Chiral Symmetry Restoration from a Boundary

Brian Tiburzi
31 July 2013

arXiv:1302.6645
Chiral Symmetry Restoration from a Boundary

• **Practical consideration**: *Dirichlet* boundary conditions in lattice QCD

• **Finite volume effect**: quantitative

• **Speculation**: modeling confinement
Symmetry Breaking and Finite Size Effects

- Spontaneous ChSB central to QCD
  - Order parameter for chiral symmetry: condensate
  \[ \langle \bar{\psi}_i R \psi_j L \rangle = -\lambda \delta_{ji} \]

- Gaussian fluctuations about vacuum configuration are pions
  \[ U = e^{i\vec{\pi} \cdot \vec{\tau} / v} = 1 + \frac{i\vec{\pi} \cdot \vec{\tau}}{v} + \ldots \]

Phenomenological theory of vacuum fluctuations (ChPT)

\[ \Delta L = -\frac{\lambda m_q}{4} \text{Tr} \left( U + U^\dagger \right) = -\lambda m_q \left( 1 - \frac{1}{2v^2} \vec{\pi} \cdot \vec{\pi} + \ldots \right) \]

- Restoration of chiral symmetry
  @ finite temperature (finite Euclidean time),
  @ finite volume
  PBCs \[ \pi(x + L) = \pi(x) \]
  IR effects use ChPT!

QM vs. QFT

\[ k_n = \frac{2\pi}{L} n \]
Dirichlet Boundary Conditions

- Frustrate spontaneous chiral symmetry breaking
  \[ \langle \bar{\psi} \psi \rangle \bigg|_{\text{boundary}} = 0 \]

**Practical Application:** addressing systematic finite-size effect in lattice QCD

**DBC**s used for

1. Chopping lattices in time
   \[ N_T \to N_T / 2 \]

2. External field calculations (not all groups)
   \[ \Delta E(\vec{E}, \vec{B}) = -\frac{1}{2} \alpha_E \vec{E}^2 - \frac{1}{2} \beta_M \vec{B}^2 \]
   \[ \text{gauge potential} \quad \vec{A} = -B y \hat{x} \]

3. Lattice QCD in rotating frames

4. Schrödinger functional **SFBC**s (inhomog**DBC**s in time)
   \[ (1 + \gamma_4)\psi(0) = 0 \]
   \[ (1 - \gamma_4)\psi(T) = 0 \]
   \[ \bar{\psi}(0)(1 - \gamma_4) = 0 \]
   \[ \bar{\psi}(T)(1 + \gamma_4) = 0 \]

Massless renormalization scheme for operators: no problem

Hadron properties: lighter quark masses without zero mode
Chiral Perturbation Theory with DBCs

Choose DBCs in one direction “x”

\( \psi(0) = \psi(L) = 0 \)

• How is spontaneous chiral symmetry breaking modified?

\[ U \in SU(2)_L \otimes SU(2)_R / SU(2)_V \]

\[ U(x) = \exp [i \bar{\pi}(x) \cdot \bar{\tau} / v] \]

\[ \bar{\pi}(x) \sim \bar{\psi}(x) \gamma_5 \bar{\tau} \psi(x) \]

\[ U(0) = U(L) = 1 \]

\[ \bar{\pi}(0) = \bar{\pi}(L) = 0 \]

• Compute chiral condensate in ChPT

\[ \langle \bar{\psi} \psi(x) \rangle = -\frac{\lambda}{4} \text{Tr} \left[ U(x) + U^\dagger(x) \right] + \ldots = -\lambda \left( 1 - \frac{1}{2v^2} \bar{\pi}(x) \cdot \bar{\pi}(x) + \ldots \right) \]

No problem when \( m_q = 0, L \neq 0 \)

\[ \langle \bar{\psi} \psi(0) \rangle = \langle \bar{\psi} \psi(L) \rangle = -\lambda \left[ 1 + 0 + \ldots \right] \]

Cannot fluctuate off coset manifold

IR effect use ChPT

\[ + \]

\[ + m^2_\pi \]

Need more d.o.f. beyond pions (~summing series to all orders)
Simplified dynamics of ChSB: Sigma Model

\[ \mathcal{L} = \frac{1}{2} \partial_\mu S \partial_\mu S + \frac{1}{2} \partial_\mu \bar{P} \cdot \partial_\mu \bar{P} - \frac{\lambda m}{v} S + \Lambda (S^2 + \bar{P}^2 - v^2)^2 \]

