The kaon mass in 2+1+1 flavor twisted mass Wilson ChPT

Oliver Bär and Ben Hörz

Humboldt Universität zu Berlin

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Introduction

- $N_f = 2$ twisted mass term: $m + i\gamma_5\sigma_3\mu$ — Frezzotti *et al* `01
- Advantages
  - Dirac operator bounded from below: $D^\dagger D \geq \mu^2$
  - Twisted mass renormalizes multiplicatively only
  - Simplified renormalization in some cases — Frezzotti, Rossi `04
  - Automatic O(a) improvement at maximal twist — Frezzotti, Rossi `04
- Drawbacks
  - Tuning to maximal twist needed
  - Parity and isospin breaking → leads to pion mass splitting

$$\Delta_\pi^2 = M_{\pi^0}^2 - M_{\pi^\pm}^2 = \mathrm{O}(a^2)$$

Introduction

- Splitting is of $O(a^2)$ and vanishes in the continuum limit
- In practice rather large:
  - $N_f = 2+1+1$
  - $a \approx 0.09$ fm (A)
    $a \approx 0.08$ fm (B)
    $a \approx 0.06$ fm (D)

Herdoiza *et al* (ETMC) 2013
arXiv:1303.3516

Ensemble	$aM_{\pi^\pm}$	aM_{π^0}	$M_{\pi^0}/M_{\pi^\pm}$
A30.32	0.1234	0.0611	0.50
A40.32	0.1415	0.0811	0.57
A40.24	0.1445	0.0694	0.48
A60.24	0.1727	0.1009	0.58
A80.24	0.1987	0.1222	0.61
A100.24	0.2215	0.157	0.71
A80.24s	0.1982	0.1512	0.76
A100.24s	0.2215	0.1863	0.84
B25.32	0.1064	0.0605	0.57
B35.32	0.1249	0.071	0.57
B55.32	0.154	0.1323	0.86
B75.32	0.1808	0.1557	0.86
B85.24	0.1931	0.1879	0.97
D15.48	0.0695	0.0561	0.81
D20.48	0.0797	0.0651	0.82
D30.48	0.0978	0.086	0.88
D45.32sc	0.1198	0.0886	0.74

Introduction

Consequences of a large pion mass splitting:

- Modified quark mass dependence because of chiral logs involving the neutral pion mass

$$M^2_{\pi^\pm,\mathrm{NLO}} = M^2_\pm\left(1 + \frac{M_0^2}{32\pi^2 f^2}\ln\frac{M_0^2}{\Lambda_3^2} + C_{M_\pm}a^2\right)$$

OB 2010

$$f_{\pi,\mathrm{NLO}} = f\left(1 - \frac{1}{32\pi^2 f^2}\left(M_\pm^2\ln\frac{M_\pm^2}{\Lambda_4^2} + M_0^2\ln\frac{M_0^2}{\Lambda_4^2}\right) + C_f a^2\right)$$

$\Lambda_3, \Lambda_4, C_{M_\pm}, C_f$: LECs of the NLO Lagrangian

➥ Impact on the chiral extrapolation for $M_0 \ll M_\pm$

- Enhanced FV corrections: 1-loop ChPT predicts FV corrections $\propto \exp(-M_\pi L)$
Colangelo et al 2010, OB 2010

Rule of thumb: Corrections small for $M_\pi L \geq 4$

Introduction

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Ensemble	$aM_{\pi^\pm}$	aM_{π^0}	$M_{\pi^0}/M_{\pi^\pm}$	$M_{\pi^\pm}L$	$M_{\pi^0}L$
A30.32	0.1234	0.0611	0.50	3.9	2.0
A40.32	0.1415	0.0811	0.57	4.5	2.6
A40.24	0.1445	0.0694	0.48	3.5	1.7
A60.24	0.1727	0.1009	0.58	4.1	2.4
A80.24	0.1987	0.1222	0.61	4.8	2.9
A100.24	0.2215	0.157	0.71	5.3	3.8
A80.24s	0.1982	0.1512	0.76	4.8	3.6
A100.24s	0.2215	0.1863	0.84	5.3	4.5
B25.32	0.1064	0.0605	0.57	3.4	1.9
B35.32	0.1249	0.071	0.57	4.0	2.3
B55.32	0.154	0.1323	0.86	4.9	4.2
B75.32	0.1808	0.1557	0.86	5.8	5.0
B85.24	0.1931	0.1879	0.97	4.6	4.5
D15.48	0.0695	0.0561	0.81	3.3	2.7
D20.48	0.0797	0.0651	0.82	3.8	3.1
D30.48	0.0978	0.086	0.88	4.7	4.1
D45.32sc	0.1198	0.0886	0.74	3.8	2.8

