Interaction of static charges in graphene within Monte-Carlo simulation

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Details of the model

$$H_{tb} = -\chi \sum_{\sigma=\uparrow,\downarrow} \sum_{<XY>} (a^+_{\sigma X} a_{\sigma Y} + a^+_{\sigma Y} a_{\sigma X})$$

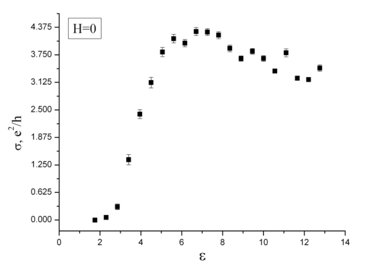
$$Z = \int D\psi D\bar{\psi} DA_0 \exp\left(-\frac{1}{2} \int d^4x (\partial_i A_0)^2 - \int d^3x \bar{\psi}_f (\gamma_0(\partial_0 - igA_0) - \gamma_i \partial_i) \psi_f \right)$$

- Instantaneous, coulomb interaction
- Effective charge $g^2 = \frac{4\pi\alpha}{v} \frac{2}{\epsilon_0 + 1}$
  (ϵ_0 substrate dielectric permitivity)
- Strongly interacting theory! ($v \sim \frac{1}{300}$, $\frac{g^2}{4\pi}|_{\epsilon_0=1} \sim 2$)

Details of Monte-Carlo simulation (Buividovich et. al., Phys.Rev. B86 (2012) 045107)

- Noncompact $(3+1)$-dimensional Abelian lattice gauge field
- $(2+1)$-dimensional staggered lattice fermions

Insulator-semiconductor phase transion



Weak coupling region ($\epsilon_0 > 5$)

- Interaction between fermions is weak! $\frac{g^2}{4\pi} < 0.7$
- Particles spectrum: massless fermions

Strong coupling region ($\epsilon_0 < 3$)

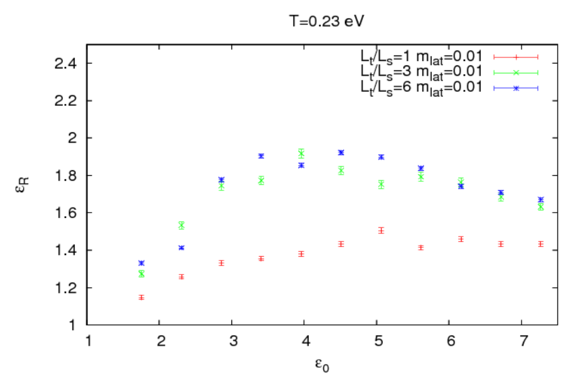
- Interaction between fermions is strong! $\frac{g^2}{4\pi} > 1$
- Particles spectrum: "π meson " ($m_\pi = 0$, $Q_\pi = 0$), heavy fermions (Y. Araki, Annals Phys. 326 (2011) 1408-1424)

Measurement of the interaction potential between static charges

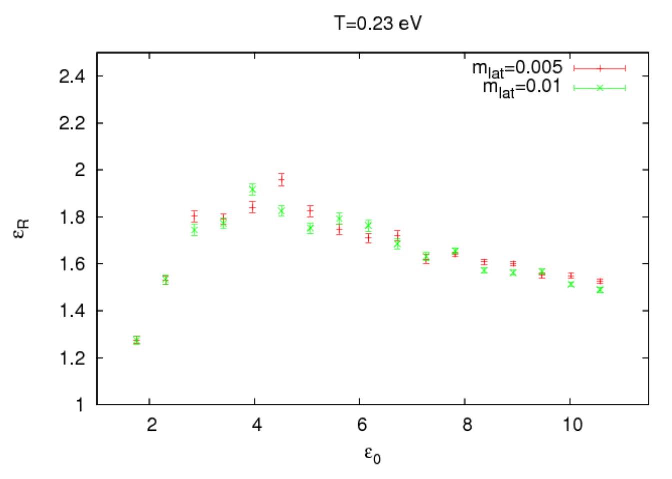
- $\langle P(0)P^{+}(\vec{r})\rangle \sim exp(-\beta \cdot V(\vec{r})), \quad \beta = \frac{1}{T}$
- Fitting procedure: $V(\vec{r}) = \frac{1}{\epsilon_R} V_c(\vec{r}) + C$
- $V_c(\vec{r}) = \frac{\pi}{L_s^3 a} \frac{2\alpha}{\epsilon_0+1} \sum_{\vec{p}} \frac{1}{\sum_{\mu} sin^2(p_{\mu} a/2)} e^{i(\vec{p}\cdot\vec{r})}$
  $\lim_{a\to 0} V_c(r) \to \frac{2\alpha}{\epsilon_0+1} \frac{1}{r}$
- $\epsilon_R = \epsilon_R(\epsilon_0, T)$

To exclude the effects of compact QED one should take the limit $\delta t \to 0$, $T = \frac{1}{\delta t \cdot L_t} = const$

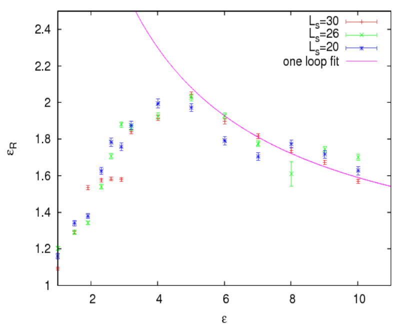
The limit $\delta t \to 0$



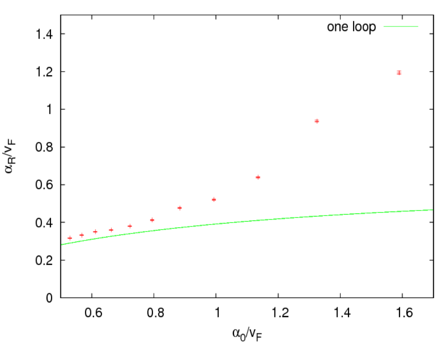
The limit of massless fermions $m_f \to 0$