Vacuum expectation value \( S_0^2 + \bar{P}_0^2 = v^2 \)

\( SO(4) \rightarrow SO(3) \)

... can derive sigma model from NJL model

Polar variables \( S + i \bar{P} \cdot \vec{\tau} = \Sigma U \)

\[ \Sigma_0 = v, \quad U_0 = 1 \]

\[ \mathcal{L} = \frac{1}{4} \text{Tr} \left[ \partial_\mu \Sigma \partial_\mu \Sigma + \Sigma^2 \partial_\mu U \partial_\mu U^\dagger \right] - \frac{\lambda m}{4v} \Sigma \text{Tr} \left[ U + U^\dagger \right] + \Lambda (\Sigma^2 - v^2)^2 \]

Expand about v.e.v.'s

\[ \Sigma \rightarrow v + \Sigma, \quad U = e^{i \vec{\pi} \cdot \vec{\tau} / v} \]

\[ \int \mathcal{D}\Sigma \rightarrow \text{ChPT} \]

\[ \langle \bar{\psi} \psi(x) \rangle = -\frac{\lambda}{4v} \Sigma(x) \text{Tr} \left[ U(x) + U^\dagger(x) \right] + \ldots \]

\[ \Sigma(x) \sim \bar{\psi}(x) \psi(x), \quad \vec{\pi}(x) \sim \bar{\psi}(x) \gamma_5 \vec{\tau} \psi(x) \]
Sigma Model with DBCs

\[ \Sigma \rightarrow \Sigma_0(x) + \ldots \]

\[ S[\Sigma_0] = \int_0^L dx \left[ \frac{1}{2} \left( \frac{d\Sigma_0}{dx} \right)^2 + \Lambda (\Sigma_0^2 - v^2)^2 \right] \]

Fixed ends: \[ \Sigma_0(0) = \Sigma_0(L) = 0 \]

Classical mechanics exercise... “solvable” in terms of elliptic integrals

Turning point: \[ \max \Sigma_0(x) = v \sqrt{1 - \xi} \]

Energetics

Asymptotics \[ @ x = \frac{L}{2} \]

\[ \max \frac{\langle \psi\psi(x) \rangle}{\langle \psi\psi \rangle} = 1 - 4 e^{-\frac{1}{2}m_\sigma L} + \ldots \]
Local Chiral Condensate

\[
\langle \bar{\psi} \psi \rangle = \frac{1}{L} \int_0^L \langle \bar{\psi} \psi(x) \rangle dx
\]

Asymptotics

\[
\frac{\langle \bar{\psi} \psi \rangle}{\langle \bar{\psi} \psi \rangle} = 1 - \frac{4 \log 2}{m_\sigma L}
\]

Considerably bad for spatial DBCs hadron physics not spatially localized

Temporal DBCs less of a crime

*Model Dependent*

I don’t know what a sigma meson is
Confinement and Chiral Symmetry Breaking

• Rephrase as: can ChSB be confined to a sphere? \[ \langle \overline{\psi} \psi(R) \rangle = 0 \]

Enforcing “bag-model” confinement \[ \text{[Chodos, Jaffe, Johnson, Thorne, Weisskopf]} \]

• In sigma model answer appears to be NO... for any \( R \! \! \! \)!

\[ S[\Sigma_0] = \frac{1}{\frac{4}{3} \pi R^3} \int_V \left[ \frac{1}{2} \nabla \Sigma_0 \cdot \nabla \Sigma_0 + \Lambda (\Sigma_0^2 - v^2)^2 \right] \]

spherical symmetry

\[ t \sim r, \quad x \sim r \Sigma_0 \]

\[ \ddot{x} = \frac{x}{2} \left( \frac{x^2}{t^2} - 1 \right) \]

Numerically appears only trivial solution

Analytical proof...

• Model points to impossibility of confining the condensate

• Reimagine: spherical droplet of symmetric matter?

Only oscillatory solution for condensate

Quark mass effects? Not likely...
Chiral Symmetry Restoration from a Boundary

- DBCs can alter the phase of QCD
- Power-law volume corrections make one worry about lattice QCD with DBCs
  - spatial, be skeptical
  - temporal, be careful
- IR effect of DBCs cannot be described in low-energy effective theory
  - \( \Sigma \) degree of freedom
  - more realistic model, lattice calculation
- Bag confinement and ChSB appear to be incompatible
  - dependence on dimensions, boundary geometry