Quantum Critical Behavior with Massless Staggered Fermions in Three Dimensions

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Based on work in collaboration with
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Motivation

- Relativistic 3d four-fermion models interesting.
  - ✳ Long history.
  - ✳ Revival due to “graphene”.
- Review of “Lattice Results” reveal some puzzles.
  - ⋆ misleading success story?
  - ⋆ too many fixed points for a given lattice symmetry?
- Fermion bag approach offers new opportunities to resolve puzzles.
  - ∗ some sign problems solvable!
  - ∗ “efficient” algorithms in the chiral limit.
- Surprise : fermion mass generation without condensate?

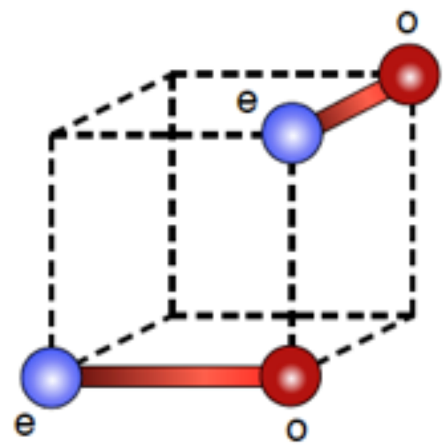
Staggered Fermion Models

$$S(\overline{\chi},\chi) = \sum_{x,y} \overline{\chi}_x \; D_{xy} \; \chi_y - \sum_{\langle xy \rangle} U_{\langle xy \rangle} \; \overline{\chi}_x \chi_x \; \overline{\chi}_y \chi_y$$

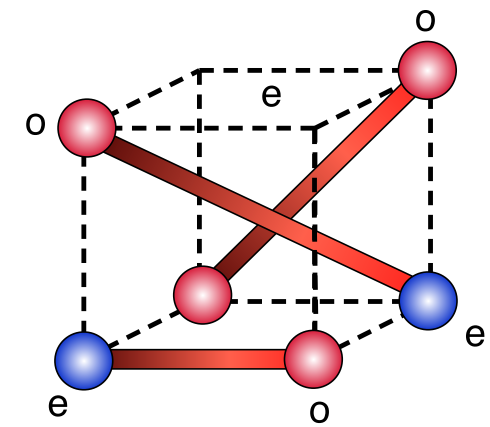
N=1 Symmetry

SU(2) x U(1)

flavor chiral



Thirring



Gross-Neveu

N=1 Symmetry

SU(2) x U(1)

