Message from Yoichi Iwasaki

I am very happy to come back to research activity in lattice community after 8 years absence.

However, unfortunately my present health condition has not allowed me to travel to Europe.

Ken-ichi Ishikawa will give a talk on behalf of me.

I hope I am able to meet you in New York next year.

Best regards.

Lattice 2013
July 31 2013

Toward the Global Structure of Conformal Theories in the SU(3) Gauge Theory

Y. Iwasaki
U.Tsukuba and KEK

In Collaboration with
K.-I. Ishikawa(U.Hiroshima)
Yu Nakayama(CalTech)
T. Yoshie(U. Tsukuba)

arXiv 1212.4343, 1301.4785 (Phys.RevD. 87.071503), 1304.4345, and in preparation

Objective (this talk)

Introducing the nomenclature ``conformal theories with an IR cutoff'',

Verify the existence of the ‘’conformal region’’ in addition to the confining region and the deconfining region and clarify the property of the conformal region

Objective (for a long term)

Clarify the whole structure of Conformal theories with an IR cutoff and thereby reveal the characteristic of each theory

Plan of Talk

- Propose the nomenclature ‘’Conformal theories with an IR cutoff’’
- Make conjecture based on RG argument
- Verify the conjecture for various cases
  - Clarify the structure of the vacuum of the conformal region
  - Clarify the phase structure including the conformal region

Stage and Tools

SU(3) gauge theories with Nf quarks in the fundamental representation

Action: one-plaqutte gauge action + Wilson fermion action

Nf = 7, 12, 16

Lattice size: 16^3 x 64

Boundary conditions: periodic boundary conditions except for an anti-periodic boundary conditions in the t direction for fermions

Algorithm: Blocked HMC for 2N and RHMC for 1 : Nf=2N + 1

Statistics: 1,000(500) +1,000(500) trajectories

Computers: U. Tsukuba: CCS HAPACS; KEK: HITAC 16000

Nomenclature

Conformal theories with an IR cutoff

Theories with an IR fixed point and with an IR cutoff

Conformal QCD (the large N_f QCD within the conformal window) with an IR cutoff

Note that all numerical simulations for Conformal QCD are with an IR cutoff

Conjecture:
based on RG argument

”Conformal region" : $m_H \leq c\,\Lambda_{IR}\,(m_q \leq m_q^c)$

“Conformal behavior”:
meson propagator G(t) behaves at large t as

$$G(t) = c\,\frac{\exp\left(-m\,t\right)}{t^{\alpha}}$$

Transition at the boundary of the conformal region is first order

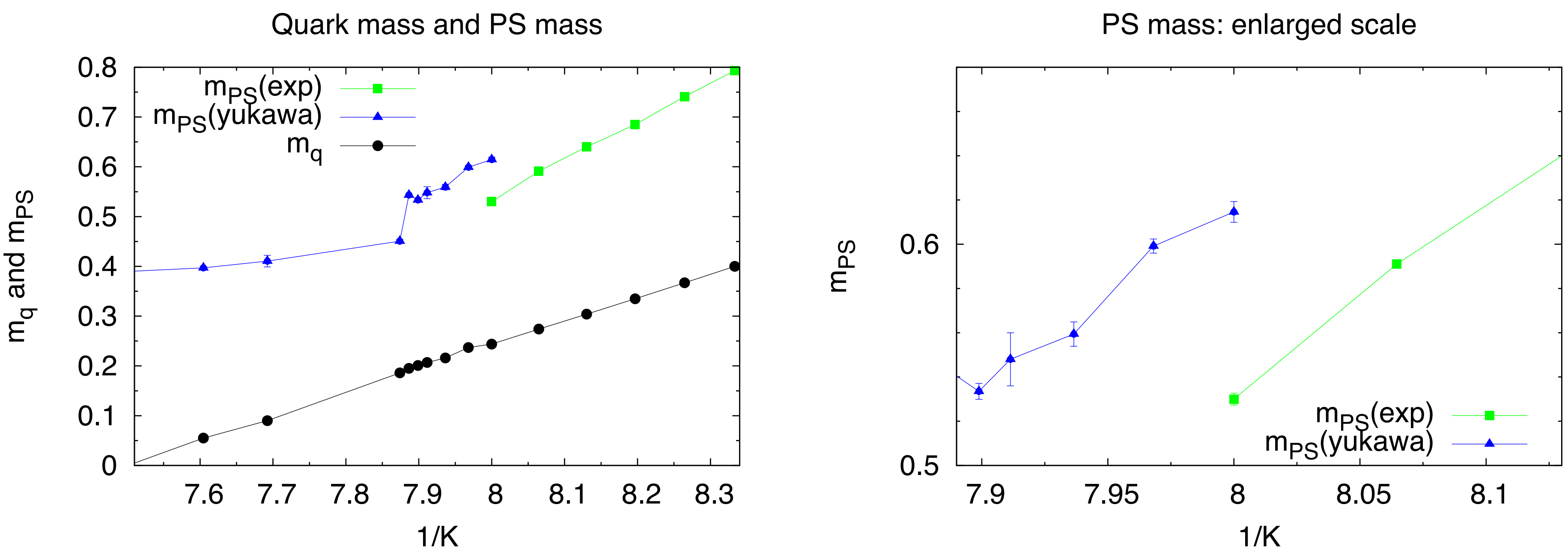
Verification of Conjecture

Nf=16

Nf=7

Nf=12

$N_f = 16; \beta = 11.5$

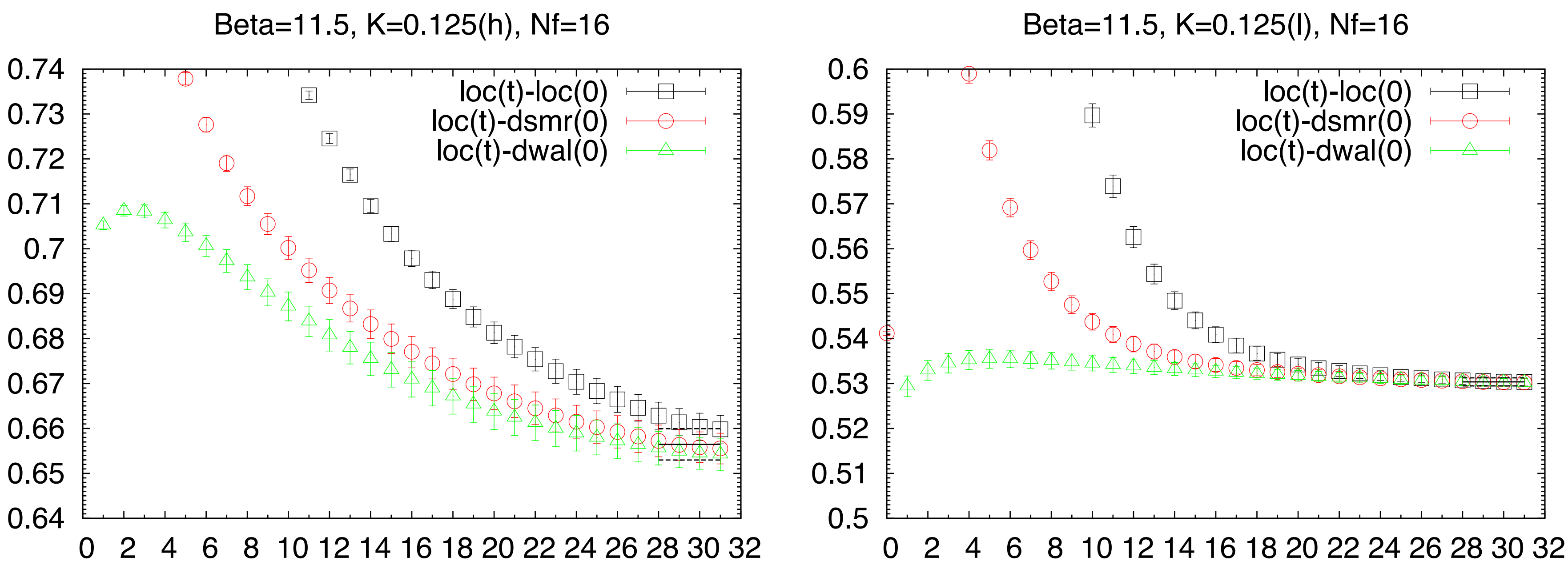


For K>0.125 yukawa-type decay:

For K<0.125 exponential-type decay

<u>Two states at K=0.125; First order transition</u>

Two states at K=0.125: effective mass



Yukawa-type decay

m→0.66

Exponential-type decay

m→ 0.53

Structure of the vacuum

Effective potential in the one-loop approximation

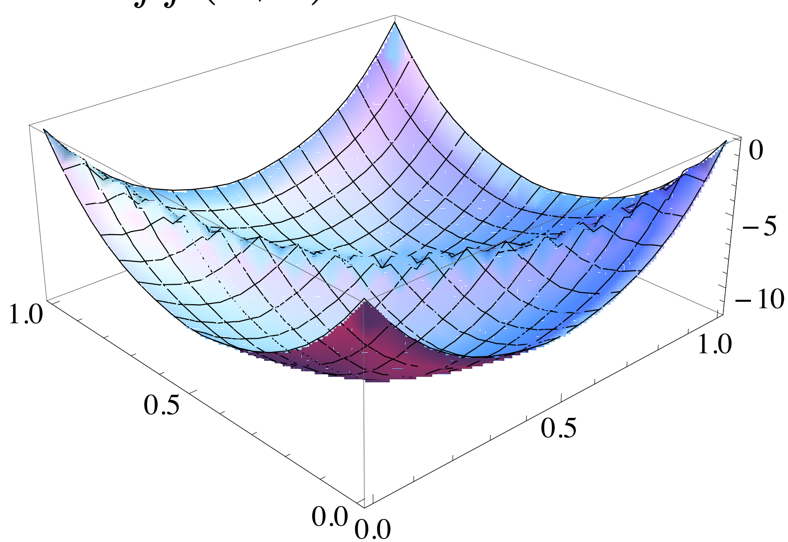
Parametrize the loop of link variables in spatial directions

