Non-perturbative renormalization of overlap quark bilinears on domain wall fermion configurations

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July 31 2013, Mainz

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Outline

- Introduction
- Lattice set up
- Numerical results
- Summary

Motivation

- Renormalization constants are needed to convert lattice results to the continuum $\overline{\mathrm{MS}}$ scheme.
- For example, quark masses, quark chiral condensate, strangeness content of nucleon etc.
- Overlap and domain wall fermions have good chiral symmetry on the lattice.
- The χQCD collaboration is using a setup of overlap on 2+1 domain wall configurations for calculations of strangeness in nucleon, strange and charm quark masses (see Yi-Bo Yang's poster) etc.
- The renormalization constants of quark bilinears are being calculated using a non-perturbative scheme (RI/MOM).

RI/MOM

- The renormalization condition in the RI-MOM scheme is [G. Martinelli et al., Nucl. Phys. B **445** (1995) 81]

$$\lim_{m_q \to 0} Z_q^{-1} Z_{\mathcal{O}} \frac{1}{12} \mathrm{Tr}[\Lambda_{\mathcal{O}}(p) \Lambda_{\mathcal{O}}^{tree}(p)^{-1}]_{p^2=\mu^2} = 1,$$

  Z_q is the quark field renormalization constant: $\psi_R = Z_q^{1/2}\psi$,
  $Z_{\mathcal{O}}$ is the renormalization constant for the operator $\mathcal{O}$: $\mathcal{O}_R = Z_{\mathcal{O}}\mathcal{O}$,
  μ is the renormalization scale.

- $\Lambda_{\mathcal{O}}(p)$ is the amputated forward Green function

$$\Lambda_{\mathcal{O}}(p) = S^{-1}(p) G_{\mathcal{O}}(p) S^{-1}(p),$$

  where $S(p)$ is the quark propagator.

- The calculation has to be done in a fixed gauge, say, Landau gauge. The method is supposed to work in the window

$$\Lambda_{QCD} \ll \mu \ll \pi/a.$$

RI/MOM

- The forward Green's function $G_{\mathcal{O}}(p)$ is computed as

$$G_{\mathcal{O}}(p) = \sum_{x,y} e^{-ip\cdot(x-y)} \langle \psi(x)\mathcal{O}(0)\bar{\psi}(y) \rangle$$

  by using point source quark propagator $S_i(x,0)$.

- The quark propagator in momentum space is given by

$$S(p) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{x} e^{-ip\cdot x} S_i(x,0)\right]$$

  from N configurations.

- For quark bilinears $\bar{\psi}\Gamma\psi'$ with $\Gamma = I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$, $\Lambda_{\mathcal{O}}^{tree}(p) = I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$.

Z_q

- In the RI scheme,

$$Z_q^{RI}(\mu) = \frac{-i}{48}\mathrm{Tr}\left[\gamma_\nu \frac{\partial S^{-1}(p)}{\partial p_\nu}\right]_{p^2=\mu^2}.$$

- In the RI' scheme, Z_q is given by

$$Z_q^{RI'}(\mu) = \frac{1}{12}\mathrm{Tr}\left[S^{-1}(p)S_f^{ov}(p)\right]_{p^2=\mu^2},$$

where $S_f^{ov}(p)$ is the free overlap quark propagator.

- We obtain the renormalization constant of the local axial vector current Z_A^{WI} from Ward Identities, which equals to Z_A^{RI} in the RI scheme. Then

$$Z_q^{RI} = Z_A^{WI}\frac{1}{12}\mathrm{Tr}\left[\Lambda_A(p)\Lambda_A^{tree}(p)^{-1}\right]_{p^2=\mu^2}.$$

Lattice setup

Table: Parameters of configurations with 2+1 flavor dynamical domain wall fermions (RBC-UKQCD). [Aoki et al. 2011]

$1/a$(GeV)	label	am_{sea}	volume	N_{conf}
1.73(3)	c005	0.005/0.04	$24^3 \times 64$	92
	c01	0.01/0.04	$24^3 \times 64$	88
	c02	0.02/0.04	$24^3 \times 64$	138
2.28(3)	f004	0.004/0.03	$32^3 \times 64$	50
	f006	0.006/0.03	$32^3 \times 64$	40
	f008	0.008/0.03	$32^3 \times 64$	50

- Eight overlap valence quark masses are used on each lattice.
- The pion masses range from about 220 MeV to about 600 MeV.