➥ Significant FV corrections expected due to small M_{π^0}

Introduction

- Natural question: Impact of the pion mass splitting on other observables?
- Rest of this talk: Kaon mass

ChPT for $N_f = 2+1+1$ flavors

- $N_f = 2+1+1$ tm QCD involves four flavors with mass term

$$M = \begin{pmatrix} M_l & 0 \\ 0 & M_h \end{pmatrix}$$ 4 x 4 matrix — Frezzotti and Rossi 2004

$$M_l = m + i\gamma_5\sigma_3\mu_l$$ light sector, up and down quark mass

$$M_h = m + i\gamma_5\sigma_1\mu_h + \sigma_3\delta$$ heavy sector, strange and charm quark mass

- Standard procedure to set up ChPT involves four flavors and treats D and Ds mesons as pseudo Goldstone bosons

  ➥ only valid for $m_c \approx m_s$, i.e. $M_D = M_K$ — Abdel-Rehim *et al* 2006

  $M_D \gtrsim M_K$ — Münster and Sudmann 2011

  ➥ not applicable to phenomenologically relevant case with $M_D \gg M_K$

How to construct charmless ChPT ?

- Illustrative example: Pion physics from SU(3) ChPT

Gasser and Leutwyler 1985

Fields:

$$\Sigma_{(3)} = \exp\left(\frac{2i}{f_{(3)}}\pi\right) \qquad \pi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ & 0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

explicit: $N_f = 3$

drop the kaon and eta fields

Mass term:

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

← the strange mass still appears !

- Comparison with SU(2) ChPT gives relations between the LECs

$$f_{(2)} = f_{(3)}\left(1 + 8\frac{B_{(3)} m_s}{f_{(3)}^2} L_4\right)$$

Shows the leading m_s dependence of $f_{(2)}$

(chiral logs ignored)

Recall:

SU(3) ChPT expands around $m_s = 0$

SU(2) ChPT expands around $m_s \neq 0$

How to construct charmless ChPT ?

Follow the same strategy to construct charmless tm WChPT:

OB and B. Hörz
in preparation

- Starting point: Chiral lagrangian for $N_f = 2+1+1$ tm QCD
- Drop the heavy D and D_s meson fields
- Absorb the remaining m_c dependence in the LECs, e.g.

$$f_{(3)} = f_{(4)} \left(1 + 8\frac{B_{(4)} m_c}{f_{(4)}^2} L_4\right)$$

as on slide before

$$W_{0,(3)} = W_{0,(4)} \left(1 + 8\frac{B_{(4)} m_c}{f_{(4)}^2} W_6\right)$$

W_0, W_6 : LECs associated with nonzero lattice spacing

Rupak and Shoresh 2002

- Final result reads (schematically)

$$\mathcal{L}^{\mathrm{SU}(4)}(f_{(4)}, B_{(4)}, W_{0,(4)}, m_c) \rightarrow \mathcal{L}^{\mathrm{SU}(3)}(f_{(3)}, B_{(3)}, W_{0,(3)})$$

no explicit m_c dependence !

