Chiral Symmetry Restoration from a Boundary

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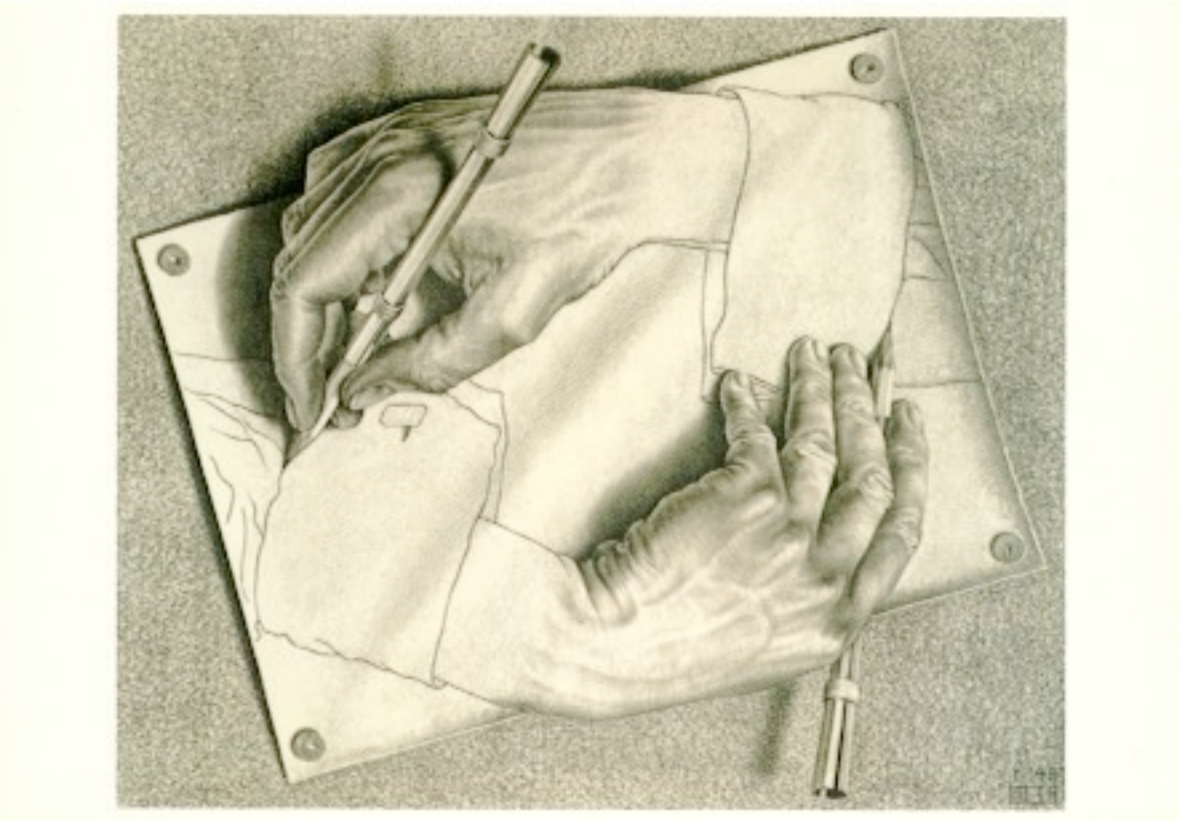
Chiral Symmetry Restoration from a Boundary

- **Practical consideration**: *Dirichlet* boundary conditions in lattice QCD
- **Finite volume effect**: quantitative
- **Speculation**: modeling confinement



Reminder:

Symmetry Breaking and Finite Size Effects

$$N_f = 2 \qquad m_u, m_d \ll \Lambda_{\mathrm{QCD}} \qquad \mathcal{L}_\psi = \overline{\psi}_L \not{D}_L \psi_L + \overline{\psi}_R \not{D}_R \psi_R$$

- Spontaneous ChSB central to QCD $\quad SU(2)_L \otimes SU(2)_R \longrightarrow SU(2)_V$

