Finite volume scaling of the electro-magnetic pion form factor in the epsilon regime

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What we do

We consider Finite Volume Effects(F.V.E.) in ε regime to the pion vector form factor within chiral perturbation theory.

What we find

We can remove dominant F.V.E. from zero-mode even in the ε regime.

Only perturbatively small F.V.E. from non-zero-mode is remained.

This study is also useful in p regime.

Plan of Presentation

1. Introduction

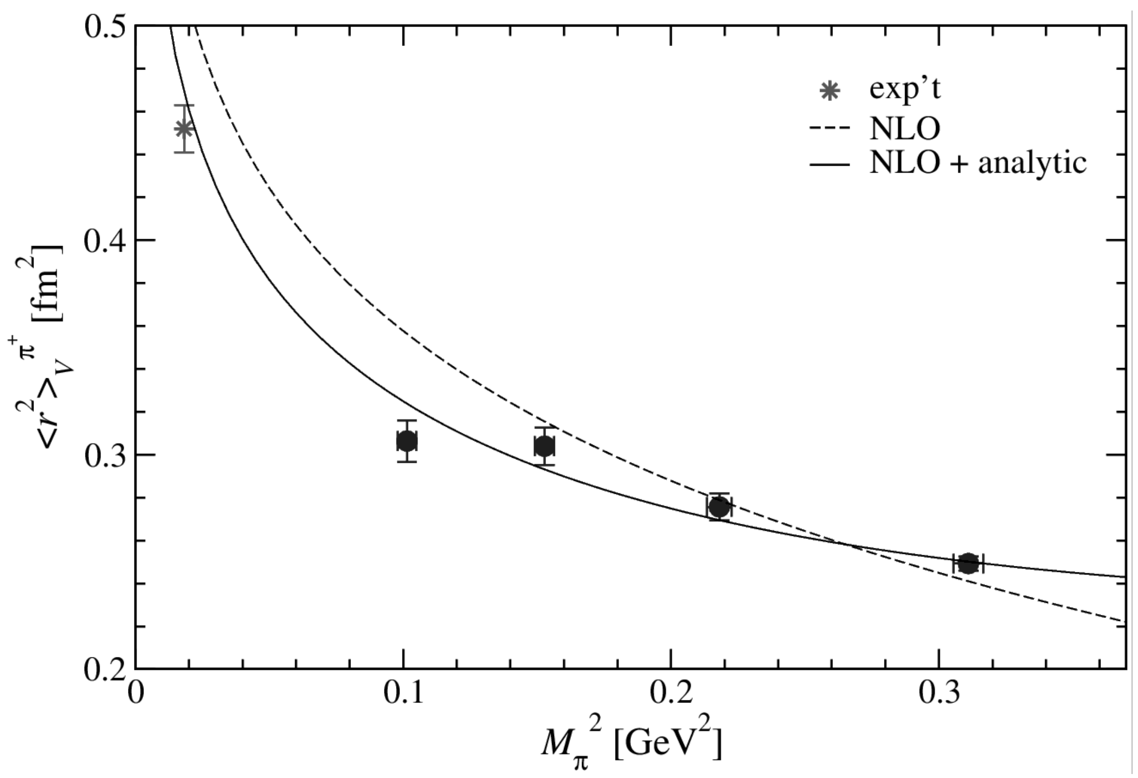
2. How to remove zero-mode's contribution
   - Non-zero Momentum insertion
   - Time subtraction
   - Ratio of correlators

