ITEP Lattice

ρ^0 and A^0 mesons in the background of strong magnetic fields

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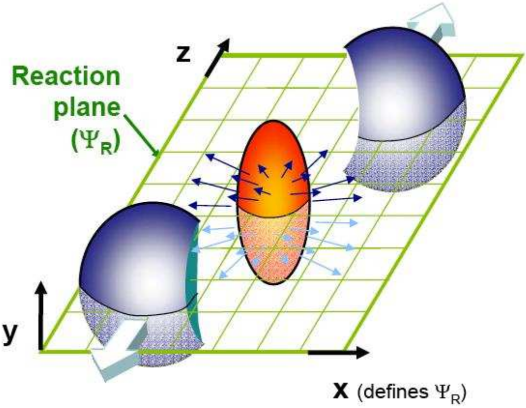
Introduction

We work in $SU(2)$ gluodynamics with chirally invariant Dirac operator in a constant external abelian magnetic field, directed along the third axes and calculate on the lattice

1. the correlator of two vector currents, axial-vector currents and pseudoscalar versus the value of magnetic field in the confinement phase,
2. perform mass extrapolation on the lattice,
3. explore the dependence of neutral ρ^0 and A^0 meson masses versus the magnetic field.

$eB \sim \Lambda^2_{QCD}$



$$B \sim 10^{15}\ T, \ \sqrt{eB} \sim 10\ Mev...400\ MeV$$

Technical details

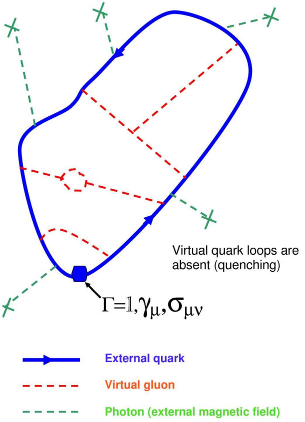
Model: $SU(2)$ gluodynamics.
Generation of A_μ: the tadpole-improved Wilson-Symanzik action.
Fermion spectrum: chirally-invariant overlap operator, 50 lowest modes for the inversion, different quark masses
$am_{lat} = 0.01 \div 0.8$.

$$\langle \psi^\dagger(x) O_1 \psi(x) \psi^\dagger(y) O_2 \psi(y) \rangle_A =$$

$$\int DA_\mu e^{-S_{YM}}[A_\mu] Tr\left(\frac{1}{D+m} O_1\right) Tr\left(\frac{1}{D+m} O_2\right) -$$

$$\int DA_\mu e^{-S_{YM}}[A_\mu] Tr\left(\frac{1}{D+m} O_1 \frac{1}{D+m} O_2\right)$$

$O_1, O_2 = \gamma_5,\ \gamma_5\gamma_\mu,\ \gamma_\mu$



We calculate $\bar{\psi}\Gamma\psi$, where $\gamma_\mu,\ \gamma_5\gamma_\mu,\ \gamma_5$ in the external abelian magnetic field and in the presence of the vacuum nonabelian gluon fields.

Technical details

To add magnetic field $F_{12} = B_3 = B$ in the overlap operator we:

- make the exchange

$$A_{\mu\, ij} \to A_{\mu\, ij} + A_\mu^{(B)} \delta_{ij}, \qquad A_\mu^{(B)}(x) = \frac{B}{2}(x_1 \delta_{\mu,2} - x_2 \delta_{\mu,1})$$

- perform the additional twist for fermions (for p.b.c).