Lattice setup

- Anti-periodic boundary condition in the time direction,

$$ap = \left(\frac{(2k_t + 1)\pi}{T}, \frac{2\pi k_x}{L}, \frac{2\pi k_y}{L}, \frac{2\pi k_z}{L}\right),$$

$L = 24$ lattice: $k_\mu = -6, -5, ..., 6$
$L = 32$ lattice: $k_t = -5, -1, ..., 6$ and $k_i = -6, -7, ..., 6$

- To reduce effects of Lorentz non-invariant discretization errors, we only analyze the momenta which satisfy

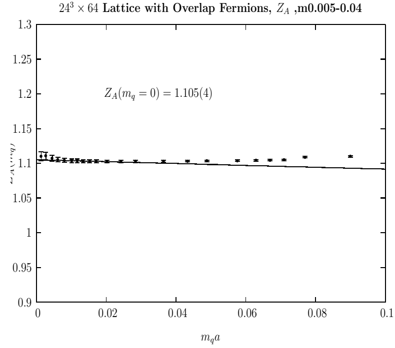
$$\frac{p^{[4]}}{(p^2)^2} < 0.32, \quad \text{where } p^{[4]} = \sum_\mu p_\mu^4, \quad p^2 = \sum_\mu p_\mu^2.$$

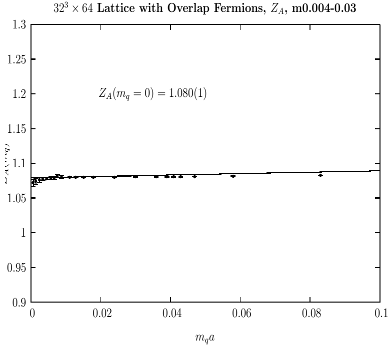
- The statistical errors are from Jackknife processes.

Z_A

- Using
$$Z_A \partial_\mu A_\mu = 2 Z_m m_q Z_P P$$
and $Z_m = Z_P^{-1}$ for overlap fermions, one has
$$Z_A \partial_\mu \langle 0|A_\mu|\pi\rangle = 2 m_q \langle 0|P|\pi\rangle.$$
- If the pion is at rest, then from the above one gets
$$Z_A = \frac{2 m_q \langle 0|P|\pi\rangle}{m_\pi \langle 0|A_4|\pi\rangle}.$$
- From 2-point functions $G_{PP}(\vec{p}=0,t) = \sum_{\vec{x}} \langle 0|P(x)P(0)|0\rangle$ and $G_{A_4 P}(\vec{p}=0,t) = \sum_{\vec{x}} \langle 0|A_4(x)P(0)|0\rangle$, one obtains
$$Z_A = \lim_{m_q \to 0, t \to \infty} \frac{2 m_q G_{PP}(\vec{p}=0,t)}{m_\pi G_{A_4 P}(\vec{p}=0,t)}.$$

