

# Nonperturbative tests of the renormalisation of mixed clover-staggered currents in lattice QCD

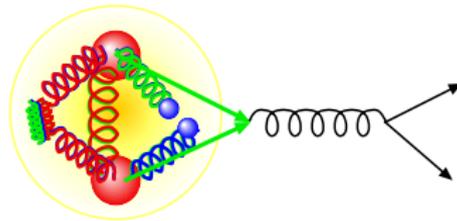
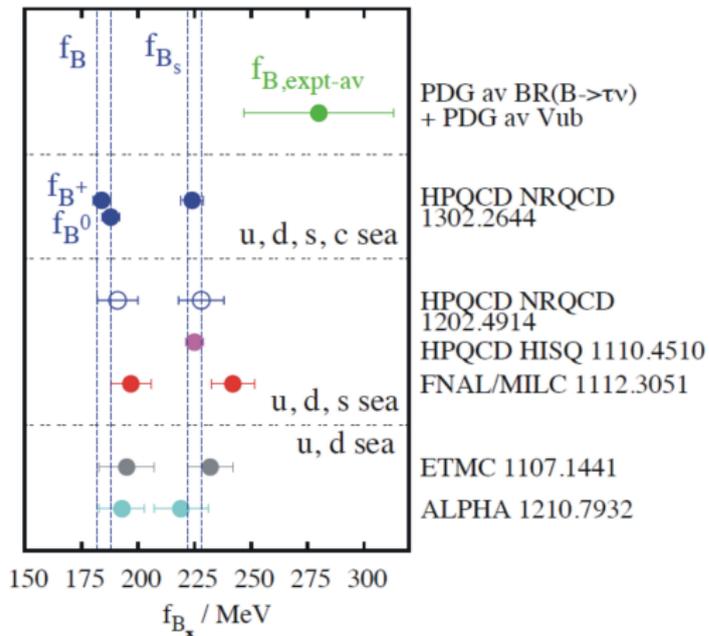
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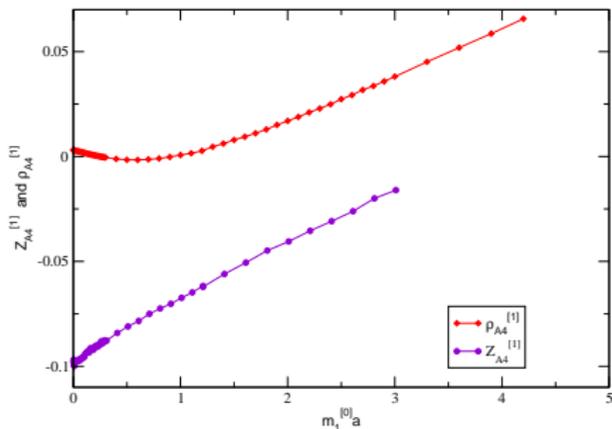
# Motivation



- The leptonic decay constants of heavy-light mesons: Important physical quantities in LQCD
- Different LQCD methods used to compare the systematic errors
- We focus on the systematic errors in the current renormalisations appearing in the Fermilab method

# Fermilab Method and Our Aim

- Fermilab used Asqtad light valence quarks and clover bottom and charm quarks ( arXiv:1112.3051)
- $Z_{A_{Qq}}^\mu$  calculated partly non-perturbatively and partly in one-loop perturbation theory
- Definition:  $Z_{A_{Qq}}^4 = \rho_{A_{Qq}}^4 \sqrt{Z_{V_{qq}}^4 Z_{V_{Qq}}^4}$  for local vector currents
- $\rho_{A_{Qq}}^4$  close to unity at 1-loop in lattice perturbation theory
- They believe  $\rho_{A_{Qq}}^4$  close to unity for higher orders too