$M.\,H.\,Al-Hashimi,\ U.\,J.\,Wiese,\ 2009.$

The magnetic field is quantized:

$$qB = \frac{2\pi k}{L^2}, \quad k \in \mathbb{Z}, \quad q = -e/3$$

Correllators and spectral functions

are extracted from the correlator of currents

$$\langle\psi^\dagger(\vec{0},n_t)O_1\psi(\vec{0},n_t)\psi^\dagger(\vec{0},0)O_2\psi(\vec{0},0)\rangle_A=\sum_k\langle 0|O_1|k\rangle\langle k|O^\dagger|0\rangle e^{-n_tE_k}.$$

The main contributions comes from $\langle 0|O_1|k\rangle\langle k|O^\dagger|0\rangle e^{-n_tE_0}$, fit the correlator by the function $C(n_t)=A\cosh(m_{eff}(n_t-N_T/2))$ at $4<n_t<N_T-4$

$$G(\tau,\vec{p})=\int d^3x\,\langle j_\mu(\tau,\vec{x})j_\mu^\dagger(0,\vec{0})\rangle e^{-i\vec{p}\vec{x}}$$

The corresponding spectral function $\rho(\omega,\vec{p})$ is defined by

$$G(\tau,\vec{p})=\int_0^\infty\frac{d\omega}{2\pi}K(\tau,\omega)\rho(\omega,\vec{p}).$$

$$K(\tau,\omega)=\frac{\omega}{2T}\frac{\cosh(\omega(\tau-\frac{1}{2T})))}{\sinh(\frac{\omega}{2T})},$$

Maximal Entropy Method

This procedure is equivalent to the minimization of the free energy $F = L - \alpha S$ or maximization of conditional probability

$$P[\rho|DH\alpha m] = exp\left(-\frac{1}{2}\chi^2 + \alpha S\right)$$

$$S = \int_0^\infty \frac{d\omega}{2\pi}\left[\rho(\omega) - m(\omega) - \rho(\omega) ln\frac{\rho(\omega)}{m(\omega)}\right]$$

χ^2 is the likelihood function, α is a parameter, S is the entropy term, H is our hypothesis. The default model is

$$\bar{m}(\omega) = m_a * \omega + m_b, \quad m_a = \frac{G(N_\tau/2)}{9,8696 * T^2}, \quad m_b = a_H \frac{3}{8\pi^2},$$

where $a_H = 1$ for scalar and pseudoscalar channel, $a_H = 2$ for vector and axial vector channel.

Use modifyed Bryan algorithm. Redefine the kernel and spectral function

$$\bar{K}(\omega,\tau) = \frac{\omega}{2T}K(\omega,\tau), \quad \bar{\rho}(\omega) = \frac{2T}{\omega}\rho(\omega)$$

then apply SVD theorem ($K = U_{N_\omega \times N} W_{N\times N} V^T_{N\times N}$) to the modified discretized kernel $\bar{K}(\omega_n,\tau_i)$ to find the spectral function

$$\bar{\rho}(\omega) = \bar{m}(\omega) exp \sum_{i=1}^{N} \bar{c}_i \bar{u}_i(\omega)$$

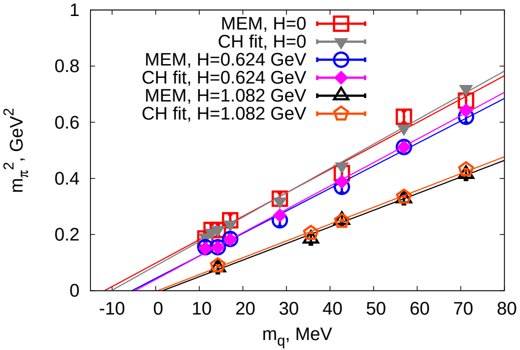
The column vectors $u_i, \ (i = 1,..,N)$ are normalized

$$\langle u_i | u_j \rangle \equiv \sum_{n=1}^{N_\omega} u_i(\omega_n) u_j(\omega_n) = \delta_{ij},$$

c_i are the coefficients and we set $\bar{K}(0,\tau) = 1$.