or

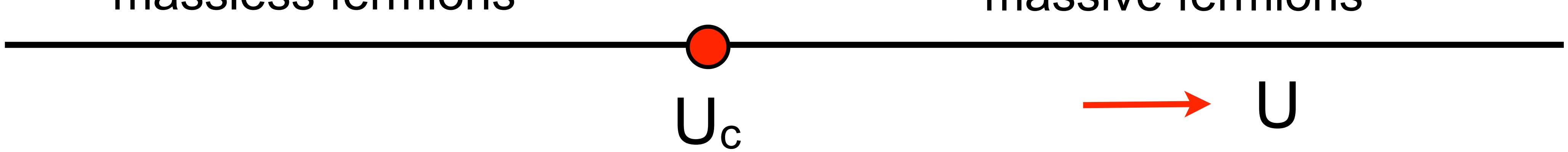
SU(2) x Z_2

no sign problems

suffer from sign problems

chirally symmetric
massless fermions

spontaneous breaking of chiral symmetry
massive fermions



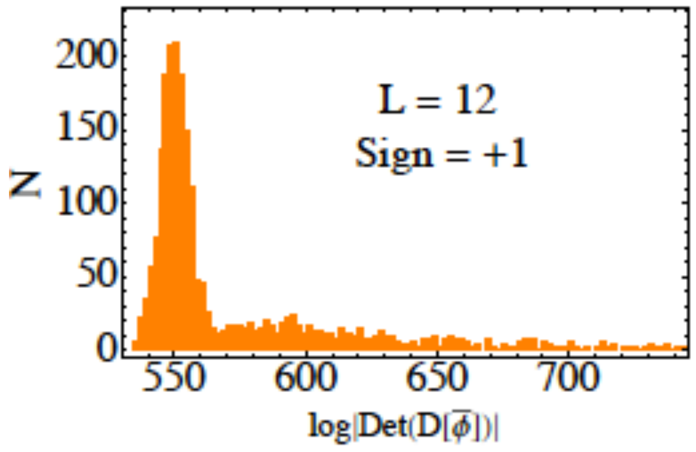
Origin of sign problem

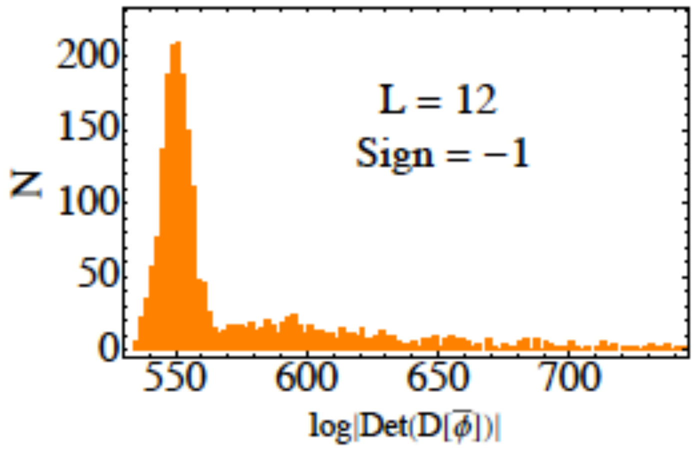
The "Hubbard-Stratanovich" field introduces a fluctuating mass term

$$D[\overline{\phi}] = \begin{pmatrix} \sigma & D \\ -D^\dagger & \varphi \end{pmatrix}$$

This can lead to negative determinants

N=1 Sign Problem
in the
SU(2) x Z_2
model





Previous Work

Model	Symmetry	Work	ν	η
N=1 Lattice-GN	SU(2) x Z_2	Karkkainen,et.al. (1994)	1.00(4)	0.756(8)
N=2 Lattice-GN	$[SU(2)]^2$ x Z_2	Christofi/Strouthos (2007)	0.99(2)	0.84(4)
N=2 Lattice-GN	$[SU(2)]^2$ x U(1)	Christofi/Strouthos (2007)	1.03(4)	0.91(4)
N = 1 Lattice-Th	SU(2)x U(1)	Debbio, et.al., (1997)	0.80(15)	0.70(15)
N = 1 Lattice-Th	SU(2)x U(1)	Barbour et. al., (1998)	0.80(20)	0.4(2)
N_f = 2 Cont-GN	U(4) x Z2	Hofling et.al., (2002)	1.017	0.754
N_f = 2 Cont-Th	U(4)	Janssen,Gies (2012)	2.4	1.4

Success story 1(?)

The staggered GN model with
SU(2) x Z_2 symmetry

Karkkainen, Lacaze and Lacock, NPB 415, 781 (1994)

The continuum U(4) x Z_2 model

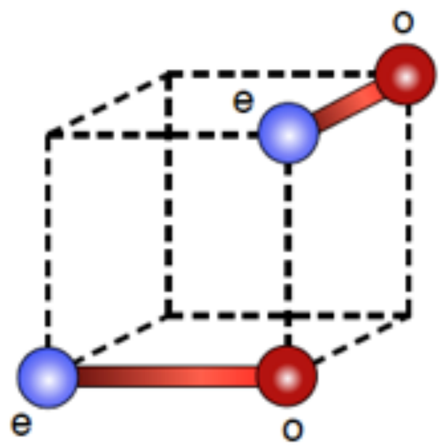
Hofling, Novac and Wetterich, PRB 66, 205111 (2002)

critical exponents in both models seem to match!

Sign problem not important??

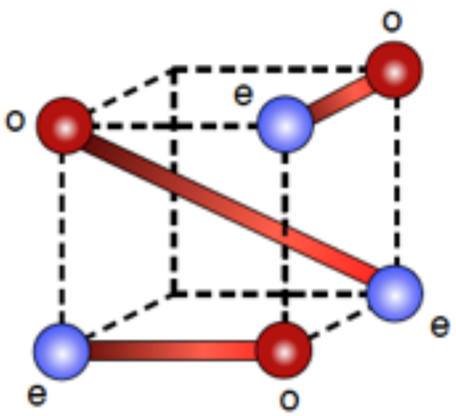
Success story 2(?)

GN and Thirring models are different



Thirring

no sign problems



Gross-Neveu

suffers from sign problems

Fermion Bag Result
SC & A.Li
(2012)

$\nu \approx 0.85(1), \;\; \eta \approx 0.65(1)$

Both models have SU(2) x U(1) symmetry??