$$U = \mathrm{diag}(e^{i2\pi a}, e^{i2\pi b}, e^{-i2\pi(a+b)})$$

x, y, z directions

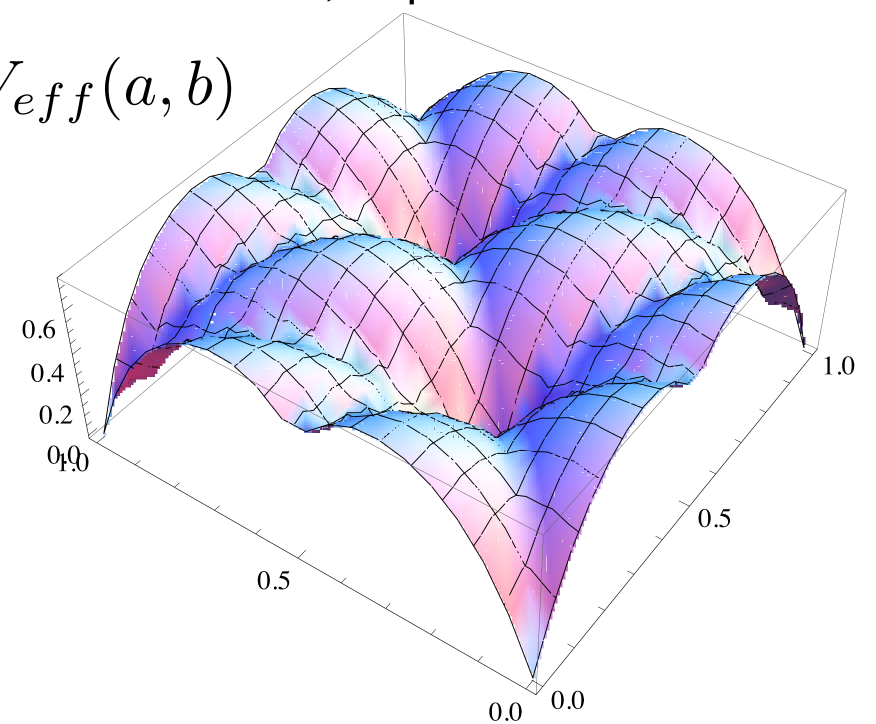
Nf=16, mq=0.0005

$V_{eff}(a,b)$



Nf=16, mq=1.0

$V_{eff}(a,b)$



Z(3) twisted vacuum

Polyakov loops in spatial directions: Px, Py, Pz

Nf=16, mq=0.0005

Lowest energy state $a = b = 1/3$ and $= 2/3 (= -1/3)$

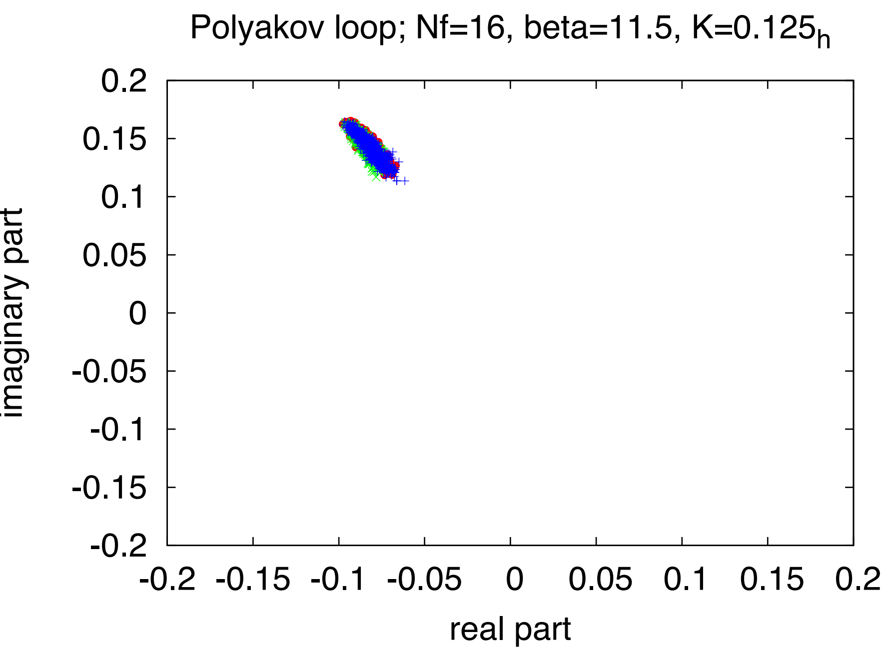
$$P_x, P_y, P_z = \exp\left(\pm i\, 2\pi/3\right)$$

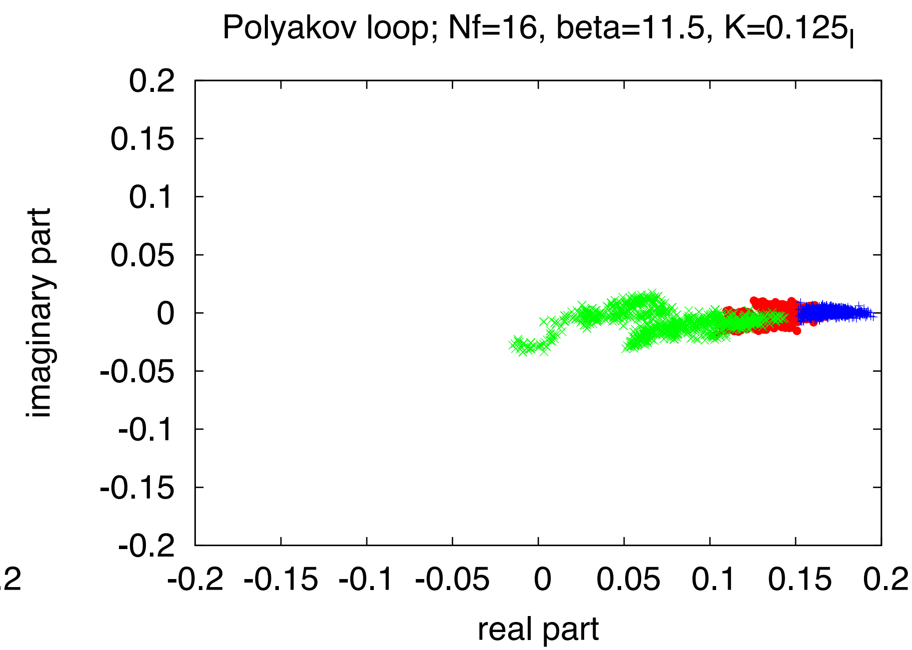
Nf=16, mq < ~ 0.15

$a = b = 0$ $\qquad P_x, P_y, P_z = 1$

Locally unstable state

Nf=16:Two states at K=0.125: Polyakov loops





$$P_x, P_y, P_z \simeq 0.2 \exp\left(\pm i\, 2\pi/3\right)$$

$$P_x, P_y, P_z \simeq 0.05 \sim 0.2$$

Close to the twisted vacuum,
But not equal

characteristic in the deconfining region

Verified in the Nf=16 case

Inside the conformal region

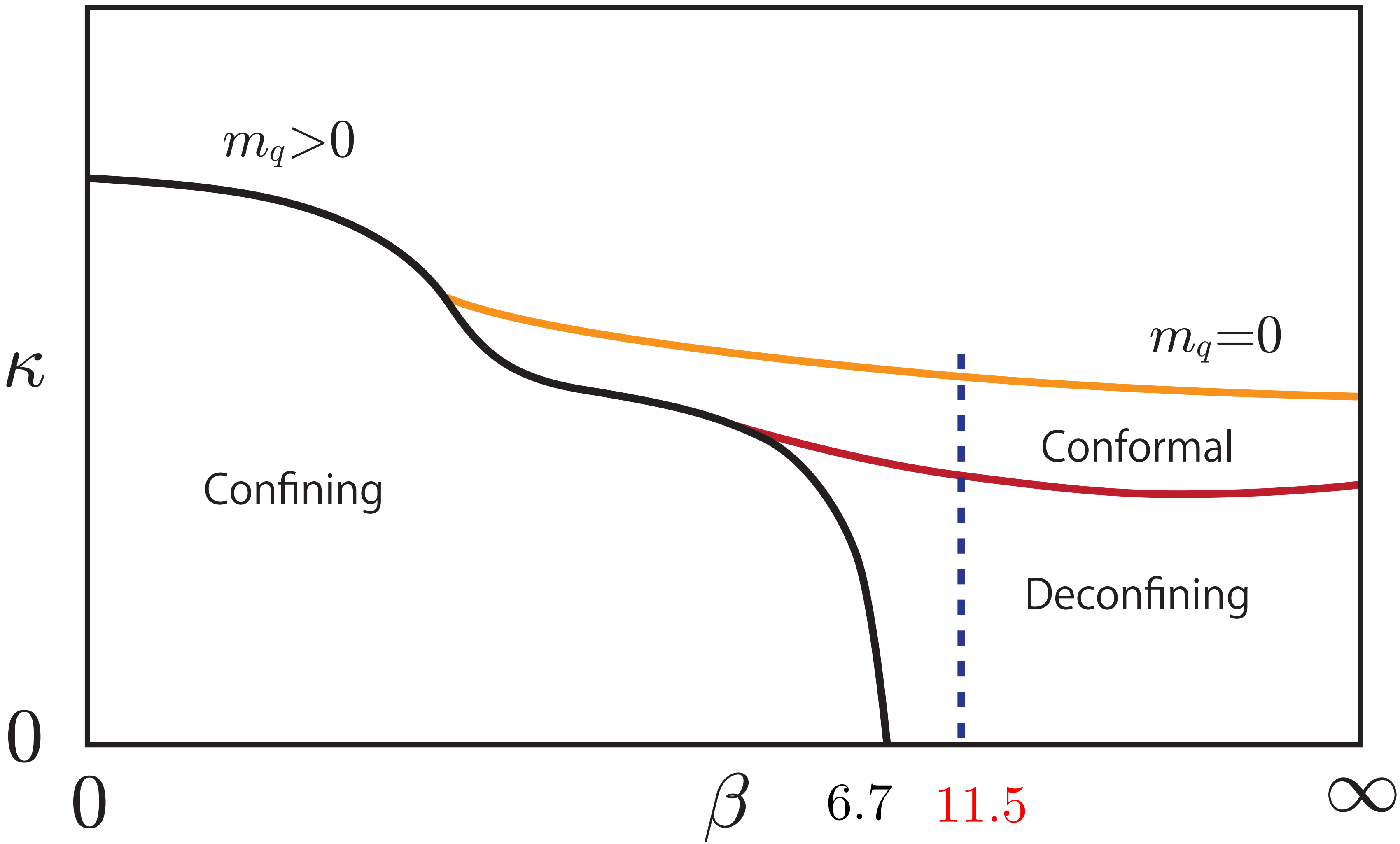
$$P_x, P_y, P_z \simeq 0.2 \exp\left(\pm i\, 2\pi/3\right)$$