Renormalization of quark bare mass on the lattice

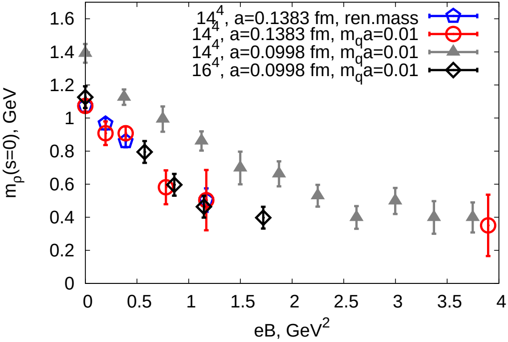
$$f_\pi^2 m_\pi^2 = 2m_q \langle\bar{\psi}\psi\rangle$$



Perform the mass extrapolation to the m_{phys} corresponding to to $m_\pi = 135\ MeV$, $m_{lat} = \delta m_{ren} + m_{phys}$. The results are obtained for the 14^4 lattice volume, lattice spacing 0.1383 fm.

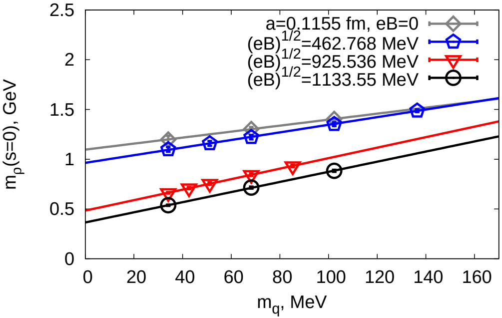
Mass of $\rho^0(s=0)$ meson. MEM procedure.

$$C_{zz}^{VV} = \langle \bar{\psi}(\vec{0}, n_t)\gamma_3\psi(\vec{0}, n_t)\bar{\psi}(\vec{0}, 0)\gamma_3\psi(\vec{0}, 0)\rangle$$



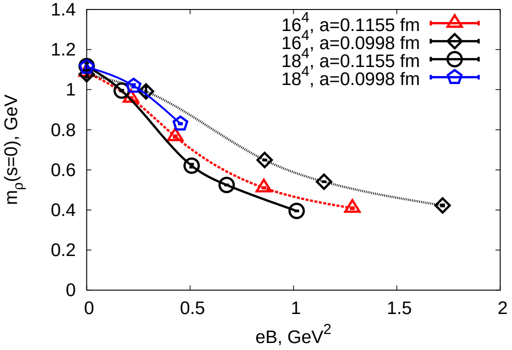
Mass of $\rho^0(s=0)$ meson versus the magnetic field. Results were obtained with Maximal Entropy method.

Mass extrapolation for $\rho^0(s=0)$ meson. Fit.



The $\rho^0(s=0)$ - meson mass versus the value of lattice quark mass. $m_{lat}(H=0) = -3.25\ MeV$, $m_{lat}(H=1) = -1.55\ MeV$, $m_{lat}(H=6) = 8.76\ MeV$.

Mass of $\rho^0(s=0)$ meson. Fit.



The $\rho^0(s=0)$ - meson mass versus the magnetic field field after mass extrapolation and lattice quark mass renormalization.

Correlators of currents in the directions perpendicular to the magnetic field

$$C_{yy}^{VV} = \langle \bar{\psi}(\vec{0}, n_t)\gamma_2\psi(\vec{0}, n_t)\bar{\psi}(\vec{0}, 0)\gamma_2\psi(\vec{0}, 0)\rangle$$

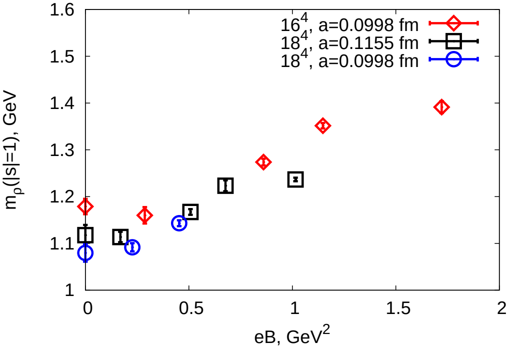
$$C_{xx}^{VV} = \langle \bar{\psi}(\vec{0}, n_t)\gamma_1\psi(\vec{0}, n_t)\bar{\psi}(\vec{0}, 0)\gamma_1\psi(\vec{0}, 0)\rangle$$

