Finite size scaling for 3 and 4-flavor QCD with finite chemical potential

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in collaboration with

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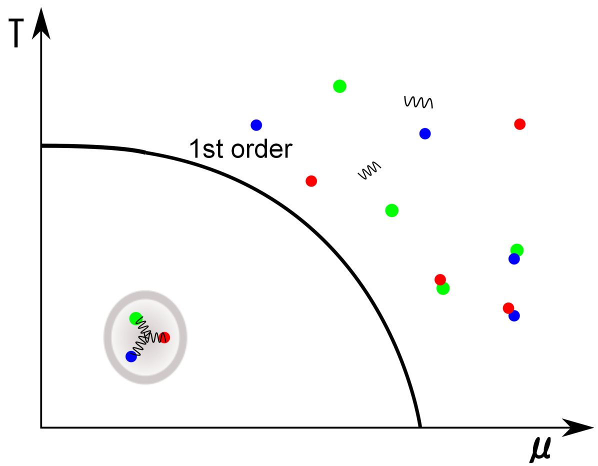
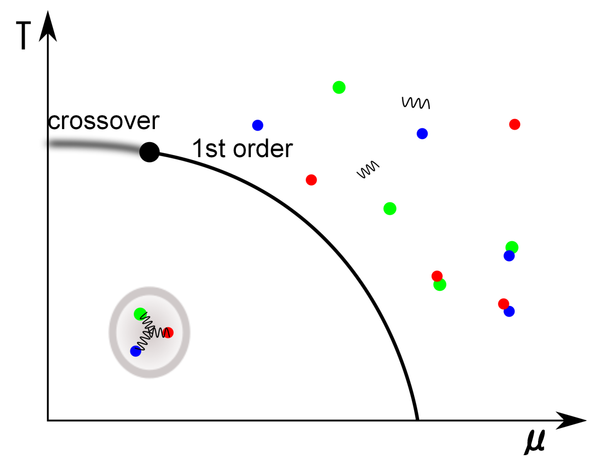
Why 4-flavor ?

- Good testing ground before 3-flavor
- Depending on the size of mass, phase diagram changes
- Reasonable cost to survey transition region



massless



large mass

Lattice study so far

- Multi-parameter reweighting Fodor & Katz 01
- Imaginary chemical potential D'Elia & Lombardo 02
- Canonical approach de Forcrand 06, Kentucky 10

It is not well investigated by finite size scaling!

What we do here

- Careful finite size scaling and high statistics ~ 10^5 conf.
- Grand canonical approach with Wilson type fermions

$$\mathcal{Z}_{\rm QCD}(T,\mu) \quad = \quad \int [dU] e^{-S_{\rm g}[U]} \det D(\mu;U)$$

← Complex

- Phase reweighting

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{iN_{\rm f}\theta} \rangle_{||}}{\langle e^{iN_{\rm f}\theta} \rangle_{||}}$$

Phase can be controlled for larger temporal size

ST, Kuramashi & Ukawa (2011)

$$\mathcal{Z}_{||}(T,\mu) \quad = \quad \int [dU] e^{-S_{\rm g}[U]} |\det D(\mu;U)|$$

- Reduction technique

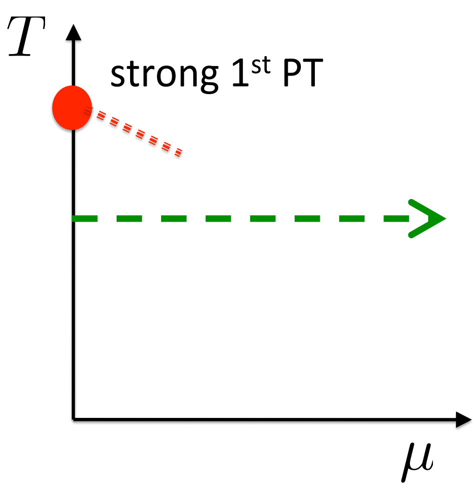
Danzer & Gattringer (2008)

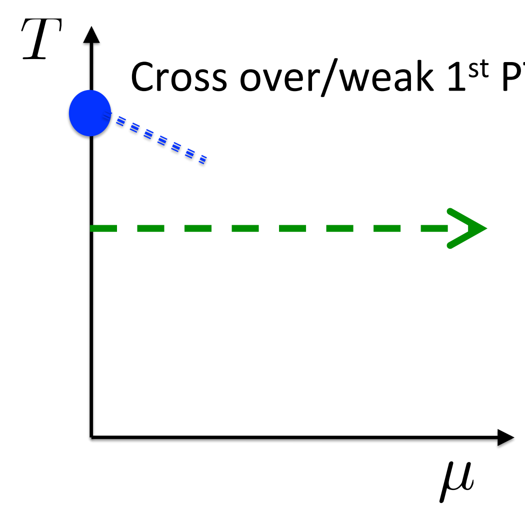
  - exact phase & quark number
  - GPGPU

Simulation parameters

- Clover fermions and Iwasaki gauge ← Kentucky group (2010) canonical approach
- Parameters:

Light mass	Heavy mass
β=1.58	β=1.60
κ=0.1385	κ=0.1371
$V=6^3$-10^3	$V=6^3$-8^3
μ=0.02-0.30	μ=0.10-0.35
m_π/m_ρ=0.822	m_π/m_ρ=0.839
T/m_ρ=0.154	T/m_ρ=0.150





$N_t=4$

μ-reweighting

$$\langle \mathcal{O}(\mu') \rangle_{\mu'} = \frac{\left\langle \mathcal{O}(\mu') \frac{\det D(\mu')^{N_\mathrm{f}}}{\det D(\mu)^{N_\mathrm{f}}} e^{iN_\mathrm{f}\theta(\mu)} \right\rangle_{||\mu}}{\left\langle \frac{\det D(\mu')^{N_\mathrm{f}}}{\det D(\mu)^{N_\mathrm{f}}} e^{iN_\mathrm{f}\theta(\mu)} \right\rangle_{||\mu}}$$

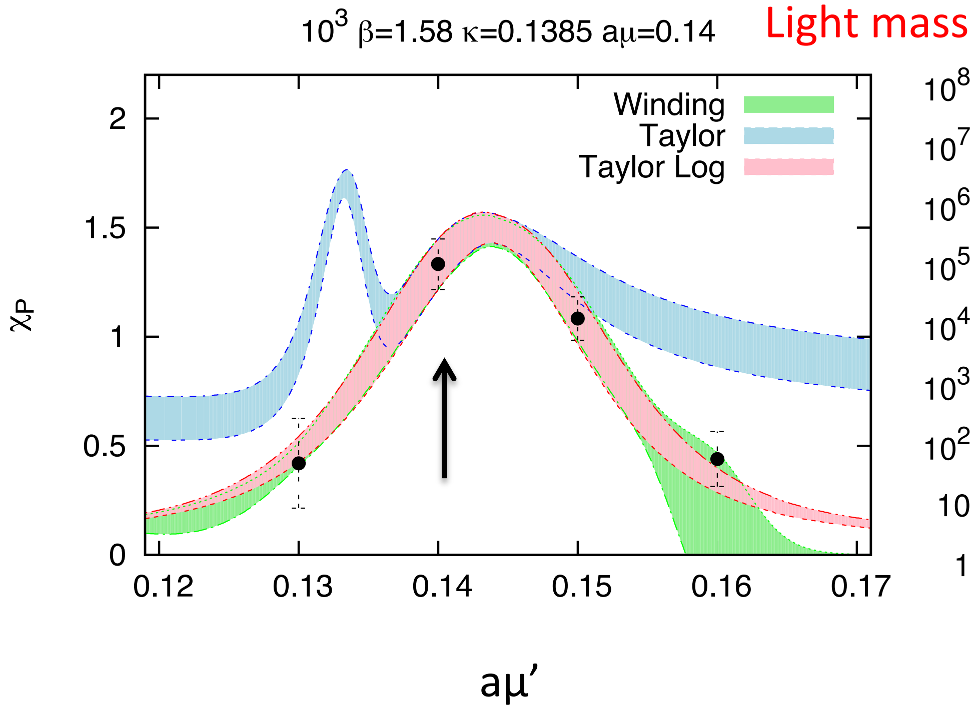
Expansion schemes to evaluate ratio of determinant