  Order parameter for chiral symmetry: condensate $\quad \langle \overline{\psi}_{iR} \psi_{jL} \rangle = -\lambda\, \delta_{ji}$

- Gaußian fluctuations about vacuum configuration are pions $\quad \delta_{ji} \to U_{ji}(x)$

$$U = e^{i\vec{\pi}\cdot\vec{\tau}/v} = 1 + \frac{i\vec{\pi}\cdot\vec{\tau}}{v} + \ldots$$

Phenomenological theory of vacuum fluctuations (ChPT) [Weinberg, Gasser & Leutwyler]

$$\Delta\mathcal{L} = -\frac{\lambda m_q}{4} \mathrm{Tr}\left(U + U^\dagger\right) = -\lambda m_q \left(1 - \frac{1}{2v^2} \vec{\pi}\cdot\vec{\pi} + \ldots\right)$$

Vacuum energy $\quad \langle \overline{\psi}\psi \rangle = -\lambda$

Pion mass $\quad m_\pi^2 = \lambda m_q \,/\, v^2$

- Restoration of chiral symmetry
  @ finite temperature (finite Euclidean time),
  @ finite volume
  IR effects use ChPT!

PBCs $\quad \pi(x+L) = \pi(x)$

[Gasser & Leutwyler]

QM vs. **QFT**

$$k_n = \frac{2\pi}{L} n$$

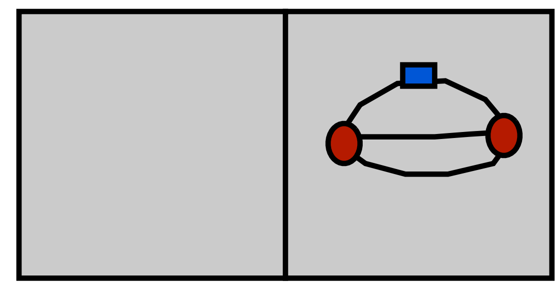
Dirichlet Boundary Conditions

- Frustrate spontaneous chiral symmetry breaking $\langle \overline{\psi}\psi \rangle \Big|_{\text{boundary}} = 0$

Practical Application*: addressing systematic finite-size effect in lattice *QCD

DBCs used for 1). Chopping lattices in time

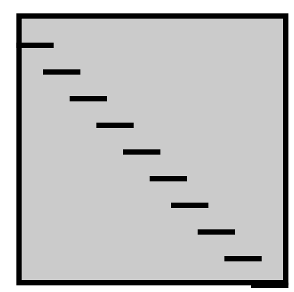
$$N_T \to N_T/2$$



2). External field calculations (not all groups)

$$\Delta E(\vec{E},\vec{B}) = -\frac{1}{2}\alpha_E \vec{E}^2 - \frac{1}{2}\beta_M \vec{B}^2$$

gauge potential $\vec{A} = -By\,\hat{x}$



spatial gradient of field

$$\vec{B} = \hat{z}B\,[1 - L\,\delta(y-L)]$$

3). Lattice QCD in rotating frames

4). Schrödinger functional **SFBC**s (inhomog**DBC**s in time)

$$(1+\gamma_4)\psi(0) = 0$$
$$(1-\gamma_4)\psi(T) = 0$$

Massless renormalization scheme for operators: no problem

Hadron properties: lighter quark masses without zero mode

$$\overline{\psi}(0)(1-\gamma_4) = 0$$
$$\overline{\psi}(T)(1+\gamma_4) = 0$$

Chiral Perturbation Theory with **DBC**s

Choose **DBC**s in *one* direction "x" $\psi(0) = \psi(L) = 0$

- How is spontaneous chiral symmetry breaking modified? ***IR*** effect use *ChPT*

