Magnetization and pressures at nonzero magnetic fields in QCD

Gergely Endrődi

University of Regensburg

[arXiv:1303.1328]

in collaboration with G. Bali, F. Bruckmann, F. Gruber, A. Schäfer

Universität Regensburg

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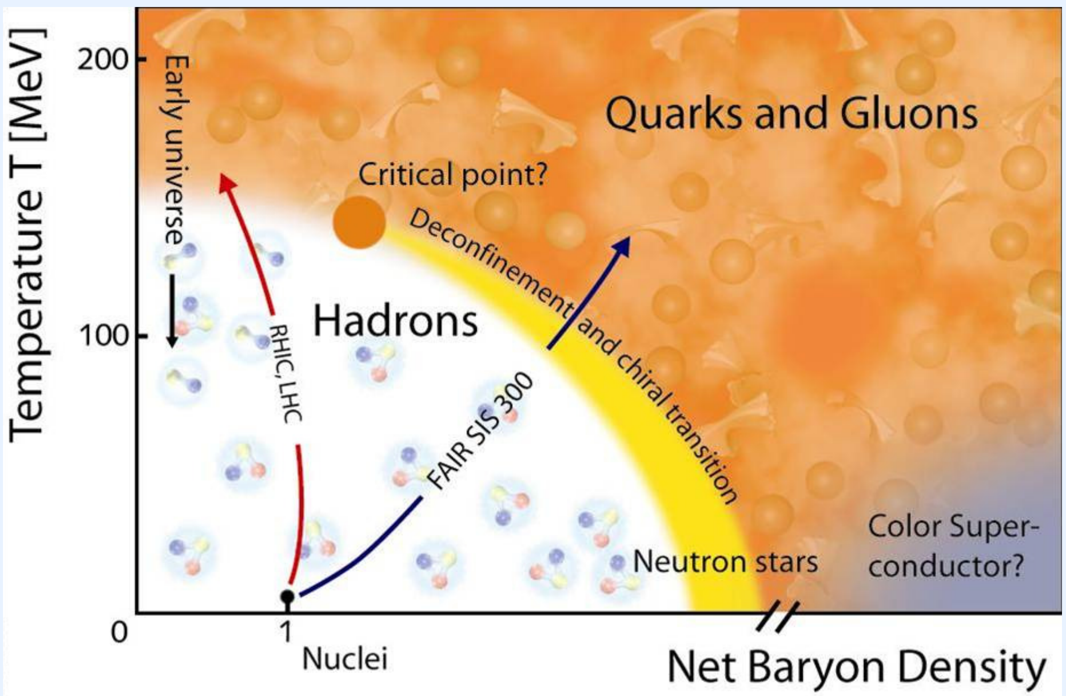
QCD and magnetic fields

- examples for systems with strongly interacting matter and magnetic fields:
  - dense neutron stars, magnetars
  - non-central heavy ion collisions
  - early universe cosmology



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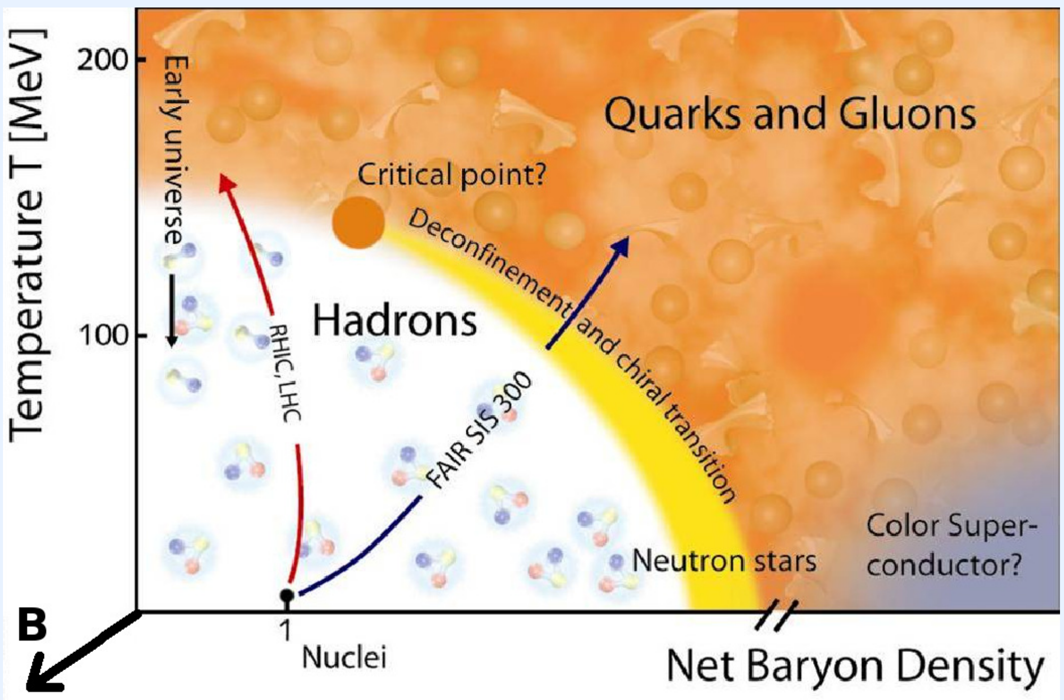