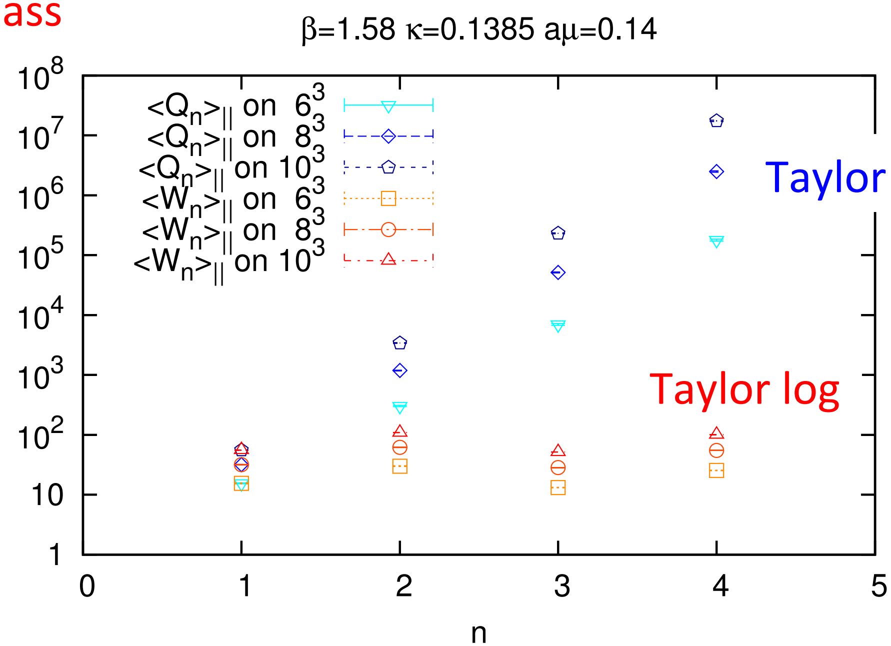
$$\Delta\mu = \mu' - \mu$$

- Taylor expansion $= 1 + \sum_{n=1}^{\infty} \frac{(\Delta\mu/T)^n}{n!} Q_n$
- Taylor expansion of logarithm $= \exp\left[\sum_{n=1}^{\infty} \frac{(\Delta\mu/T)^n}{n!} W_n\right]$
- Winding expansion (fugacity expansion of logarithm)

$$= \exp\left[\sum_{n\in\mathbb{Z}} V_n \left(e^{n\mu'/T} - e^{n\mu/T}\right)\right]$$

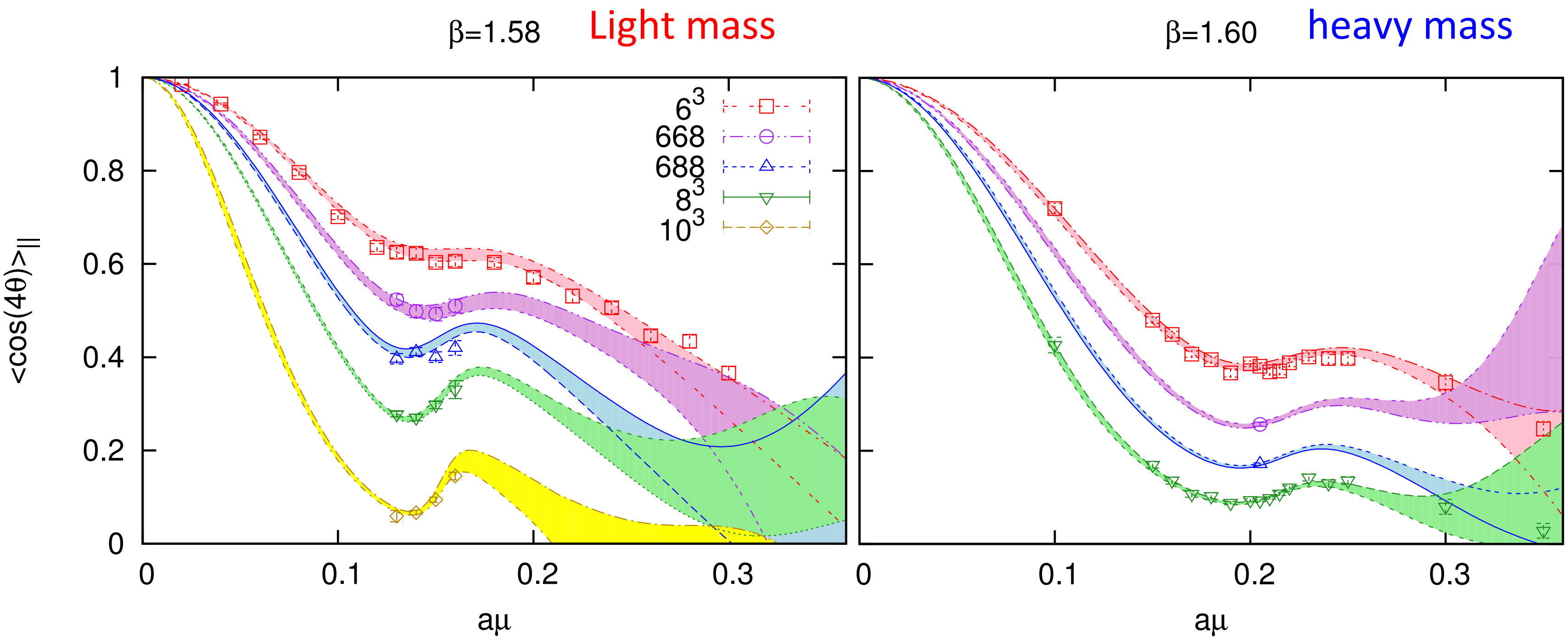
Which expansion is better?





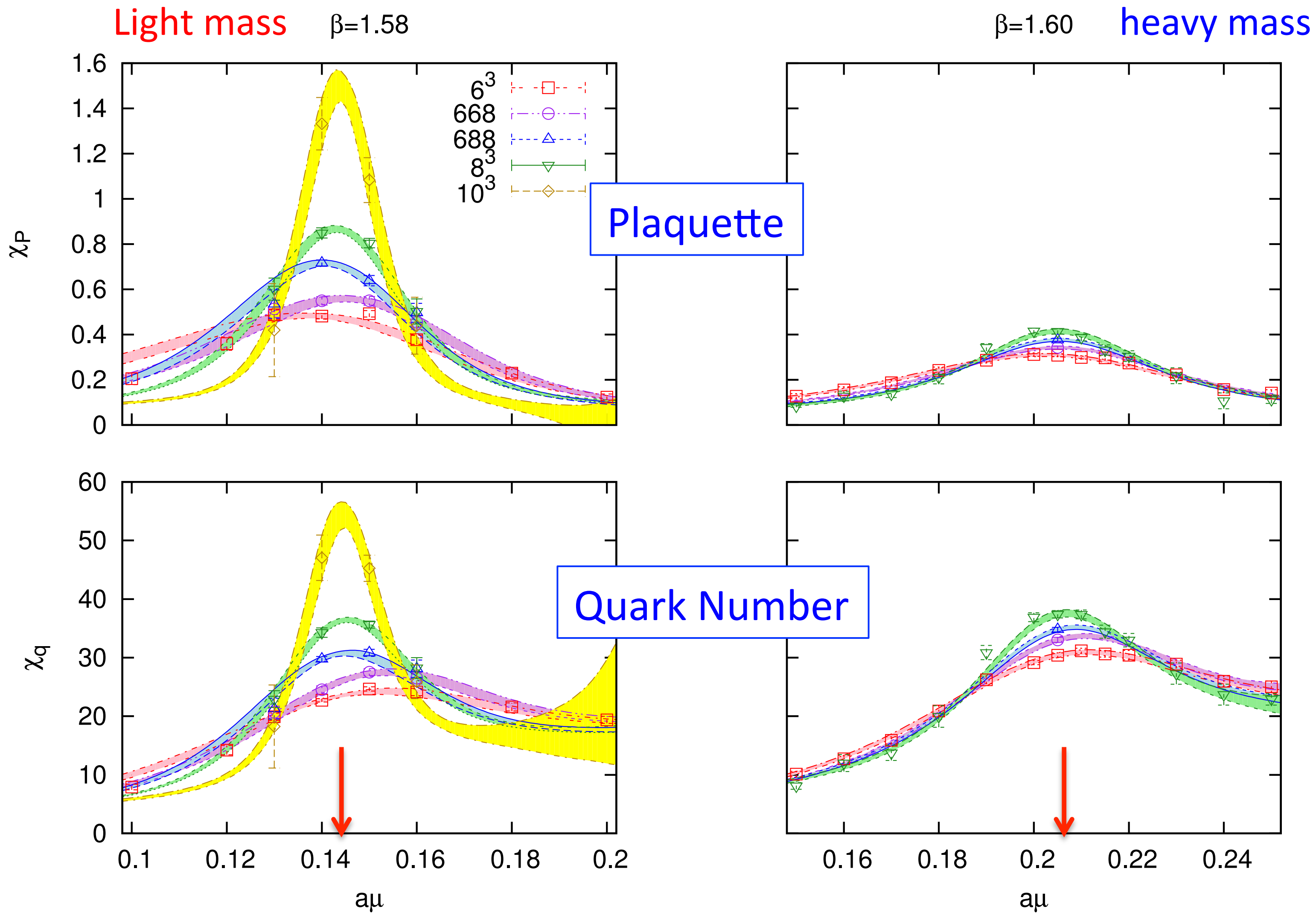
Phase-reweighting factor

$$\langle e^{i4\theta} \rangle_{||} = \frac{\mathcal{Z}_{\mathrm{QCD}}}{\mathcal{Z}_{||}}$$