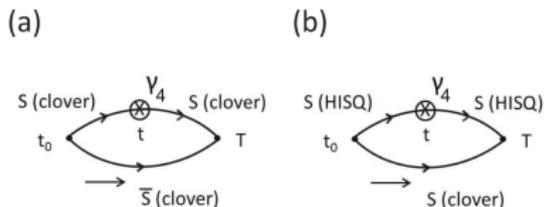
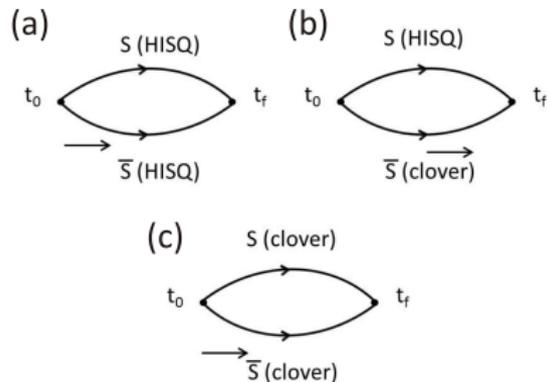


(Aida X. El-Khadra et al., arXiv:0710.1437)

Our Aim : To test the generality of this claim fully non-perturbatively using HISQ-HISQ, Clover-Clover and mixed Clover-HISQ currents made of strange valence quarks

# Our method: Non-perturbative Calculation of $Z_{A^4}$ and $Z_{V^4}$

- Valence strange quark masses tuned to  $m_{\eta_s} = 688.5$  Mev (R.J.Dowdall et.al., arXiv:1303.1670)
- The  $\eta_s$  decay constant:  $\langle 0|A^4|\eta_s(0)\rangle = m_{\eta_s} f_{\eta_s}$
- Absolutely normalized HISQ-HISQ temporal axial vector current
- $Z_{A^4_{Cl-H}}$  for Clover-HISQ discretization:  $Z_{A^4_{Cl-H}} f_{\eta_s}^{Cl-H} = f_{\eta_s}^{H-H}$



- Normalization at zero momentum transfer:  $1 = Z_{V^4_{qq}} \langle H_q | V_{qq}^4 | H_q \rangle$
- Need to use local vector currents
- Need to use untagged (clover) quark at the bottom of the three point functions for HISQ-HISQ local vector current

# Lattice Configurations and Parameters

- Three MILC lattice ensembles:  $a \approx 0.15\text{fm}$  (very coarse),  $0.12\text{fm}$  (coarse),  $0.09\text{fm}$  (fine)
- $2 + 1 + 1$  flavors of HISQ sea quarks:  $m_l/m_s \approx 0.2$

Set	$\beta$	$a(\text{fm})$	$am_l$	$am_s$	$am_c$	$L_s/a$	$L_t/a$	$u_0$
VC	5.80	0.1543(8)	0.013	0.065	0.838	16	48	0.85535
C	6.00	0.1241(7)	0.0102	0.0509	0.635	24	64	0.86372
F	6.30	0.0907(5)	0.0074	0.0507	0.4400	32	96	0.87417

- Source: Delta function random color wall with subset corner mask
- Point sink

Set	$am_s^{\text{HISQ, val}}$	$\kappa_s^{\text{Clover, val}}$	$n_{\text{cfg}}$	$n_t$
VC	0.0705(9)(4)	0.14082	1021	12
C	0.0541(6)(3)	0.13990	527	16
F	0.0376(5)(2)	0.13862	504	16

