\( \rho^0 \) and \( A^0 \) mesons in the background of strong magnetic fields

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We work in \( SU(2) \) gluodynamics with chirally invariant Dirac operator in a constant external abelian magnetic field, directed along the third axes and calculate on the lattice

1. the correlator of two vector currents, axial-vector currents and pseudoscalar versus the value of magnetic field in the confinement phase,

2. perform mass extrapolation on the lattice,

3. explore the dependence of neutral \( \rho^0 \) and \( A^0 \) meson masses versus the magnetic field.
$\rho^0$ and $A^0$ mesons in the background of strong magnetic fields

\[ eB \sim \Lambda_{QCD}^2 \]

B \sim 10^{15} \ T, \ \sqrt{eB} \sim 10 \ MeV \ldots 400 \ MeV
Model: $SU(2)$ gluodynamics.

Generation of $A_\mu$: the tadpole-improved Wilson-Symanzik action.

Fermion spectrum: chirally-invariant overlap operator, 50 lowest modes for the inversion, different quark masses

$$am_{\text{lat}} = 0.01 \div 0.8 .$$

$$\langle \psi^\dagger(x)O_1 \psi(x)\psi^\dagger(y)O_2 \psi(y) \rangle_A =$$

$$\int DA_\mu e^{-SYM[A_\mu]} \text{Tr} \left( \frac{1}{D+m}O_1 \right) \text{Tr} \left( \frac{1}{D+m}O_2 \right) -$$

$$\int DA_\mu e^{-SYM[A_\mu]} \text{Tr} \left( \frac{1}{D+m}O_1 \frac{1}{D+m}O_2 \right)$$

$O_1, O_2 = \gamma_5, \gamma_5\gamma_\mu, \gamma_\mu$
We calculate $\bar{\psi} \Gamma \psi$, where $\gamma_\mu$, $\gamma_5 \gamma_\mu$, $\gamma_5$ in the external abelian magnetic field and in the presence of the vacuum nonabelian gluon fields.
Technical details

To add magnetic field $F_{12} = B_3 = B$ in the overlap operator we:

- make the exchange

$$A_{\mu}^{ij} \rightarrow A_{\mu}^{ij} + A^{(B)}_{\mu} \delta_{ij}, \quad A^{(B)}_{\mu}(x) = \frac{B}{2}(x_1 \delta_{\mu,2} - x_2 \delta_{\mu,1})$$

- perform the additional twist for fermions (for p.b.c).

_M. H. Al-Hashimi, U. J. Wiese, 2009._

The magnetic field is quantized:

$$qB = \frac{2\pi k}{L^2}, \quad k \in \mathbb{Z}, \quad q = -e/3$$
Correlators and spectral functions

are extracted from the correlator of currents

$$\langle \bar{\psi}(0, n_t) O_1 \psi(0, n_t) \bar{\psi}(0, 0) O_2 \psi(0, 0) \rangle_A = \sum_k \langle 0 \mid O_1 \mid k \rangle \langle k \mid O_1^\dagger \mid 0 \rangle e^{-n_t E_k}.$$  

The main contributions comes from $$\langle 0 \mid O_1 \mid k \rangle \langle k \mid O_1^\dagger \mid 0 \rangle e^{-n_t E_0}$$, fit the correlator by the function $$C(n_t) = A \cosh(m_{\text{eff}}(n_t - N_T/2))$$ at $$4 < n_t < N_T - 4$$

$$G(\tau, \vec{p}) = \int d^3 x \langle j_\mu(\tau, \vec{x}) j_\mu^\dagger(0, \vec{0}) \rangle e^{-i\vec{p}\vec{x}}$$

The corresponding spectral function $$\rho(\omega, \vec{p})$$ is defined by

$$G(\tau, \vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho(\omega, \vec{p}).$$

$$K(\tau, \omega) = \frac{\omega}{2T} \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})},$$
\( \rho^0 \) and \( A^0 \) mesons in the background of strong magnetic fields

Maximal Entropy Method

This procedure is equivalent to the minimization of the free energy \( F = L - \alpha S \) or maximization of conditional probability

\[
P[\rho|DH\alpha m] = \exp \left( -\frac{1}{2} \chi^2 + \alpha S \right)
\]

\[
S = \int_0^\infty \frac{d\omega}{2\pi} \left[ \rho(\omega) - m(\omega) - \rho(\omega) \ln \frac{\rho(\omega)}{m(\omega)} \right]
\]