Volume dependence



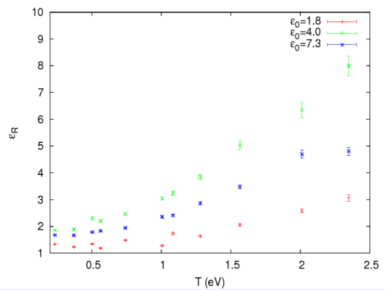
Renormalized coupling α_R at temperature $T = 0.23$ eV



- $\frac{\alpha_R}{\alpha_0} = \frac{1}{1 - D_{00}\Pi_{00}}$
- At one loop $\frac{\alpha_R}{\alpha_0} = \frac{1}{1 + \frac{\pi}{2}\frac{\alpha_0}{v_F}}$

Good agreement with one loop results!

The temperature dependence: $\epsilon_0 = const, \epsilon_R = \epsilon_R(T)$



- The larger the temperature the worse fit by the Coulomb potential
- The most strong temperature dependence of the ϵ_R is in the region of phase transition

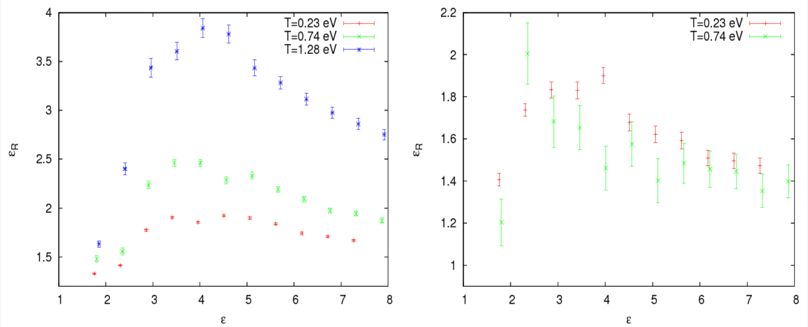
Debye screening:

- $n_\pm(r) = n \exp\left(\mp \frac{e\varphi(r)}{T}\right)\delta(z)$, $n_\pm(r)$ three-dimensional density, n two-dimensional density
- Charge density $\rho(r) = Q\delta^3(\vec r) + e(n_+ - n_-) \simeq e\delta^3(r) - 2e^2 n \frac{\varphi}{T}\delta(z)$
- Maxwell equation $\Delta\varphi(r) = -4\pi\rho, \quad -\Delta\varphi(r) + \frac{8\pi n e^2}{T}\varphi(r)\delta(z) = 4\pi Q\delta^3(\vec r)$
- Solution

$$\varphi(r) = \frac{Q}{r}\int_0^\infty d\xi \frac{e^{-(m_D r)\xi}}{(1+\xi^2)^{3/2}}\xi, \quad \frac{m_D}{T} = \frac{4\pi e^2 n}{T} = \frac{2\pi^2}{3}\frac{e^2}{v_F^2},$$

$$\varphi(r) = \begin{cases} \frac{Q}{r}, & (rm_D) \ll 1 \\ \frac{Q}{r}\frac{1}{(m_D r)^2}, & (rm_D) \gg 1. \end{cases}$$

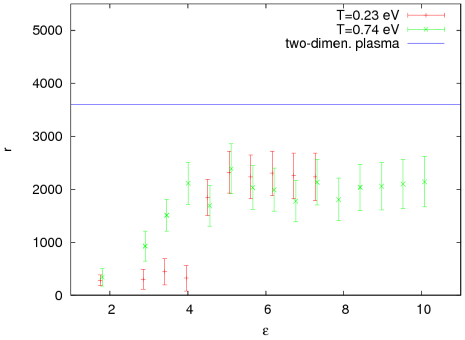
Temperature dependence of the ϵ_R



The temperature dependence is extracted from ϵ_R and put to m_D

Debye mass

- $m_D \sim \alpha_R \frac{n}{T}$ (The usage of the α_R instead of α_0 is important)
- Ratio $r = \frac{m_D e^2}{T \alpha_R} \sim \frac{n}{T^2}$
- For weakly interacting two-dimensional plasma $r = \frac{2\pi^2}{3} \frac{e^2}{v_F^2}$



- Sharp insulator-semimetal phase transition at $\epsilon_0 \sim 4$ and low T
- In the weak coupling regime ($\epsilon > 4$) $n \neq 0$, $M_f = 0$, excitations form two-dimensional plasma
- The difference in r can be attributed to renormalization of the v_F
- In the strong coupling regime ($\epsilon < 4$) $n \sim 0$, the interaction potential is Coulomb, $M_f \neq 0$
- At larger temperatures the transition becomes milder

Conclusion

- The dielectric permitivity of graphene has been calculated in the region $\epsilon_0 \in (1, 10), \quad T \in (0.2, 1.5)$ eV
- The dielectric permitivity in the week coupling region can be decribed by one loop result
- The interaction potential of static charges in graphene is Debye screening potential
- In the weak coupling region the excitations form two-dimentional plasma
- Debye screening radius $\sim 1/m_D$ is small

  $(1/(m_D a) \sim 20$ at room temperature)

THANK YOU