarts and I. O. Stamatescu, arXiv:0809.5227 and its references for earlier work).
- Why Taylor series expansion? — i) Ease of taking continuum and thermodynamic limit & ii) Better control of systematic errors.

Detail of Expansion

Standard definitions yield various number densities and susceptibilities :

$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i} \text{ and } \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations, $\lambda_s \ldots$) (S. Datta's talk)

Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure P has the expansion in μ:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!}\left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!}\left(\frac{\mu_d}{T}\right)^{n_d} \qquad (1)$$

Detail of Expansion

Standard definitions yield various number densities and susceptibilities :

$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i} \text{ and } \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations, $\lambda_s \ldots$) (S. Datta's talk)

Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure P has the expansion in μ:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!}\left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!}\left(\frac{\mu_d}{T}\right)^{n_d} \tag{1}$$

- From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence obtained from the coefficients of the series using $\sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}T^2}}$ or $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}T^2}\right)^{1/n}$. We use both definitions and terms up to 8th order in μ.

- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

- We (Gavai-Gupta '05, '09) use up to 8th order. Bielefeld-RBC so far has up to 4th-6th order.

- 10th & even 12th order may be possible : Ideas to extend to higher orders are emerging (Gavai-Sharma PRD 2012 & PRD 2010) which save up to 60 % computer time.

- From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence obtained from the coefficients of the series using $\sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}T^2}}$ or $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}T^2}\right)^{1/n}$. We use both definitions and terms up to 8th order in μ.

- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

- We (Gavai-Gupta '05, '09) use up to 8^{th} order. Bielefeld-RBC so far has up to 4^{th}-6^{th} order.

- 10th & even 12th order may be possible : Ideas to extend to higher orders are emerging (Gavai-Sharma PRD 2012 & PRD 2010) which save up to 60 % computer time.

- From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence obtained from the coefficients of the series using $\sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}T^2}}$ or $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}T^2}\right)^{1/n}$. We use both definitions and terms up to 8th order in μ.

- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

- We (Gavai-Gupta '05, '09) use up to 8th order. Bielefeld-RBC so far has up to 4th-6th order.

- 10th & even 12th order may be possible : Ideas to extend to higher orders are emerging (Gavai-Sharma PRD 2012 & PRD 2010) which save up to 60 % computer time.

Our Simulations & Results

- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.
- $m_\pi/m_\rho = 0.31 \pm 0.01$; Kept the same as $a \to 0$ (on all N_t).
- Earlier Lattice : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24 (Gavai-Gupta, PRD 2005)
  Finer Lattice : 6 $\times N_s^3$, $N_s =$ 12, 18, 24 (Gavai-Gupta, PRD 2009).