\( \chi^2 \) is the likelihood function, \( \alpha \) is a parameter, \( S \) is the entropy term, \( H \) is our hypothesis. The default model is

\[
\tilde{m}(\omega) = m_a \ast \omega + m_b, \quad m_a = \frac{G(N_T/2)}{9,8696 \ast T^2}, \quad m_b = a_H \frac{3}{8\pi^2},
\]

where \( a_H = 1 \) for scalar and pseudoscalar channel, \( a_H = 2 \) for vector and axial vector channel.
Use modified Bryan algorithm. Redefine the kernel and spectral function

\[ \tilde{K}(\omega, \tau) = \frac{\omega}{2T} K(\omega, \tau), \quad \tilde{\rho}(\omega) = \frac{2T}{\omega} \rho(\omega) \]

then apply SVD theorem \((K = U_{N_\omega \times N} W_{N \times N} V_{N \times N}^T)\) to the modified discretized kernel \(\tilde{K}(\omega_n, \tau_i)\) to find the spectral function

\[ \tilde{\rho}(\omega) = \tilde{m}(\omega) \exp \sum_{i=1}^{N} \tilde{c}_i \tilde{u}_i(\omega) \]

The column vectors \(u_i, \ (i = 1, \ldots, N)\) are normalized

\[ \langle u_i | u_j \rangle \equiv \sum_{n=1}^{N_\omega} u_i(\omega_n) u_j(\omega_n) = \delta_{ij}, \]

\(c_i\) are the coefficients and we set \(\tilde{K}(0, \tau) = 1.\)
Perform the mass extrapolation to the $m_{\text{phys}}$ corresponding to $m_\pi = 135 \text{ MeV}, m_{\text{lat}} = \delta m_{\text{ren}} + m_{\text{phys}}$. The results are obtained for the $14^4$ lattice volume, lattice spacing 0.1383 fm.
Mass of $\rho^0(s=0)$ meson. MEM procedure.

$$C^{VV}_{zz} = \langle \bar{\psi}(\vec{0}, n_t) \gamma_3 \psi(\vec{0}, n_t) \bar{\psi}(\vec{0}, 0) \gamma_3 \psi(\vec{0}, 0) \rangle$$

Mass of $\rho^0(s=0)$ meson versus the magnetic field. Results were obtained with Maximal Entropy method.
The $\rho^0(s = 0)$ - meson mass versus the value of lattice quark mass. $m_{\text{lat}}(H = 0) = -3.25$ MeV, $m_{\text{lat}}(H = 1) = -1.55$ MeV, $m_{\text{lat}}(H = 6) = 8.76$ MeV.
\( \rho^0 \) and \( A^0 \) mesons in the background of strong magnetic fields

\( \rho^0(s = 0) \) meson

Mass of \( \rho^0(s = 0) \) meson. Fit.

The \( \rho^0(s = 0) \) - meson mass versus the magnetic field field after mass extrapolation and lattice quark mass renormalization.
Correlators of currents in the directions perpendicular to the magnetic field

\[
C^{VV}_{yy} = \langle \bar{\psi}(\vec{0}, n_t) \gamma_2 \psi(\vec{0}, n_t) \bar{\psi}(\vec{0}, 0) \gamma_2 \psi(\vec{0}, 0) \rangle
\]

\[
C^{VV}_{xx} = \langle \bar{\psi}(\vec{0}, n_t) \gamma_1 \psi(\vec{0}, n_t) \bar{\psi}(\vec{0}, 0) \gamma_1 \psi(\vec{0}, 0) \rangle
\]

We extract masses with \( s = \pm 1 \) from the combinations

\[
C^{VV}(s = 1) = \frac{1}{\sqrt{2}} (C^{VV}_{xx} + iC^{VV}_{yy})
\]

\[
C^{VV}(s = -1) = -\frac{1}{\sqrt{2}} (C^{VV}_{xx} - iC^{VV}_{yy})
\]
The $\rho^0(s = \pm 1)$ meson mass with nonzero spin projection to the direction of the magnetic field after mass extrapolation and lattice quark mass renormalization.
$\rho^0$ and $A^0$ mesons in the background of strong magnetic fields

$A^0(s = 0)$ meson

$A^0(s = 0)$ meson mass. MEM procedure.

Mass of $A^0(s = 0)$ meson versus the magnetic field for lattice quark mass $m_qa = 0.01$ and for renormalized mass.
$\rho^0$ and $A^0$ mesons in the background of strong magnetic fields

$A^0(s = 0)$ meson

$A^0(s = 0)$ meson mass. Fit.

$$C_{zz}^{AA} = \langle \bar{\psi}(\vec{0}, n_t) \gamma_5 \gamma_3 \psi(\vec{0}, n_t) \bar{\psi}(\vec{0}, 0) \gamma_5 \gamma_3 \psi(\vec{0}, 0) \rangle$$

The $A^0(s = 0)$ meson mass after mass extrapolation and lattice quark mass renormalization.
$\rho^0$ and $A^0$ mesons in the background of strong magnetic fields

$A^0(s = 0)$ meson

$A^0(s = \pm 1)$ meson mass. Fit.

The $A^0(s = \pm 1)$ meson mass with nonzero spin projection to the direction of the magnetic field after mass extrapolation and lattice quark mass renormalization.
In $SU(2)$ gluodynamics the masses of $\rho^0$ and $A^0$ mesons with spin projection $s = 0$ to the direction of the magnetic field decrease with the value of the field.

The masses of neutral mesons with spin projection $s = \pm 1$ increase with the magnetic field.