3. Result

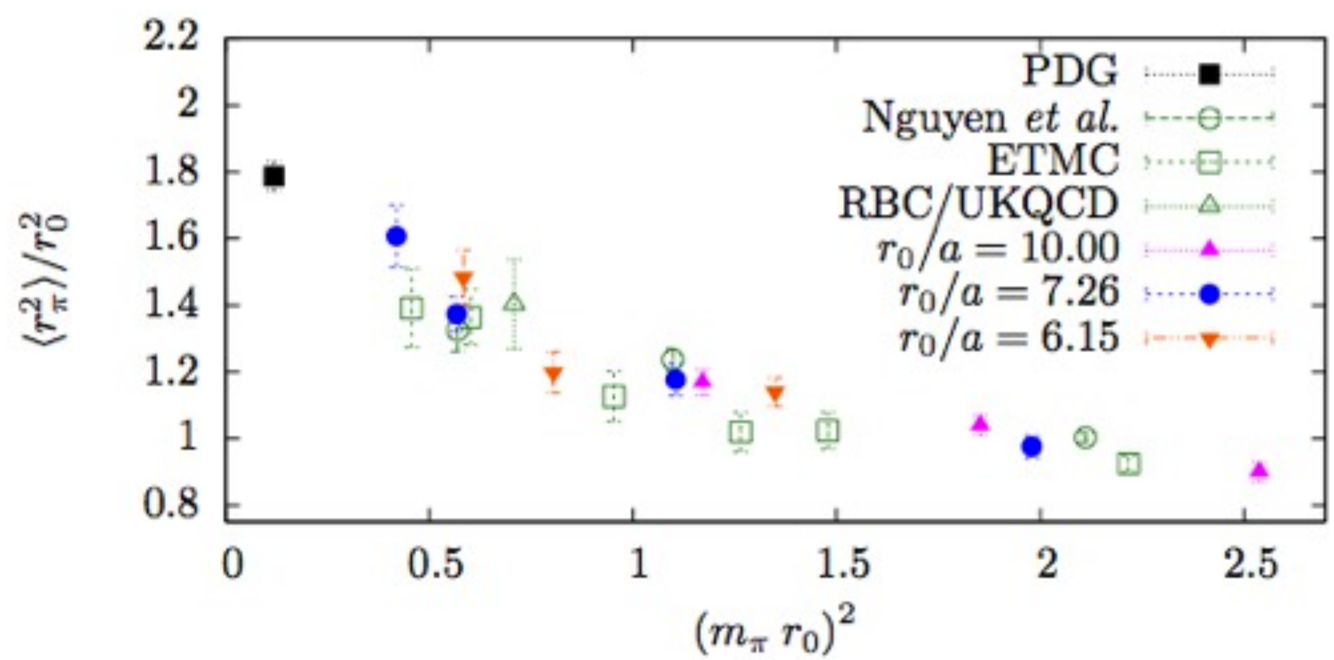
4. Summary

Introduction

[JLQCD and TWQCD Collaboration], Phys. Rev. D **80**, 034508 (2009)



[Bastian B. Brandt, et al], arXiv:1306.2916v1



- Talk by Jonna Koponen on Monday

We need simulation near the physical point, but finite V effects become large.

Introduction

Questions

Near the physical point,

Is there any way to reduce finite V effects?

and

reliably extract the pion form factor ?

Especially, in the worst case : epsilon regime?

zero-mode $\sim \mathcal{O}(1)$

Introduction

Answer

Yes.

By removing the dominant finite V effects
which come from zero-mode,
we can extract reliable pion form factor
even in the epsilon regime.

Our analysis is also useful in the p regime.

Introduction

ε regime and vacuum

	p regime	ε regime
$U(x) \in SU(N_f)$	$U(x) = \underline{1} \exp\left(\frac{i\sqrt{2}}{F}\xi(x)\right)$	$U(x) = U_0 \exp\left(\frac{i\sqrt{2}}{F}\xi(x)\right)$
vacuum	fixed	not fixed

In ε regime

vacuum = zero-mode U_0 = dynamical variable

should be treated non-perturbatively.

non-zero-mode $\xi(x)$ can be treated perturbatively.

Introduction

ε expansion

Chiral Perturbation Theory(ChPT) J. Gasser and H. Leutwyler

$$\mathcal{L}_{\mathrm{ChPT}} = \frac{F^2}{4}\mathrm{Tr}[(\partial_\mu U(x))^\dagger(\partial^\mu U(x))] - \frac{\Sigma}{2}\mathrm{Tr}[\mathcal{M}^\dagger U(x) + U(x)^\dagger \mathcal{M}] + \cdots$$

$$U(x) = U_0 \exp\left(\frac{i\sqrt{2}}{F}\xi(x)\right)$$

ε expansion

$U_0 \sim \mathcal{O}(1)$: non-perturbative

$\partial_\mu \sim \frac{1}{V^{1/4}} \sim m_\pi^{1/2} \sim m^{1/4} \sim \xi(x) \sim \mathcal{O}(\epsilon)$: perturbative

Introduction

ε expansion

$$\mathcal{L}_{\mathrm{ChPT}} = -\frac{\Sigma}{2}\mathrm{Tr}[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}]$$

zero-mode = Matrix model

$$+\frac{1}{2}\mathrm{Tr}[\partial_\mu \xi \partial^\mu \xi](x)$$

non-zero-mode = massless boson

$$+\frac{\Sigma}{2F^2}\mathrm{Tr}\left[\left(\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}\right)\xi^2\right](x) + \cdots$$

interaction between U_0 and $\xi(x)$

Introduction

ε expansion

$$\mathcal{L}_{\mathrm{ChPT}} = -\frac{\Sigma}{2}\mathrm{Tr}[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}]$$

zero-mode = Matrix model

dominant F.V.E.

$$+\frac{1}{2}\mathrm{Tr}[\partial_\mu \xi \partial^\mu \xi](x)$$

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interaction between U_0 and $\xi(x)$

Introduction

ε expansion

$$\mathcal{L}_{\mathrm{ChPT}} = -\frac{\Sigma}{2}\mathrm{Tr}[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}]$$

zero-mode = Matrix model

dominant F.V.E.