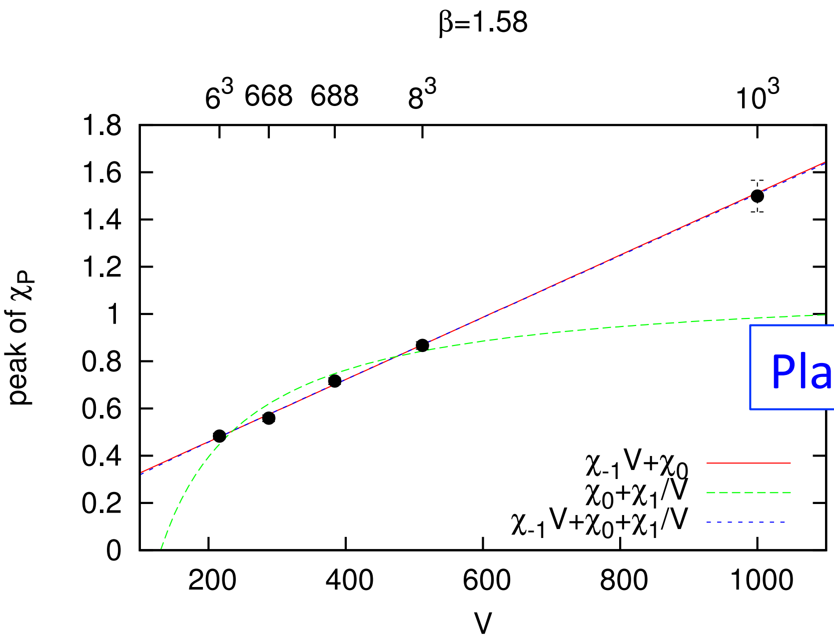


$$a\mu_c = am_\pi/2 \sim 0.7$$

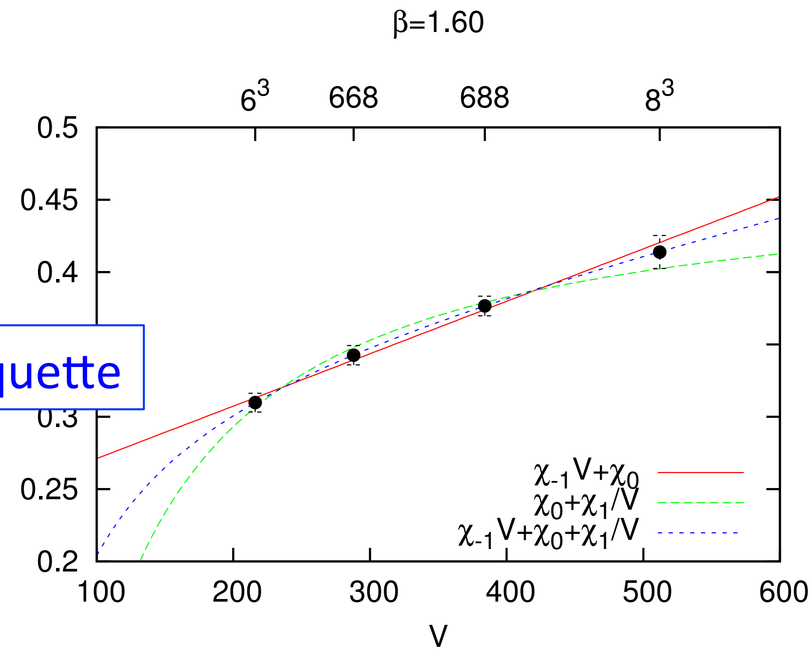
Susceptibility



Volume scaling of susceptibility peak



Plaquette



1st order phase transition

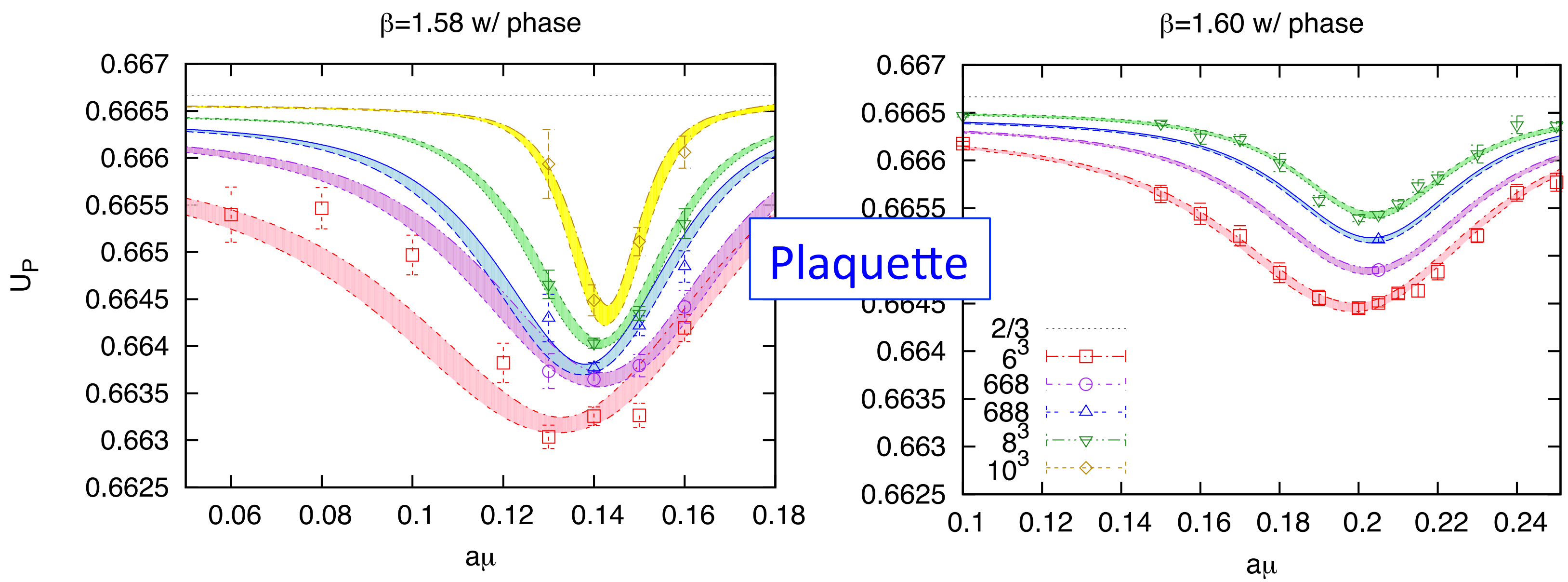
Cross over/weak 1st order PT

Challa Landau Binder cumulant

$$U_X = 1 - \frac{1}{3}\frac{\langle X^4 \rangle}{\langle X^2 \rangle^2}$$

Challa, Landau & Binder 86

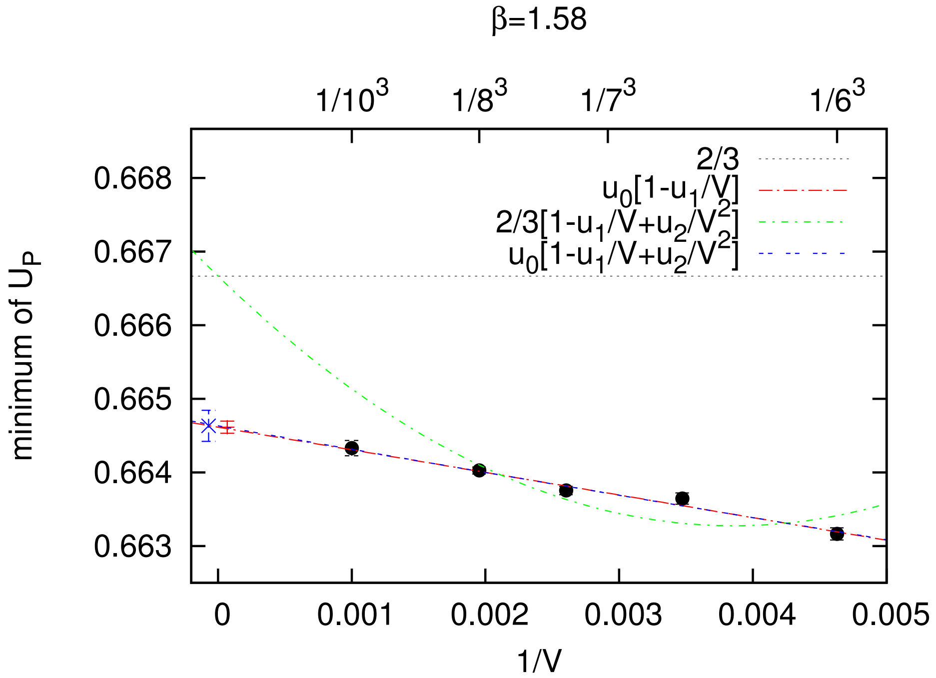
Fukugita, Okawa & Ukawa 89



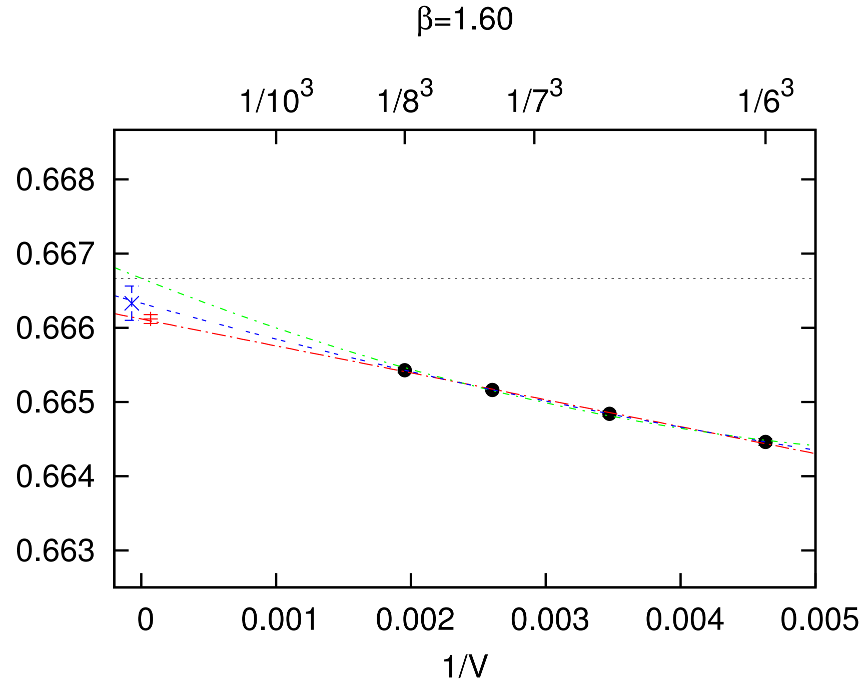
$$\lim_{V\to\infty} U_{\rm min} = \begin{cases} 2/3 & \text{cross over} \\ \text{others} & \text{1st order} \end{cases}$$