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- examples for systems with strongly interacting matter and magnetic fields:
  - dense neutron stars, magnetars
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  - early universe cosmology
- QCD vacuum: charged quarks and neutral gluons
  $\Rightarrow$ external B-field acts as probe of QCD vacuum:
  - affects chiral symmetry breaking [Gusynin et al '96]
  - changes the hadron spectrum
  - phase diagram structure [Bruckmann's talk, Kovács's talk]
  - broken Lorentz symmetry $\rightarrow$ new order parameter(s) [Smilga et al '84]
  - para- or diamagnetism [Bali, Bruckmann, GE et al '12, '13]
  - equation of state

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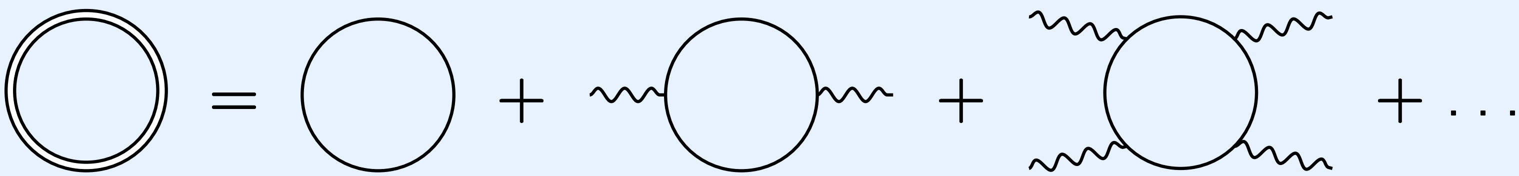
  - equation of state: preliminary lattice results

Two comments in advance

- renormalization in magnetic fields
- definition of pressure in magnetic fields

Renormalization in magnetic fields

- thermodynamic potential schematically (1-loop):



$$\log \mathcal{Z}_B = \log \mathcal{Z}_0 + \beta_1 (eB)^2 \log a + c(eB)^4 \cdot \text{finite} + \ldots$$

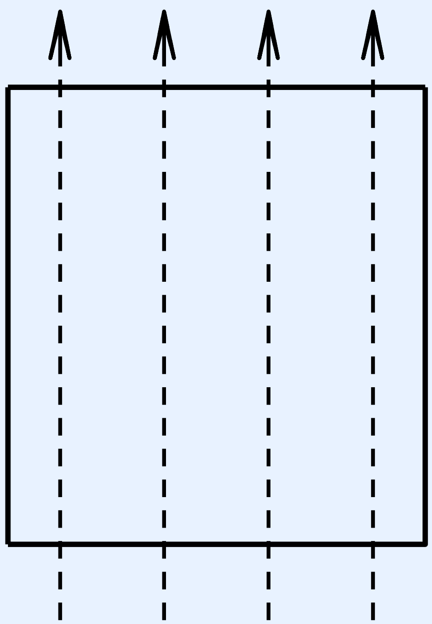
$$\beta_1 (eB)^2 \log a + \frac{B^2}{2} = \frac{B_r^2}{2}$$