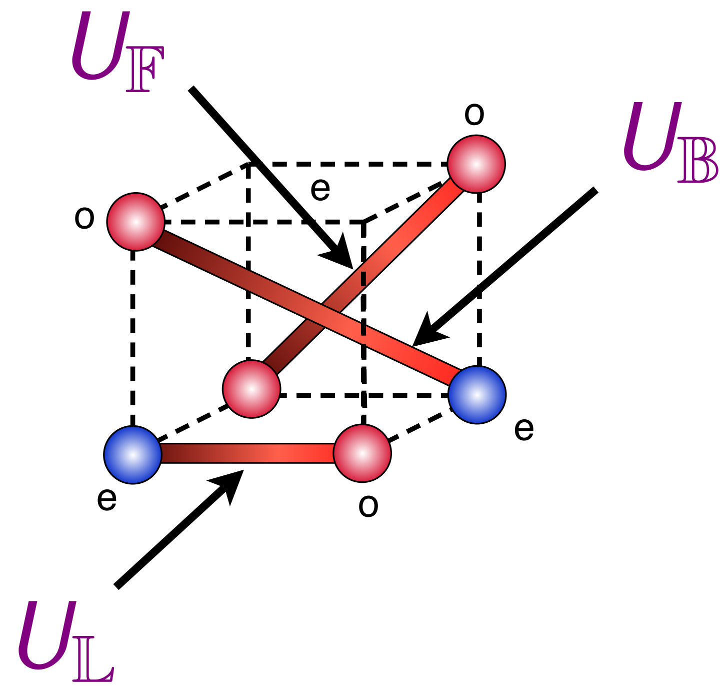
Lattice Gross-Neveu Models

SC & A.Li PRD (Rapid Comm) (2013)

$$S(\overline{\chi},\chi) = \sum_{x,y} \overline{\chi}_x\ D_{xy}\ \chi_y - \sum_{\langle xy\rangle} U_{\langle xy\rangle}\ \overline{\chi}_x\chi_x\ \overline{\chi}_y\chi_y$$

SU(2) x Z_2 Model

$$U_{\mathbb{L}} = 2U_{\mathbb{F}} = 4U_{\mathbb{B}} = U$$



SU(2) x U(1) Model

$$U_{\mathbb{L}} = 4U_{\mathbb{B}} = U, \quad U_{\mathbb{F}} = 0$$

suffer from sign problems in conventional formulations

no sign problems in the fermion bag approach

Results

We expect

$$\langle \overline{\chi}\chi \rangle = 0, \quad U < U_c$$

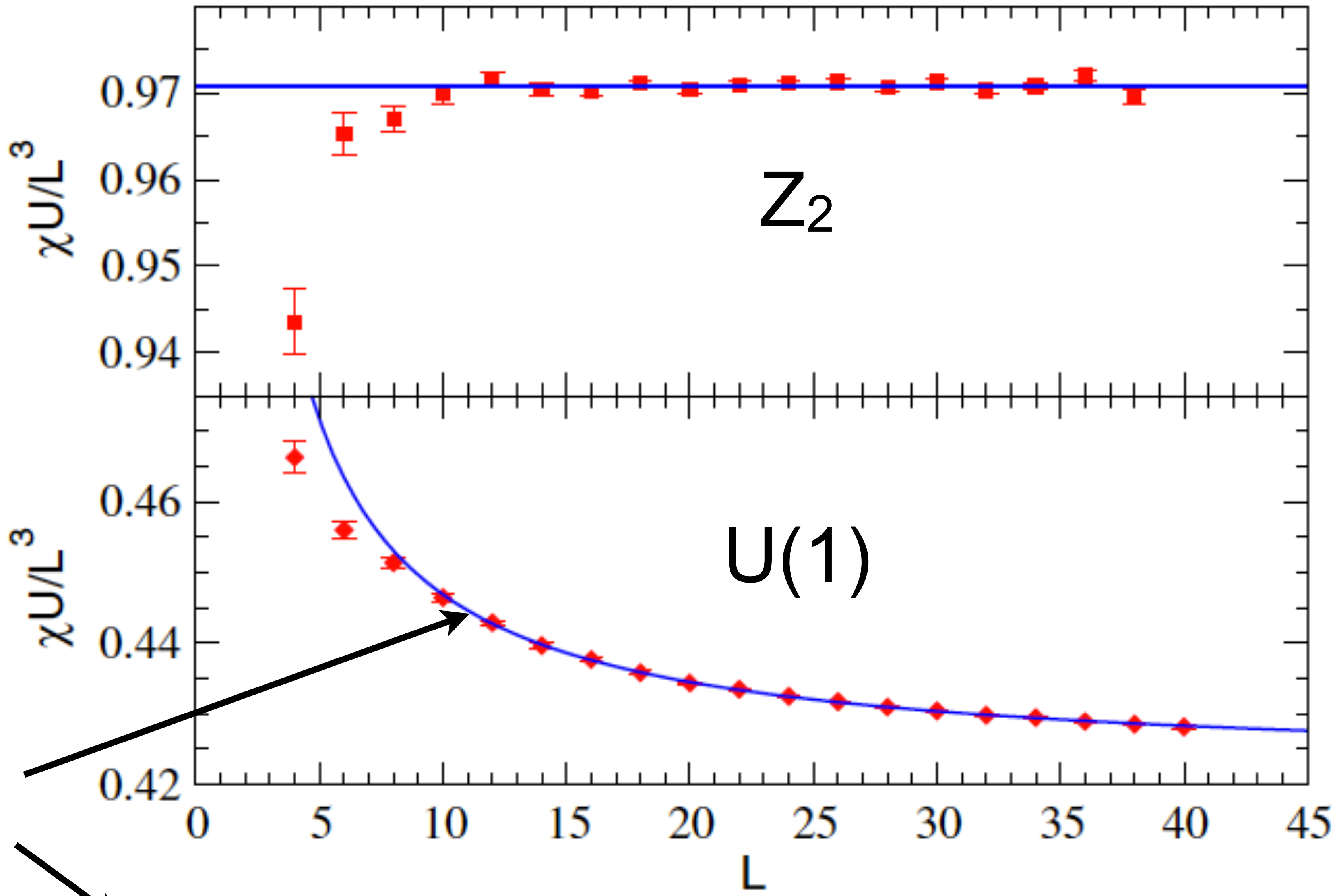
$$\langle \overline{\chi}\chi \rangle \neq 0, \quad U > U_c$$