Can we remove this effects?

$$+\frac{1}{2}\mathrm{Tr}[\partial_\mu \xi \partial^\mu \xi](x)$$

non-zero-mode = massless boson

$$+\frac{\Sigma}{2F^2}\mathrm{Tr}\left[\left(\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}\right)\xi^2\right](x) + \cdots$$

interaction between U_0 and $\xi(x)$

Introduction

Two types of zero-mode's contributions

Constant
(x-independent)

Overall factor
on x-dependent part

Introduction

Two types of zero-mode's contributions

Constant (x-independent)

Overall factor on x-dependent part

We can remove these by

Non-zero momentum insertion
or
Time subtraction

Taking ratio of correlators

Introduction

This work

Our main target is the pion vector form factor.

F.V.E. in the ε regime

dominant part
from zero-mode

perturbative part
from non-zero-mode

Introduction

This work

Our main target is the pion vector form factor.

F.V.E. in the ε regime

dominant part
from zero-mode

perturbative part
from non-zero-mode

In this calculation to one loop order

can be removed

is perturbatively small

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2. How to remove zero-mode's contribution
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4. Summary

Two point function

- Momentum insertion

$$\langle P(x)P(y)\rangle = A + B\frac{1}{V}\sum_{q\neq 0}\frac{e^{iq(x-y)}}{q^2} + C\frac{1}{V}\sum_{q\neq 0}\frac{e^{iq(x-y)}}{q^4} + \cdots$$

$\sum_{q\neq 0}(\cdots)$: non-zero-mode contribution

A, B, C : zero-mode contribution

⇒ dominant F.V.E.

We want to remove these.

Two point function

- Momentum insertion

$$\int d^3x\, e^{-i\mathbf{p}\cdot\mathbf{x}} \langle P(x) P(y) \rangle = A + B \frac{1}{V} \sum_{q\neq 0} \frac{e^{iq(x-y)}}{q^2} + C \frac{1}{V} \sum_{q\neq 0} \frac{e^{iq(x-y)}}{q^4} + \cdots$$

$$\mathbf{p} \neq 0$$

Two point function

- Momentum insertion

$$\int d^3x\, e^{-i\mathbf{p}\cdot\mathbf{x}} \langle P(x)P(y)\rangle = A + B\frac{1}{V}\sum_{q\neq 0}\frac{e^{iq(x-y)}}{q^2} + C\frac{1}{V}\sum_{q\neq 0}\frac{e^{iq(x-y)}}{q^4} + \cdots$$

$\mathbf{p} \neq 0$

$$\boxed{P(x_0 : \mathbf{p}) \equiv \int d^3x e^{-i\mathbf{p}\cdot\mathbf{x}} P(x)}$$

Two point function

- Momentum insertion

$$\langle P(x_0 : \mathbf{p}) P(y_0 : \mathbf{p}') \rangle \quad = A + B \frac{1}{V} \sum_{q \neq 0} \frac{e^{iq(x-y)}}{q^2} + C \frac{1}{V} \sum_{q \neq 0} \frac{e^{iq(x-y)}}{q^4} + \cdots$$

$$\mathbf{p} \neq \mathbf{0},\ \mathbf{p}' \neq \mathbf{0}$$

$$P(x_0 : \mathbf{p}) \equiv \int d^3 x e^{-i\mathbf{p}\cdot\mathbf{x}} P(x)$$

Two point function

- Momentum insertion

$$\langle P(x_0 : \mathbf{p}) P(y_0 : \mathbf{p}') \rangle = \int d^3x \, d^3y \, e^{-i\mathbf{p}\cdot\mathbf{x}} e^{-i\mathbf{p}'\cdot\mathbf{y}} A$$

$$+B \, \frac{L^3}{T} \delta_{\mathbf{p},-\mathbf{p}'} \sum_{q^0} \frac{e^{iq^0 t}}{(q^0)^2 + \mathbf{p}^2} + \cdots$$

$\mathbf{p} \neq \mathbf{0}, \ \mathbf{p}' \neq \mathbf{0}$

$t \equiv x_0 - y_0$

$$P(x_0 : \mathbf{p}) \equiv \int d^3x e^{-i\mathbf{p}\cdot\mathbf{x}} P(x)$$

Two point function

- Momentum insertion

$$\langle P(x_0 : \mathbf{p}) P(y_0 : \mathbf{p}') \rangle = \int d^3x \, d^3y \, e^{-i\mathbf{p}\cdot\mathbf{x}} e^{-i\mathbf{p}'\cdot\mathbf{y}} A$$

$$+ B \frac{L^3}{T} \delta_{\mathbf{p},-\mathbf{p}'} \sum_{q^0} \frac{e^{iq^0 t}}{(q^0)^2 + \mathbf{p}^2} + \cdots$$

$\mathbf{p} \neq \mathbf{0}, \; \mathbf{p}' \neq \mathbf{0}$

$t \equiv x_0 - y_0$

$E(\mathbf{p}) = \sqrt{\mathbf{p}^2}$

$$= 0 + \delta_{\mathbf{p},-\mathbf{p}'} B \frac{\cosh(E(\mathbf{p})(t - T/2))}{E(\mathbf{p}) \sinh(E(\mathbf{p})T/2)} + \cdots$$

$$P(x_0 : \mathbf{p}) \equiv \int d^3x e^{-i\mathbf{p}\cdot\mathbf{x}} P(x)$$

Constant A is removed
by inserting non-zero momentum.