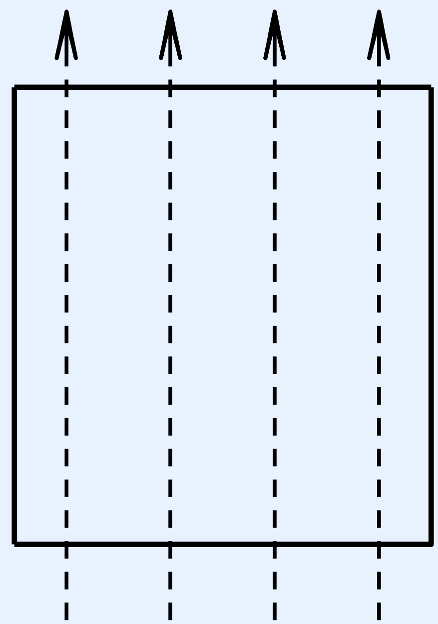
- coefficient of $\mathcal{O}(B)^2$ contribution equals the leading coeff of QED β-function [Schwinger '51]
  $\rightarrow$ background field method [Abbott '81]
- renormalization at $T = 0 \Leftrightarrow$ subtract $\mathcal{O}((eB)^2)$ term from the free energy

Definition of pressure in magnetic fields

- free energy $\mathcal{F} = -T \log \mathcal{Z}$
- consider a finite volume $V = L_x L_y L_z$, traversed by a magnetic flux $\Phi = eBL_xL_y$

$$p_i = -\frac{1}{V} L_i \frac{\mathrm{d}\mathcal{F}}{\mathrm{d}L_i}, \qquad M = -\frac{1}{V}\frac{1}{e}\frac{\partial \mathcal{F}}{\partial B}$$





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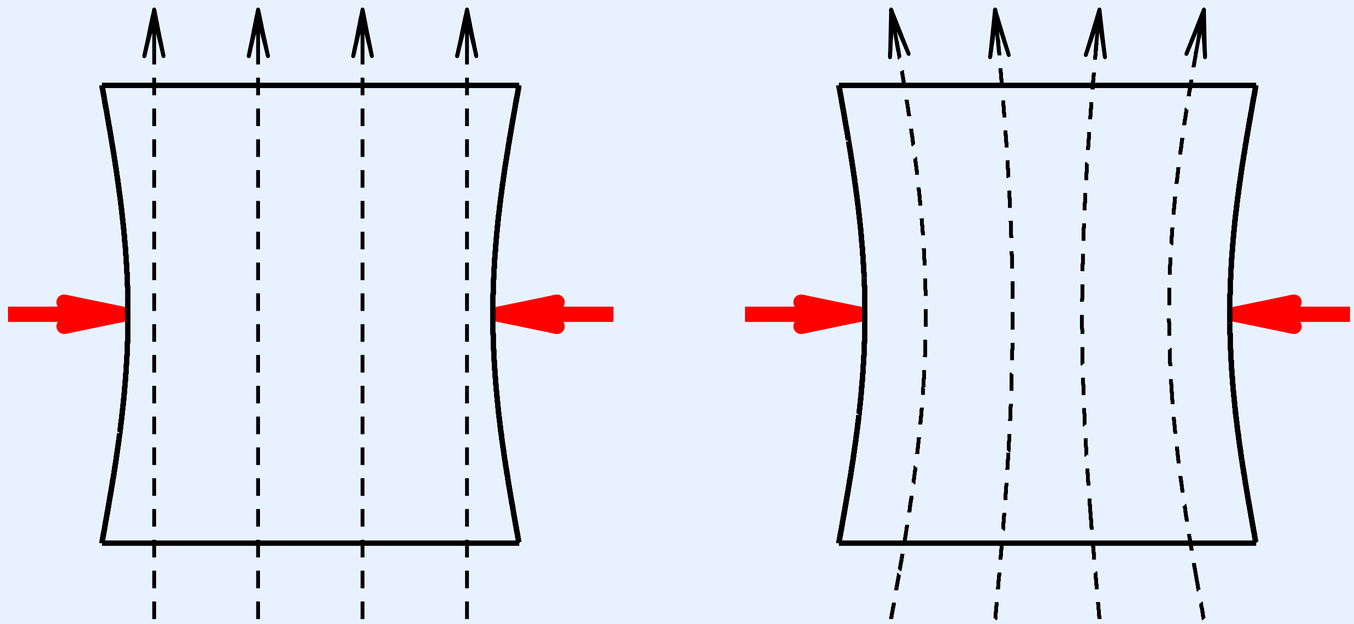

