COMBINED NNLO LATTICE/CONTINUUM ANALYSIS OF THE FLAVOR ud V-A CORRELATOR

with

P. Boyle, L. Del Debbio, N. Garron, R. Hudspith, E. Kerrane, J. Zanotti

+ D. Boito, M. Golterman, M. Jamin, S. Peris

Lattice 2013, Mainz, July 31/13

OUTLINE

- $\Pi_{ud;V-A}(Q^2)$: *from data, the lattice, and at NNLO*
- *A problem for the continuum NNLO* L_{10}^r *determination*
- *Resolving the problem with lattice data*
- *Further improvement with chiral sum rule input*

THE V-A CORRELATOR

- $J = 0, 1$ scalar correlators $\Pi^{(J)}_{V-A;ud}(Q^2)$
  - Continuum (Minkowski):

$$\Pi^{\mu\nu}_{V/A}(q^2) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0| T \left(J^{ud,\mu}_{V/A}(x) J^{ud\dagger\,\nu}_{V/A}(0) \right) |0\rangle$$

$$= \left(q^\mu q^\nu - q^2 g^{\mu\nu} \right) \Pi^{(1)}_{V/A}(q^2) + q^\mu q^\nu\, \Pi^{(0)}_{V/A}(q^2)$$

  - Euclidean (lattice) version:

$$\Pi^{\mu\nu}_{V/A}(Q^2) = \left(Q^2 \delta^{\mu\nu} - Q^\mu Q^\nu \right) \Pi^{(1)}_{V/A}(Q^2) - Q^\mu Q^\nu \Pi^{(0)}_{V/A}(Q^2)$$

- For LEC determinations, convenient to focus on π-pole-subtracted $J = 0 + 1$ combination

$$\Delta\bar{\Pi}(Q^2) \equiv \Pi^{(0+1)}_{V-A}(Q^2) + \frac{2f_\pi^2}{m_\pi^2 + Q^2}$$

  - $J = 0 + 1$ sum avoids kinematic singularities
  - Dispersive representation for physical m_q from experimentally accessible continuum ud, $V-A$ spectral function, $\Delta\rho(s) \equiv \rho^{cont}_{V-A}(s)$
  - Ensemble f_π, m_π for pole subtraction in lattice cases

- Continuum (physical m_q) $\Delta\bar{\Pi}(Q^2)$ results
  - Dispersive representation

$$\Delta\Pi(Q^2) = \int_0^\infty ds \, \frac{\Delta\rho(s)}{s+Q^2}$$

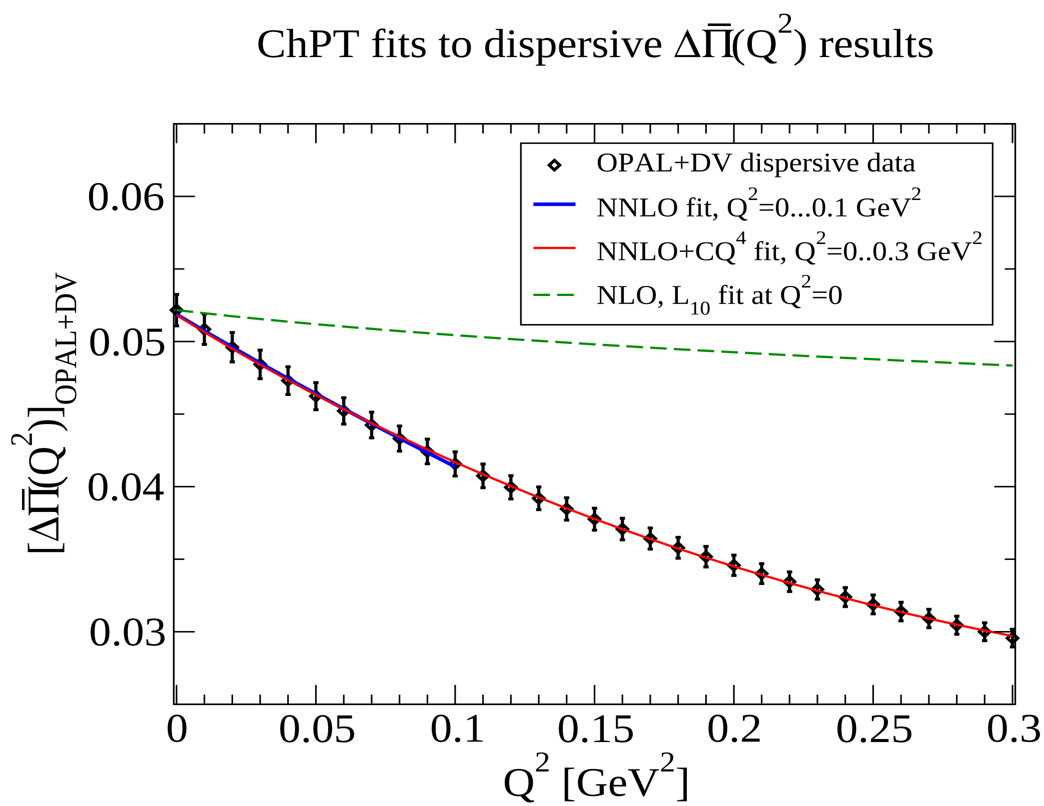
  - $\Delta\rho(s)$ from
    * OPAL hadronic τ decay data, $s < m_\tau^2$
    * Higher s: physical DV ansatz, fitted to V, A τ decay data [PRD85 (2012) 093015 for details]
  - **$\Delta\bar{\Pi}(Q^2)$ at low Q^2 of interest for LEC determination STRONGLY data dominated**