Two point function
- Time subtraction

Even $\mathbf{p} = \mathbf{0}$

leading zero-mode contribution can be removed

subtracting by different time-slice:

$$\Delta P(x_0 : \mathbf{0}) \equiv P(x_0 : \mathbf{0}) - P(x_0^{\mathrm{ref}} : \mathbf{0})$$

$$\langle \Delta P(x_0 : \mathbf{0}) P(y_0 : \mathbf{0}) \rangle = 0 + B \left[T h_1(t/T) - h_1(t^{\mathrm{ref}}/T) \right]$$

$$h_1(\tau) \equiv \frac{1}{2}\left(\tau - \frac{1}{2}\right)^2 - \frac{1}{24}$$

$$t = x_0 - y_0,\ t^{\mathrm{ref}} = x_0^{\mathrm{ref}} - y_0$$

Two point function
- Ratio of correlators

Constant (x-independent) can be removed

$$\langle P(x_0:\mathbf{p})P(y_0:\mathbf{p}')\rangle = B\,\frac{\cosh(E(\mathbf{p})(t-T/2))}{E(\mathbf{p})\sinh(E(\mathbf{p})T/2)} + \cdots$$

$$\langle \Delta P(x_0:\mathbf{0})P(y_0:\mathbf{0})\rangle = B\left[Th_1(t/T) - h_1(t^{\mathrm{ref}}/T)\right] + \cdots$$

Two point function

- Ratio of correlators

Constant (x-independent) can be removed

$$\langle P(x_0 : \mathbf{p}) P(y_0 : \mathbf{p}') \rangle = \boxed{B} \frac{\cosh(E(\mathbf{p})(t - T/2))}{E(\mathbf{p}) \sinh(E(\mathbf{p})T/2)} + \cdots$$

$$\langle \Delta P(x_0 : \mathbf{0}) P(y_0 : \mathbf{0}) \rangle = \boxed{B} \left[T h_1(t/T) - h_1(t^{\mathrm{ref}}/T) \right] + \cdots$$

Overall factor
on x-dependent part :
zero-mode contribution

Two point function

- Ratio of correlators

Constant (x-independent) can be removed

$$\langle P(x_0:\mathbf{p})P(y_0:\mathbf{p}')\rangle = \boxed{B}\,\frac{\cosh(E(\mathbf{p})(t-T/2))}{E(\mathbf{p})\sinh(E(\mathbf{p})T/2)} + \cdots$$

$$\langle \Delta P(x_0:\mathbf{0})P(y_0:\mathbf{0})\rangle = \boxed{B}\,[Th_1(t/T) - h_1(t^{\mathrm{ref}}/T)] + \cdots$$

Overall factor can be removed

$$\frac{\langle P(x_0:\mathbf{p})P(y_0:\mathbf{p}')\rangle}{\langle \Delta P(x_0:\mathbf{0})P(y_0:\mathbf{0})\rangle} = \frac{\left(\frac{\cosh(E(\mathbf{p})(t-T/2))}{E(\mathbf{p})\sinh(E(\mathbf{p})T/2)}\right)}{[Th_1(t/T) - Th_1(t^{\mathrm{ref}}/T)]} + \cdots$$

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✔ 2. How to remove zero-mode's contribution

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Three point function<PVP>

Three point function<PVP>

$$\langle P^{12}(x_0 : -\mathbf{p}_f) V_0^{ii}(y_0 : \mathbf{q}) P^{21}(z_0 : \mathbf{p}_i) \rangle$$

$$= 0 + \frac{L^3 \Sigma_{\mathrm{eff}}^2}{4F_{\mathrm{eff}}} \langle \mathcal{C}(U_0) \rangle_{U_0} \delta_{\mathbf{q},\mathbf{p}_f - \mathbf{p}_i} Z_k F_V(q_0, \mathbf{q})$$

$$\times [iE(\mathbf{p}_i) c(\mathbf{p}_f, t) s(\mathbf{p}_i, t') + iE(\mathbf{p}_f) s(\mathbf{p}_f, t) c(\mathbf{p}_i, t')]$$

$$+ \cdots$$

where

$$c(\mathbf{p}, t) = \frac{\cosh [E(\mathbf{p})(t - T/2)]}{2E(\mathbf{p}) \sinh [E(\mathbf{p})(t - T/2)]} \qquad s(\mathbf{p}, t) = \frac{\sinh [E(\mathbf{p})(t - T/2)]}{2E(\mathbf{p}) \sinh [E(\mathbf{p})(t - T/2)]}$$

$$E(\mathbf{p}) = \sqrt{M^2 + \mathbf{p}^2} \qquad M^2 = 2m\Sigma/F$$

Three point function<PVP>

$$\langle P^{12}(x_0 : -\mathbf{p}_f) V_0^{ii}(y_0 : \mathbf{q}) P^{21}(z_0 : \mathbf{p}_i)\rangle$$