$eB = \text{fix}$ $\qquad$ $\Phi = eB \cdot L_xL_y = \text{fix}$

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- extensivity of $\mathcal{F}$ in a large homogeneous system

$$\mathcal{F}(L_i, B) = L_x L_y L_z \cdot f(B)$$
$$\mathcal{F}(L_i, \Phi) = L_x L_y L_z \cdot f(\Phi / L_x L_y)$$

- compression with B or Φ constant?

$$p_i^{(B)} = -\frac{1}{V}\frac{\partial \mathcal{F}}{\partial \log L_i}, \qquad p_i^{(\Phi)} = -\frac{1}{V}\frac{\partial \mathcal{F}}{\partial \log L_i} - \frac{1}{V}\frac{\partial \mathcal{F}}{\partial B} \cdot \left.\frac{\partial B}{\partial \log L_i}\right|_{\Phi},$$

- define B- and Φ-schemes

$$p_{x,y}^{(B)} = p_z^{(B)}, \qquad p_{x,y}^{(\Phi)} = p_z^{(\Phi)} - M \cdot eB$$

Magnetization on the lattice

- let's calculate $M \sim \partial \log \mathcal{Z} / \partial B$
- quantization of magnetic flux in a finite volume with periodic boundary conditions ['t Hooft '79]

$$\Phi = eB \cdot L_x L_y = 2\pi N_b, \qquad\qquad N_b \in \mathbb{Z}$$

$\Rightarrow$ B-derivative ill-defined! [DeTar's talk]

$\Rightarrow$ naturally corresponds to the Φ-scheme

- instead, determine magnetization from

$$p_x - p_z = -M \cdot eB$$

$\to$ consider anisotropic lattice $\xi = a/a_\alpha$ [Karsch '82]

$$p_\alpha = -\xi^2 \frac{T}{V} \left. \frac{\mathrm{d} \log \mathcal{Z}}{\mathrm{d}\xi} \right|_a ,$$

- p_α contains certain components of the action

$\to$ $M \cdot eB$ contains *anisotropies* of the action

Magnetization from anisotropies

- dominant contribution comes from fermions:

$$M \cdot eB \approx \sum_f A(\mathcal{C}_f)$$

- with the fermionic action

$$S = \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f,$$

separating into components

$$\mathcal{C}_{\mu,f} = \bar{\psi}_f \not{D}_{(\mu)} \psi_f$$

building up the anisotropy

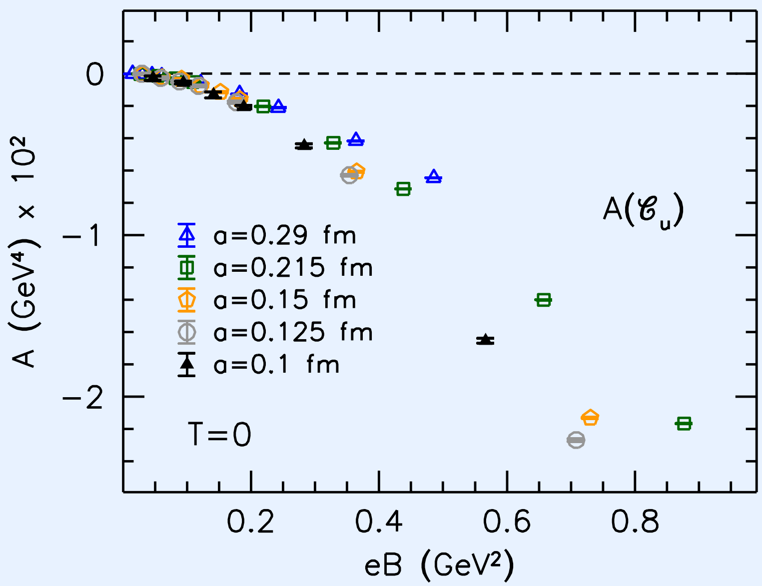
$$A(\mathcal{C}_f) = \frac{\left\langle \mathcal{C}_{x,f} \right\rangle + \left\langle \mathcal{C}_{y,f} \right\rangle}{2} - \left\langle \mathcal{C}_{z,f} \right\rangle$$