Outside the conformal region

$$P_x, P_y, P_z \simeq 0.05 \sim 0.2$$

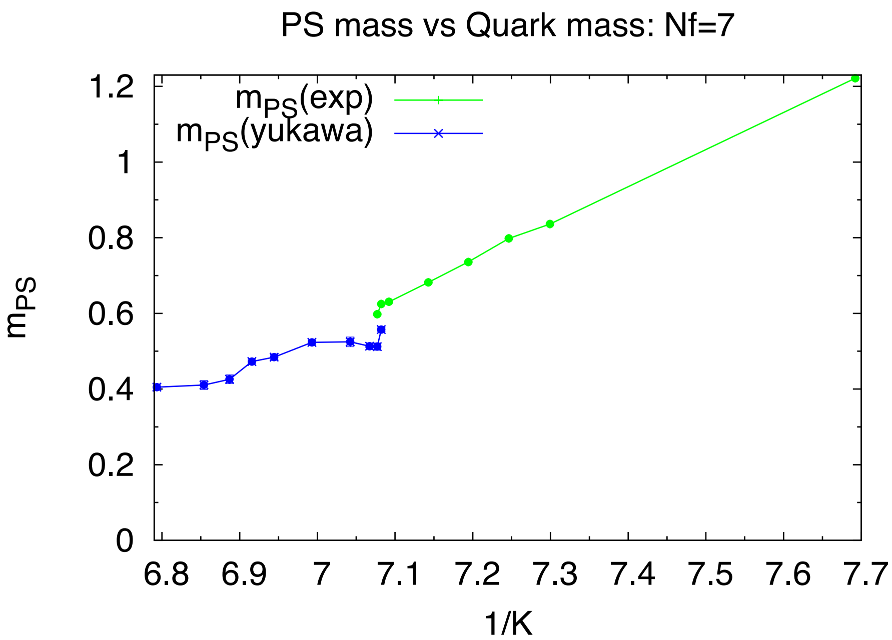
Boundary is that between different vacua and therefore first order

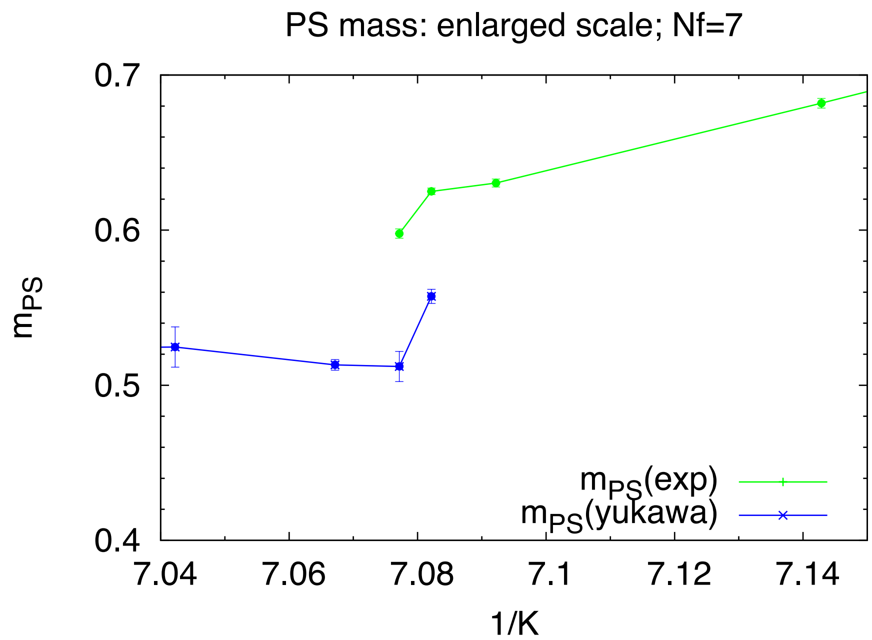
Phase structure on a finite lattice: Nf=16



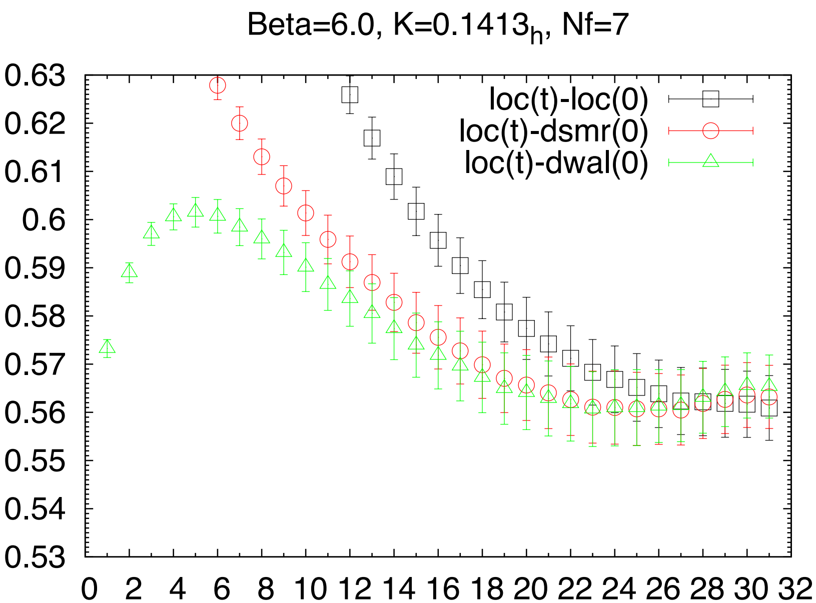
15

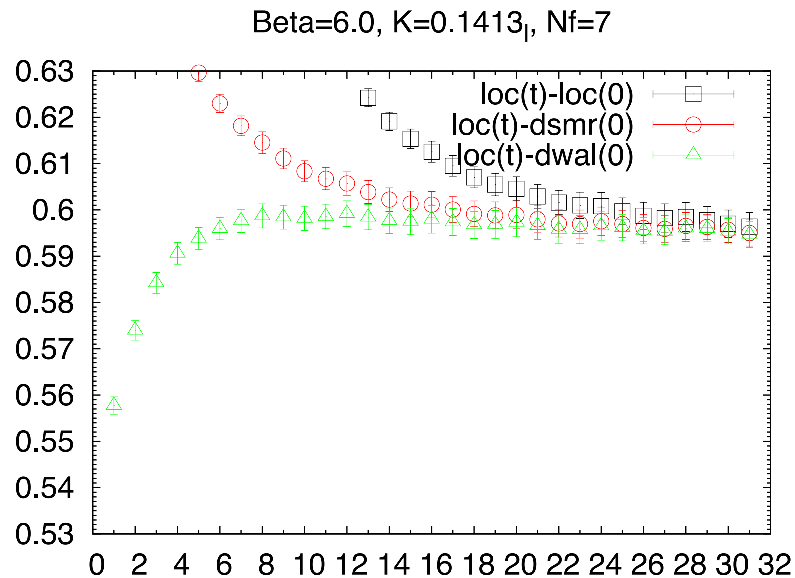
N_f=7: beta = 6.0: Two states at K=0.1413