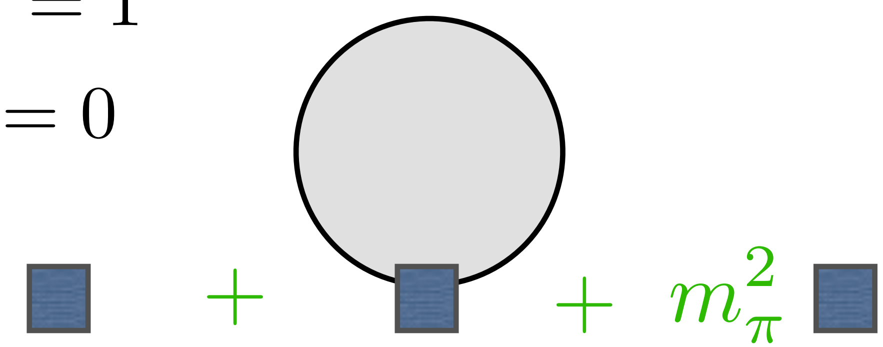
  $U \in SU(2)_L \otimes SU(2)_R / SU(2)_V$

$U(x) = \exp\left[i\vec{\pi}(x)\cdot\vec{\tau}/v\right]$

$\vec{\pi}(x) \sim \overline{\psi}(x)\gamma_5\vec{\tau}\,\psi(x)$ → $U(0) = U(L) = 1$

$\vec{\pi}(0) = \vec{\pi}(L) = 0$

- Compute chiral condensate in *ChPT*



$$\langle\overline{\psi}\psi(x)\rangle = -\frac{\lambda}{4}\mathrm{Tr}\left[U(x) + U^\dagger(x)\right] + \ldots \; = -\lambda\left(1 - \frac{1}{2v^2}\vec{\pi}(x)\cdot\vec{\pi}(x) + \ldots\right)$$

No problem when $m_q = 0, L \neq 0$

$$\frac{1}{(vL)^2}\sum_{n=1}^{\infty}\frac{\sin^2\left(\frac{n\pi x}{L}\right)}{(n\pi)^2 + (m_\pi L)^2}$$

$\langle\overline{\psi}\psi(0)\rangle = \langle\overline{\psi}\psi(L)\rangle = -\lambda\left[1 + 0 + \ldots\right]$

Cannot fluctuate off coset manifold

Need more d.o.f. beyond pions

(~summing series to all orders)

Simplified dynamics of *ChSB*: Sigma Model

$$\mathcal{L} = \frac{1}{2}\partial_\mu S\,\partial_\mu S + \frac{1}{2}\partial_\mu \vec{P}\cdot\partial_\mu \vec{P} - \frac{\lambda m}{v} S + \Lambda(S^2 + \vec{P}^2 - v^2)^2$$

Vacuum expectation value $S_0^2 + \vec{P}_0{}^2 = v^2$ $\quad SO(4) \to SO(3)$

... can derive sigma model from NJL model

Polar variables $S + i\vec{P}\cdot\vec{\tau} = \Sigma U$ $\quad SU(2)\otimes SU(2) \to SU(2)$

$$\Sigma_0 = v,\; U_0 = 1$$

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left[\partial_\mu\Sigma\partial_\mu\Sigma + \Sigma^2\partial_\mu U\partial_\mu U^\dagger\right] - \frac{\lambda m}{4v}\Sigma\,\mathrm{Tr}\left[U + U^\dagger\right] + \Lambda(\Sigma^2 - v^2)^2$$

Expand about v.e.v.'s

$$\Sigma \to v + \Sigma,\; U = e^{i\vec{\pi}\cdot\vec{\tau}/v}$$

$$\langle\overline{\psi}\psi(x)\rangle = -\frac{\lambda}{4v}\Sigma(x)\mathrm{Tr}\left[U(x) + U^\dagger(x)\right] + \ldots$$

$\int \mathcal{D}\Sigma \longrightarrow$ *ChPT*

$$\Sigma(x) \sim \overline{\psi}(x)\psi(x), \quad \vec{\pi}(x) \sim \overline{\psi}(x)\gamma_5\vec{\tau}\,\psi(x)$$