# Our Results

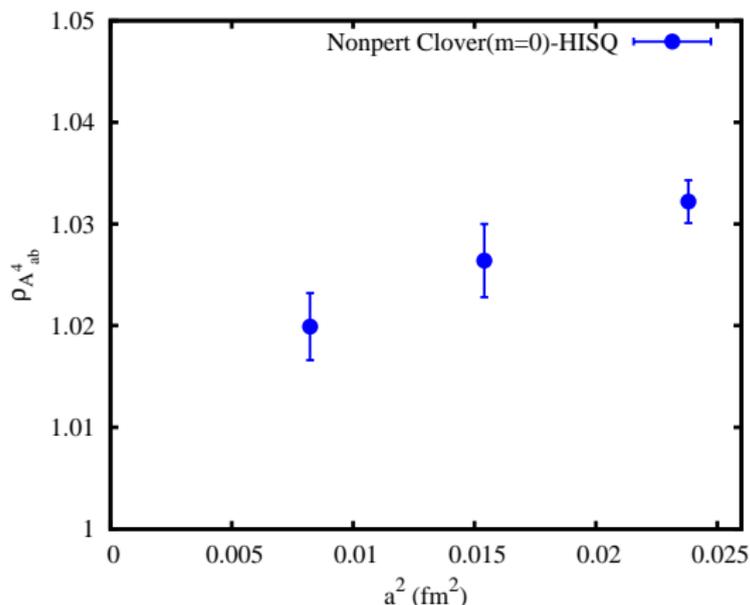
- $Z_{A^4_{Cl-H}} = \rho_{A^4_{Cl-H}} \sqrt{Z_{V^4_{Cl-Cl}} Z_{V^4_{H-H}}}$

Set	Combinations	$Z_{A^4}$	$Z_{V^4}$	$\rho_{A^4_{ab}}$
VC	H-H	1.000	0.9887(20)	-
	Cl-Cl	0.2046(4)	0.2045(3)	-
	Cl-H	0.4642(6)	-	1.0322(21)
C	H-H	1.000	0.9938(17)	-
	Cl-Cl	0.2096(4)	0.2071(4)	-
	Cl-H	0.4656(4)	-	1.0263(36)
F	H-H	1.000	0.9944(10)	-
	Cl-Cl	0.2152(4)	0.2116(4)	-
	Cl-H	0.4679(7)	-	1.0199(33)

- $\rho_{A^4_{Cl-H}}$  indeed close to 1.0 up to all orders of lattice perturbation theory
- $Z_{V^4_{Cl-Cl}}$  and  $Z_{A^4_{Cl-Cl}}$  close too

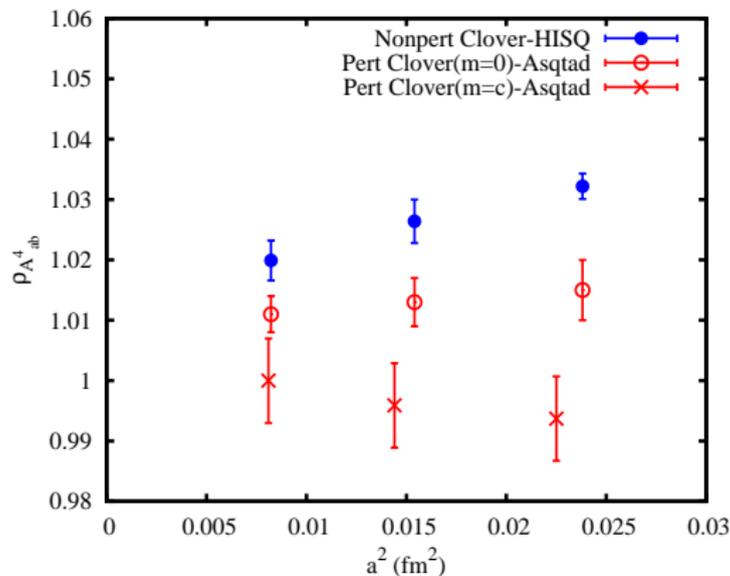
# Our results for $\rho_{A^4_{Cl-H}}$

- $\rho_{A^4_{Cl-H}}$  is close to 1.0 up to all orders of lattice perturbation theory with a maximum deviation of  $\sim 3\%$  on very coarse lattice