N_f=7: beta = 6.0: effective masses at K=0.1413





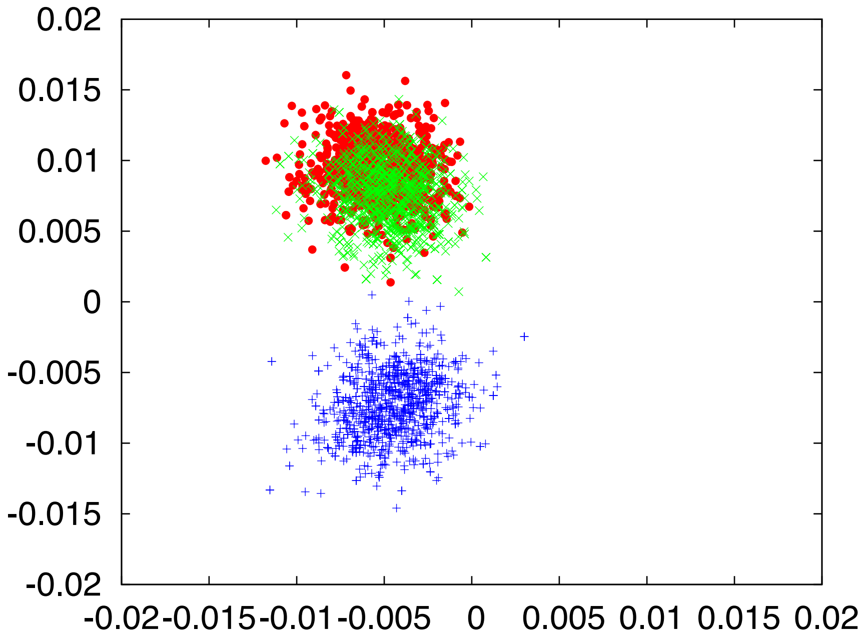
Yukawa-type decay

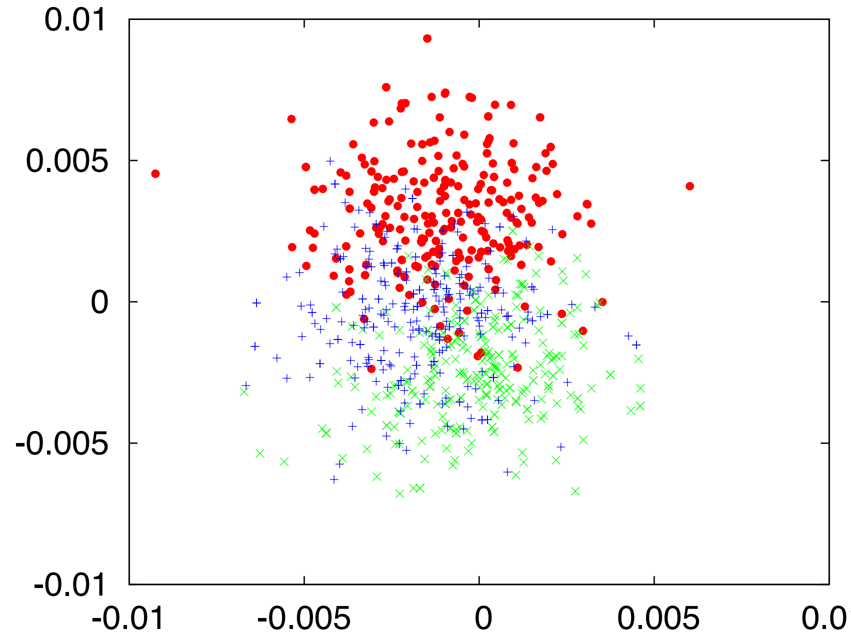
m→0.56

Exponential-type decay

m→0.59

Polyakov loops : Nf=7



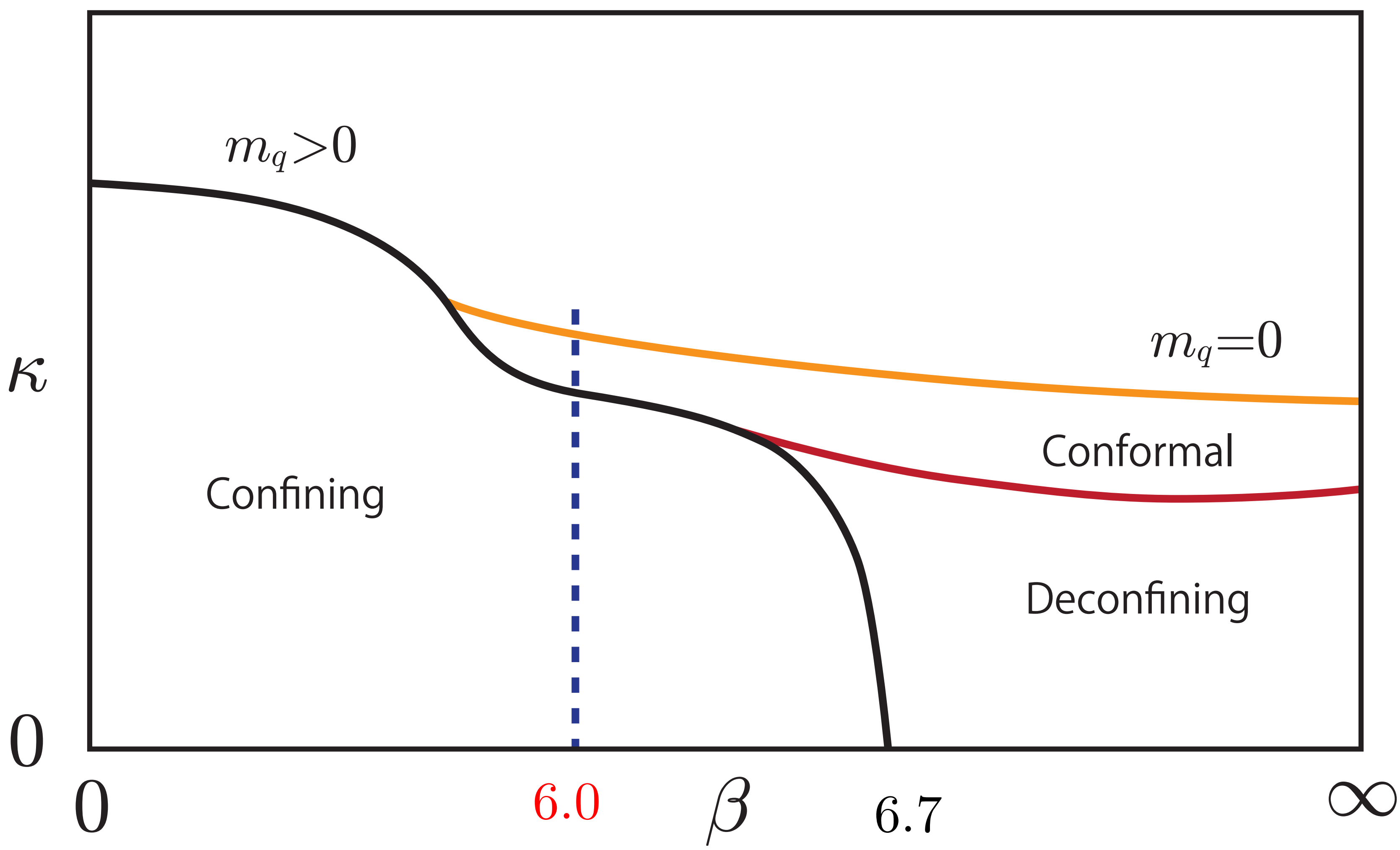


Inside the conformal region

$$P_x, P_y, P_z \simeq 0.01 \exp\left(\pm i\, 2\pi/3\right)$$

Outside the conformal region
Typical in the confining region

Phase structure on a finite lattice: Nf=7



Nf=12: beta=6.0 and beta=8.0

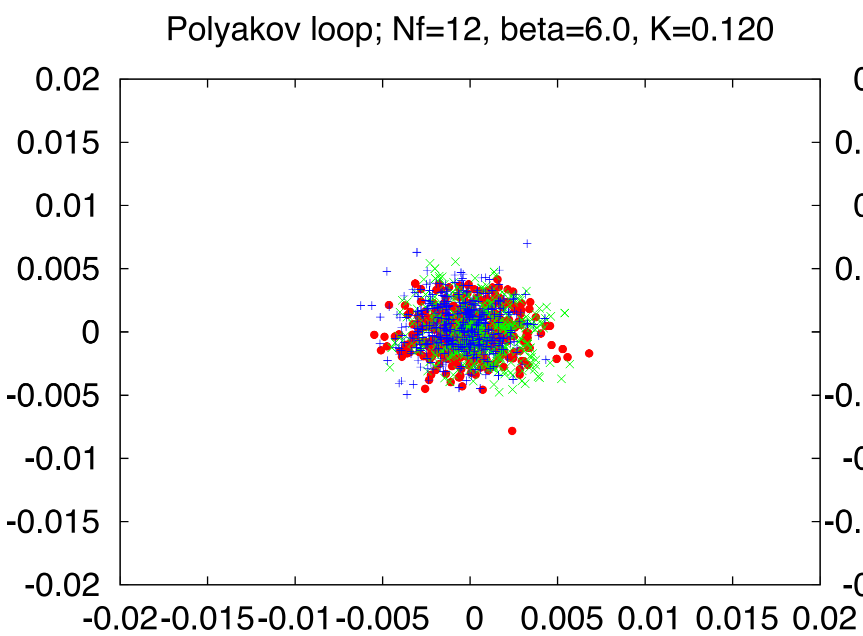
Investigate the state outside the conformal region