Sigma Model with **DBC**s

$\Sigma \to \Sigma_0(x) + \ldots$

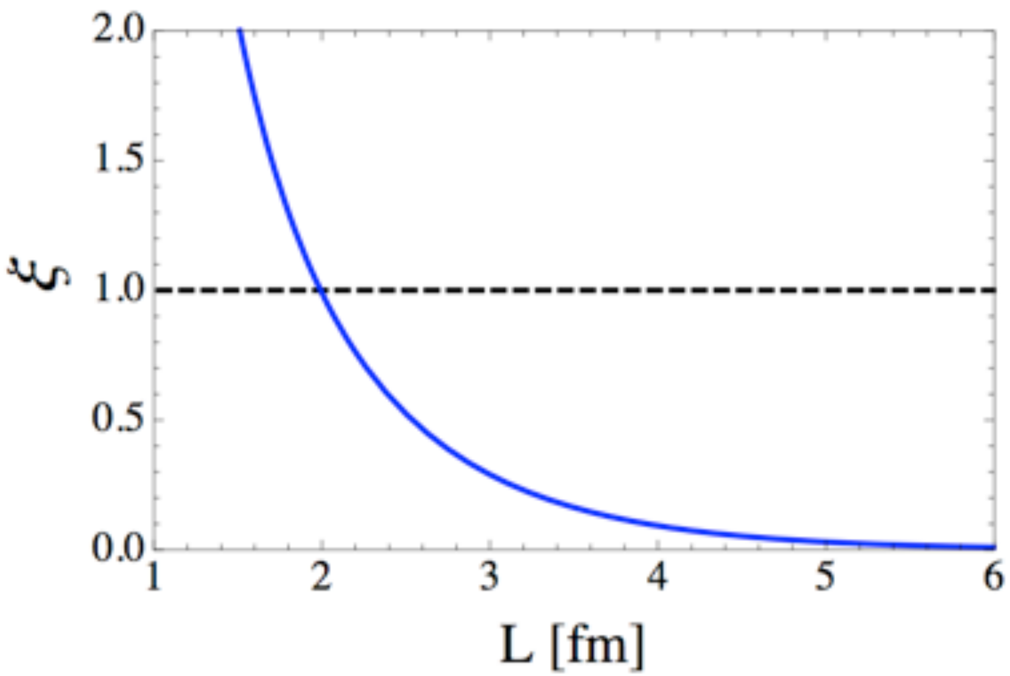
$$S[\Sigma_0] = \int_0^L dx \left[\frac{1}{2} \left(\frac{d\Sigma_0}{dx} \right)^2 + \Lambda(\Sigma_0^2 - v^2)^2 \right]$$

