Magnetic catalysis of (electric) charge

Tin Sulejmanpašić
Regensburg University
In collaboration with Falk Bruckmann and Pavel Buividovich

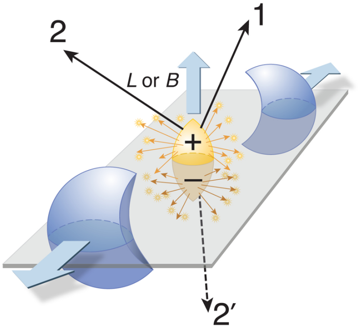
arXiv:1303.1710 [hep-th]

Lattice 2013, Mainz

July 31, 2013

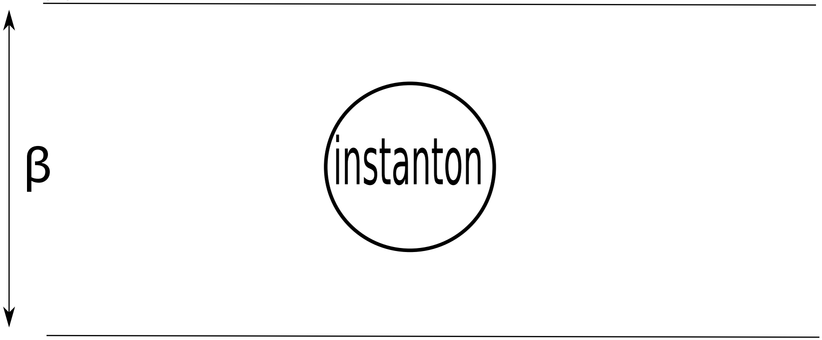
Why magnetic field?

- Heavy ion collisions produce strong magnetic fields ($eB \sim (100\mathrm{MeV})^2 >< ?$)
- Phase diagram of QCD in magnetic field
- Application in graphene (valleytronics)
- It's fun!