The condensate $\langle \overline{\chi}\chi \rangle$ breaks Z_2 or U(1) symmetry but is invariant under the SU(2) symmetry.

The U(1) model contains a Goldstone boson
in the broken phase
while the Z_2 model does not!

Evidence of Goldstone bosons

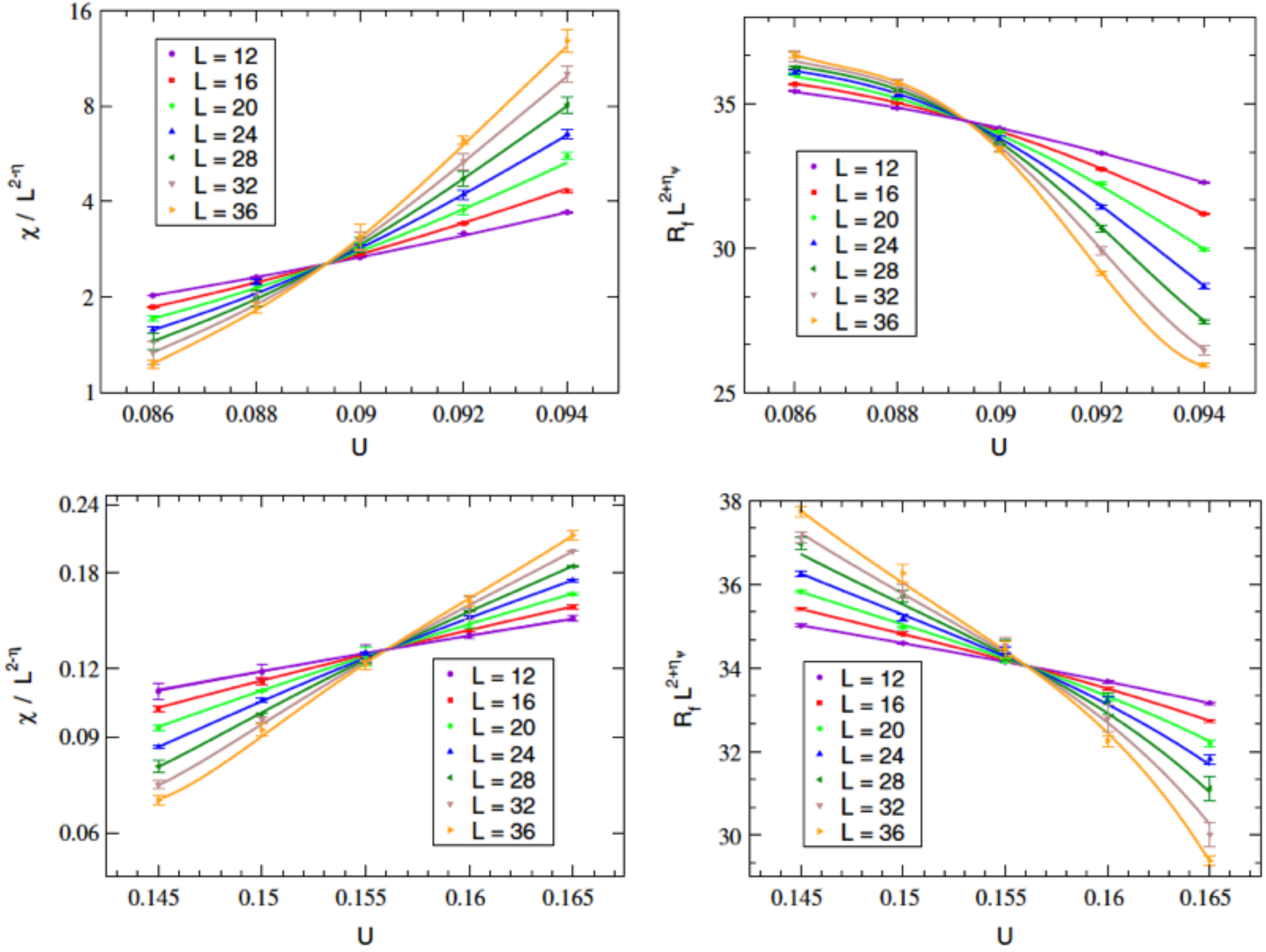
at U = ∞



ChPT fit

$$\chi/L^3 = \frac{\Sigma^2}{2}(1 + 0.224/(\rho_s L)),$$

Quantum Critical Behavior



combined fit results

$$\chi/L^{2-\eta} = \sum_{k=0}^{4} f_k[(U-U_c)L^{\frac{1}{\nu}}]^k,$$

$$R_f L^{2+\eta_\psi} = \sum_{k=0}^{4} p_k[(U-U_c)L^{\frac{1}{\nu}}]^k,$$

U_c	ν	η	η_ψ	f_0	f_1	f_2	f_3	f_4	p_0	p_1	p_2	p_3	p_4	χ^2/d.o.f
0.0893(1)	0.83(1)	0.62(1)	0.38(1)	2.54(7)	9.33(5)	27.3(3)	55.3(1)	48.67(3)	34.4(1)	−18.2(7)	−51.2(6)	7.4(4)	259.2(10)	1.8