- Lattice data for $\Delta\bar{\Pi}(Q^2)$
  - From RBC/UKQCD $n_f = 2+1$ DWF ensembles
  - **FINE:** $32^3 \times 64 \times 16_5$, $1/a = 2.31$ GeV, Iwasaki gauge, $L \sim 2.7\ fm$, $m_\pi = 293$, 349, 399 MeV [PRD83 (2011) 074508 for details]
  - **COARSE:** $32^3 \times 64 \times 32_5$, $1/a = 1.37$ GeV, Iwasaki + DSDR, $L \sim 4.6\ fm$, $m_\pi = 171$, 248 MeV [PRD87 (2012) 094514 for details]
  - *Limited low-Q^2 coverage for fine ensembles, improved for coarse*

A few key observations re lattice, continuum results

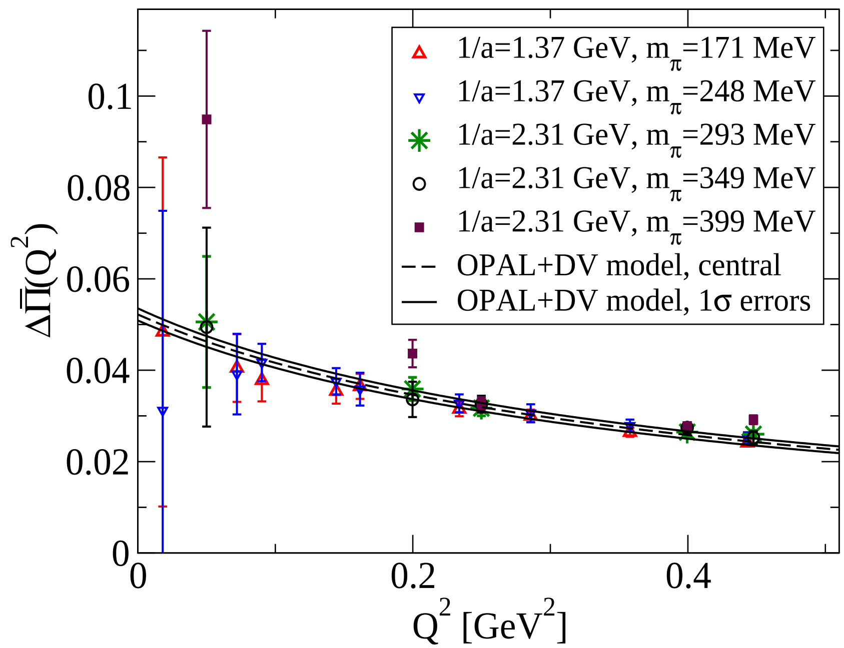
- NNLO analysis of continuum $\Delta\bar{\Pi}(Q^2)$ [D. Boito et al. PRD87 (2013) 094008 for details] shows [Figure]
  - NNLO contributions significant $\Rightarrow$ **old continuum, lattice NLO L^r_{10} determinations NOT reliable**
  - $Q^2 < 0.3\ GeV^2$ for NNLO+ analysis to follow