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$$\times \left[iE(\mathbf{p}_i) c(\mathbf{p}_f, t) s(\mathbf{p}_i, t') + iE(\mathbf{p}_f) s(\mathbf{p}_f, t) c(\mathbf{p}_i, t')\right]$$

$$+ \cdots$$

Overall factor: zero-mode contribution

where

$$c(\mathbf{p}, t) = \frac{\cosh\left[E(\mathbf{p})(t - T/2)\right]}{2E(\mathbf{p}) \sinh\left[E(\mathbf{p})(t - T/2)\right]} \qquad s(\mathbf{p}, t) = \frac{\sinh\left[E(\mathbf{p})(t - T/2)\right]}{2E(\mathbf{p}) \sinh\left[E(\mathbf{p})(t - T/2)\right]}$$

$$E(\mathbf{p}) = \sqrt{M^2 + \mathbf{p}^2} \qquad M^2 = 2m\Sigma/F$$

Three point function<PVP>

zero-mode = Matrix model

$$\langle \mathcal{C}(U_0) \rangle_{U_0} = \frac{\int \mathcal{D}U_0 \, \mathcal{C}(U_0) \, e^{\frac{\Sigma_{\rm eff} V}{2} \mathrm{Tr}[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}]}}{\int \mathcal{D}U_0 \, e^{\frac{\Sigma_{\rm eff} V}{2} \mathrm{Tr}[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}]}}$$

where

$$\begin{aligned}\mathcal{C}(U_0) = 2(\delta_{i2} - \delta_{i1}) & \left(1 + [U_0]_{11}[U_0]_{22} + [U_0^\dagger]_{11}[U_0^\dagger]_{22} + [U_0]_{ii}[U_0^\dagger]_{ii}\right) \\ & +(1 - \delta_{i2}) \left([U_0]_{2i}[U_0^\dagger]_{i2} + [U_0]_{i2}[U_0^\dagger]_{2i}\right) \\ & -(1 - \delta_{i1}) \left([U_0]_{1i}[U_0^\dagger]_{i1} + [U_0]_{i1}[U_0^\dagger]_{1i}\right)\end{aligned}$$

Three point function<PVP>

$$\langle P^{12}(x_0:-\mathbf{p}_f)V_0^{ii}(y_0:\mathbf{q})P^{21}(z_0:\mathbf{p}_i)\rangle = \frac{L^3\Sigma_{\rm eff}^2}{4F_{\rm eff}}\langle\mathcal{C}(U_0)\rangle_{U_0}\delta_{\mathbf{q},\mathbf{p}_f-\mathbf{p}_i}Z_kF_V(q_0,\mathbf{q})$$
$$\times\left[iE(\mathbf{p}_i)c(\mathbf{p}_f,t)s(\mathbf{p}_i,t')+iE(\mathbf{p}_f)s(\mathbf{p}_f,t)c(\mathbf{p}_i,t')\right]$$

$$\langle \Delta P^{12}(x_0:\mathbf{0})V_0^{ii}(y_0:\mathbf{q})P^{21}(z_0:\mathbf{p}_i)\rangle = \frac{L^3\Sigma_{\rm eff}^2}{4F_{\rm eff}}\langle\mathcal{C}(U_0)\rangle_{U_0}\delta_{\mathbf{q},-\mathbf{p}_i}Z_kF_V(q_0,\mathbf{q})$$
$$\times\left[iE(\mathbf{p}_i)\Delta c(\mathbf{0},t)s(\mathbf{p}_i,t')+iE(\mathbf{0})\Delta s(\mathbf{0},t)c(\mathbf{p}_i,t')\right]$$

$$\langle \Delta P^{12}(x_0:\mathbf{0})V_0^{ii}(y_0:\mathbf{q})\Delta P^{21}(z_0:\mathbf{0})\rangle = \frac{L^3\Sigma_{\rm eff}^2}{4F_{\rm eff}}\langle\mathcal{C}(U_0)\rangle_{U_0}\delta_{\mathbf{q},\mathbf{0}}Z_kF_V(0,\mathbf{0})$$
$$\times\left[iE(\mathbf{0})\Delta c(\mathbf{0},t)\Delta s(\mathbf{0},t')+iE(\mathbf{0})\Delta s(\mathbf{0},t)\Delta c(\mathbf{0},t')\right]$$

$$\Delta f(x_0,\mathbf{0}) = f(x_0,\mathbf{0}) - f(x_0^{\rm ref},\mathbf{0})$$

Three point function<PVP>

$$\langle P^{12}(x_0:-\mathbf{p}_f)V_0^{ii}(y_0:\mathbf{q})P^{21}(z_0:\mathbf{p}_i)\rangle = \frac{L^3\Sigma_{\text{eff}}^2}{4F_{\text{eff}}}\langle\mathcal{C}(U_0)\rangle_{U_0}\delta_{\mathbf{q},\mathbf{p}_f-\mathbf{p}_i}Z_kF_V(q_0,\mathbf{q})$$
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Three point function<PVP>