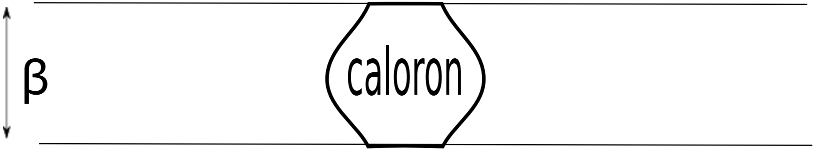
KvBLL Calorons

SU(2) Yang Mills on $R^3 \times S^1$



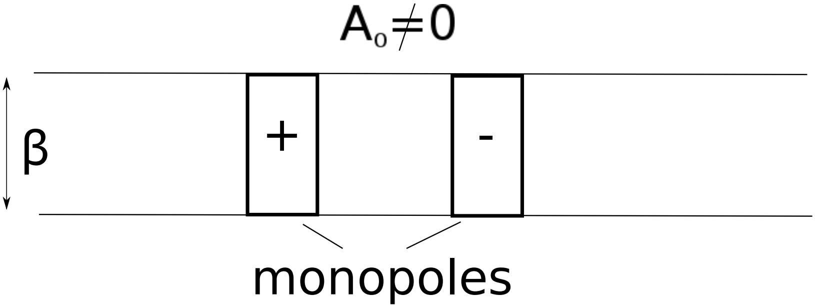
KvBLL Calorons

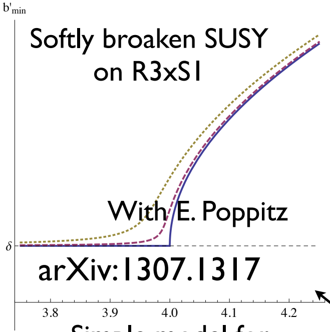
SU(2) Yang Mills on $R^3 \times S^1$

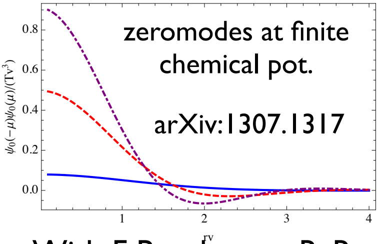


KvBLL Calorons

SU(2) Yang Mills on $R^3 \times S^1$

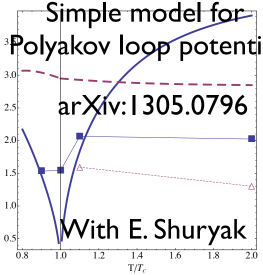






With F. Bruckmann R. Roedl

Fri, 17:10, Seminar Room B -- Parallels 10B



My world of
instanton-monopoles

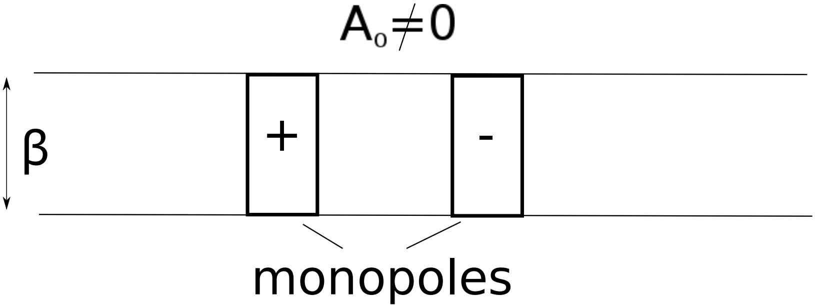
- Deformed YM, QCD(adj), Large N volume independence (M. Unsal, et. al), (S)YM and Confinement (talk by T. Schaefer) renormalons, etc.

-Monopols on the lattice: E.M. Ilgenfritz, M. Muller-Preusker,

-Models of ChiralSB: P. Faccioli, E. Shuryak, TS

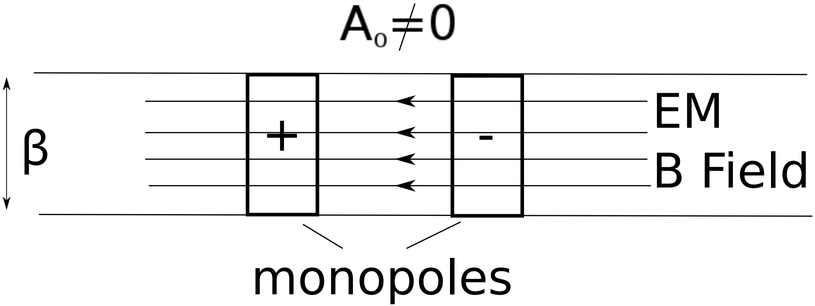
What motivated this work: Caloron in magnetic field

SU(2) Yang Mills on $R^3 \times S^1$



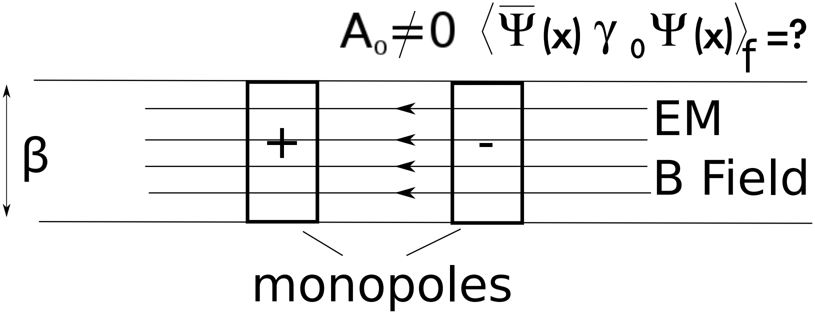
What motivated this work: Caloron in magnetic field