- Lattice $\Delta\bar{\Pi}(Q^2)$ errors comparable to continuum for $Q^2 >\sim 0.3\ GeV^2$ BUT larger for Q^2 in range of NNLO analysis (especially for lowest Q^2) [FIGURE]



ChPT fits to dispersive $\Delta\overline{\Pi}(Q^2)$ results
OPAL+DV dispersive data
NNLO fit, Q^2=0...0.1 GeV^2
NNLO+CQ^4 fit, Q^2=0..0.3 GeV^2
NLO, L_{10} fit at Q^2=0
$[\Delta\overline{\Pi}(Q^2)]_{OPAL+DV}$
0.06
0.05
0.04
0.03
0
0.05
0.1
0.15
0.2
0.25
0.3
Q^2 [GeV^2]

Comparison of Lattice to continuum $\Delta\bar{\Pi}(Q^2)$



$\Delta\bar{\Pi}(Q^2)$ to NNLO in the chiral expansion

- The NNLO representation [ABT, NPB568 (2000) 319]

$$\Delta\bar{\Pi}(Q^2) = \mathcal{R}(Q^2) + C_9(Q^2)\, L_9^r - 16Q^2\, C_{87}^r$$
$$+ [-8 + 32\,(2\mu_\pi + \mu_K)]\; L_{10}^r + \mathcal{C}_0 + \mathcal{C}_1$$

with $\mu_P = \frac{m_P^2}{32\pi^2 f_\pi^2}\, log\left(\frac{m_P^2}{\mu^2}\right)$

$$\begin{aligned}
\mathcal{C}_0 &\equiv 32 m_\pi^2\, (C_{12}^r - C_{61}^r + C_{80}^r) \equiv 32 m_\pi^2\, \hat{\mathcal{C}}_0 \\
\mathcal{C}_1 &\equiv 32\left(m_\pi^2 + 2m_K^2\right)(C_{13}^r - C_{62}^r + C_{81}^r) \\
&\equiv 32\left(m_\pi^2 + 2m_K^2\right)\hat{\mathcal{C}}_1
\end{aligned}$$

and $C_9(Q^2)$, $\mathcal{R}(Q^2)$ completely known in terms of the chiral renormalization scale μ and PS masses $\{m_P\}$

- Existing input/features of the NNLO representation
  - L_9^r well known from NNLO π charge radius analysis [Bijnens, Talavera, JHEP 0203 (2002) 046]
  - NNLO correction to L_{10}^r coefficient (-4.1650 for physical $\{m_P\}$) c.f. NLO contribution -8
  - NNLO LEC combination $\hat{\mathcal{C}}_0 = C_{12}^r - C_{61}^r + C_{80}^r$ LO in $1/N_c$, $C_{12,61}^r$ experimentally accessible, C_{80}^r from RChPT (m_π^2 prefactor makes $\mathcal{C}_0$ safely negligible)
  - NNLO LEC combination $\hat{\mathcal{C}}_1 = C_{13}^r - C_{62}^r + C_{81}^r$ NLO in $1/N_c$, NOT experimentally accessible
  - RChPT estimate for $\mathcal{C}_1$ unavailable (resonant contributions to $C_{13,62,80}^r$ absent in RChPT)