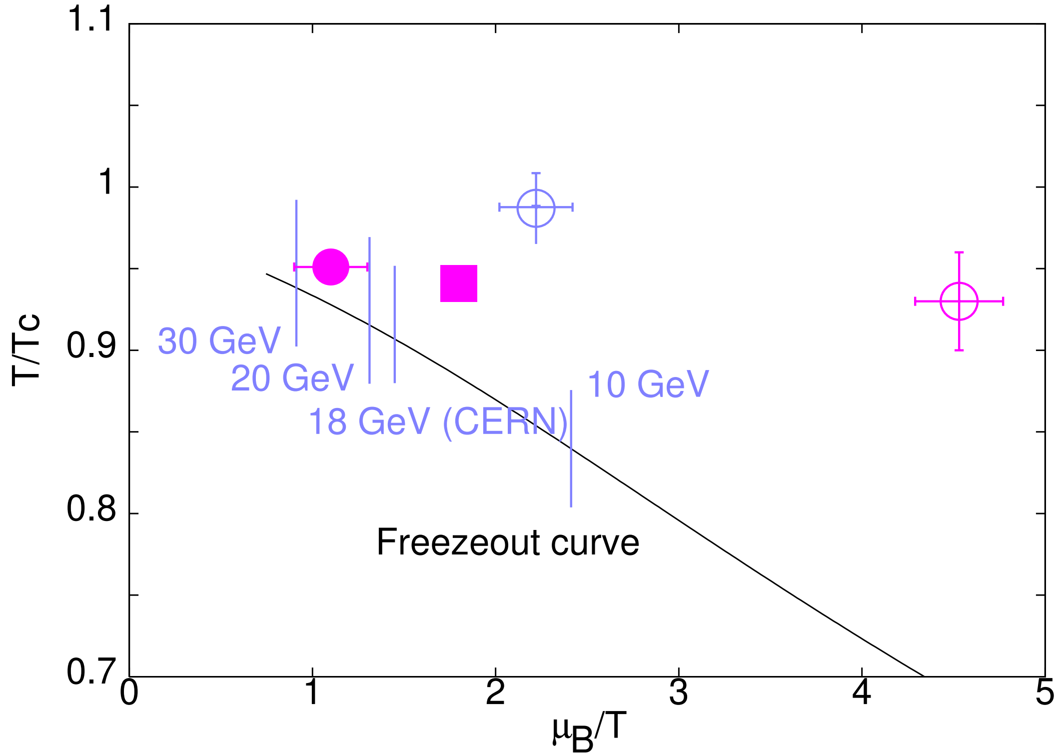
Our Simulations & Results

- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.
- $m_\pi/m_\rho = 0.31 \pm 0.01$; Kept the same as $a \to 0$ (on all N_t).
- Earlier Lattice : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24 (Gavai-Gupta, PRD 2005)
  Finer Lattice : 6 $\times N_s^3$, $N_s =$ 12, 18, 24 (Gavai-Gupta, PRD 2009).
- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$ for finer lattice: Our earlier coarser lattice result was $\mu_B^E/T^E = 1.3 \pm 0.3$. Infinite volume result: $\downarrow$ to 1.1(1)

Our Simulations & Results

- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.
- $m_\pi/m_\rho = 0.31 \pm 0.01$; Kept the same as $a \to 0$ (on all N_t).
- Earlier Lattice : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24 (Gavai-Gupta, PRD 2005)
  Finer Lattice : 6 $\times N_s^3$, $N_s =$ 12, 18, 24 (Gavai-Gupta, PRD 2009).
- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$ for finer lattice: Our earlier coarser lattice result was $\mu_B^E/T^E = 1.3 \pm 0.3$. Infinite volume result: $\downarrow$ to 1.1(1)
- T_c — defined by the peak of Polyakov loop susceptibility.
- Even finer Lattice : 8 $\times 32^3$ — This Talk
  Aspect ratio, N_s/N_t, maintained four to reduce finite volume effects.

Critical Point : Story thus far



♠ $N_f = 2$ (magenta) and 2+1 (blue) (Fodor-Katz, JHEP '04).

♡ $N_t = 4$ Circles (GG '05 & Fodor-Katz JHEP '02), $N_t = 6$ Box (GG '09).

Computing The Susceptibilities

All susceptibilities can be written as traces of products of M^{-1} and various derivatives of M.

At leading order,

$$\chi_{20} = \left(\frac{T}{V}\right) [\langle \mathcal{O}_2 + \mathcal{O}_{11} \rangle], \qquad \chi_{11} = \left(\frac{T}{V}\right) [\langle \mathcal{O}_{11} \rangle]$$

Here $\mathcal{O}_2 = \mathrm{Tr}\ M^{-1}M'' - \mathrm{Tr}\ M^{-1}M'M^{-1}M'$, and $\mathcal{O}_{11} = (\mathrm{Tr}\ M^{-1}M')^2$, and the traces are estimated by a stochastic method (Gottlieb et al., PRL '87):

$\mathrm{Tr}\ A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\mathrm{Tr}\ A)^2 = 2 \sum_{i>j=1}^{L} (\mathrm{Tr}\ A)_i (\mathrm{Tr}\ A)_j / L(L-1)$, where R_i is a complex vector from a set of N_v subdivided in L independent sets.

Computing The Susceptibilities

All susceptibilities can be written as traces of products of M^{-1} and various derivatives of M.