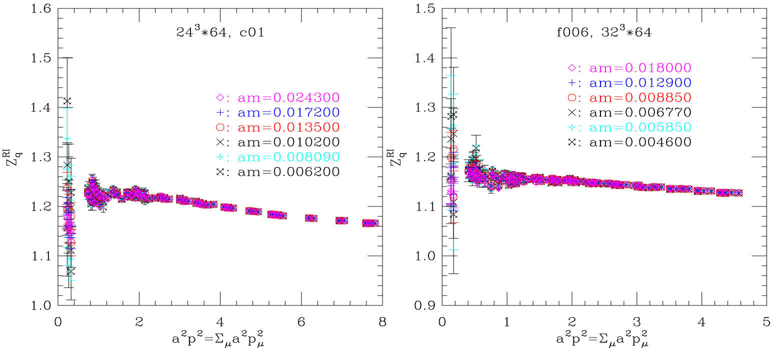
Z_A





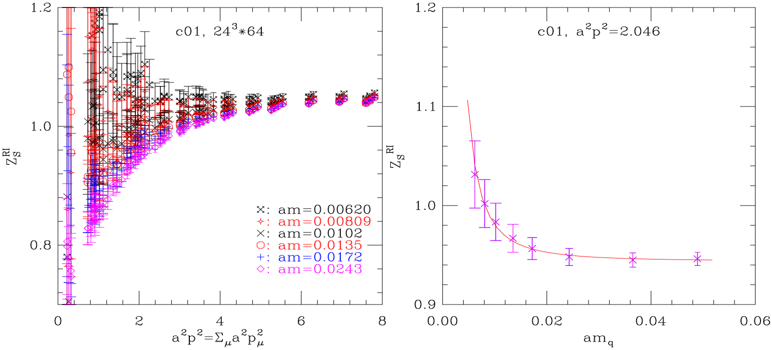
- A straight line fit in $am_q \in [0.00809, 0.02430]$ on the coarse lattice.
- $am_q \in [0.00585, 0.01520]$ on the fine lattice.

Z_q



- Examples of Z_q^{RI} as a function of the momentum scale.
- Small valence quark mass dependence.
- In Landau gauge, the anomalous dimension at 1-loop is zero.

Z_S

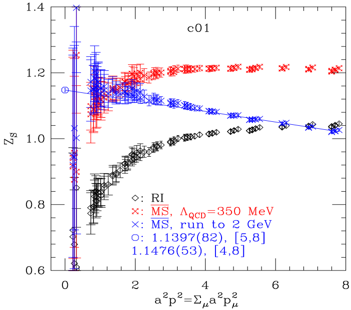


- To go to the chiral limit, we use the following to fit and take B_s:

$$Z_S = \frac{A_s}{(am_q)^2} + B_s + C_s \cdot (am_q)$$

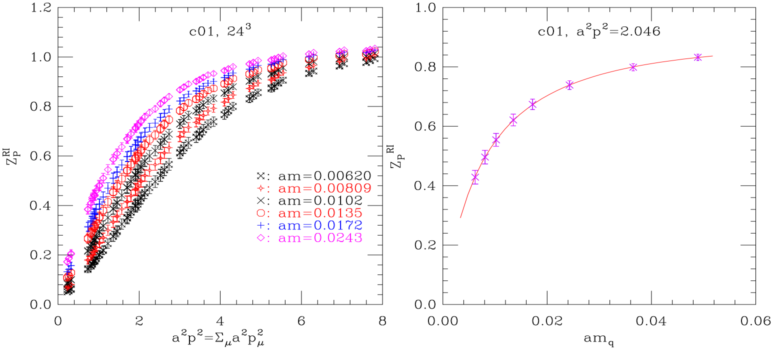
[Blum et al. 2001, Aoki et al. 2007]

Z_S



- The conversion ratio from RI to $\overline{\text{MS}}$ scheme is to 3-loops. [Chetyrkin and Retey 1999]
- The running of $Z_S^{\overline{\text{MS}}}$ uses anomalous dimension to 4-loops.
- $Z_S = A + B \cdot a^2p^2$ to extrapolate away $\mathcal{O}(a^2p^2)$ discretization errors.

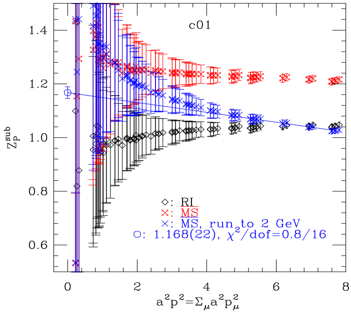
Z_P



- To remove the contamination from the Goldstone bosons, we use the following to fit and take Z_P^{sub} as the chiral limit value:

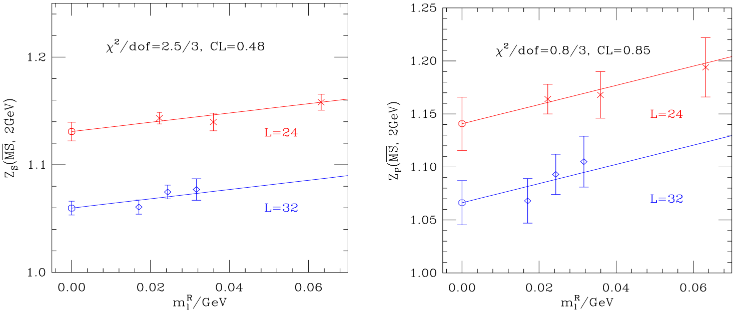
$$Z_P^{-1} = \frac{A_p}{am_q} + (Z_P^{sub})^{-1} + C_p \cdot (am_q)$$