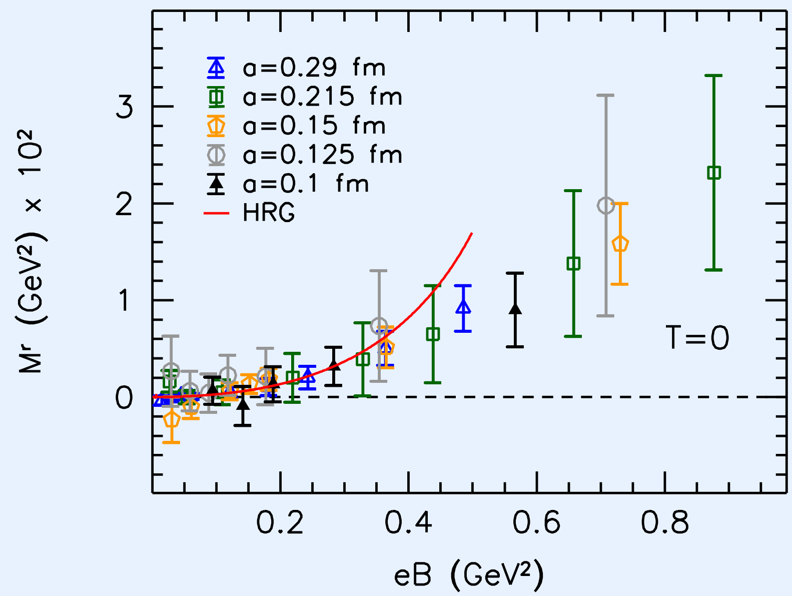
- see details in [Bali, Bruckmann, GE et al '13]

Magnetization from anisotropies

- charge renormalization for the magnetization

$$M^r \cdot eB = M \cdot eB - (eB)^2 \cdot \lim_{eB \to 0} \frac{M \cdot eB}{(eB)^2}.$$



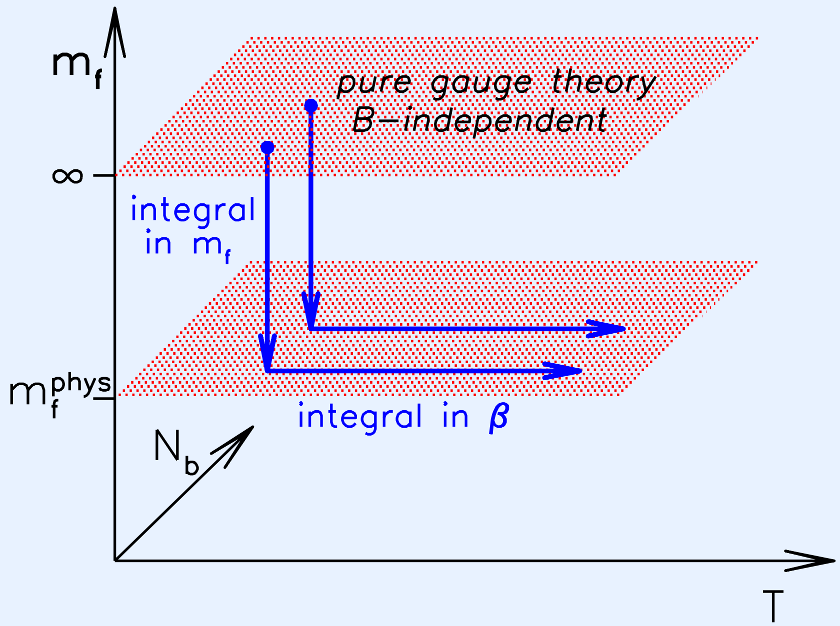


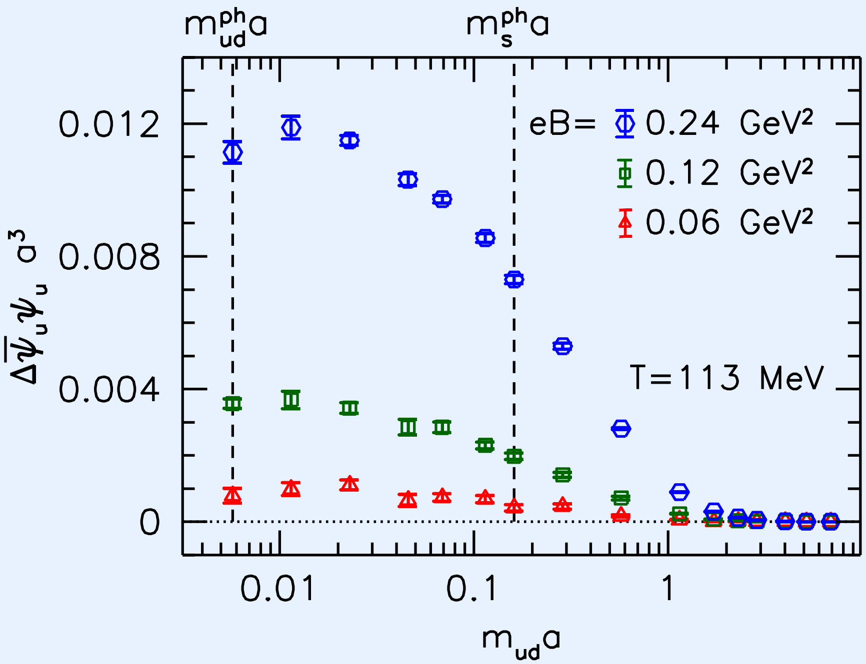
- QCD vacuum is a paramagnet!
  cf. [Bonati's talk, DeTar's talk]