Scaling for the min of CLB cumulant

$$\lim_{V\to\infty} U_{\min} = \begin{cases} 2/3 & \text{cross over} \\ \text{others} & \text{1st order} \end{cases}$$



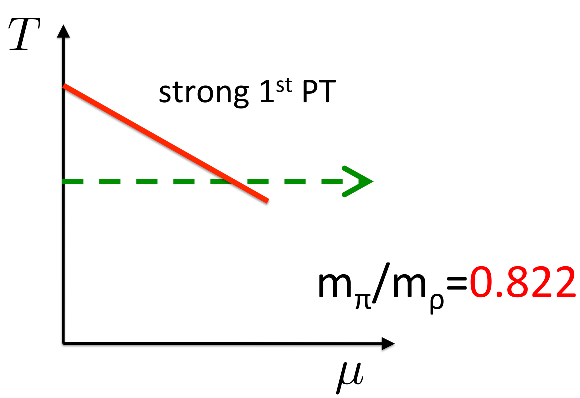
1st PT

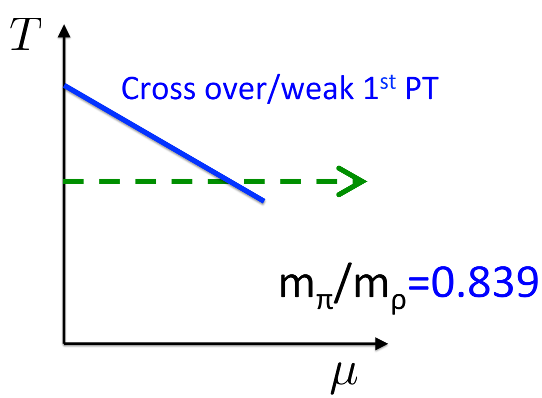


Cross over/weak 1st PT

Summary for N_f=4

- μ-reweighting works very well
- Taylor expansion of logarithm of determinant is a good approximation
- Moments analysis shows that





- Lee-Yang zero analysis will be presented by X-Y. Jin after this talk

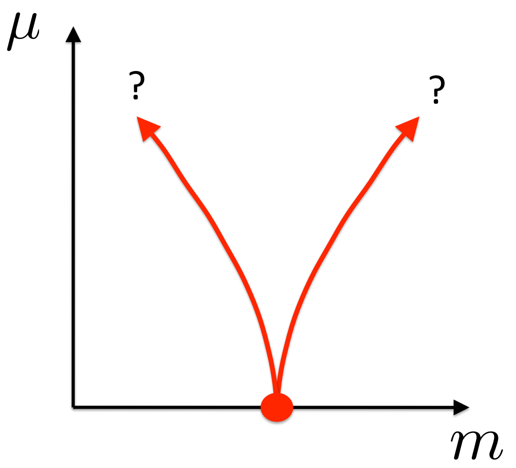
N_f=3 finite density QCD

- Purpose :

  Tracing critical end point

  in (m,μ) plane

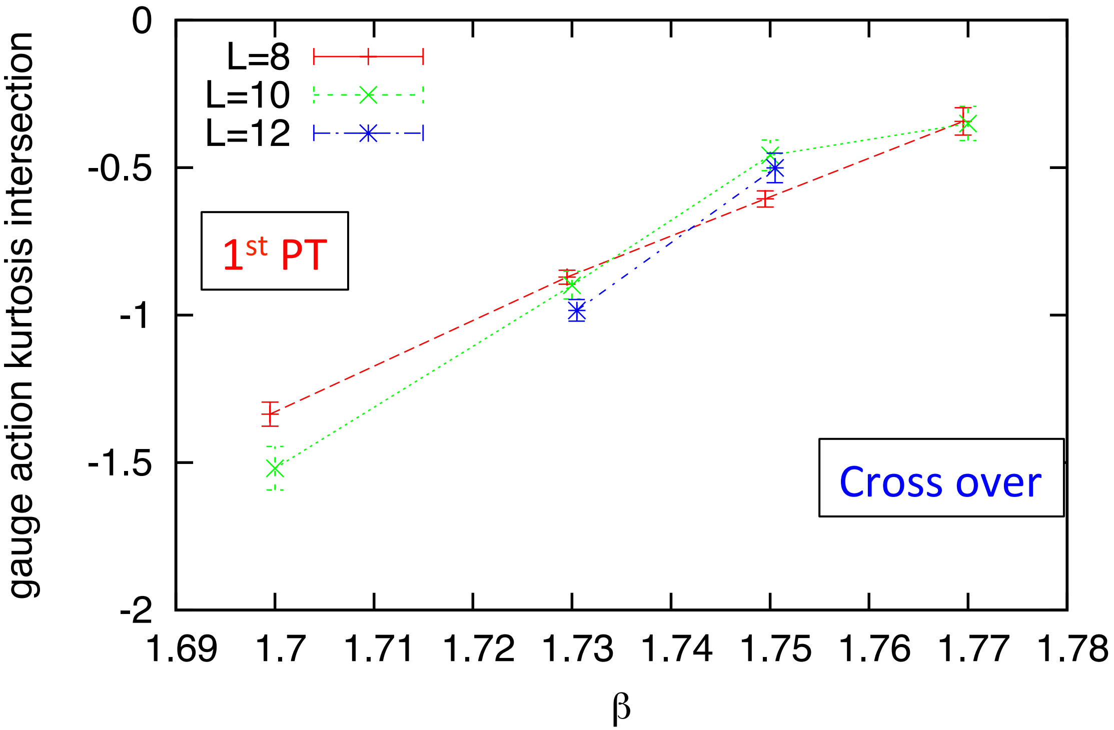
  de Forcrand & Philipsen 2006



- Procedure
  - Critical end point is estimated by Binder cumulant (kurtosis) intersection method Karsch et al. 2001
  - μ=0 is discussed by Nakamura on Thu.

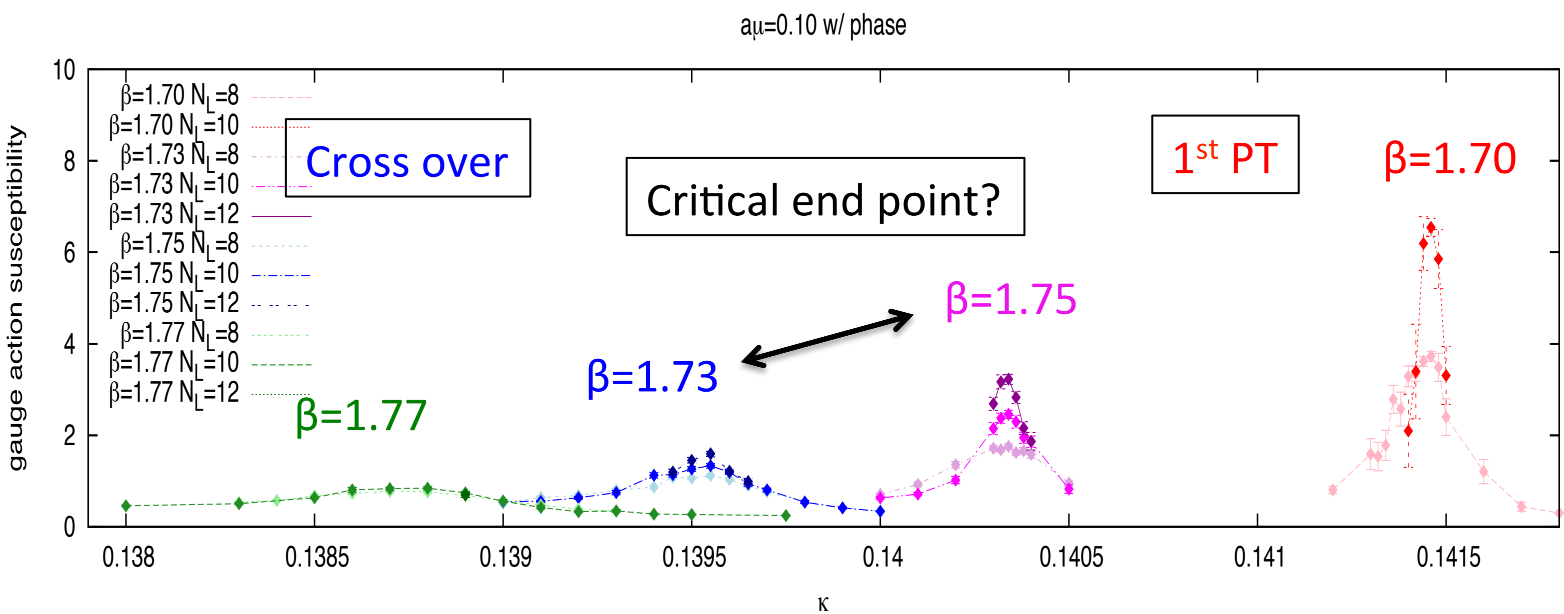
Kurtosis intersection

- Iwasaki gauge & clover fermions
- Grand canonical & phase reweighting
- N_T=6 N_L=8, 10, 12   aμ=0.1 (μ/T=0.6)