Fixed ends: $\Sigma_0(0) = \Sigma_0(L) = 0$

Classical mechanics exercise... “solvable” in terms of elliptic integrals

Turning point: $\max \Sigma_0(x) = v\sqrt{1-\xi}$

Energetics

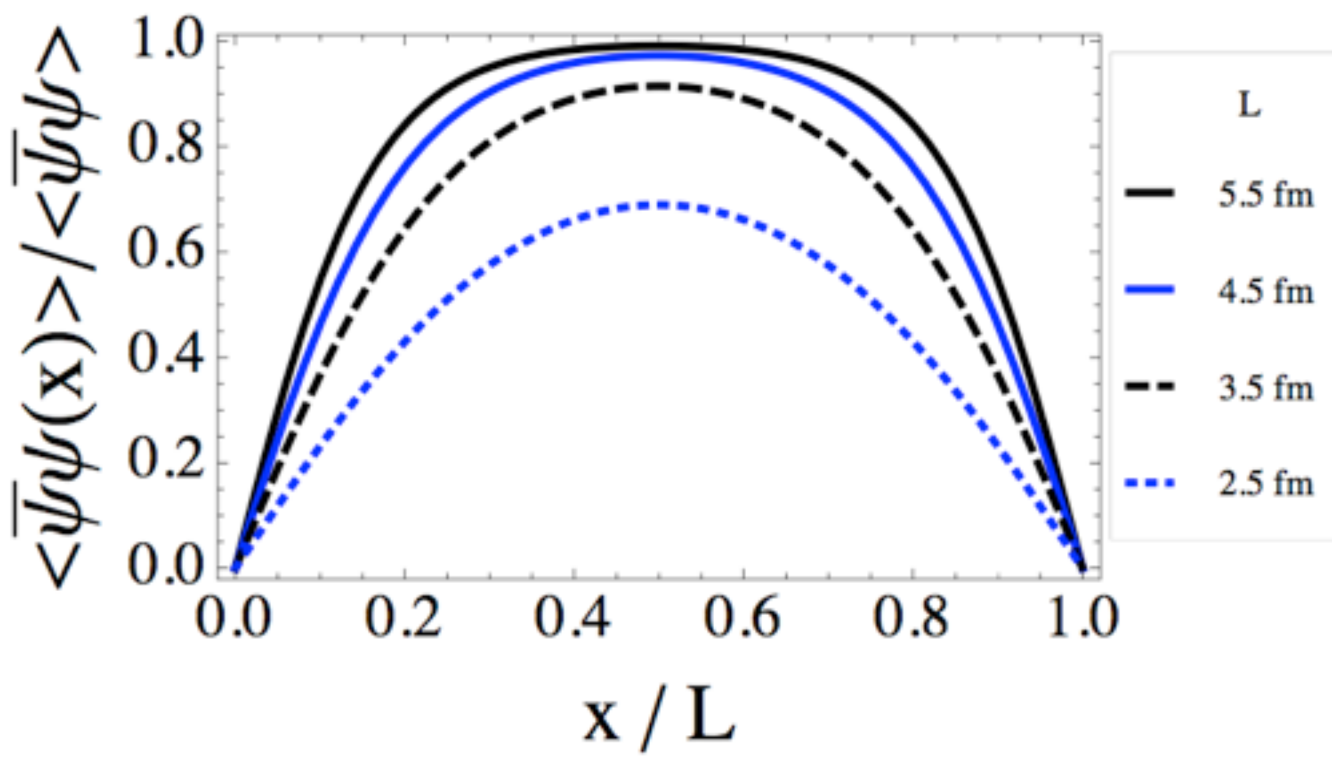


Asymptotics

$@ \; x = \frac{L}{2}$

$$\max \frac{\langle \overline{\psi}\psi(x) \rangle}{\langle \overline{\psi}\psi \rangle} = 1 - 4\, e^{-\frac{1}{2} m_\sigma L} + \ldots$$

Local Chiral Condensate



$$\overline{\langle\overline{\psi}\psi\rangle} = \frac{1}{L}\int_0^L \langle\overline{\psi}\psi(x)\rangle dx$$