0.1560(4)	0.82(2)	0.62(2)	0.37(1)	0.13(1)	0.09(1)	0.02(1)	0.004(1)	0.02(1)	34.0(1)	−4.5(3)	−1.4(3)	−1.8(8)	−0.5(2)	0.88

critical exponents between Z_2 and U(1) indistinguishable

The difference in chiral symmetry seems irrelevant (?).

Results disagree with Karkkainen et. al. who got

$\nu \approx 1.00$ and $\eta \approx 0.750$

N=1 Lattice GN model and Lattice Thirring model belong to the same universality class.

Summary: Old vs New

Model	Symmetry	Work	ν	η	η_ψ
N=1 Lattice-GN	SU(2) x Z_2	Karkkainen,et.al. (1994)	1.00(4)	0.756(8)	-
N=1 Lattice GN	SU(2) x Z_2	SC & Li (2012)	0.83(1)	0.62(1)	0.38(1)
N = 1 Lattice-Th	SU(2)x U(1)	Debbio, et.al., (1997)	0.80(15)	0.70(15)	-
N = 1 Lattice-Th	SU(2)x U(1)	Barbour et. al., (1998)	0.80(20)	0.4(2)	-
N=1 Lattice-(GN/Th)	SU(2) x U(1)	SC & Li (2013)	0.849(8)	0.633(8)	0.373(3)

Surprise with N = 2 (?)