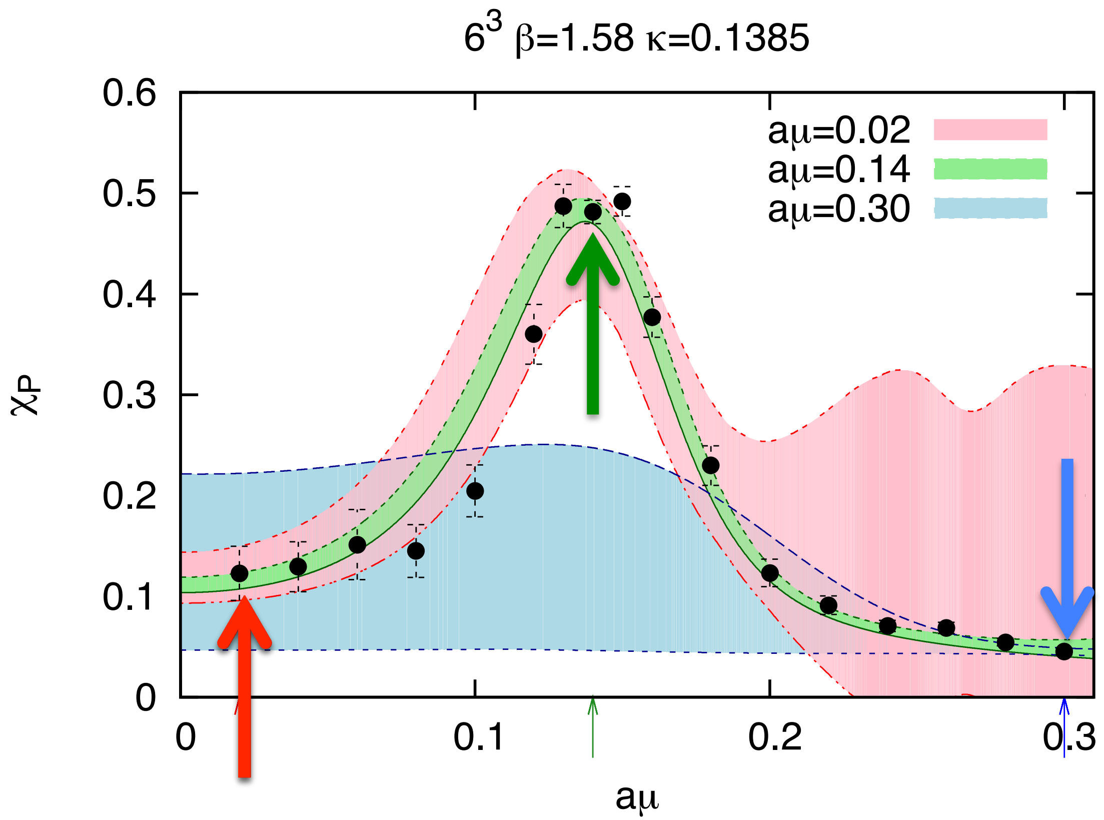
Need more Statistics, β points and N_L=14

Gauge action susceptibility

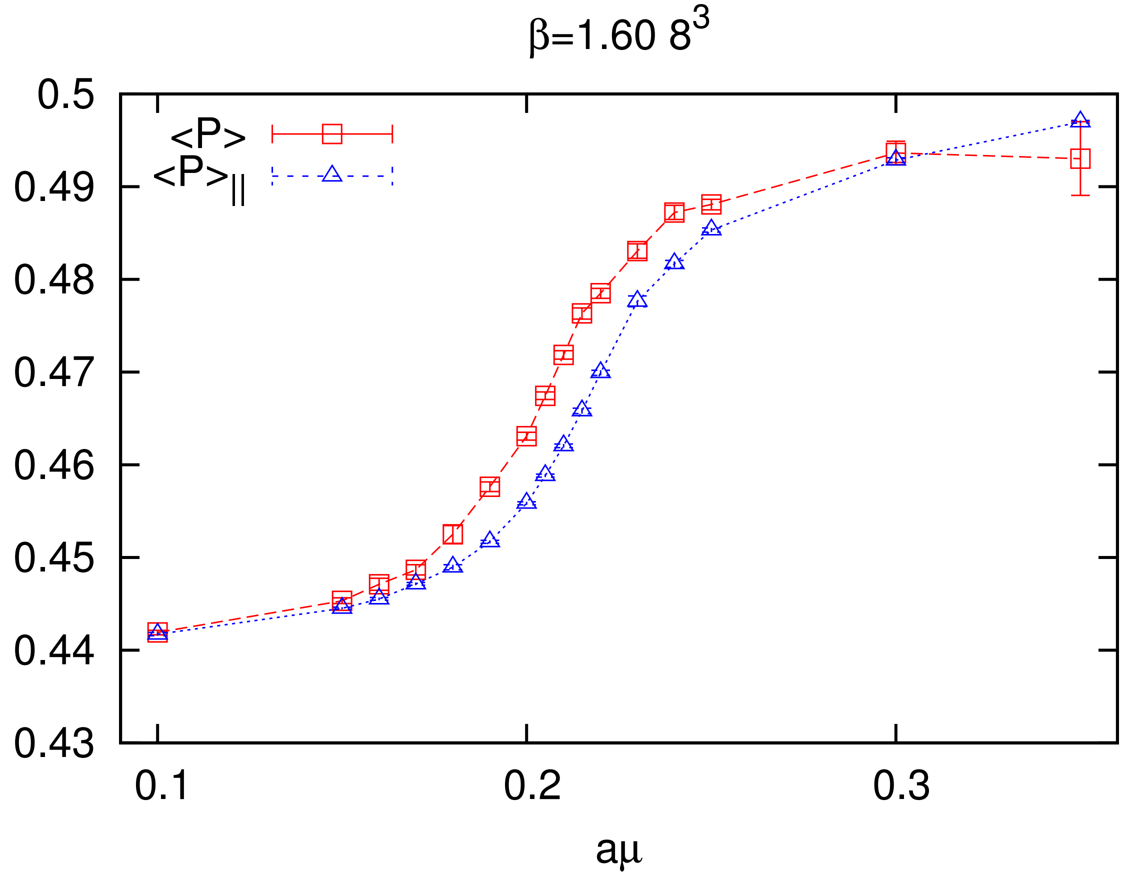


BACK UP SLIDES

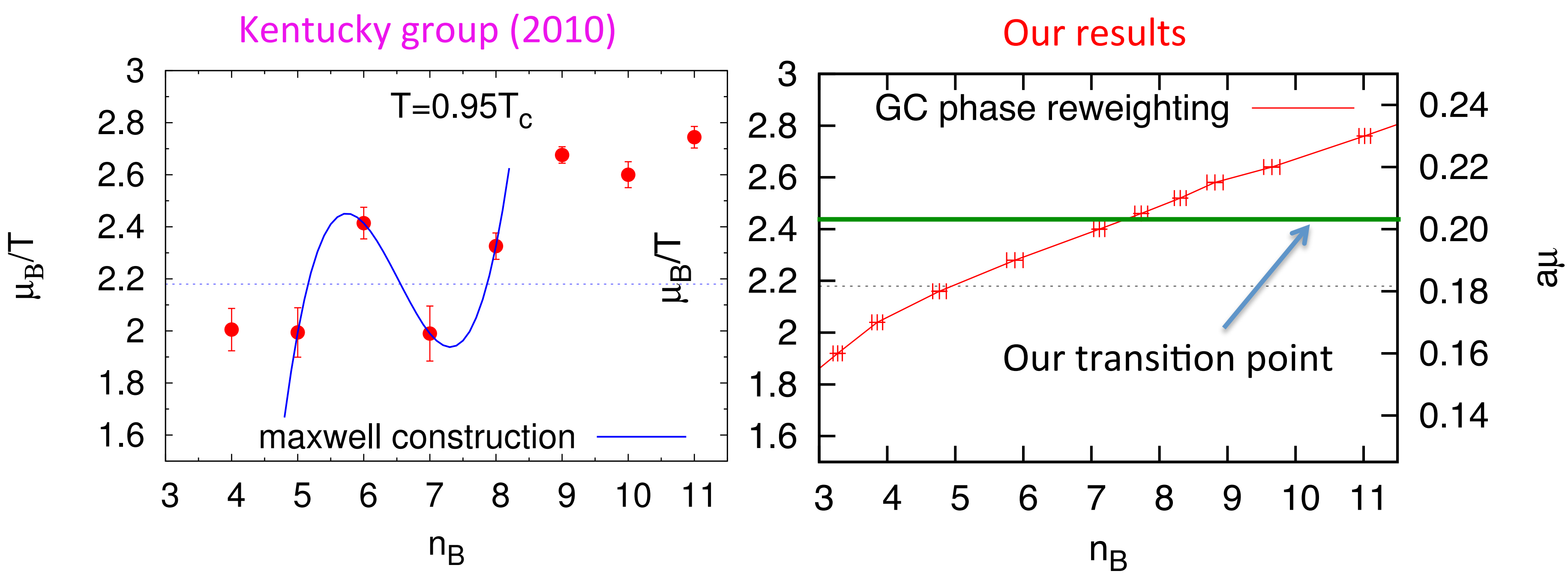
Applicable range (Taylor log)