Z_P



- The conversion from RI to $\overline{\rm MS}$ scheme and the running of $Z_P^{sub}(\overline{\rm MS})$ are similar to those of $Z_S^{\overline{\rm MS}}$.
- $Z_P^{sub} = A + B \cdot a^2p^2$ to extrapolate away $\mathcal{O}(a^2p^2)$ discretization errors.

Z_S and Z_P in the light sea quark massless limit



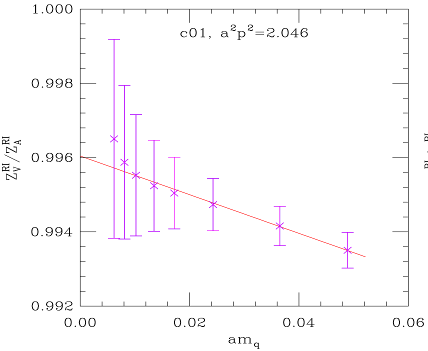
$$Z(m_l^R) = Z(0) + c \cdot m_l^R, \quad \text{where } m_l^R = (m_l + m_{res})Z_m^{sea}$$

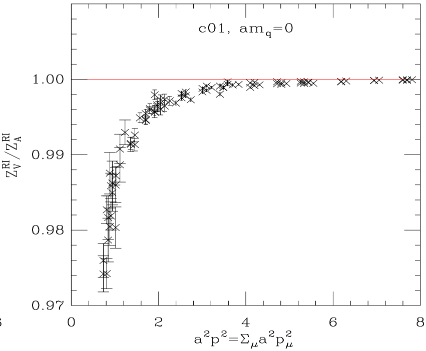
ensemble	c005	c01	c02	$m_l^R = 0$
$Z_S^{\overline{\rm MS}}$(2 GeV)	1.1433(54)	1.1397(82)	1.1581(74)	1.1308(87)
$Z_P^{\overline{\rm MS}}$(2 GeV)	1.164(14)	1.168(22)	1.194(28)	1.141(25)
ensemble	f004	f006	f008	$m_l^R = 0$
$Z_S^{\overline{\rm MS}}$(2 GeV)	1.0607(66)	1.0747(64)	1.077(10)	1.0597(64)
$Z_P^{\overline{\rm MS}}$(2 GeV)	1.068(21)	1.093(19)	1.105(24)	1.066(21)

Error budget of $Z_S^{\overline{\mathrm{MS}}}$

Source	Error (%,L=24)	Error (%,L=32)
Statistical	0.8	0.6
Truncation (RI to $\overline{\mathrm{MS}}$)	2.2	2.1
Coupling constant	1.5	1.5
Perturbative running	<0.02	<0.02
Lattice spacing	0.5	0.5
Fit range of a^2p^2	0.5	<0.1
Extrapolation in m_l	0.18	1.8
Total	2.9	3.2

Z_V/Z_A





- A linear extrapolation of Z_V/Z_A in am_q is used for the chiral limit.
- At large momentum scale $Z_V^{RI}/Z_A^{RI} = 1$, i.e., $Z_V^{RI} = Z_A^{RI}$ is satisfied as expected.

Summary

- Using the RI-MOM scheme, we calculate the renormalization constants for overlap quark bilinears on domain wall configurations.
- $Z_S = Z_P$ and $Z_V = Z_A$ are confirmed as expected.
- The conversion from the RI scheme to the $\overline{\mathrm{MS}}$ scheme is the main source of error for Z_S.
- $Z_m = 1/Z_S$ is used for the determination of strange and charm quark masses.
- We find $m_s^{\overline{\mathrm{MS}}}$(2 GeV)=0.104(9) GeV and $m_c^{\overline{\mathrm{MS}}}$(2 GeV)=1.107(38) GeV. (See Yi-Bo Yang's poster)

Thank you.