SU(2) Yang Mills on $R^3 \times S^1$

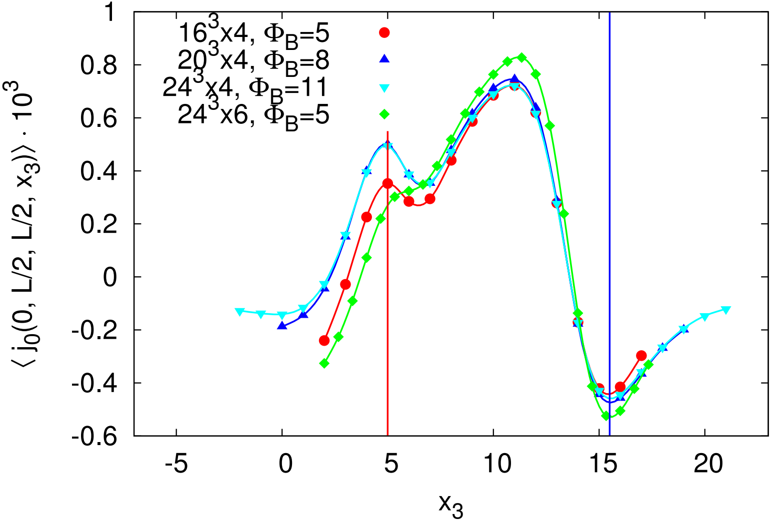


What motivated this work: Caloron in magnetic field

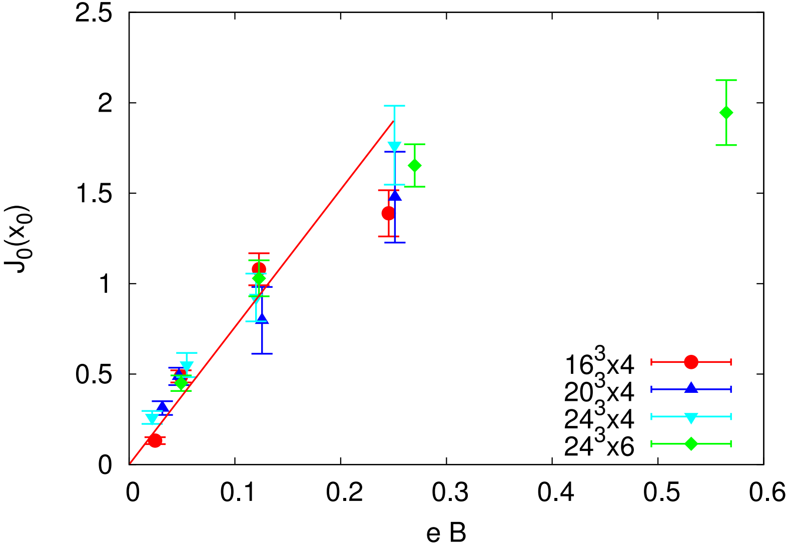
SU(2) Yang Mills on $R^3 \times S^1$



What motivated this work: Caloron in magnetic field



What motivated this work: Caloron in magnetic field



Introduction

- Generation of charge by the interplay of two things
  - Combination of two magnetic fields of $U_{(1)}(1) \times U_{(2)}(1)$ type
  - Chemical potential for one of the charges (say for charge (1))
- This combination would induce a spectrum of fermions which is charged under (2).
- The origin of this spectrum is rooted in the degeneracy of of states in magnetic field

Fermions in 2D

Consider the Dirac equation in (2+1) dimensions

$$H\psi_n = E_n \psi_\lambda \ , \qquad H = i\sigma_1 D_x + i\sigma_2 D_y$$

Introduce an isospin quantum number, and consider $U(1) \times U(1)$ theory with

$$D_\mu = \partial_\mu - iA_\mu - i\mathcal{A}_\mu \tau^3 \ .$$

Fermions decouple into two sectors $\tau^3 = \pm 1$.

Fermions in 2D

Consider the Dirac equation in (2+1) dimensions

$$H\psi_n = E_n\psi_\lambda \ , \qquad H = i\sigma_1 D_x + i\sigma_2 D_y$$

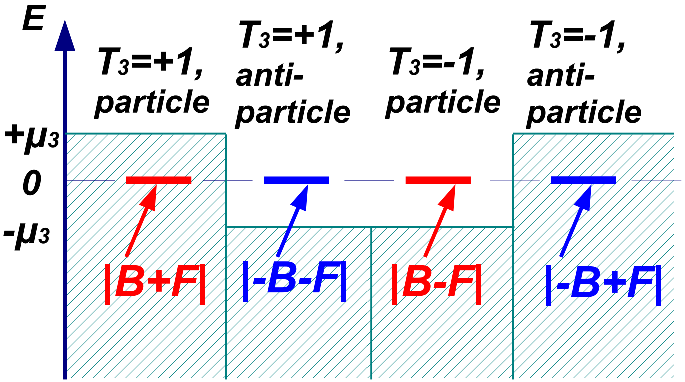
Introduce an isospin quantum number, and consider $U(1) \times U(1)$ theory with

$$D_\mu = \partial_\mu - iA_\mu - i\mathcal{A}_\mu\tau^3 \ .$$

Fermions decouple into two sectors $\tau^3 = \pm 1$.
Now introduce magnetic field $\mathcal{B} = B + F\tau^3$ and τ^3 chemical potential $\mu_3\tau^3$ so partition function