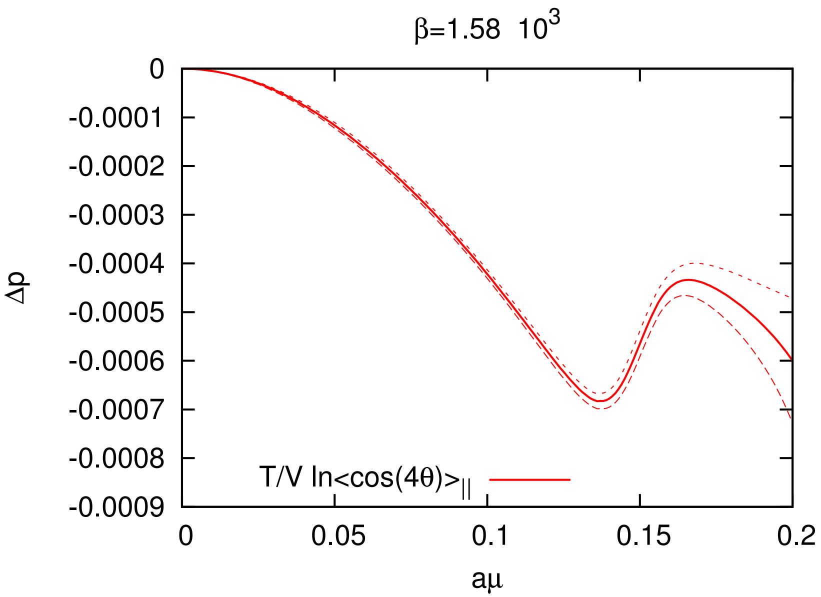
Comparison between QCD and phase quenched QCD

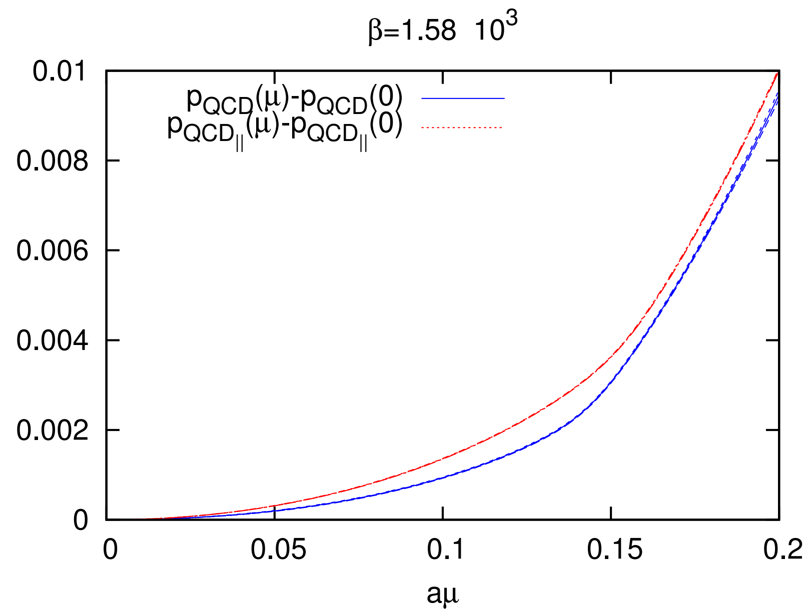


Comparison between Grand Canonical and Canonical approach



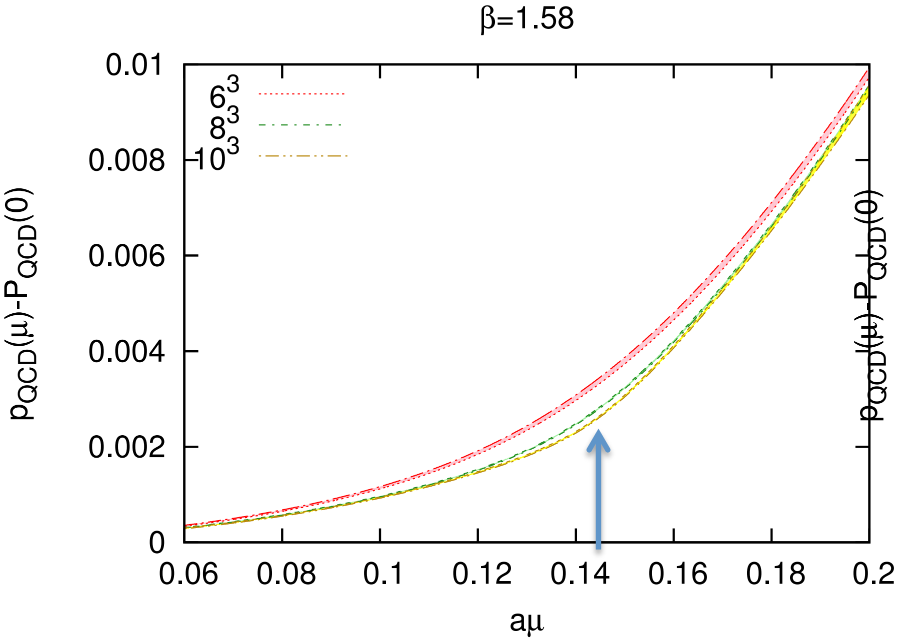
Pressure of QCD and phase quenched QCD

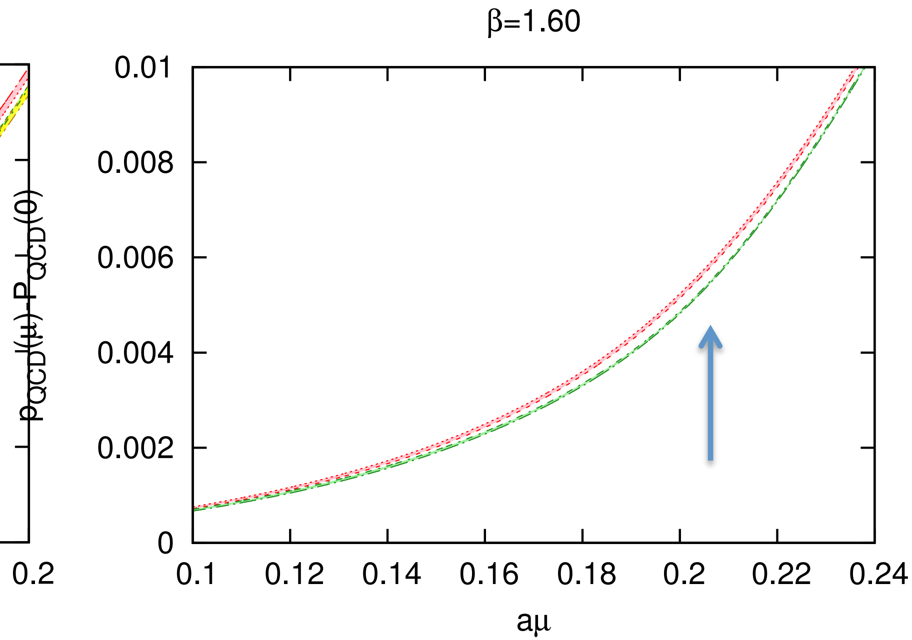




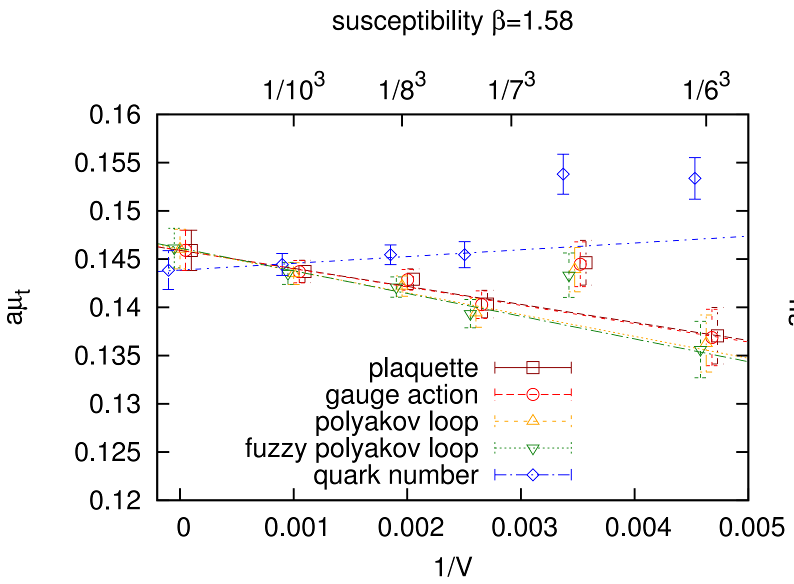
$$\langle \cos(4\theta) \rangle_{||} = \exp\left[\frac{V}{T}\left(p_{QCD}(\mu) - p_{QCD_{||}}(\mu)\right)\right] = \exp\left[\frac{V}{T}\Delta p(\mu)\right]$$