Application: The kaon mass in the LCE regime

- LO Lagrangian: $\mathcal{L}_{\rm LO} = \mathcal{L}_2 + \mathcal{L}_{a^2}$
  Note:
  - We assume maximal twist → no O(a) terms
  - LCE regime
- Tree level masses:

  charged pion $M_\pm^2 = 2B\mu_l$

  neutral pion $M_0^2 = 2B\mu_l + 2c_2a^2$

  all kaons $M_K^2 = B(\mu_l + \underbrace{\mu_h - \delta}_{m_s})$
- Key features of computation of M_K to NLO:
  - Additional vertices from $\mathcal{L}_{a^2}$
  - Keep track of $M_\pm^2 \neq M_0^2$ in pion propagators

Application: The kaon mass in the LCE regime

$$M_{K,\mathrm{NLO}}^2 = M_K^2 \left[1 + \frac{1}{48\pi^2} \frac{M_\eta^2}{f^2} \ln \frac{M_\eta^2}{\mu^2} + 8L_{46} \frac{M_\pm^2}{f^2} + 8(2L_{46} + L_{58}) \frac{M_K^2}{f^2} \right]$$

$$+ \hat{a}^2 W'_{78} \frac{M_K^2}{4\pi^2 f^2} \ln \frac{M_K^2}{\mu^2} - \Delta M_\pi^2 \frac{M_0^2}{64\pi^2 f^2} \ln \frac{M_0^2}{\mu^2} \quad + \text{ analytic terms}$$

$\uparrow$ $\hat{a} = 2W_0 a$

$\uparrow$ $M_0^2 - M_\pm^2 \propto a^2$

Comments

- Converges to the known Gasser-Leutwyler result in the continuum limit $a \to 0$
- $a \neq 0$: Two additonal chiral logs involving the kaon and neutral pion mass
- Neutral pion log contains no unknown LECs, size determined by mass splitting
- Neutral pion logs is prone to large FV effects

How large is this correction ?

Pion log correction: $$\frac{\Delta M_K}{M_K} = \frac{\Delta M_\pi^2}{2M_K^2}\frac{M_0^2}{64\pi^2 f^2}\left[\ln\frac{M_0^2}{\mu^2} + \delta_{\rm FV}(M_0 L)\right]$$

Estimates from data:

Ensemble	$aM_{\pi^\pm}$	aM_{π^0}	M_K	af	L/a	$\frac{\Delta M_K}{M_K}$ [%]	$\frac{\Delta M_{K,\rm stat}}{M_K}$ [%]
A30.32	0.1234	0.0611	0.25150(29)	0.046	32	0.02	0.12
A40.32	0.1415	0.0811	0.25666(23)	0.048	32	0.12	0.09
A40.24	0.1445	0.0694	0.25884(43)	0.046	24	0.10	0.16
A60.24	0.1727	0.1009	0.26695(52)	0.051	24	0.14	0.19
A80.24	0.1987	0.1222	0.27706(61)	0.054	24	0.27	0.22
A100.24	0.2215	0.1570	0.28807(34)	0.056	24	0.37	0.12
A80.24s	0.1982	0.1512			24		
A100.24s	0.2215	0.1863	0.26502(90)	0.055	24	0.32	0.34
B25.32	0.1064	0.0605	0.21240(50)	0.040	32	0.003	0.24
B35.32	0.1249	0.0710	0.21840(28)	0.043	32	0.09	0.13
B55.32	0.1540	0.1323	0.22799(34)	0.046	32	0.17	0.15
B75.32	0.1808	0.1557	0.23753(32)	0.049	32	0.23	0.13
B85.24	0.1931	0.1879	0.24476(44)	0.049	24	0.06	0.18
D15.48	0.0695	0.0561			48		
D20.48	0.0797	0.0651			48		
D30.48	0.0978	0.0860			48		
D45.32sc	0.1198	0.0886			32		

Herdoiza *et al* (ETMC) 2013, Baron *et al* (ETMC) 2010

Conclusions and Outlook

- Current twisted mass simulations have a sizable pion mass splitting
- WChPT can accomodate the large splitting and provides (modified) ChPT formulae
- Presented here:
  - “Charmless” chiral Lagrangian for $N_f = 2+1+1$ tm QCD
  - Kaon mass to NLO (in the LCE regime)
- Next step: Kaon decay constant f_K
  (requires “charmless” expression of the axial vector current ...)