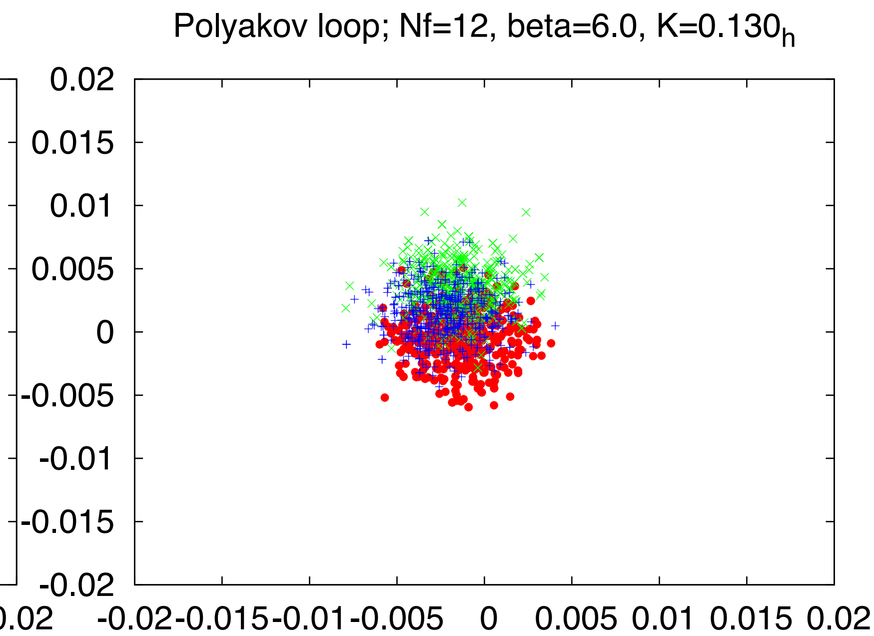
First, Polyakov loops

Then, the mq dependence of m_{PS}

Polyakov loops outside the conformal region :Nf=12

Beta=6.0



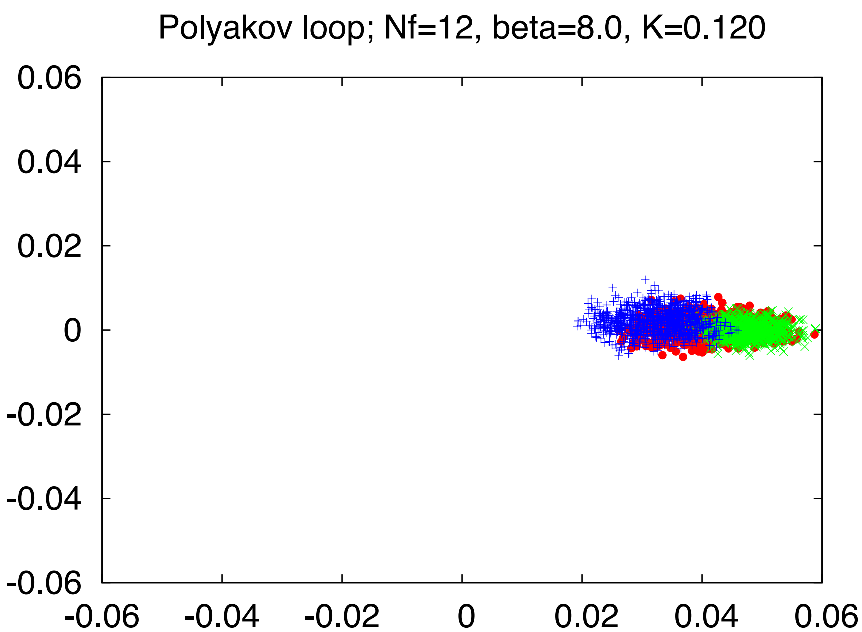


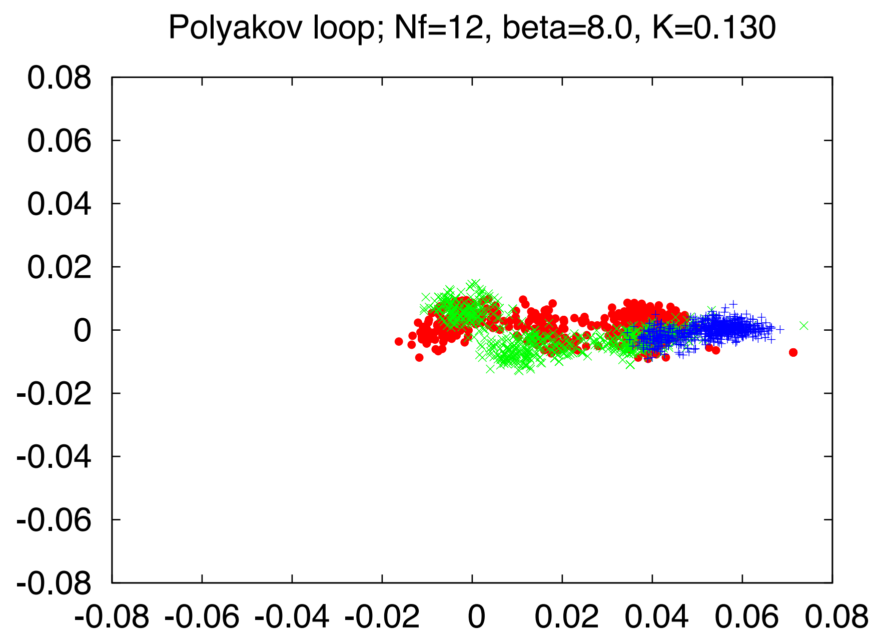
$$P_x, P_y, P_z \simeq 0$$

Characteristic in the confining region

Polyakov loops outside the conformal region :Nf=12

Beta=8.0



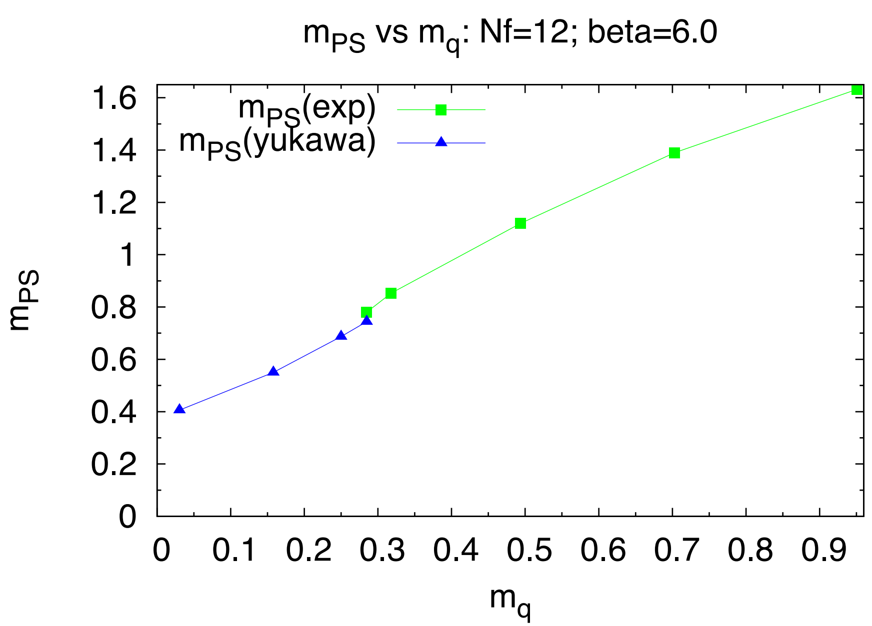


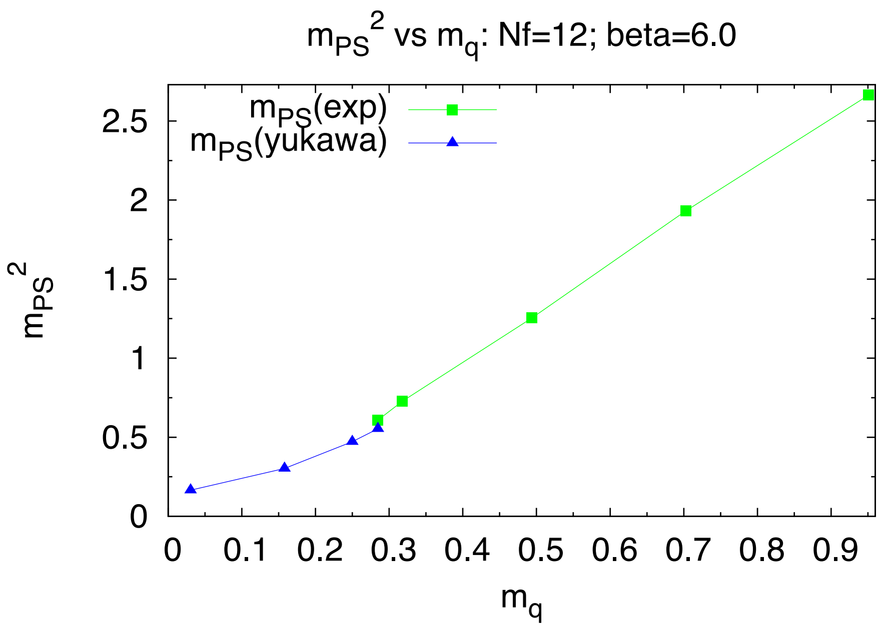
$$P_x, P_y, P_z \simeq 0.01 \sim 0.06$$

Characteristic in the deconfining region

Mq dependence of m_{PS} outside the conformal region

Beta=6.0

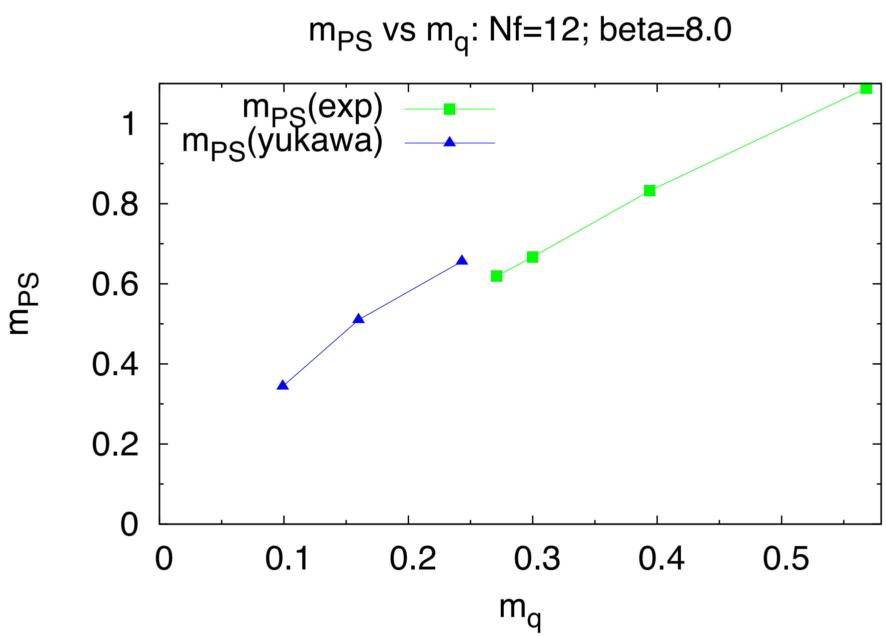


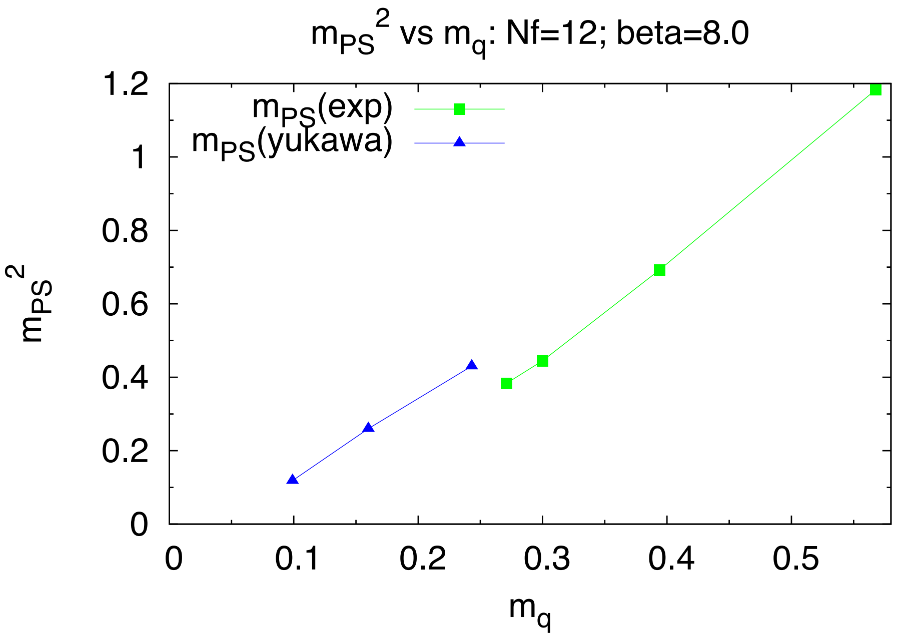


$m_{PS}^2 \sim m_q$ is favorable as expected

Mq dependence of m_{PS} outside the conformal region

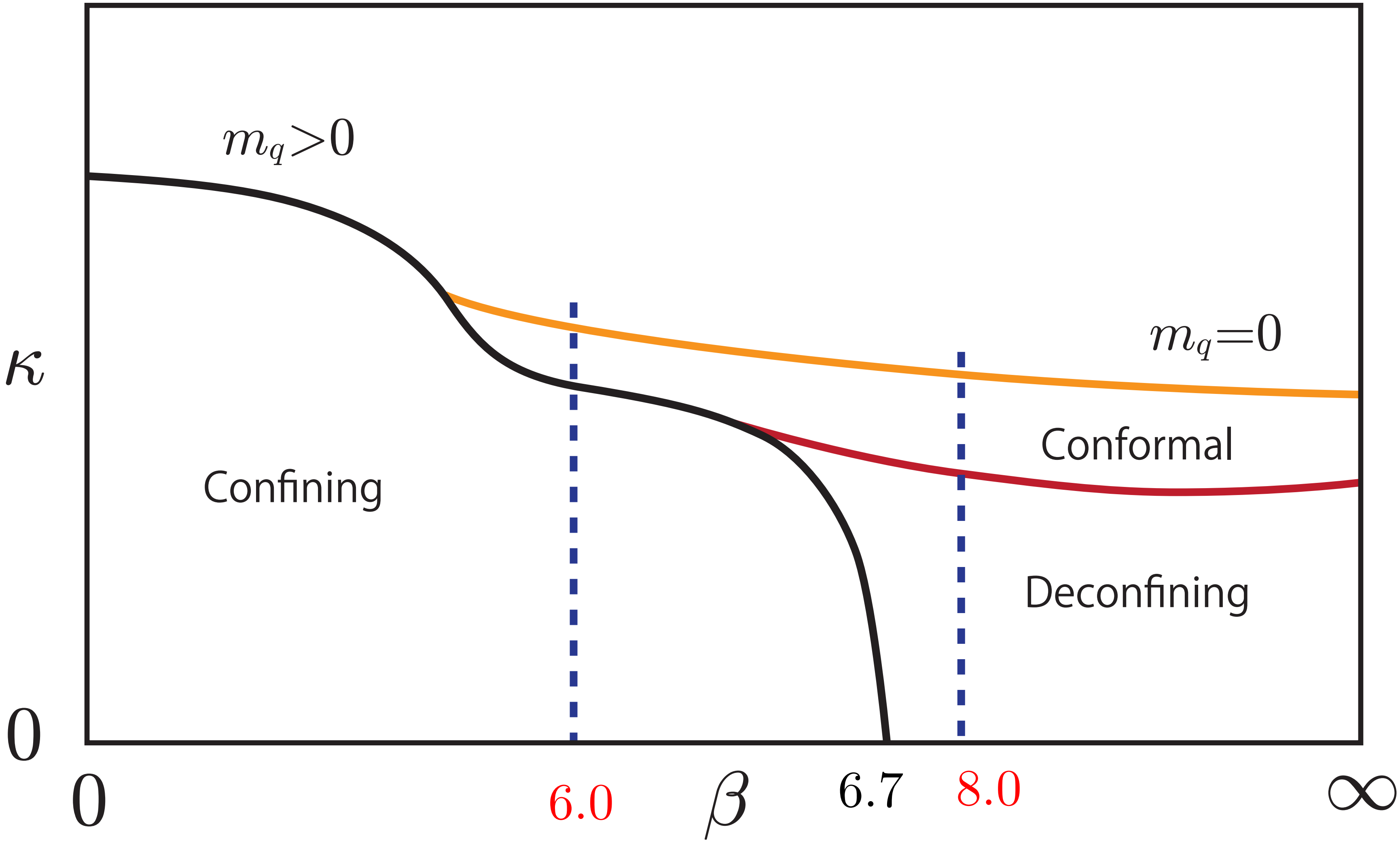
Beta=8.0