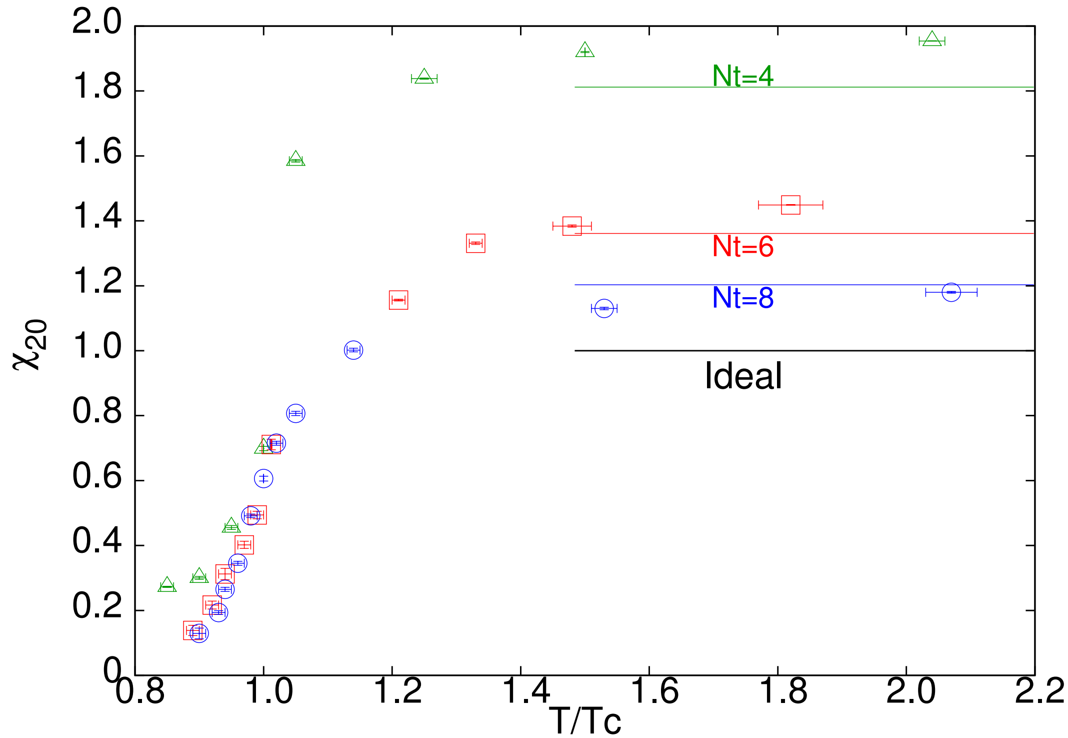
At leading order,

$$\chi_{20} = \left(\frac{T}{V}\right)[\langle \mathcal{O}_2 + \mathcal{O}_{11} \rangle], \qquad \chi_{11} = \left(\frac{T}{V}\right)[\langle \mathcal{O}_{11} \rangle]$$

Here $\mathcal{O}_2 = \mathrm{Tr}\ M^{-1}M'' - \mathrm{Tr}\ M^{-1}M'M^{-1}M'$, and $\mathcal{O}_{11} = (\mathrm{Tr}\ M^{-1}M')^2$, and the traces are estimated by a stochastic method (Gottlieb et al., PRL '87):

$\mathrm{Tr}\ A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\mathrm{Tr}\ A)^2 = 2\sum_{i>j=1}^{L} (\mathrm{Tr}\ A)_i (\mathrm{Tr}\ A)_j / L(L-1)$, where R_i is a complex vector from a set of N_v subdivided in L independent sets.