- The continuum NNLO L_{10}^r determination problem
  - L_{10}^r, $\mathcal{C}_0$, $\mathcal{C}_1$ contributions all Q^2-independent $\Rightarrow$ separation of term involving L_{10}^r impossible
  - Mass enhancement $\left(m_\pi^2+2m_K^2\right)/m_\pi^2$ of $\mathcal{C}_1$ relative to $\mathcal{C}_0$ ($\simeq$ 26 for physical $\{m_P\}$) more than undoes the $1/N_c$ LEC suppression
  - Previous NNLO continuum L_{10}^r determination [GAPP, PRD78 (2010) 116012] uses (non-conservative) guess $\hat{\mathcal{C}}_1 = 0 \pm |\hat{\mathcal{C}}_0|/3$ (*DANGER: cancellations in* $\hat{\mathcal{C}}_0$)
  - Resulting L_{10}^r error entirely dominated by assumed (non-conservative) $\mathcal{C}_1$ range

- Lattice input to the continuum NNLO problem
  - Separation of L^r_{10}, $\mathcal{C}_0$, $\mathcal{C}_1$ contributions from differing $\{m_P\}$ dependences via ensembles with a range of m_q ($\{m_P\}$)
  - FIRST PASS: L^r_{10}, $\mathcal{C}_0$, $\mathcal{C}_1$ fit using
    - * NNLO constraint associated with accurate continuum (physical m_q) $\Delta\bar{\Pi}(0)$ determination
    - * Ensemble-dependent constraints on L^r_{10}, $\mathcal{C}_0$, $\mathcal{C}_1$ from DWF ensembles noted above
  - SECOND PASS: Further improvement using additional continuum constraint (from new FB $ud - us$, $V - A$ chiral sum rule)

- A bit more on the FIRST PASS analysis

  - The continuum $\Delta\bar{\Pi}(0)$ constraint ($\mu_{ch} = 0.77\ GeV$) [Boito et al. PRD87 (2013) 094008]

$$L_{10}^r - 0.0822(\mathcal{C}_0 + \mathcal{C}_1) = -0.00410(6)_{exp}(7)_{L_9^r}$$

  - Ensemble-dependent continuum/lattice constraints from fixed-Q^2 differences

$$\Delta\left(\Delta\bar{\Pi}(Q^2)\right) \equiv \left[\Delta\bar{\Pi}(Q^2)\right]_{latt} - \left[\Delta\bar{\Pi}(Q^2)\right]_{cont}$$

through the NNLO representation

$$\Delta\left(\Delta\bar{\Pi}(Q^2)\right) = \Delta\mathcal{R}(Q^2) + \Delta c_{10} L_{10}^r + \delta_0 \mathcal{C}_0 + \delta_1 \mathcal{C}_1$$

* $\Delta\mathcal{R}(Q^2)$, Δc_{10}, $\delta_{0,1}$ fixed by μ_{ch}, physical and ensemble $\{m_P\}$ and L_9^r ($\Delta\mathcal{R}$ only)

* $\Delta\left(\Delta\bar{\Pi}(Q^2)\right) - \Delta\mathcal{R}(Q^2)$: self-consistency-checked constraint on ensemble-dependent, Q^2-*independent* combination $\Delta c_{10}\, L_{10}^r + \delta_0 \mathcal{C}_0 + \delta_1 \mathcal{C}_1$

- Fit results for $\mu = \mu_{ch}$

$$\begin{aligned} L_{10}^r &= -0.0031(8) \\ \mathcal{C}_0 &= -0.00081(82) \\ \mathcal{C}_1 &= \ 0.014(11) \end{aligned}$$

- First NNLO L_{10}^r result with NNLO LEC uncertainties under control, BUT L_{10}^r error larger than ideal