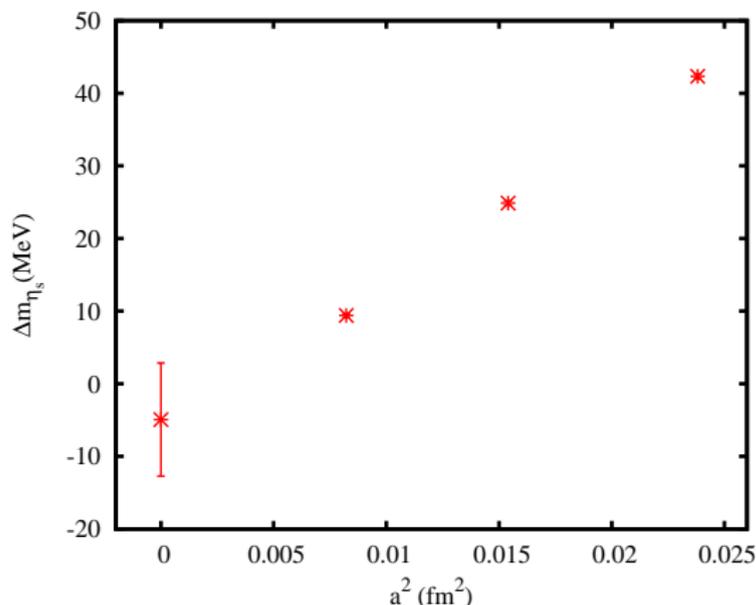
# Comparison of $\rho_{A^4_{Cl-H}}$ values

- Our numbers are not actually as close to 1.0 as Fermilab's 1-loop Clover-Asqtad results
- May be sensible to increase errors from the one loop calculation up to 2.5% to encompass our numbers



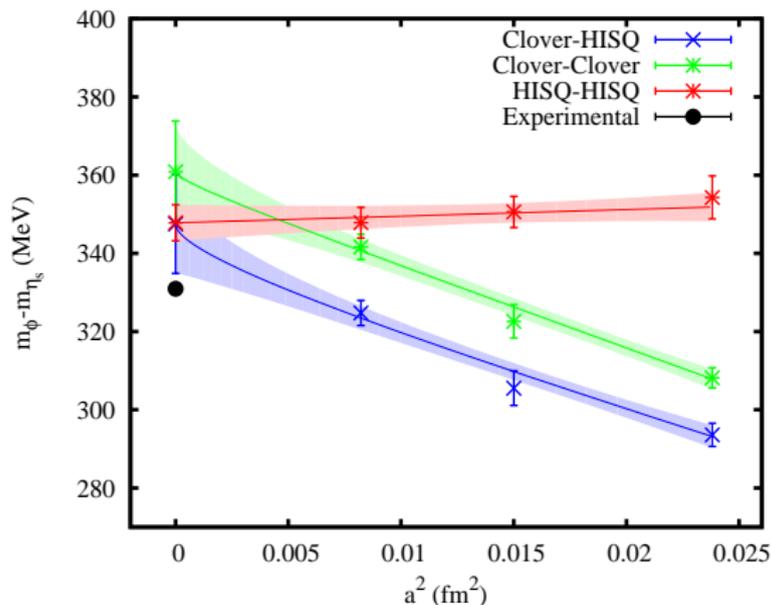
# Discretization effects

- The difference of the  $\eta_s$  mass obtained from H-H and CI-H methods is a discretization effect.



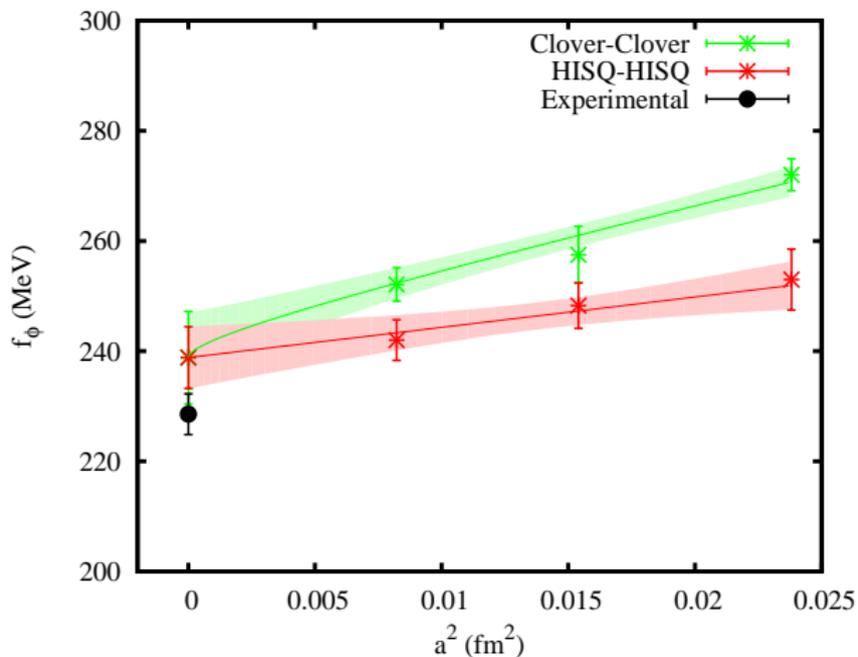
# Discretization Errors: $m_\phi - m_{\eta_s}$ extrapolation to $a = 0$