$$\langle P^{12}(x_0:-\mathbf{p}_f)V_0^{ii}(y_0:\mathbf{q})P^{21}(z_0:\mathbf{p}_i)\rangle = \frac{L^3\Sigma_{\rm eff}^2}{4F_{\rm eff}}\underline{\langle\mathcal{C}(U_0)\rangle_{U_0}}\delta_{\mathbf{q},\mathbf{p}_f-\mathbf{p}_i}Z_kF_V(q_0,\mathbf{q})$$
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$$\times\left[iE(\mathbf{0})\Delta c(\mathbf{0},t)\Delta s(\mathbf{0},t')+iE(\mathbf{0})\Delta s(\mathbf{0},t)\Delta c(\mathbf{0},t')\right]$$

Dominant part of F.V.E. $\langle\mathcal{C}(U_0)\rangle_{U_0}$ can be removed by taking appropriate ratio.

Result

$$\frac{\langle P^{12}(x_0:-\mathbf{p}_f)V_0^{ii}(y_0:\mathbf{p}_f-\mathbf{p}_i)P^{21}(z_0:\mathbf{p}_i)\rangle}{\langle \Delta P^{12}(x_0:\mathbf{0})V_0^{ii}(y_0:\mathbf{0})\Delta P^{21}(z_0:\mathbf{0})\rangle}$$

$$= F_V(q_0,\mathbf{p}_f-\mathbf{p}_i)\frac{iE(\mathbf{p}_i)c(\mathbf{p}_f,t)s(\mathbf{p}_i,t')+iE(\mathbf{p}_f)s(\mathbf{p}_f,t)c(\mathbf{p}_i,t')}{iE(\mathbf{0})\Delta c(\mathbf{0},t)\Delta s(\mathbf{0},t')+iE(\mathbf{0})\Delta s(\mathbf{0},t)\Delta c(\mathbf{0},t')}$$

$$\frac{\langle \Delta P^{12}(x_0:\mathbf{0})V_0^{ii}(y_0:-\mathbf{p}_i)P^{21}(z_0:\mathbf{p}_i)\rangle}{\langle \Delta P^{12}(x_0:\mathbf{0})V_0^{ii}(y_0:\mathbf{0})\Delta P^{21}(z_0:\mathbf{0})\rangle}$$

$$= F_V(q_0,-\mathbf{p}_i)\frac{iE(\mathbf{p}_i)\Delta c(\mathbf{0},t)s(\mathbf{p}_i,t')+iE(\mathbf{0})\Delta s(\mathbf{0},t)c(\mathbf{p}_i,t')}{iE(\mathbf{0})\Delta c(\mathbf{0},t)\Delta s(\mathbf{0},t')+iE(\mathbf{0})\Delta s(\mathbf{0},t)\Delta c(\mathbf{0},t')}$$

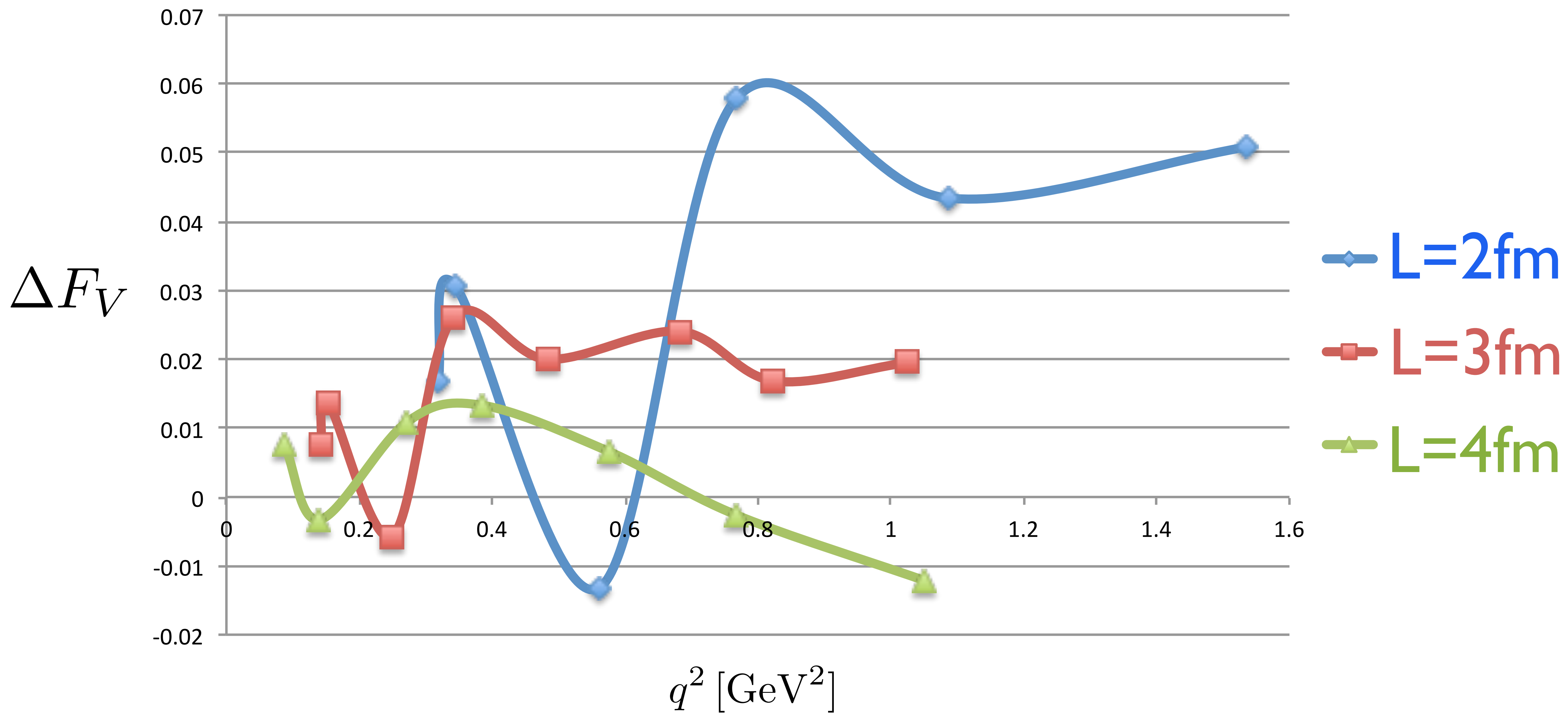
There is no more zero-mode integral.