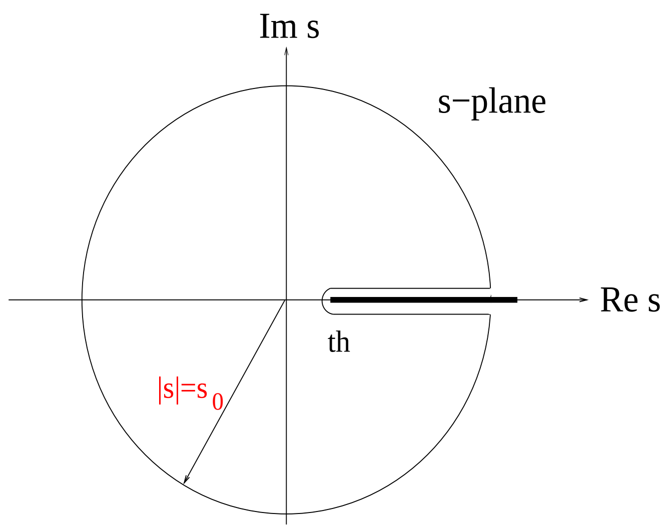
- Improving L_{10}^r with new chiral sum rule input
  - New constraint on L_{10}^r, $\mathcal{C}_0$ from $Q^2 = 0$ value of FB difference of π-, K-pole-subtracted ud, us V-A correlators

$$\bar{\Pi}_{ud-us}^{V-A}(Q^2) \equiv \bar{\Pi}_{ud;V-A}^{(0+1)}(Q^2) - \bar{\Pi}_{us;V-A}^{(0+1)}(Q^2)$$

  - $\bar{\Pi}_{ud-us}^{V-A}(0)$ determinable from Inverse Moment (Chiral) Sum Rules (IMSR) (here: FESRs with weight $1/s \times$ polynomial)
  - Convenient choice: generalization of Durr-Kambor ud-us V channel analysis [PRD61 (2000) 114025]

- Basic IMSR relation (Cauchy's Theorem) for polynomial $w(s)$, kinematic-singularity-free $\Pi(Q^2)$

$$w(0)\,\Pi(0) = \frac{1}{2\pi i}\int_{|s|=s_0} ds\,\frac{w(s)}{s}\,\Pi(Q^2) + \int_{th}^{s_0} ds\,\frac{w(s)}{s}\,\rho(s)$$



- Here: IMSR with $\Pi = \Pi^{(0+1)}_{ud-us;V-A}$, $w(s) = w_{DK}(y)$ $[y = s/s_0,\ w_{DK}(y) = (1-y)^3\left(1 + y + \frac{1}{2}y^2\right)]$

- Separating continuum from π-, K-pole term contributions ($y_{\pi,K} \equiv m^2_{\pi,K}/s_0$)

$$\bar{\Pi}^{V-A}_{ud-us}(0) = \frac{1}{2\pi i}\int_{|s|=s_0} ds\, \frac{w_{DK}(y)}{s}\,\Pi^{(0+1)}_{ud-us;V-A}(Q^2) + \int_{th}^{s_0} ds\, \frac{w_{DK}(y)}{s}\left[\rho^{(0+1)}_{ud;V-A}(s) - \rho^{(0+1)}_{us;V-A}(s)\right]_{cont} + \frac{2f_K^2}{m_K^2}\left[w_{DK}(y_K) - 1\right] - \frac{2f_\pi^2}{m_\pi^2}\left[w_{DK}(y_\pi) - 1\right]$$

- RHS: line 1: OPE; lines 2, 3: data

- RHS contributions (skipping MANY details):
  - Residual π, K pole terms accurately known
  - PDG input for OPE contour integral
  - OPE contribution numerically small (leading $D = 2, 4$ terms $O(\alpha_s)$ and chirally suppressed: $D = 2$ $O(\alpha_s m_s^2)$; $D = 4$ $O(\alpha_s m_s \langle \bar{q}q \rangle)$)
  - ud V-A spectral function from OPAL non-strange differential τ-decay distribution data, covariances (updated for current branching fractions)

* us V-A spectral function from sum over strange exclusive mode differential τ-decay distributions