$m_{PS} \sim m_q$ is favorable as expected

Phase structure on a finite lattice: Nf=12



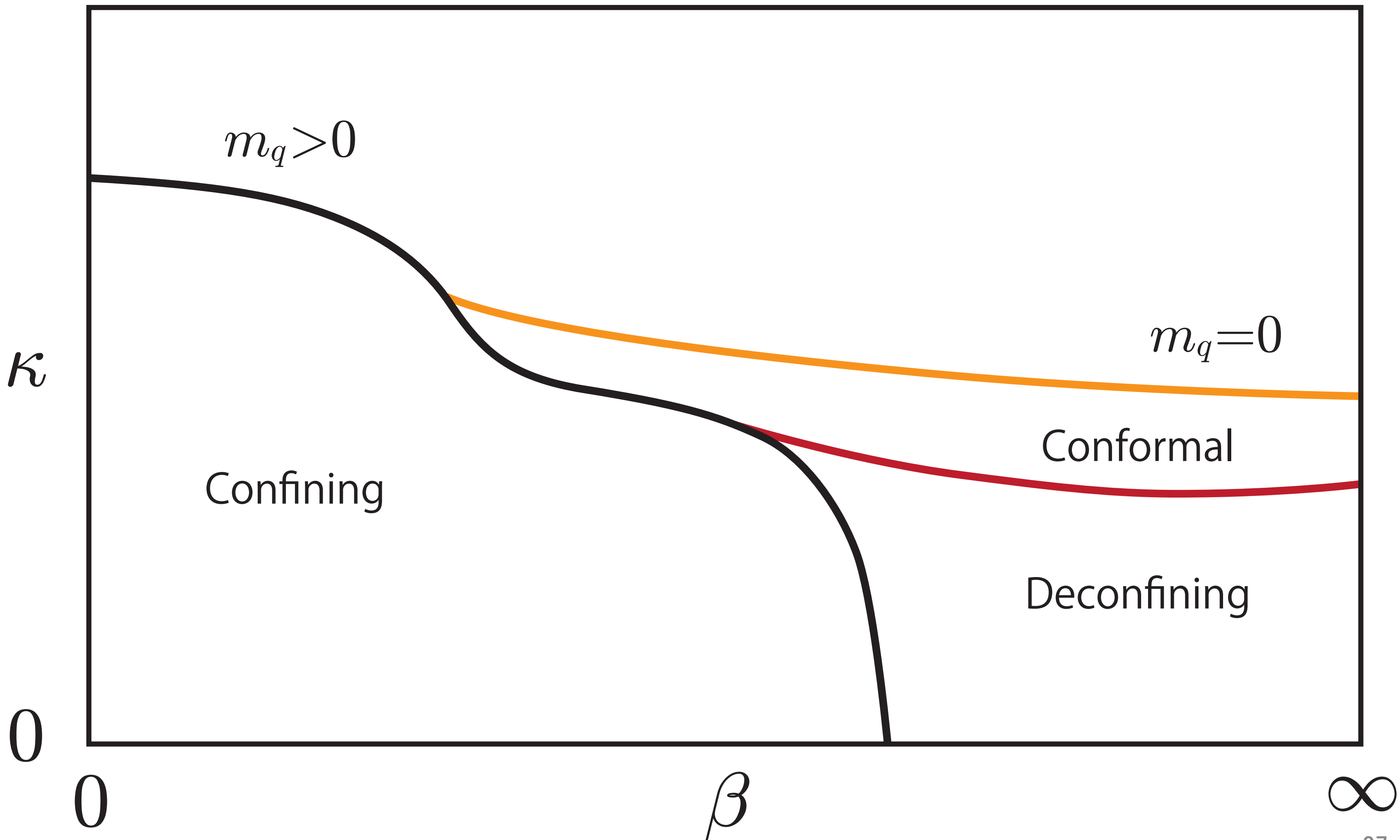
Cautious remark

The mq dependence of m_{PS} outside the conformal region is determined by the lattice size and the beta

It is irrelevant to the conformal behavior

In order to obtain conformal properties, one should be inside the conformal region

Coclusion: Phase structure on a finite lattice



Summary

We have verified

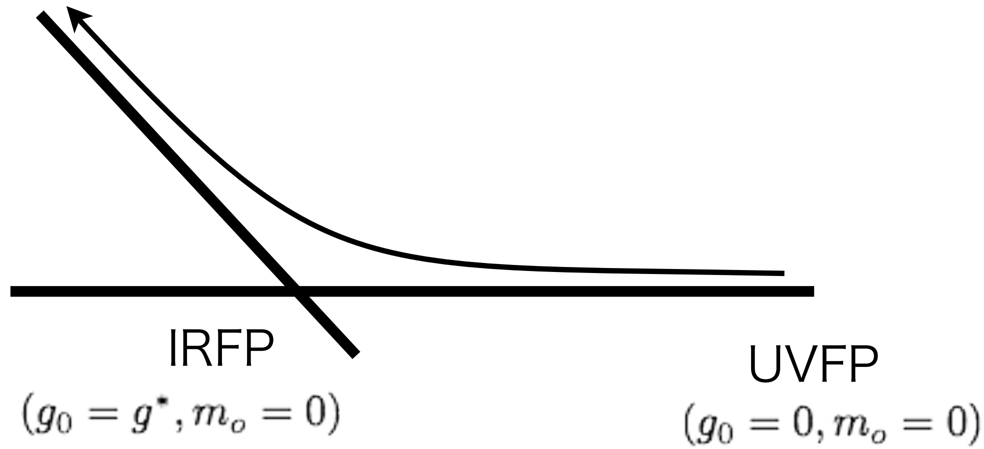
- The existence of the conformal region in Conformal QCD (Nf=7, 12, 16) on a 16^3 x 64 lattice
- The vacuum in the conformal region is close to the Z(3) twisted vacuum
- The boundary between the conformal region and the confining (deconfining) region is first order transition
- The mq dependence of m_{PS} outside the conformal region is irrelevant to the conformal behavior