Remaining F.V.E. in F_V is perturbative.

Result

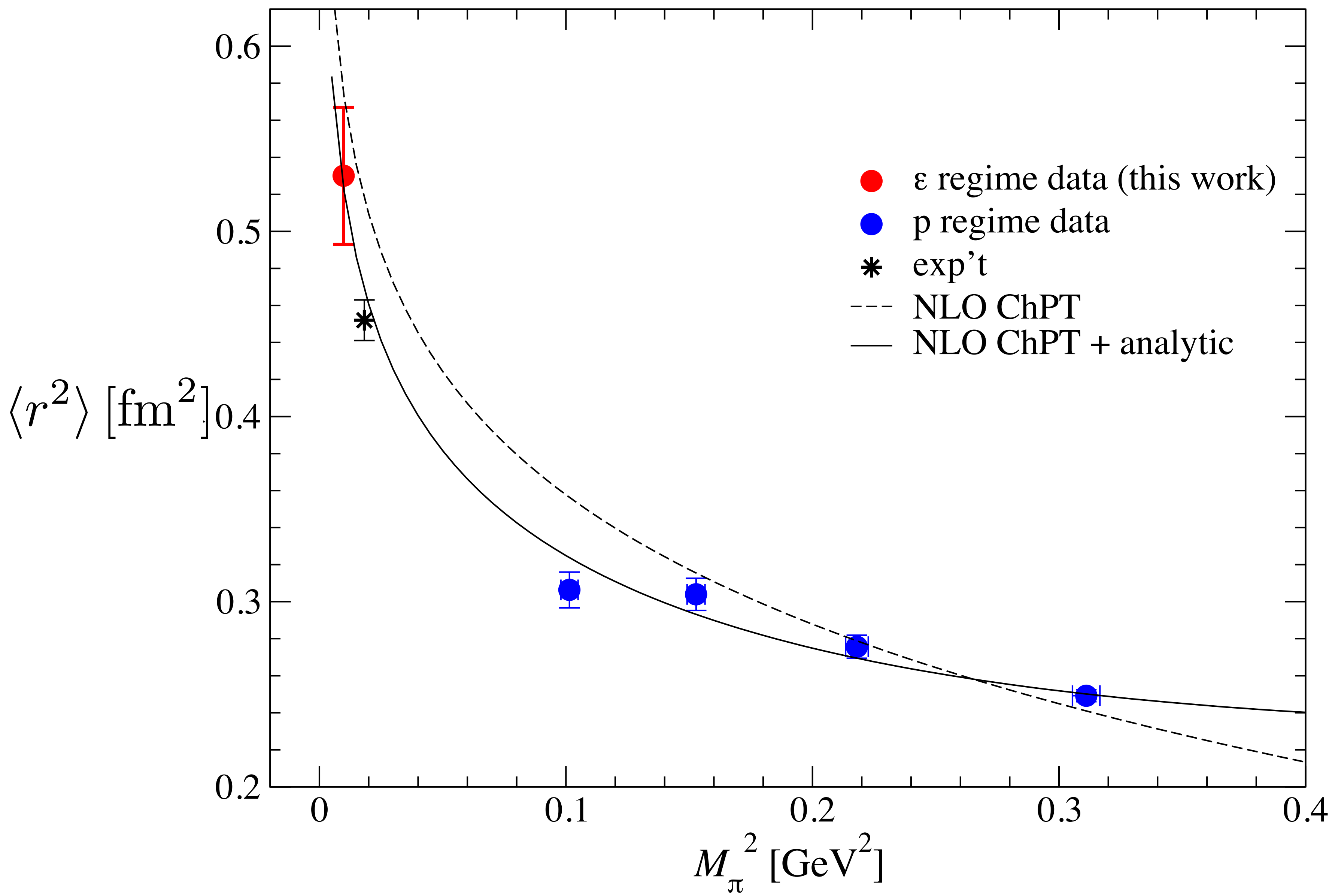
$$\Delta F_V = F_V^{\mathrm{finite}} - F_V^{\infty}$$

Remaining perturbative F.V.E.