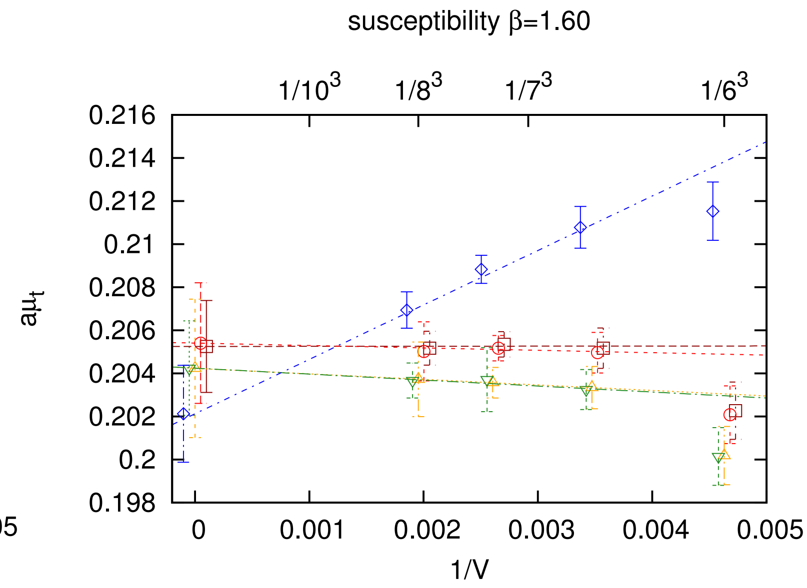
Pressure


β=1.58
0.01
0.008
0.006
0.004
0.002
0
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
aμ
$p_{QCD}(\mu)-P_{QCD}(0)$
6^3
8^3
10^3

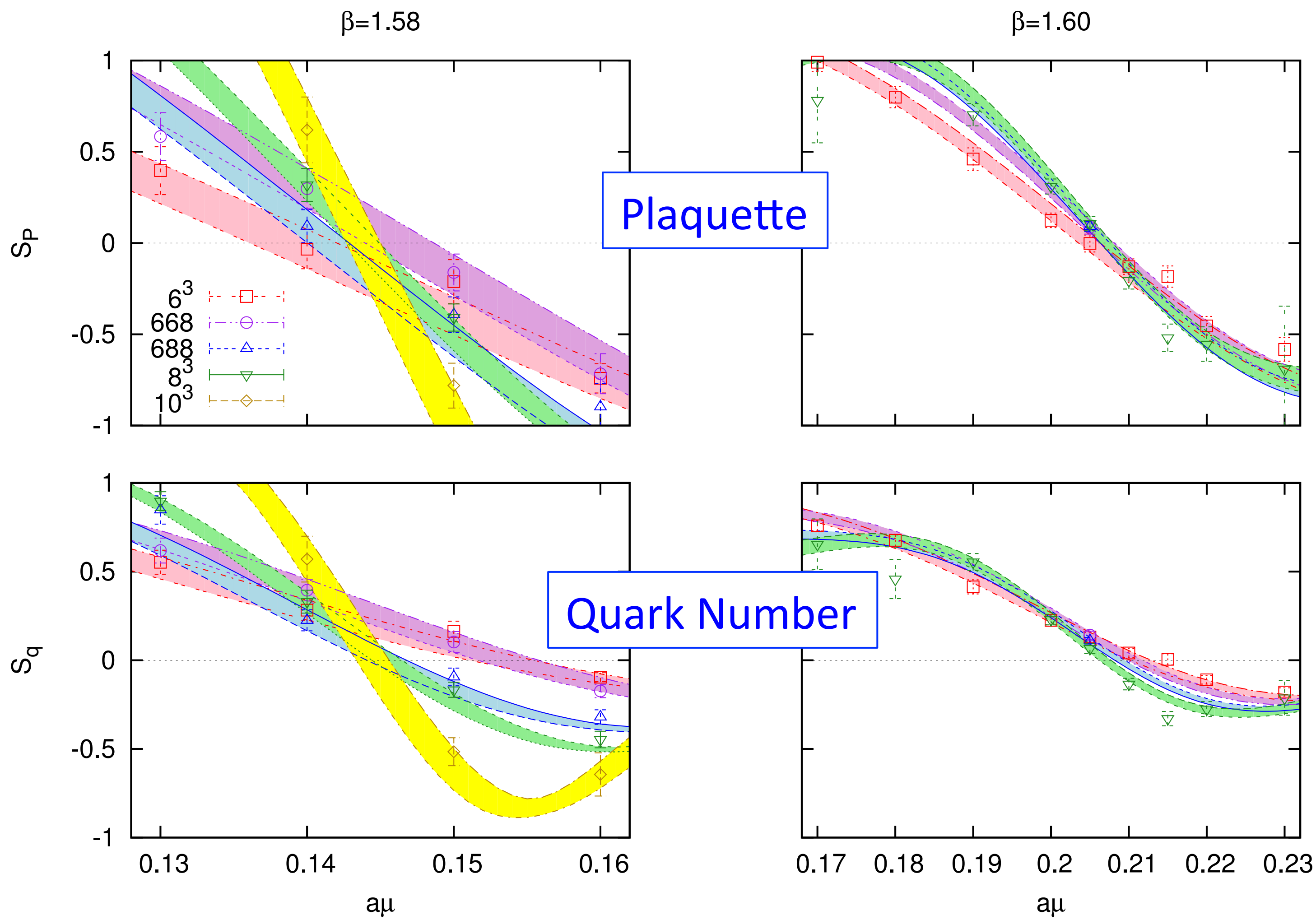

β=1.60
0.01
0.008
0.006
0.004
0.002
0
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
aμ
$p_{QCD}(\mu)-P_{QCD}(0)$