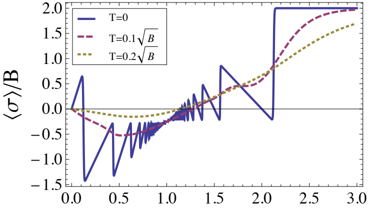
$$\ln Z = \sum_{n,q=\pm 1} g_{n,q} \ln(1 + e^{(q\mu_3 - E_n^q)\beta})$$

Charge catalysis

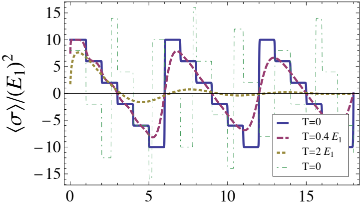


$$Q = 2A\,\mathrm{min}(F, B)\,\tanh(\mu_3\beta/2)\ ,$$

Influence of higher levels

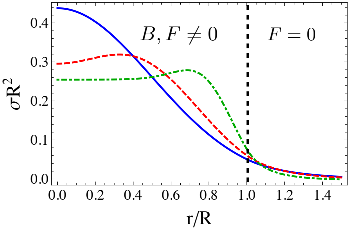


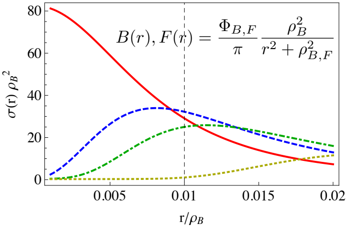
Fixed μ_3



Fixed F

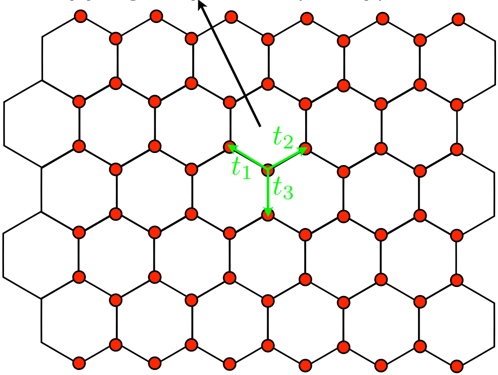
Inhomogeneous fields





Application to graphene

Hopping amplitude (localy) different



$$\mathcal{A}_x = \frac{1}{2}(2t_2 - t_1 - t_3)\ ,$$

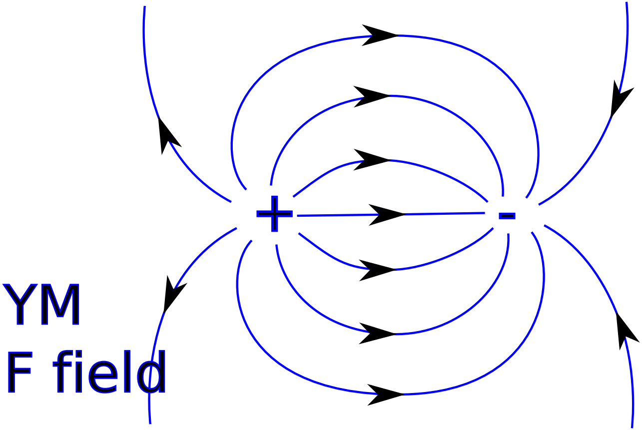
$$\mathcal{A}_y = \frac{\sqrt{3}}{2}(t_3 - t_1)$$