Result

JLQCD, PoS LATTICE2012 (2012) 198 [arXiv:1211.0743]



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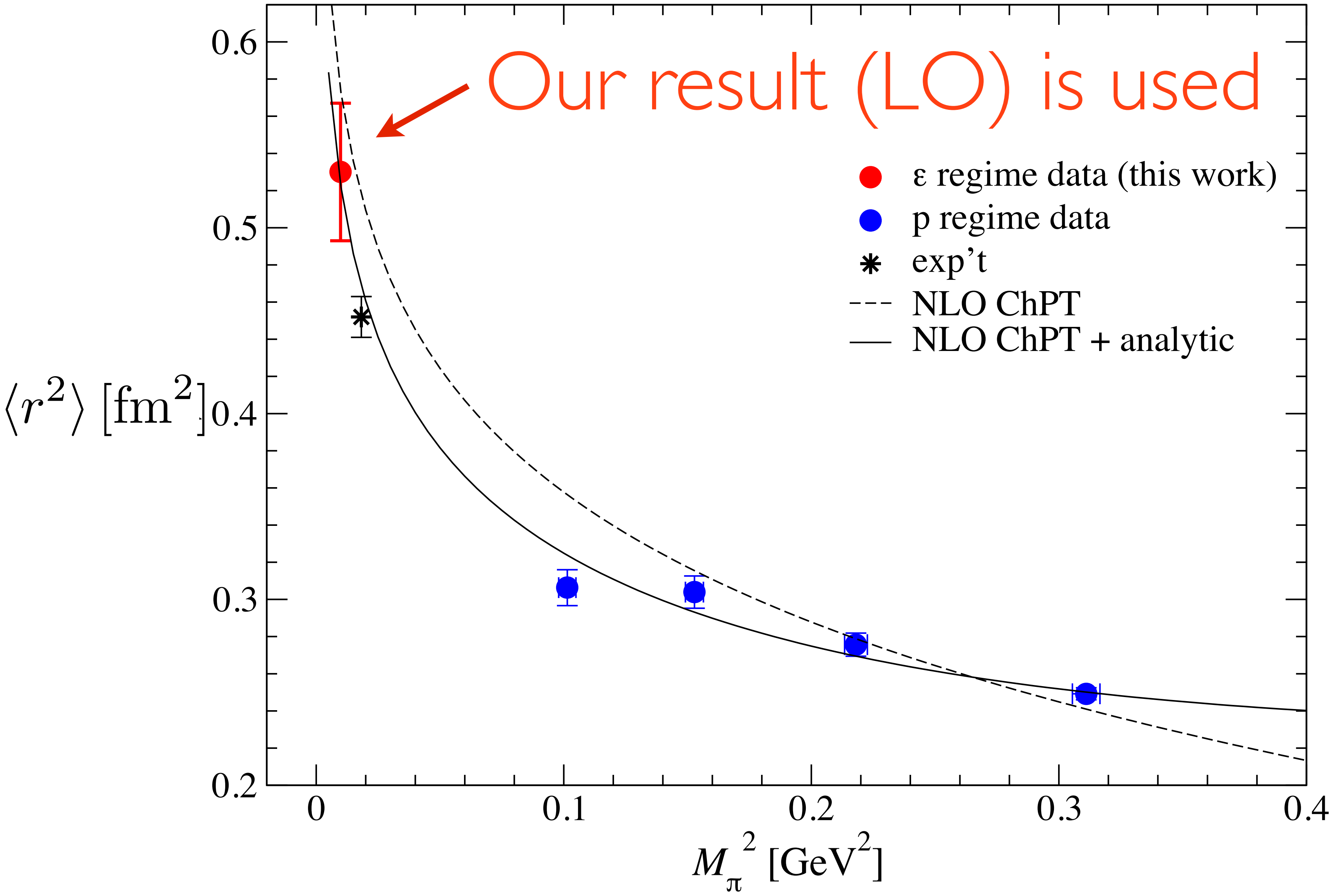

Summary

We consider F.V.E. to the pion vector form factor
in the ε regime.

ε regime

Large F.V.E.

Zero-mode should be treated non-perturbatively

Reliable pion form factor can be extracted
even in the ε regime
by inserting non-zero momentum, time subtraction,
and taking ratio:

Dominant part of F.V.E. can be removed.

Remaining F.V.E. is only perturbatively small.

This study is also useful for p regime.

Thank you for your kind attention.