- comparison with HRG [GE '13]

Another approach to the EoS

- generalized integral method in $\{\beta, m_f, N_b\}$ space: use that B has no effect in pure gauge theory!





$$\log \mathcal{Z}_2 - \log \mathcal{Z}_1 = \int_1^2 \left(\partial_\beta \log \mathcal{Z} \;\; \partial_{m_f} \log \mathcal{Z}\right) \begin{pmatrix} \mathrm{d}\beta \\ \mathrm{d}m_f \end{pmatrix}$$

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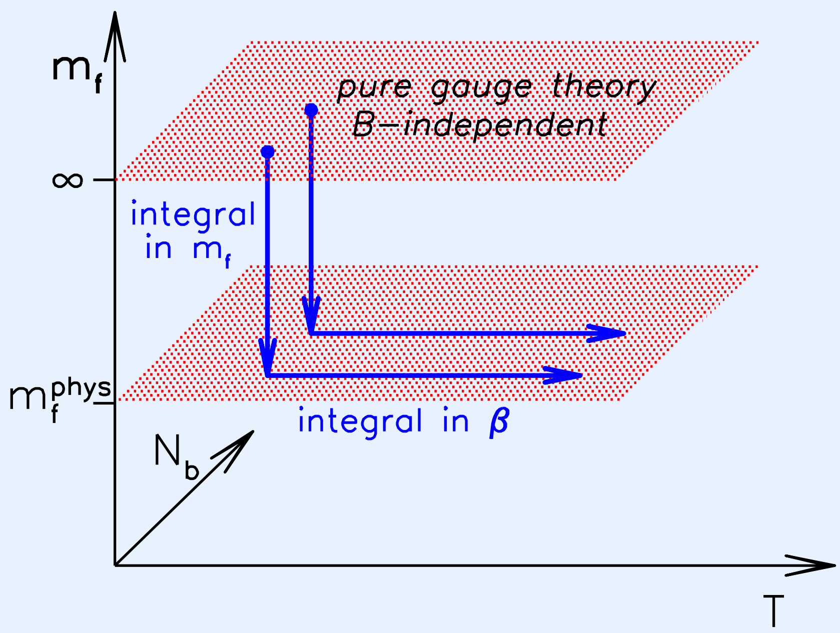
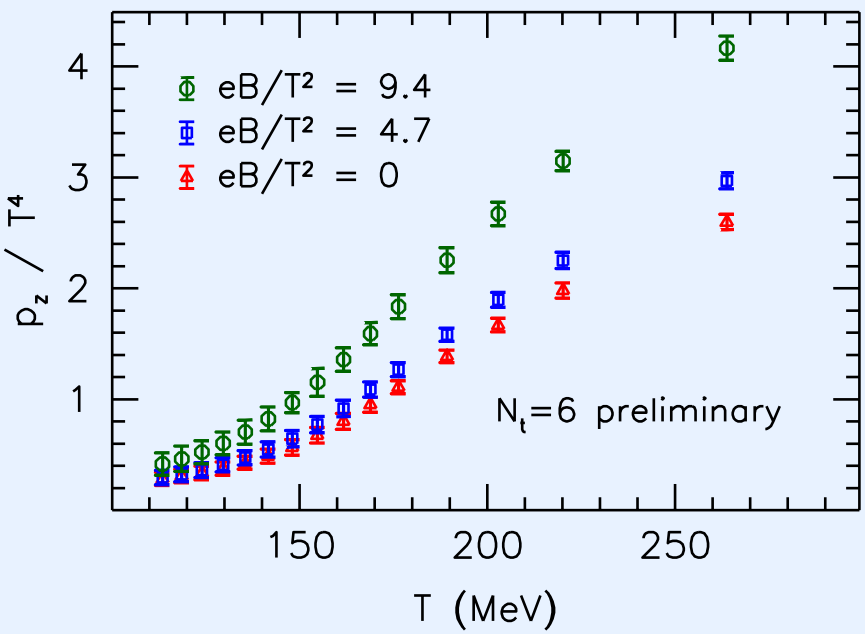