$$S(\overline{\chi},\chi) = \sum_{x,y} \begin{pmatrix} \overline{u}_x & \overline{d}_x \end{pmatrix} D_{xy} \begin{pmatrix} u_y \\ d_y \end{pmatrix} - U \sum_{\langle xy \rangle} \overline{u}_x u_x \overline{d}_x d_x$$

t'Hooft vertex

Symmetries: SU(4) x Z_2

massless fermions
zero chiral condensate

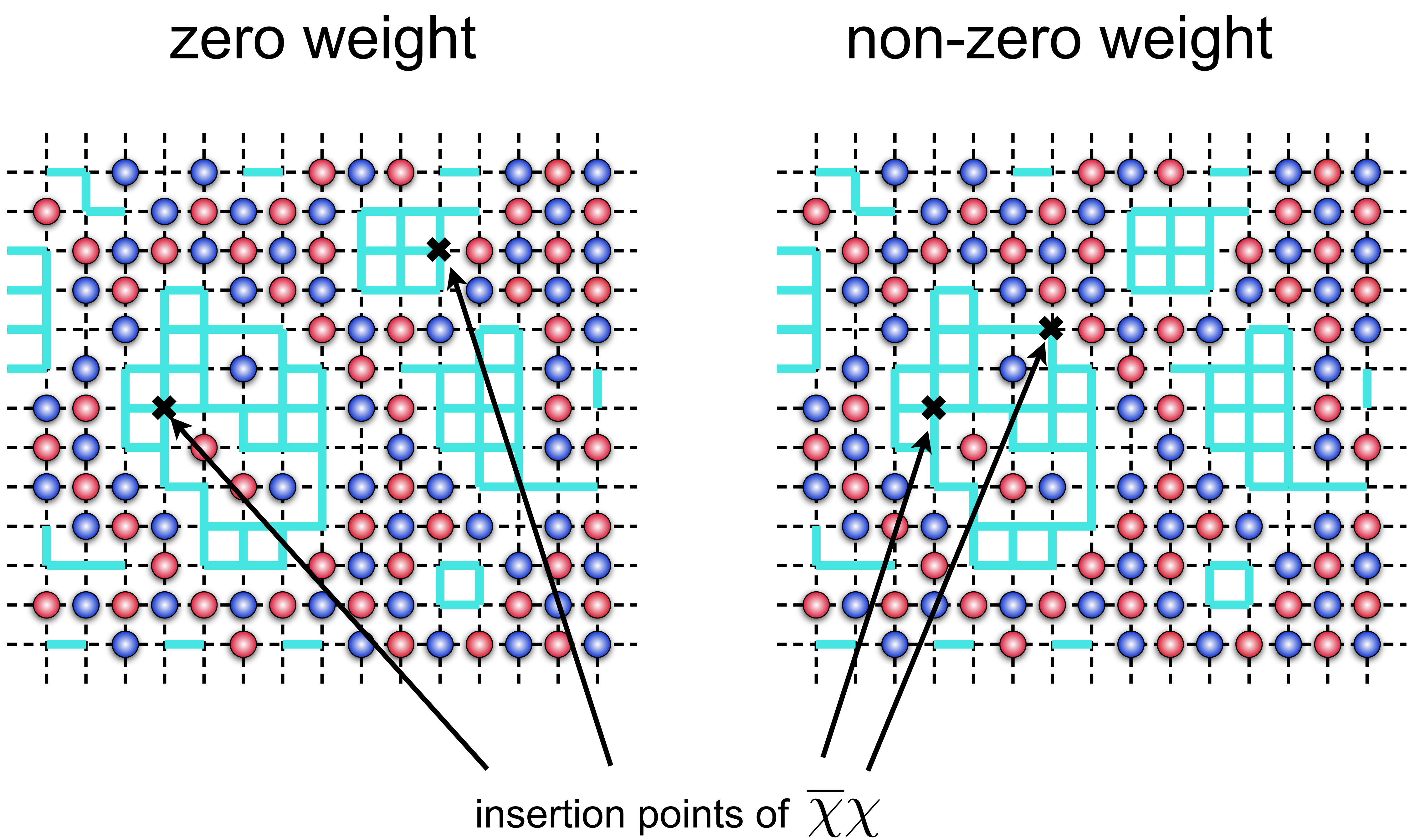
massive fermions
zero chiral condensate!

U_c

U

Puzzle : what symmetry is spontaneously broken?

Analytic Proof



The condensate correlation is non-zero only within each bag

At large U, small bags means the condensate correlation decays exponentially

Conclusions

- Critical exponents in N=1 lattice Four-Fermion models with staggered fermions have been computed with precision using the fermion bag approach.
- Find that previous calculations that ignored the sign problem are wrong(?).
- Learn that Z_2 and U(1) exponents are very similar. More precision is needed to distinguish between them.
- A new phase with massive fermions without a condensate seems to exist with N=2.