Transition point





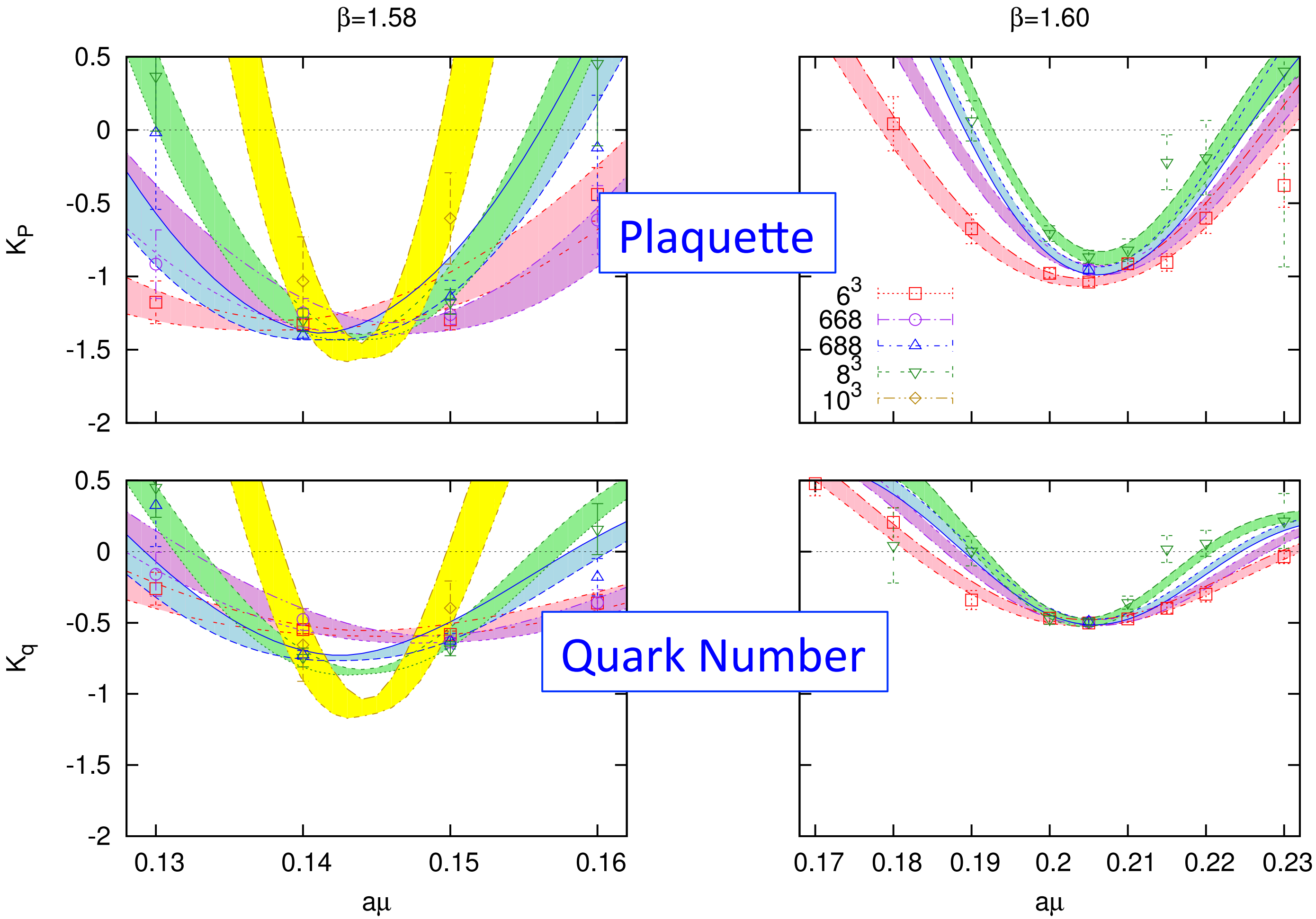
Skewness



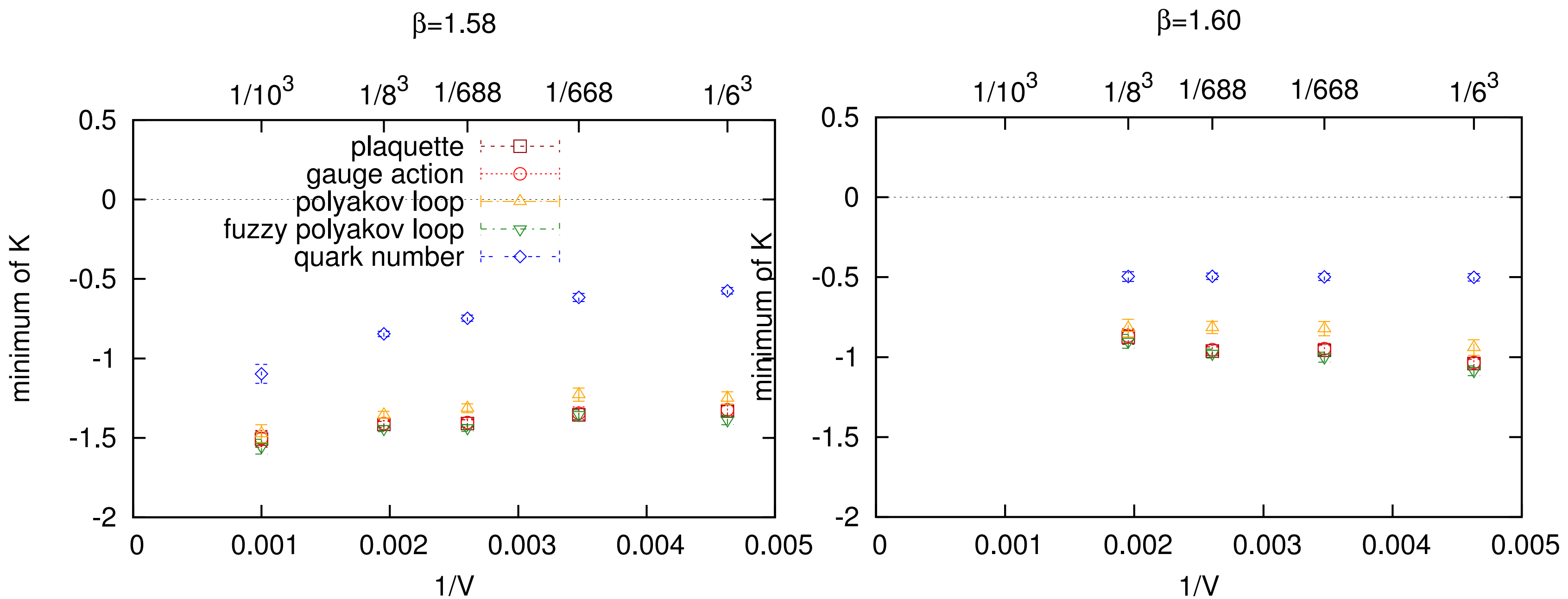
β=1.58
β=1.60
Plaquette
Quark Number
S_P
S_q
6^3
668
688
8^3
10^3
1
0.5
0
-0.5
-1
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
0.21
0.22
0.23
aμ
aμ

Kurtosis

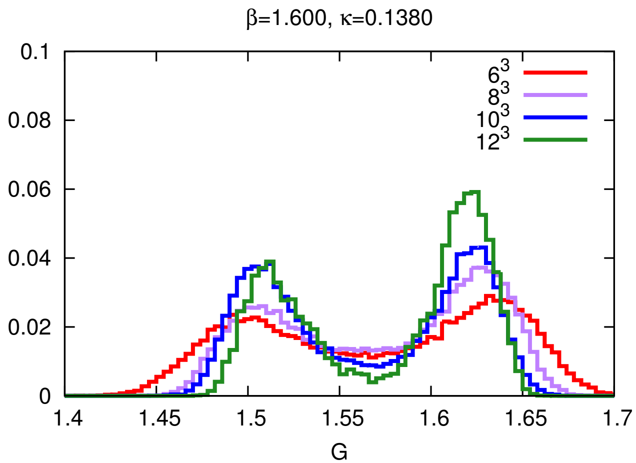
$B_4 = K + 3$



Scaling of min of kurtosis



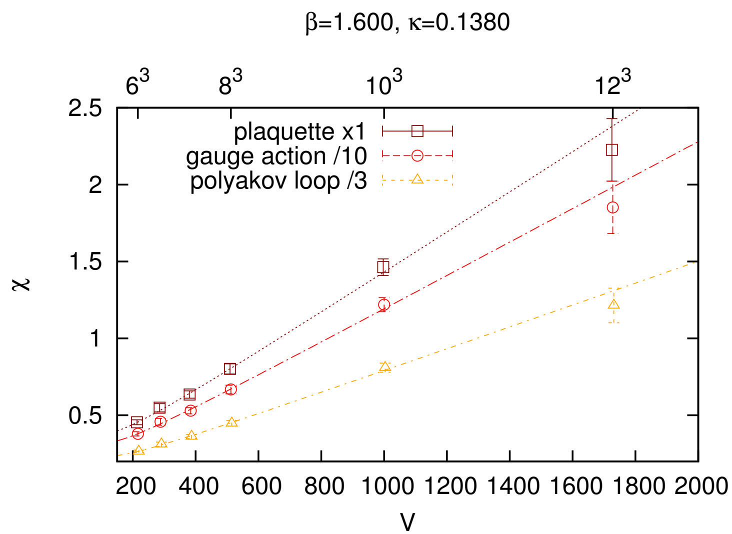
Zero density simulation 1

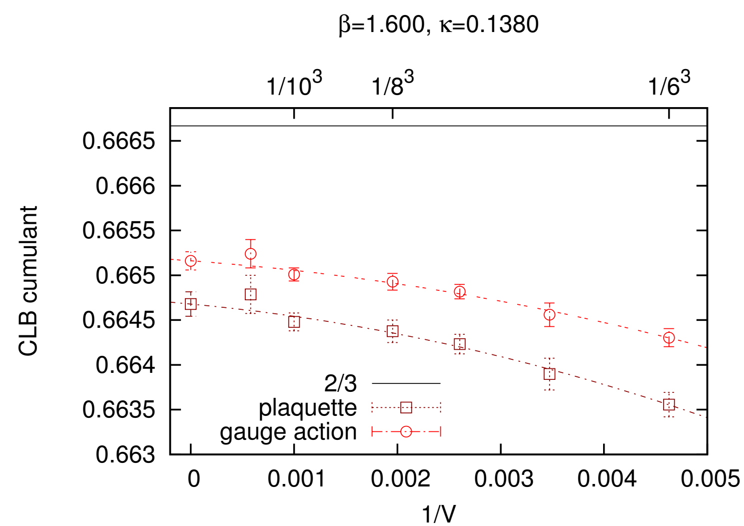


$\beta=1.60$
$\kappa=0.1380$
$V=6^3\text{-}12^3$

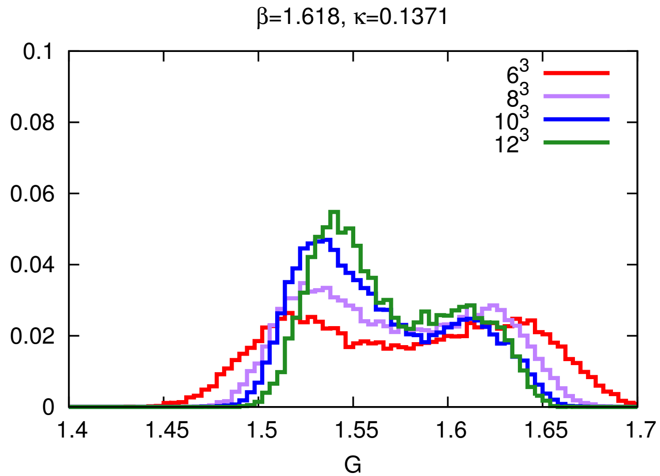
$m_\pi/m_\rho=0.825$
$T/m_\rho=0.155$

Strong 1st PT





Zero density simulation 2



β=1.618
κ=0.1371
V=6^3-12^3

m_π/m_ρ=0.834
T/m_ρ=0.155

Cross over
Or
Weak 1st PT