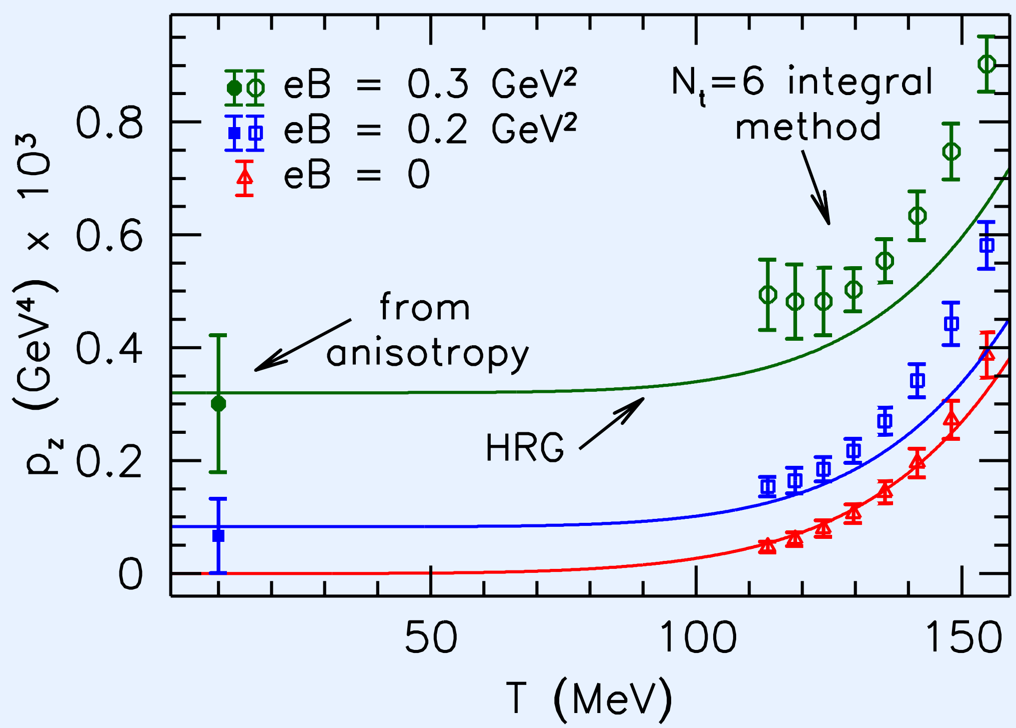




- result: pressure along fixed flux, i.e. fixed eB/T^2
one more interpolation to get $p_z(B,T)$

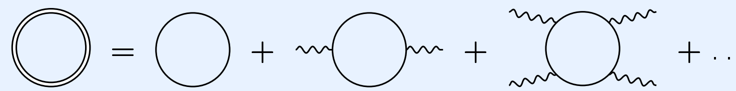
Comparison of the two methods

- two completely different ways to get $p_z(T,B)$

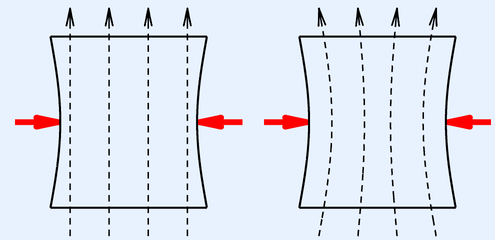


Summary

- renormalization:
  $\mathcal{O}(B^2)$ subtraction at $T = 0$

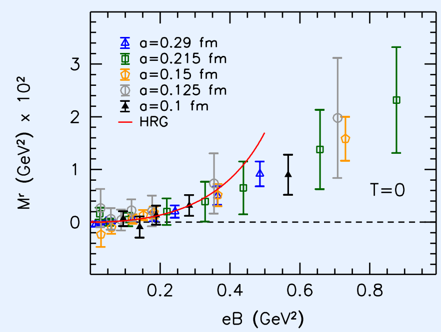


- distinct pressure definitions:
  B-scheme (isotropy) vs.
  Φ-scheme (anisotropy)