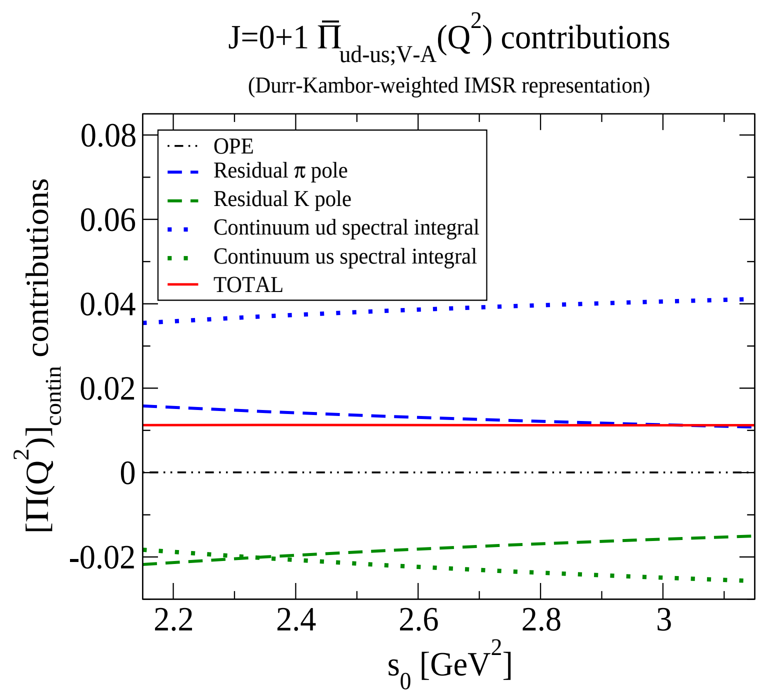
  - ◁ New precision BaBar, Belle $K\pi$ (pure V), $K\pi\pi$ (mixed V,A) results crucial to accuracy

  - ◁ Modes with no BaBar/Belle update: ALEPH 1999 distributions, rescaled for modern BFs

  - ◁ 50±50%, 100% anticorrelated V/A split for contributions where separation ambiguous

  - ◁ Good errors in spite of higher-multiplicity-mode V/A separation ambiguities due to strong higher-s suppression ($1/s$ weighting, 3^{rd}-order zero at $s = s_0$ in $w_{DK}(y)$)

* NOTE: all terms on RHS s_0-dependent, LHS s_0-independent $\Rightarrow$ s_0-stability cross-check



- Very good s_0-stability for sum [Figure]

$$\bar{\Pi}^{V-A}_{ud-us}(0) = 0.01126\,(136)_{exp,OPE}\,(5)_{s_0}$$

- Implementing known terms in NNLO representation $\Rightarrow$ $\mu_{ch} = 0.77$ GeV version of IMSR constraint

$$2.125\,L^r_{10} - 11.61\,\mathcal{C}_0 = -0.00346(149)$$

- Combining with earlier constraints yields the improved $\mu = \mu_{ch}$ results

	L^r_{10}	$\mathcal{C}_0$	$\mathcal{C}_1$
1^{st} Pass	$-0.0031(8)$	$-0.00081(82)$	$0.0136(106)$
2^{nd} Pass	$-0.00346(29)$	$-0.00034(12)$	$0.0081(31)$

CONCLUSIONS

- Pure continuum NNLO L_{10}^r determination problematic (no input on key NNLO LEC combination $\mathcal{C}_1$)

- Lattice errors at low Q^2 too large at present to allow pure lattice NNLO determination

- Nonetheless, lattice data allows $\mathcal{C}_1$ determination, especially in combination with new IMSR $\mathcal{C}_0$ constraint

- Final result, $L_{10}^r(\mu_{ch}) = -0.00346(29)$, is only determination with NNLO LEC errors under actual control

- Note: additional L_{10}^r uncertainty from missing $O(p^8)$ and higher contributions potentially $\sim 10\%$ in view of $\sim 30\%$ shift between NLO and NNLO

- Values for other LECs determined along the way [C_{87}^r, C_{80}^r, updated C_{61}^r] reported elsewhere

- Determination of $\mathcal{C}_1$ allows finalization of Gasser et al. [PLB652 (2007) 21] NNLO relation between ℓ_5^r, L_{10}^r

$$\ell_5^r(\mu_{ch}) = 1.362\, L_{10}^r(\mu_{ch}) - 0.00031(8)_{L_9^r}(39)_{\mathcal{C}_1}$$

(c.f. NLO version $\ell_5^r(\mu_{ch}) = L_{10}^r(\mu_{ch}) + 0.00003$)