Thank you

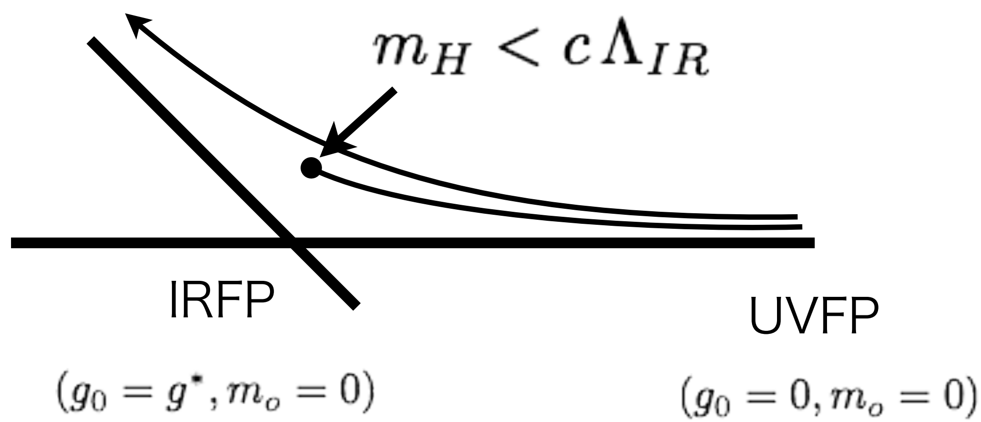
Appendix

RG argument

$\Lambda_{IR} = 0$

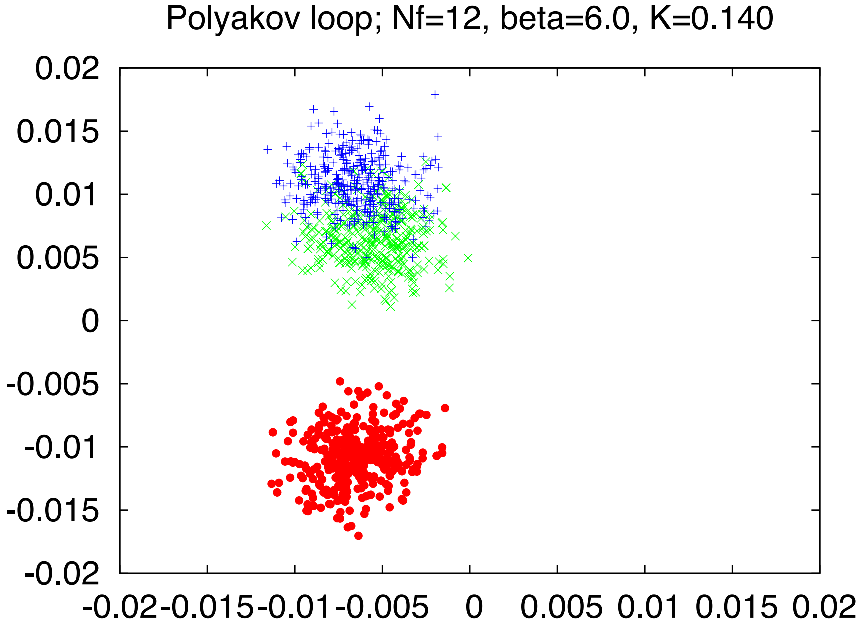


$\Lambda_{IR} = finite$

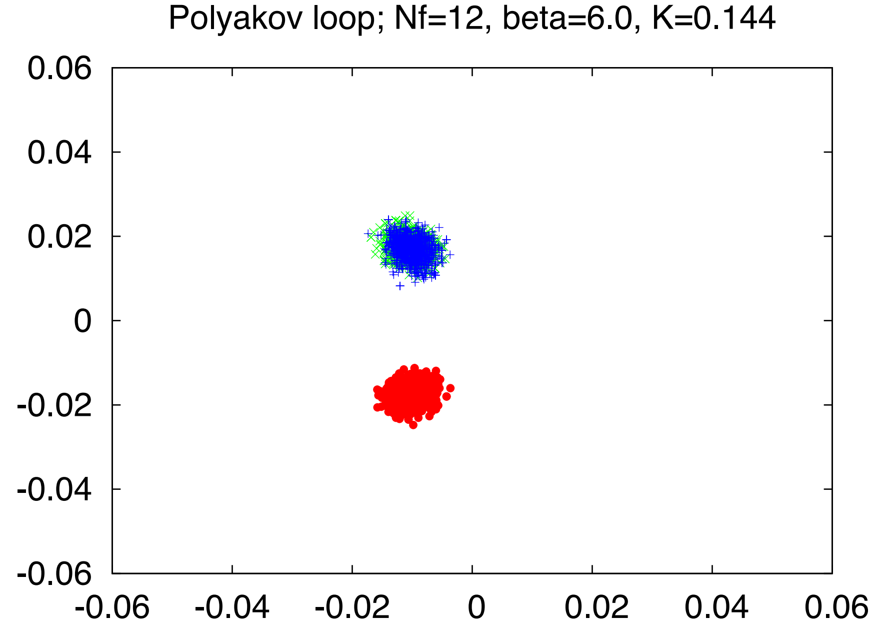


Nf=12 inside the conformal region

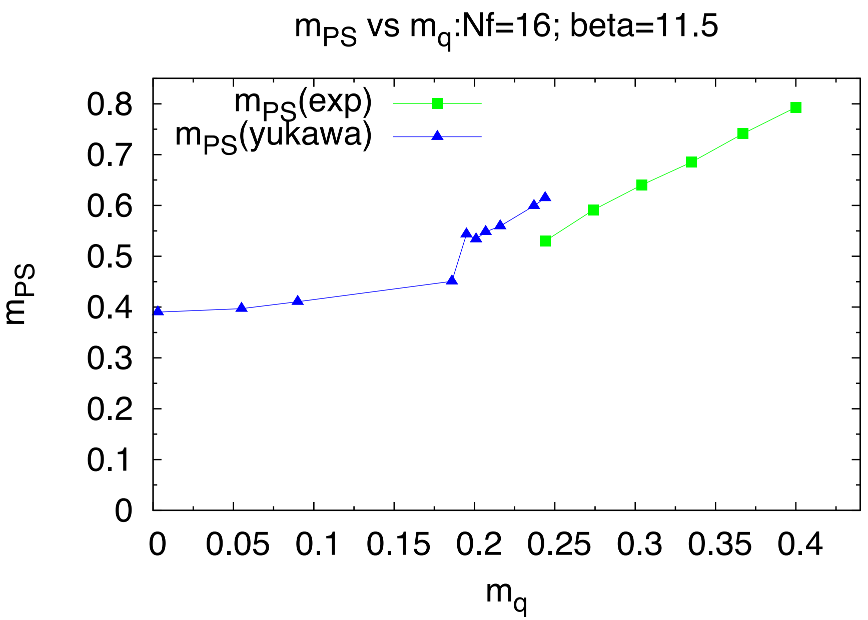
beta=6.0



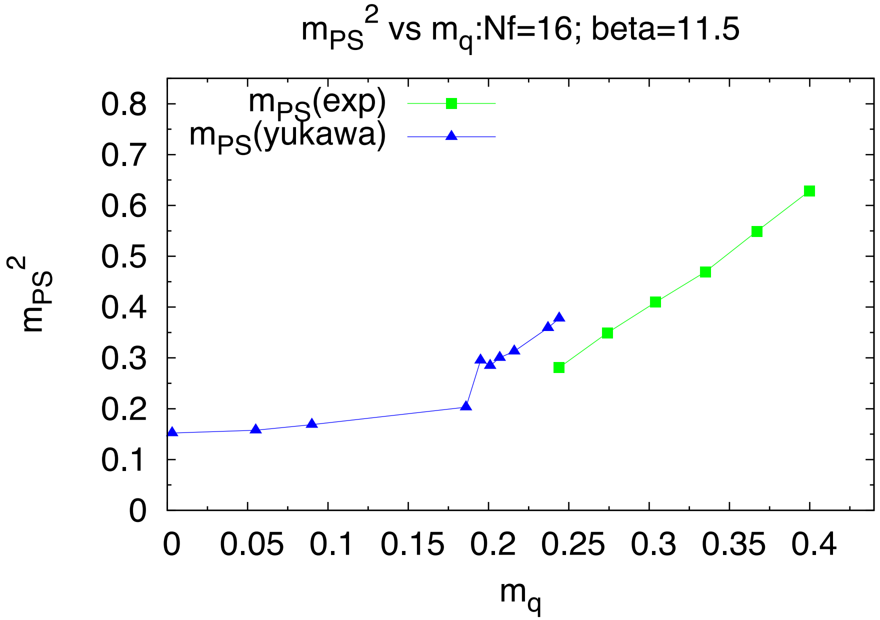
beta=8.0



Nf=!6: mq vs. m_{PS}

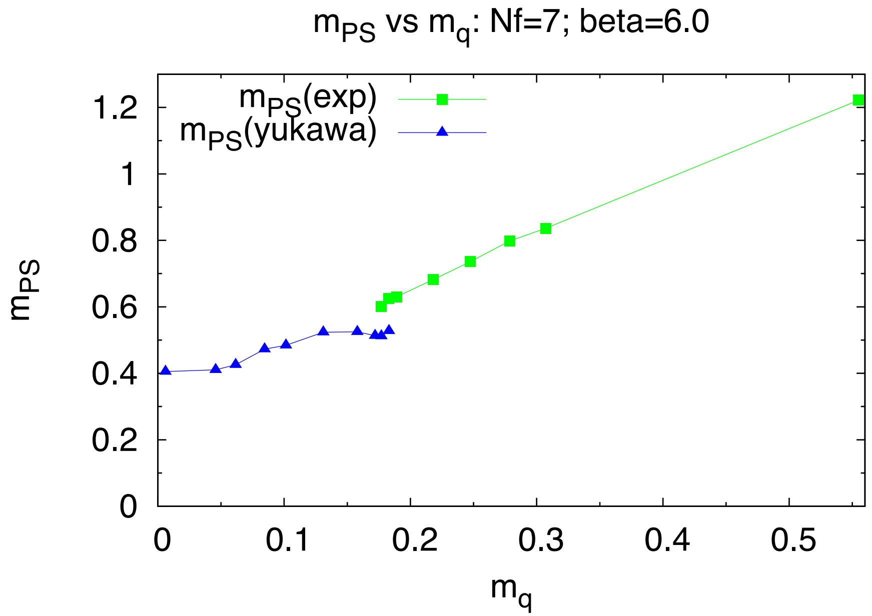


m_{PS} vs m_q:Nf=16; beta=11.5
m_{PS}(exp)
m_{PS}(yukawa)
m_{PS}
m_q
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.15
0.25
0.35