$$\partial_\mu + iA_\mu \pm \frac{i}{v_f}\mathcal{A}_\mu$$

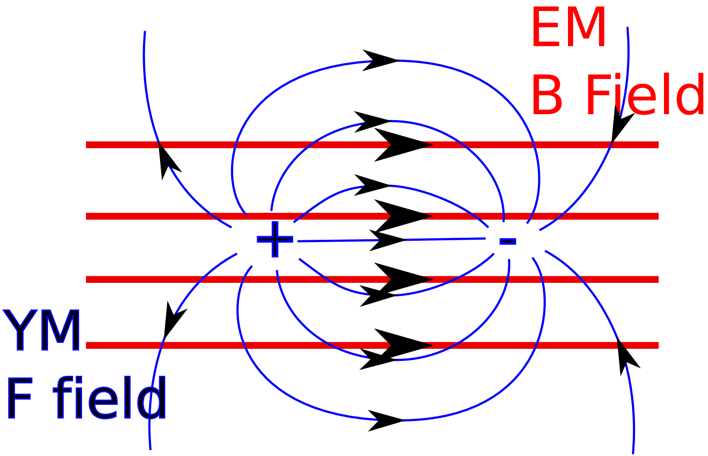
So-called valley index

$$\mathcal{B}_{tot} = B(A) + \tau^3 F(\mathcal{A})$$

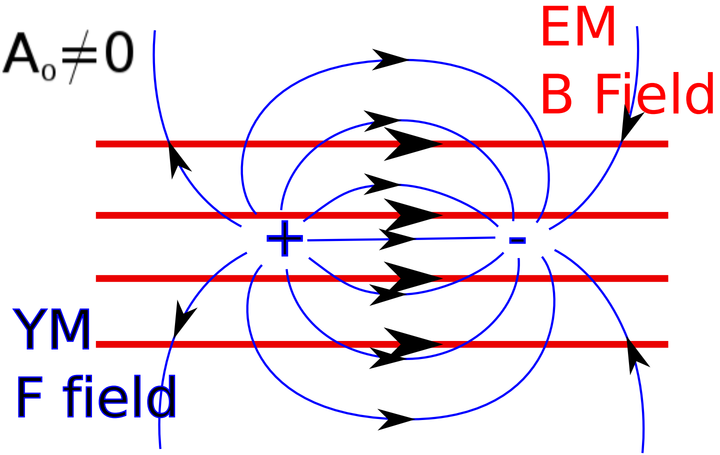
Application in QCD (Caloron revisited)



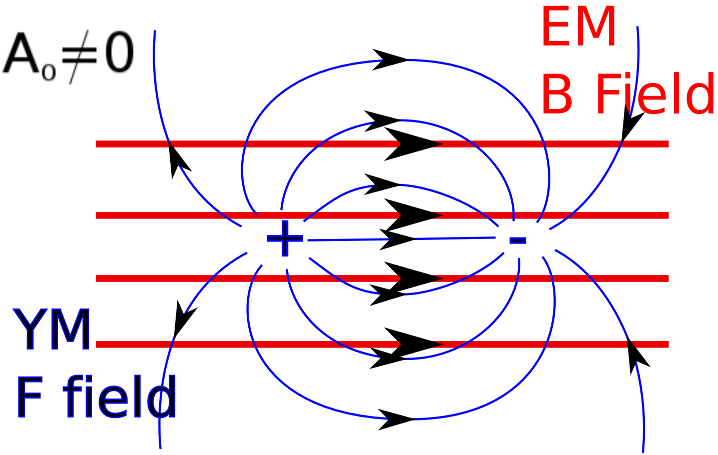
Application in QCD (Caloron revisited)



Application in QCD (Caloron revisited)

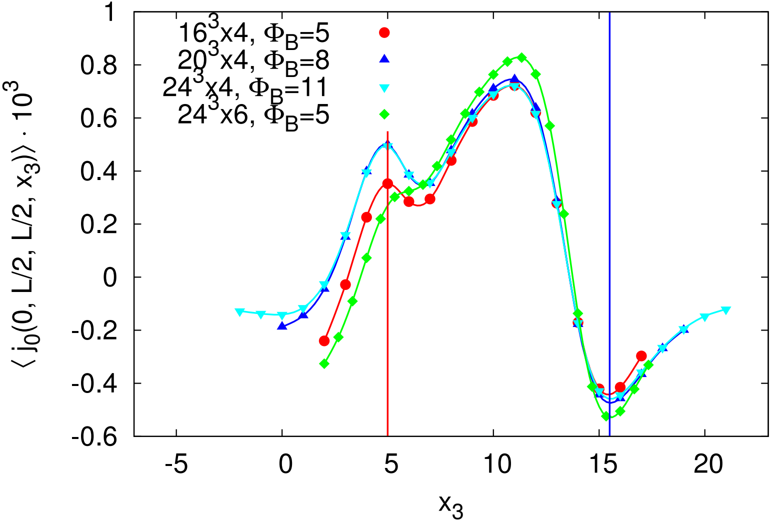


Application in QCD (Caloron revisited)



$A_0 \approx v\tau^3/2$ acts like an *imaginary* μ_3 chemical potential!