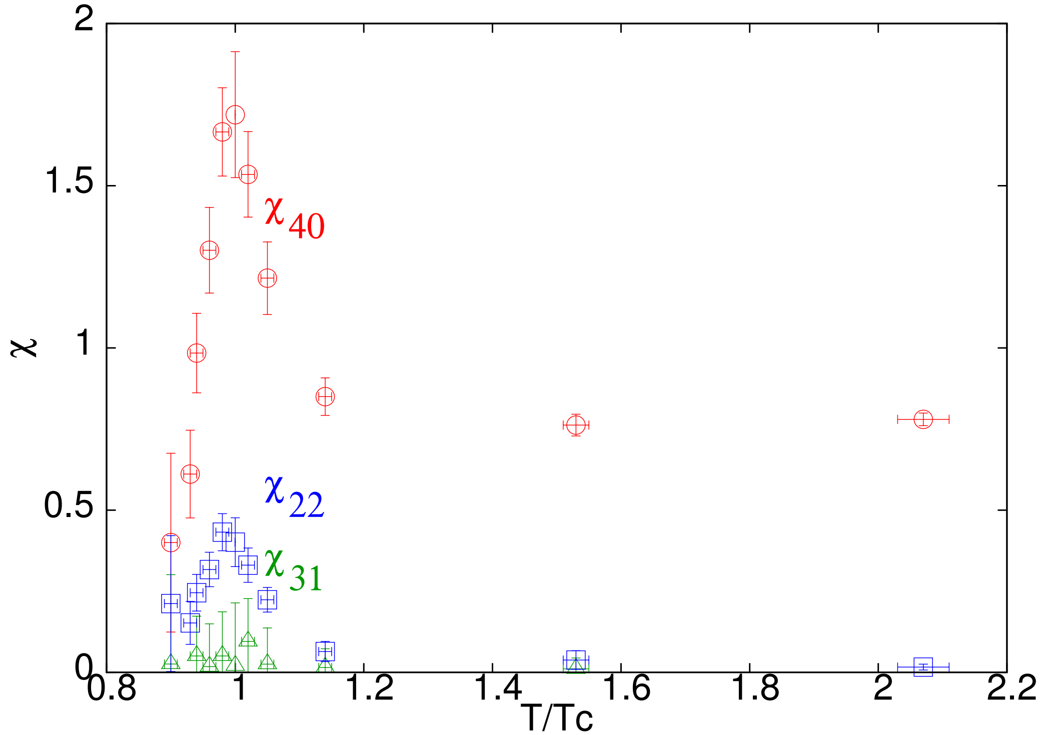
χ_2 for $N_t = 8$, 6, and 4 lattices



♠ Our $N_t = 8$ (Lattice 2013) and $N_t = 6$ (GG, PRD '09) results agree.

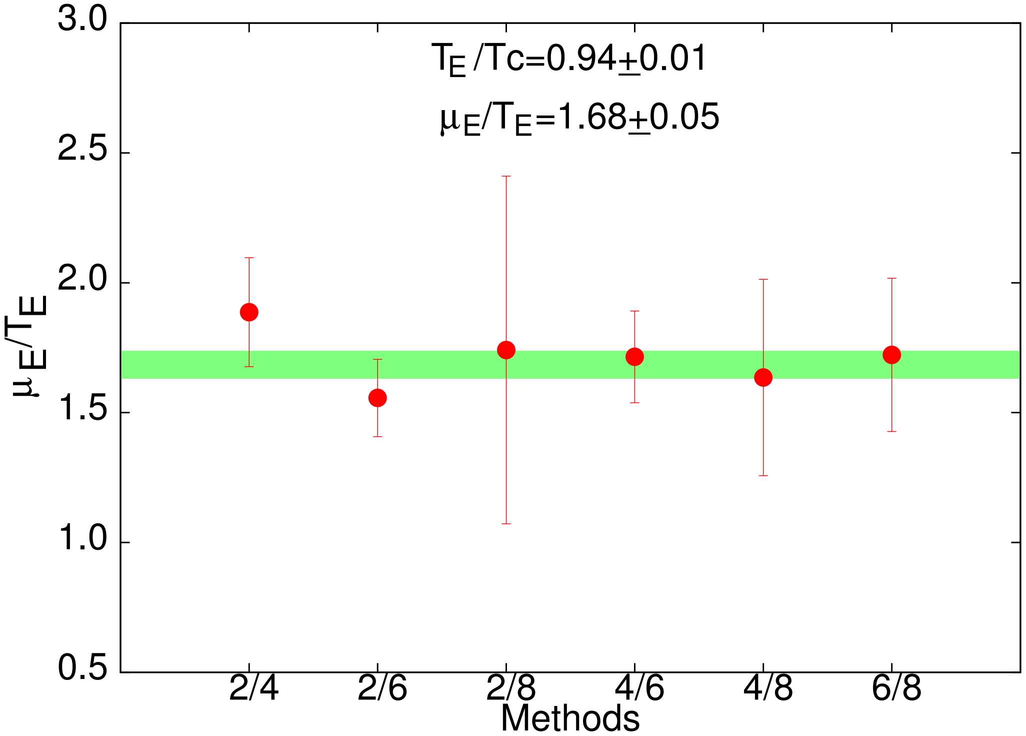
♡ Our estimate of $\beta_c(N_t = 8)$ agrees with Gottlieb et al. PR D47,1993.