- All three methods of calculating correlators agree in the continuum limit.
- H-H discretization errors much smaller than CI-CI and CI-H discretizations
- Lattice results higher than the experimental result; But  $\phi$  non-gold-plated

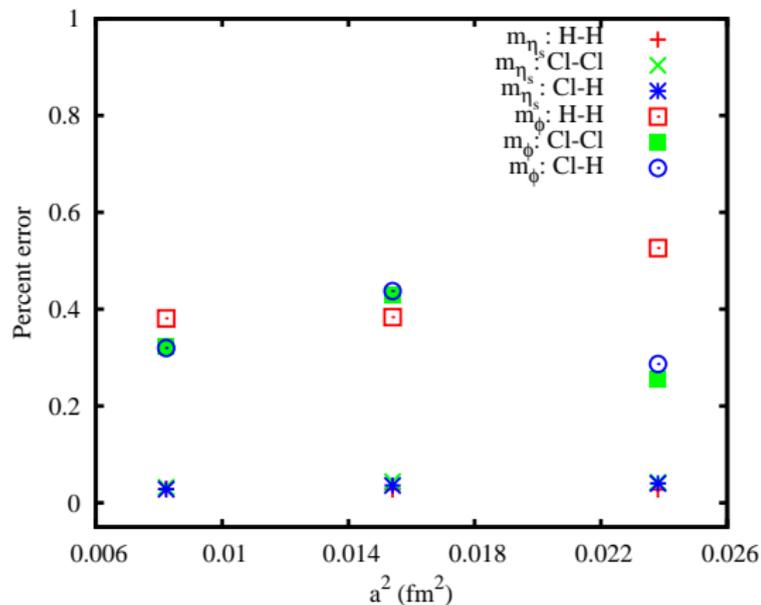


# Discretization Errors: $f_\phi$ extrapolation to $a = 0$

- $Z_{V^4_{Cl-Cl}}$  and  $Z_{V^4_{H-H}}$  used in  $f_\phi$
- Lattice  $f_\phi$  results match with experimental result up to  $1.5\sigma$



# Comparing Statistical Errors



- One clover propagator costs  $\sim 4$  times as much as one HISQ propagator
- One Clover propagator 16 times bigger than one HISQ propagator
- Statistical errors similar for both methods on coarse and fine lattices

# Conclusion and Ongoing Work

- Conclusions:

- $\rho_{A^4_{Cl-H}}$  close to 1.0 with a maximum  $\sim 3\%$  deviation using the non-perturbative lattice calculations with pure HISQ, pure Clover and mixed Clover-HISQ currents.
- Increasing Fermilab's errors from 0.7% to 2.5% in one loop perturbative calculation recommended
- Discretization errors much smaller from HISQ than Clover
- Statistical errors similar for Clover and HISQ on coarse and fine lattices, but Clover lot more costly

- Ongoing works:

- To look at  $\rho_{V^4_{Cl-H}}$  using  $Z_{V^4_{Cl-H}}$
- To study vector meson  $\phi$  on the physical point lattices