Motivation



16^3x4, Φ_B=5
20^3x4, Φ_B=8
24^3x4, Φ_B=11
24^3x6, Φ_B=5
$\langle j_0(0, L/2, L/2, x_3)\rangle \cdot 10^3$
x_3
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-5
0
5
10
15
20

Application in QCD (Caloron revisited)

Uniform fields with $A_0 = v\tau^3$ have

$$Q = V \frac{iv}{(2\pi)^2} \begin{cases} B & B \leq F \\ F & B > F \end{cases} \tag{1}$$

- Charge is imaginary!?
- It rises linearly with B and saturates at $B = F$
- It vanishes if any of the three B, F or v are zero

Application in QCD (Caloron revisited)

Uniform fields with $A_0 = v\tau^3$ have

$$Q = V \frac{iv}{(2\pi)^2} \begin{cases} B & B \leq F \\ F & B > F \end{cases} \tag{1}$$

- Charge is imaginary!?
- It rises linearly with B and saturates at $B = F$
- It vanishes if any of the three B, F or v are zero
- **This is because A_0 is not a physical field, it is a Lagrange multiplier enforcing Gauss law.**

Application in QCD (Caloron revisited)

Uniform fields with $A_0 = v\tau^3$ have

$$Q = V \frac{iv}{(2\pi)^2} \begin{cases} B & B \leq F \\ F & B > F \end{cases} \tag{1}$$

- Charge is imaginary!?
- It rises linearly with B and saturates at $B = F$
- It vanishes if any of the three B, F or v are zero
- **This is because A_0 is not a physical field, it is a Lagrange multiplier enforcing Gauss law.**
- Integration over v would yield vanishing charge, however the effect is still present in charge fluctuations

The Charge fluctuations in uniform fields F and B

$$\left\langle Q^2 \right\rangle_v = (\text{connected}) + (\text{disconnected})$$

For $F > B$

$$\left\langle Q^2 \right\rangle_v = \frac{VT}{\pi^2} F - \frac{V^2}{(2\pi)^2} v^2 B$$

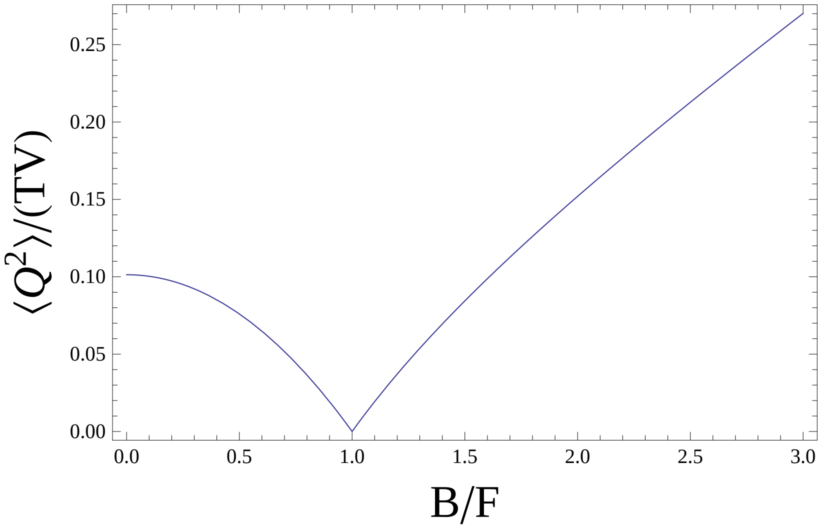
The partition function for uniform fields

$$Z_v = \text{const} \times e^{-\frac{VF}{8\pi^2 T} v^2}$$

Averiging over v

$$\left\langle Q^2 \right\rangle = \frac{VT}{\pi^2} F \left[1 - \frac{B^2}{F^2} \right]$$

The Charge fluctuations in uniform fields



Summary

- Two magnetic fields of a $U(1) \times U(1)$ theory have nontrivial properties due to different degenerecies of the two sectors $\tau^3 = \pm 1$ (charge induction, charge oscillations with magnetic field/chemical potential, charge halos)
- QCD monopole-antimonopole should exhibit this effect (lattice calculations of a KvBLL calaron show this explicitly)
- The possibility of "imaginary" charge induced in lattice configurations which tend to reduce charge fluctuations. (In the case of uniform fields when $F = B$ it completely kills all charge fluctuations due to magnetic field).
- Direct application in graphene: induction of charge via valley chemical potential, and vice versa
- Could this effect be seen in theoretically tractable confining theories such as QCD(adj) (M. Ünsal et al.)?