4^{th} Order Susceptibilities for $N_t = 8$ lattice



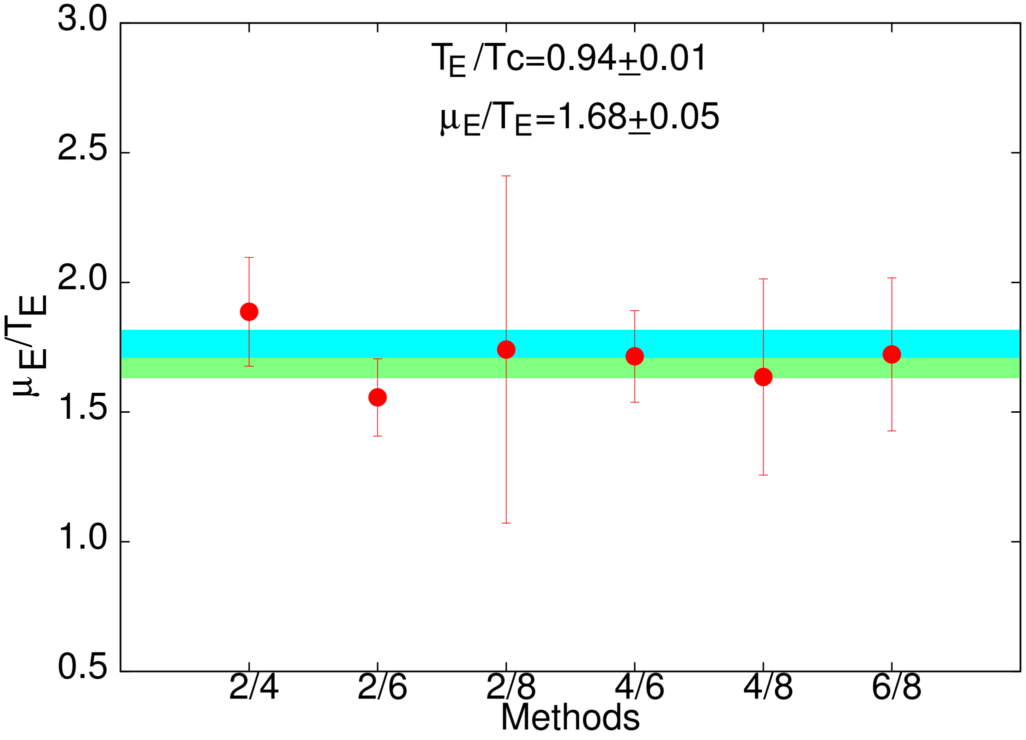
♠ ∼200 configurations separated by a a few autocorrelation lengths employed.
♠ 2000 Gaussian random vectors employed at each point.

Radius of Convergence result



♡ As earlier for smaller N_t, constancy of the ratios $\rightsquigarrow$ (T_E, μ_E) for $N_t = 8$ to be $(0.94 \pm 0.01, 1.68 \pm 0.05)$.

Radius of Convergence result



♠ Same T_E as for $N_t = 6$, with the corresponding band for μ_E (1.8 ± 0.1) displaying consistency as well.

Summary

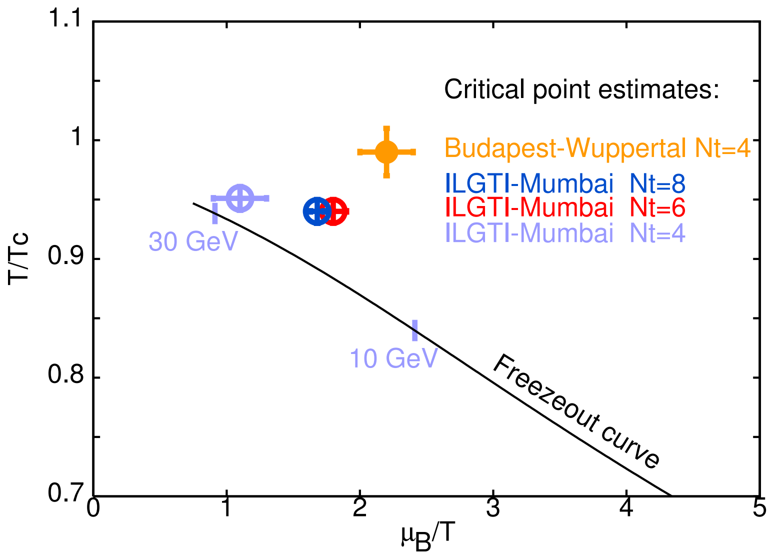
- The method we advocated, and employed for $N_t = 4$ and 6, works for $N_t = 8$ as well, yielding similar qualitative picture.

- Our results (T_E, μ_E) for $N_t = 8$ are the first to approach the continuum limit.

Summary

- The method we advocated, and employed for $N_t = 4$ and 6, works for $N_t = 8$ as well, yielding similar qualitative picture.

- Our results (T_E, μ_E) for $N_t = 8$ are the first to approach the continuum limit.



Critical Point location appears to be the same for $N_t = 8$ and 6 at temperature $\frac{T^E}{T_c} = 0.94 \pm 0.01$. Slight shift in $\mu_B^E/T = 1.68(5)$ for $N_t = 8$; Agrees with $N_t = 6$ within errors.