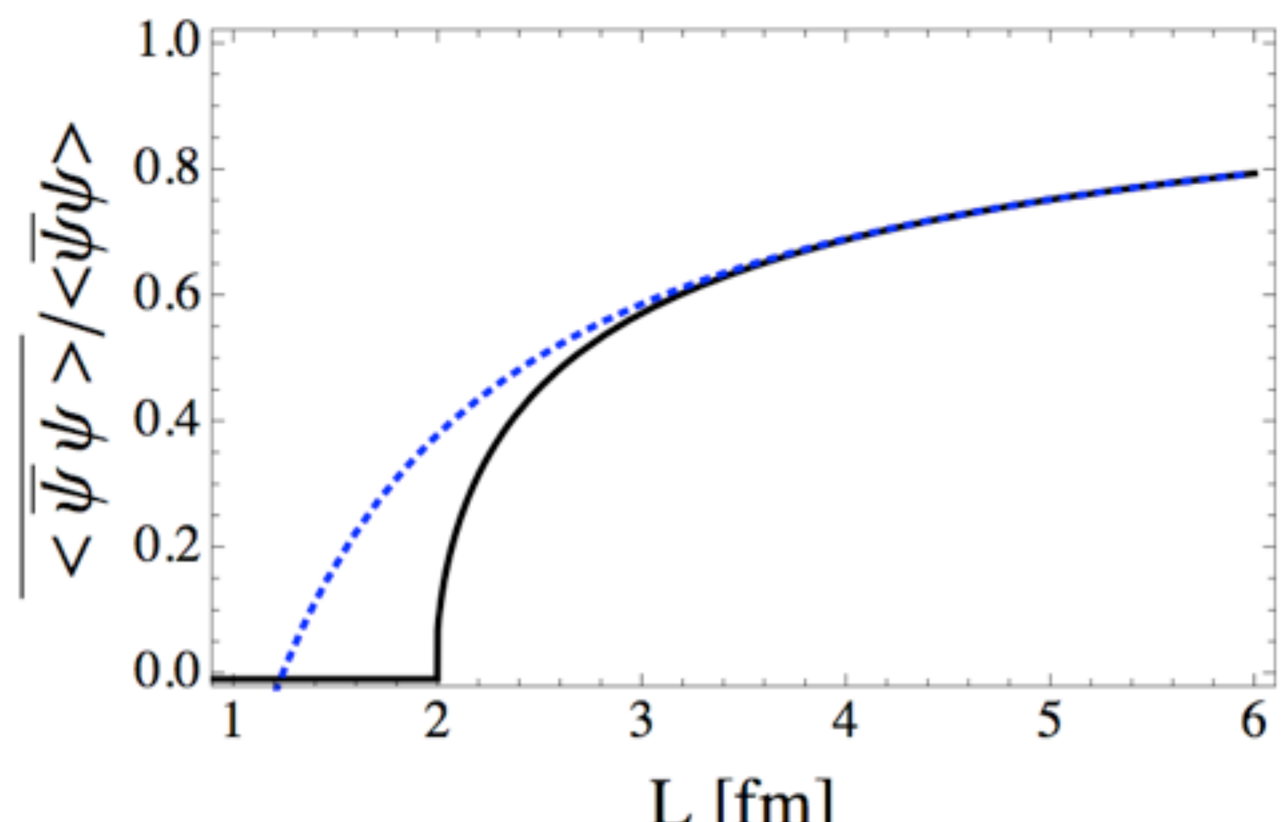
Asymptotics

$$\overline{\langle\overline{\psi}\psi\rangle}/\langle\overline{\psi}\psi\rangle = 1 - \frac{4\log 2}{m_\sigma L}$$

Considerably bad for spatial **DBCs**
hadron physics not spatially localized

Temporal **DBCs** less of a crime

Model Dependent
I don't know what a sigma meson is



Confinement and Chiral Symmetry Breaking

- Rephrase as: can *ChSB* be confined to a sphere? $\langle\overline{\psi}\psi(R)\rangle = 0$

  Enforcing "bag-model" confinement [Chodos, Jaffe, Johnson, Thorne, Weisskopf]

- In sigma model answer appears to be *NO*... for *any R*!

$$S[\Sigma_0] = \frac{1}{\frac{4}{3}\pi R^3}\int_V \left[\frac{1}{2}\vec{\nabla}\Sigma_0 \cdot \vec{\nabla}\Sigma_0 + \Lambda(\Sigma_0^2 - v^2)^2\right]$$

$\langle\overline{\psi}\psi\rangle = ?$

spherical symmetry

$t \sim r, \quad x \sim r\Sigma_0$

$$\ddot{x} = \frac{x}{2}\left(\frac{x^2}{t^2} - 1\right)$$

Numerically appears only trivial solution

Analytical proof...

- Model points to impossibility of confining the condensate

- Reimagine: spherical droplet of *symmetric* matter?

$\langle\overline{\psi}\psi\rangle = 0$ $\langle\overline{\psi}\psi\rangle = ?$

Only oscillatory solution for condensate

Quark mass effects? Not likely...

Chiral Symmetry Restoration from a Boundary

- **DBC**s can alter the phase of *QCD*
- Power-law volume corrections make one worry about lattice *QCD* with **DBC**s

spatial, be skeptical

temporal, be careful

- *IR* effect of **DBC**s cannot be described in *low-energy* effective theory

Σ degree of freedom

more realistic model, lattice calculation

- Bag confinement and *ChSB* appear to be incompatible

dependence on dimensions, boundary geometry