η_8 mass to NLO

$M^2_{\eta_8,\mathrm{NLO}} =$ continuum result

$$+ \hat{a}^2 W_7' \frac{2}{3\pi^2} \frac{M_0^2}{f^2} \ln \frac{M_0^2}{\mu^2}$$

$$- \Delta M_\pi^2 \frac{1}{96\pi^2} \frac{2M_0^2 + M_\pm^2}{f^2} \ln \frac{M_0^2}{\mu^2}$$ + analytic terms

The kaon mass to NLO

$M^2_{K,\mathrm{NLO}} =$ continuum result

$$+\,\hat{a}^2 W'_{78} \frac{M_K^2}{4\pi^2 f^4} \ln \frac{M_K^2}{\mu^2} + \hat{a}^2 W'_{68} \frac{M_0^2}{2\pi^2 f^4} \ln \frac{M_0^2}{\mu^2}$$ + analytic

$$W'_{78} = 2W'_7 + W'_8 \qquad W'_{68} = 2W'_6 + W'_8$$

The charmless chiral lagrangian (in physical basis)

$$\tilde{\Sigma}_l = \begin{pmatrix} \Sigma_l & 0 \\ 0 & \cos\omega_h \end{pmatrix}$$ vacuum

$$\begin{aligned}
\mathcal{L}_a &= -\frac{f^2}{4}\rho\langle\Sigma_3\tilde{\Sigma}_l + \tilde{\Sigma}_l^\dagger\Sigma_3^\dagger\rangle \\
&+W_4\rho\langle\partial_\mu\Sigma_3\partial_\mu\Sigma_3^\dagger\rangle\langle\Sigma_3\tilde{\Sigma}_l + \tilde{\Sigma}_l^\dagger\Sigma_3^\dagger\rangle \\
&+W_5\rho\,\langle\partial_\mu\Sigma_3\partial_\mu\Sigma_3^\dagger(\Sigma_3\tilde{\Sigma}_l + \tilde{\Sigma}_l^\dagger\Sigma_3^\dagger)\rangle \\
&-W_6\rho\,\langle\chi_3(\Sigma_3 + \Sigma_3^\dagger)\rangle\langle\Sigma_3\tilde{\Sigma}_l + \tilde{\Sigma}_l^\dagger\Sigma_3^\dagger\rangle \\
&-W_7\rho\,\langle\chi_3(\Sigma_3 - \Sigma_3^\dagger)\rangle\langle\Sigma_3\tilde{\Sigma}_l - \tilde{\Sigma}_l^\dagger\Sigma_3^\dagger\rangle \\
&-W_8\rho\,\langle\chi_3(\Sigma_3\tilde{\Sigma}_l\Sigma_3 + \Sigma_3^\dagger\tilde{\Sigma}_l^\dagger\Sigma_3^\dagger)\rangle \\
&+2W_4\rho\cos\omega_h\langle\partial_\mu\Sigma_3\partial_\mu\Sigma_3^\dagger\rangle \\
&-2W_6\rho\cos\omega_h\langle\chi_3(\Sigma_3 + \Sigma_3^\dagger)\rangle
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{a^2} &= -W_6'\rho^2\,\langle\Sigma_3\tilde{\Sigma}_l + \tilde{\Sigma}_l^\dagger\Sigma_3^\dagger\rangle^2 \\
&-W_7'\rho^2\,\langle\Sigma_3\tilde{\Sigma}_l - \tilde{\Sigma}_l^\dagger\Sigma_3^\dagger\rangle^2 \\
&-W_8'\rho^2\langle(\Sigma_3\tilde{\Sigma}_l)^2 + (\tilde{\Sigma}_l^\dagger\Sigma_3^\dagger)^2\rangle \\
&-4W_6'\rho^2\cos\omega_h\langle\Sigma_3\tilde{\Sigma}_l + \tilde{\Sigma}_l^\dagger\Sigma_3^\dagger\rangle \\
&-2W_8'\sin^2\omega_h\rho^2\langle P_s(\Sigma_3 + \Sigma_3^\dagger)\rangle\,.
\end{aligned}$$