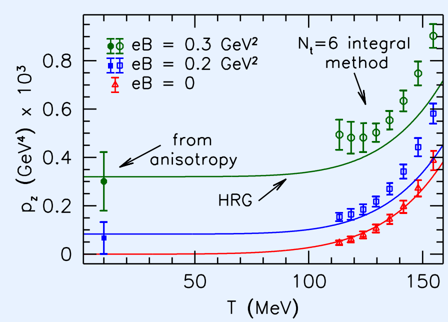


- B-dependence of QCD pressure determined

  - magnetization from lattice anisotropies [arXiv:1303.1328]



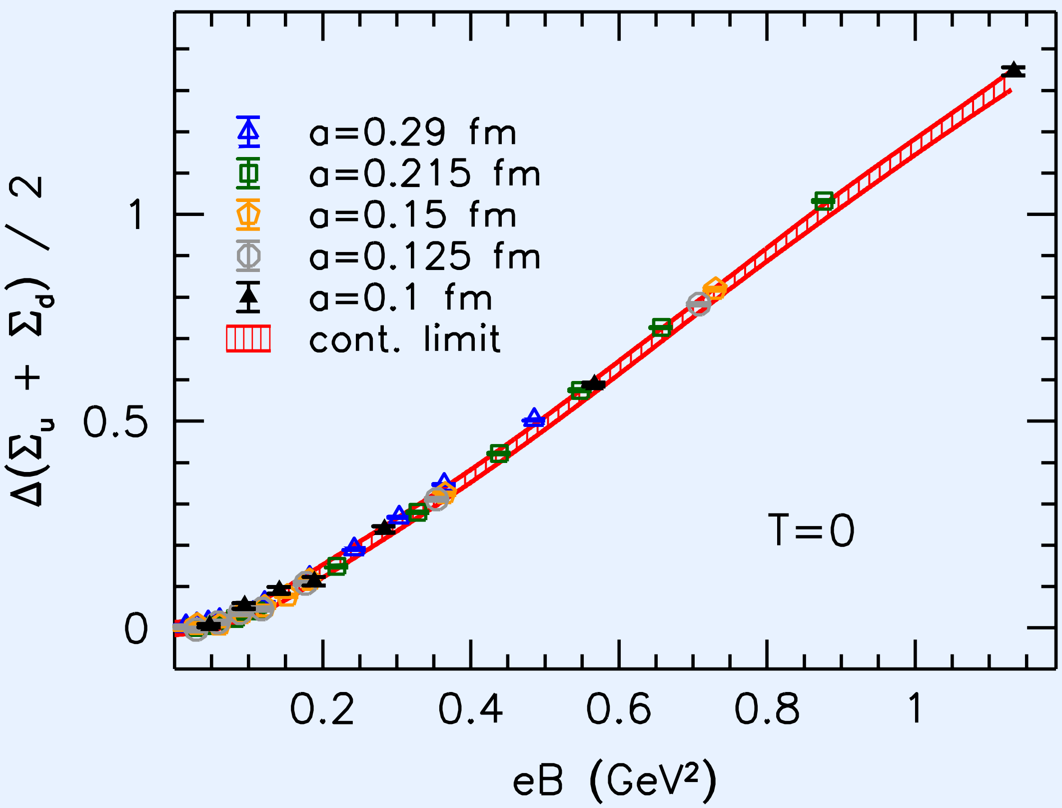
  - pressure through generalized integral method [preliminary]



The condensate has it all

$$\Delta \log \mathcal{Z} = -\int_{m_{\text{phys}}}^{\infty} \mathrm{d}m \Delta\bar{\psi}\psi, \qquad P = \text{project } \mathcal{O}(B^2)$$

$$\underbrace{P\Delta \log \mathcal{Z}}_{\sim -\beta_1} + \underbrace{(1-P)\Delta \log \mathcal{Z}}_{\sim M^r} = -\int_{m_{\text{phys}}}^{\infty} \mathrm{d}m \left[P\Delta\bar{\psi}\psi + (1-P)\Delta\bar{\psi}\psi\right]$$



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