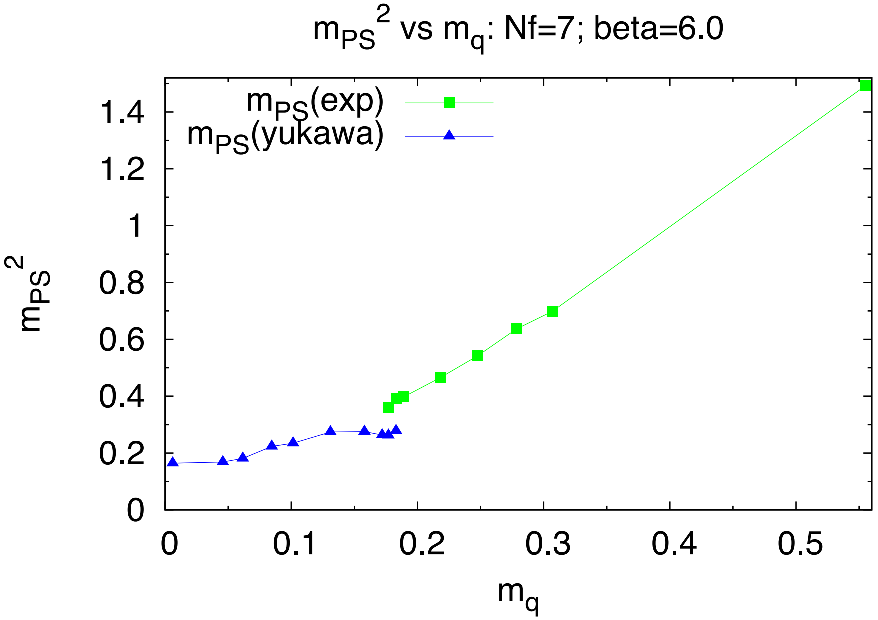


m_{PS}^2 vs m_q:Nf=16; beta=11.5
m_{PS}(exp)
m_{PS}(yukawa)
m_{PS}^2
m_q

Nf=7: mq vs. m_{PS}



mPS vs mq: Nf=7; beta=6.0
mPS(exp)
mPS(yukawa)
mPS
mq
0
0.2
0.4
0.6
0.8
1
1.2
0
0.1
0.2
0.3
0.4
0.5



mPS2 vs mq: Nf=7; beta=6.0
mPS(exp)
mPS(yukawa)
mPS2
mq
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.1
0.2
0.3
0.4
0.5

1 vacuum

1.1 Periodic boundary condition

In general quantum mechanics, the one-loop corrections to the zero-temperature vacuum energy (which is same as $\pm\mathrm{Tr}\log(D)$) is obtained by

$$E = \sum_{boson} \frac{E_B}{2} - \sum_{fermion} \frac{E_F}{2} \ . \tag{1}$$

The most generic Wilson line (in fundamental rep of $SU(3)$) would be

$$\begin{aligned} U_x &= \exp(i\int A_x dx) = \mathrm{diag}(e^{i2\pi a_x}, e^{i2\pi b_x}, e^{i2\pi c_x}) \\ U_y &= \exp(i\int A_y dy) = \mathrm{diag}(e^{i2\pi a_y}, e^{i2\pi b_y}, e^{i2\pi c_y}) \\ U_z &= \exp(i\int A_z dz) = \mathrm{diag}(e^{i2\pi a_z}, e^{i2\pi b_z}, e^{i2\pi c_z}) \end{aligned} \tag{2}$$

with $a_x + b_x + c_x = 1$ and so on (in general it can be integer) from the unitary condition. Note that $a_x = b_x = c_x = \frac{1}{3}, \frac{2}{3}$ gives a non-trivial center of the gauge group.

For free Wison fermion, the energy can be obtained by

$$\begin{aligned} k^2[k_x, k_y, k_z] &= (\sin(k_x)^2 + \sin(k_y)^2 + \sin(k_z)^2) \\ m^2[k_x, k_y, k_z] &= (m_q + 3 - \cos(k_x) - \cos(k_y) - \cos(k_z))^2 \end{aligned} \tag{3}$$

where m_q is the quark mass in the action, with the implicit form:

$$\cosh(E[k_x, k_y, k_z]) = 1 + \frac{k^2 + m^2}{2(1+m)} \ . \tag{4}$$

If we do the singular gauge transformation, the Wilson-line can be encoded in the twisted boundary condition for the quark field, which in turn changes momentum quantization in the summation. Therefore the one-loop potential (note there are three differently colored quarks) is obtained by

$$\begin{aligned} -V(\vec{a}, \vec{b}, \vec{c}) = &\sum_{n_x=a_x}^{N-1+a_x} \sum_{n_y=a_y}^{N-1+a_y} \sum_{n_z=a_z}^{N-1+a_z} E[2\pi n_x/N, 2\pi n_y/N, 2\pi n_z/N] \\ + &\sum_{n_x=b_x}^{N-1+b_x} \sum_{n_y=b_y}^{N-1+b_y} \sum_{n_z=b_z}^{N-1+b_z} E[2\pi n_x/N, 2\pi n_y/N, 2\pi n_z/N] \end{aligned}$$

$$+\sum_{n_x=c_x}^{N-1+c_x}\sum_{n_y=c_y}^{N-1+c_y}\sum_{n_z=c_z}^{N-1+c_z} E[2\pi n_x/N, 2\pi n_y/N, 2\pi n_z/N] \ . \qquad (5)$$

The summation is taken for $n_x = a_x, a_x+1, a_x+2, \cdots$. In the figure, I subtracted $V(0,0,0)$ since the absolute value is unphysical.

The gauge field contribution can be estimated as follow. For $SU(3)$, the adjoint representation (octet) of the gauge group obtains the shift of momentum in $(a-b), (b-a), (c-a), (a-c), (b-c), (c-b), 0, 0$. This can be understood as follows. Set $A_\mu = A_\mu^0 + \delta A_\mu$, where A_μ^0 is the background field that gives the specified Wilson line. The gauge transformation is given by $A_\mu^0 \to U^\dagger A_\mu^0 U + U^\dagger \partial_\mu U$ and $\delta A_\mu \to U^\dagger \delta A_\mu U$ (with $\psi \to U\psi$ for fundamental matter). Now, we use the singular gauge transformation to get rid of A_μ^0. Then the matrix U acts as the twisted boundary condition for the fluctuation δA_μ (and matter field ψ), which transforms as adjoint representation of the gauge group. This leads to the above mentioned shift of momentum. The path integral over δA_μ gives the one-loop energy from the one-loop determinant (it is instructive to see that when $a = b = c = 1/3$, there is no contribution to the potential due to center symmetry).

One can compute the one-loop shift of energy (vacuum energy) by using the similar formula to the above by the momentum shift for the adjoint representation.

$$+V(\vec{a},\vec{b},\vec{c}) = \sum_{n_x=a_x-b_x}^{N-1+a_x-b_x}\sum_{n_y=a_y-b_y}^{N-1+a_y-b_y}\sum_{n_z=a_z-b_z}^{N-1+a_z-b_z} E[2\pi n_x/N, 2\pi n_y/N, 2\pi n_z/N] + \cdots \ . \qquad (6)$$

Here $E(\vec{k})$ should be determined from the pole of the propagator of the gauge fields. I used massless limit of (4) in the potential computation.

The effective energy in terms of (a_x, b_y) is shown in Fig.??? The minimum is at $(a_x = 1/3, b_y = 1/3)$.

1.2 Antiperiodic boundary condition

We could instead use the anti-periodic boundary condition for the quarks. With the above Wilson line introduced, the one-loop potential for quark fields becomes

$$-V(\vec{a},\vec{b},\vec{c}) = \sum_{n_x=a_x+1/2}^{N-1+a_x+1/2}\sum_{n_y=a_y+1/2}^{N-1+a_y+1/2}\sum_{n_z=a_z+1/2}^{N-1+a_z+1/2} E[2\pi n_x/N, 2\pi n_y/N, 2\pi n_z/N]$$

$$+\sum_{n_x=b_x+1/2}^{N-1+b_x+1/2}\sum_{n_y=b_y+1/2}^{N-1+b_y+1/2}\sum_{n_z=b_z+1/2}^{N-1+b_z+1/2} E[2\pi n_x/N, 2\pi n_y/N, 2\pi n_z/N]$$
$$+\sum_{n_x=c_x+1/2}^{N-1+c_x+1/2}\sum_{n_y=c_y+1/2}^{N-1+c_y+1/2}\sum_{n_z=c_z+1/2}^{N-1+c_z+1/2} E[2\pi n_x/N, 2\pi n_y/N, 2\pi n_z/N] \ . \tag{7}$$

The one-loop potential from gauge field does not change. We realize that $a_i = b_i = c_i = 0$ is the minimum of the total potential (by computing the potential with mathematica).