We extract masses with $s = \pm 1$ from the combinations

$$C^{VV}(s=1) = \frac{1}{\sqrt{2}}(C_{xx}^{VV} + iC_{yy}^{VV})$$

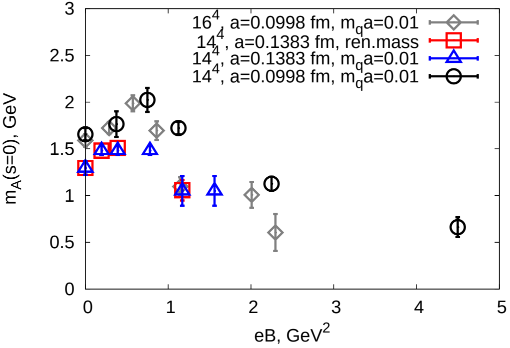
$$C^{VV}(s=-1) = -\frac{1}{\sqrt{2}}(C_{xx}^{VV} - iC_{yy}^{VV})$$

$\rho^0(s=\pm1)$ meson mass. Fit.



The $\rho^0(s=\pm1)$ meson mass with nonzero spin projection to the direction of the magnetic field after mass extrapolation and lattice quark mass renormalization.

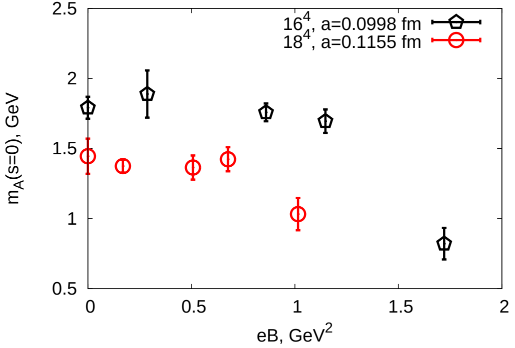
$A^0(s=0)$ meson mass. MEM procedure.



Mass of $A^0(s=0)$ meson versus the magnetic field for lattice quark mass $m_q a = 0.01$ and for renormalized mass.

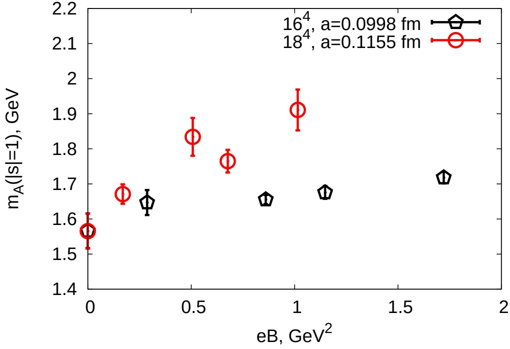
$A^0(s=0)$ meson mass. Fit.

$$C^{AA}_{zz} = \langle \bar{\psi}(\vec{0}, n_t)\gamma_5\gamma_3\psi(\vec{0}, n_t)\bar{\psi}(\vec{0}, 0)\gamma_5\gamma_3\psi(\vec{0}, 0)\rangle$$



The $A^0(s=0)$ meson mass after mass extrapolation and lattice quark mass renormalization .

$A^0(s = \pm 1)$ meson mass. Fit.



The $A^0(s = \pm 1)$ meson mass with nonzero spin projection to the direction of the magnetic field after mass extrapolation and lattice quark mass renormalization.

Conclusions

1. In $SU(2)$ gluodynamics the masses of ρ^0 and A^0 mesons with spin projection $s = 0$ to the direction of the magnetic field decrease with the value of the field.
2. The masses of neutral mesons with spin projection $s = \pm 1$ increase with the magnetic field.