Hadron spectra and leptonic decay constants with overlap fermions on HISQ gauge configurations

Nilmani Mathur
Department of Theoretical Physics
Tata Institute of Fundamental Research, India

Collaborators: S. Basak, S. Datta, A. Lytle, P. Majumdar, M. Padmanath

(Indian Lattice Gauge Theory Initiative)

Lattice 2013, Mainz, Germany
Overlap Fermions

- **Some desirable features:**
  - No $O(a)$ error.
  - Multi-mass algorithm (more than 20 masses
    - 8-12% overhead
  - Renormalization is presumed to be relatively simple (e.g. with chiral Ward identity).

- **Undesirable feature:**
  - -- Cost†
Overlap fermions on 2+1+1 Flavors
HISQ Configurations

Lattices used for this study:
HISQ gauge configurations from MILC

- 32^3 \times 96, \ a = 0.089 \text{ fm}, \ m_l/m_s = 1/5, \ m_\pi L = 4.5, \ m_\pi = 312 \text{ MeV}
- 48^3 \times 144, \ a = 0.058 \text{ fm}, \ m_l/m_s = 1/5, \ m_\pi L = 4.51, \ m_\pi = 319 \text{ MeV}

HYP smearing on gauge fields
- Both point source and coulomb gauge fixed wall source are used
- No of eigenvectors projected: 350 (a = 0.089 \text{ fm}) and 75 (a = 0.058 \text{ fm})
- Preliminary results on our ongoing study will be reported here
Rest mass Vs Kinetic mass

Charm mass is tuned by meson kinetic mass and not from rest mass
……a la FermiLab formulation

Expanding the energy momentum relation in powers of $pa$

$$E^2 = M_1^2 + \frac{M_1}{M_2} p^2 + \cdots$$

$$= M_1^2 + c^2 p^2$$

Rest mass : $M_1 = E(0)$

Kinetic mass : $M_2 = M_1/c^2$
Dispersion relation (at charm mass)

\[ E^2(p) = E^2(p = 0) + p^2 c^2 \]

Finite momentum wall source is used to project to particular momentum state which reduce errorbars substantially.
Lattice spacings and tuning of charm and strange masses

Lattice spacings are calculated by Omega(sss) mass = 1672 GeV

48³ x 144 : 0.0582(5) fm
32³ x 96 : 0.0877(10) fm which are quite consistent with lattice spacings determined by MILC

- Strange mass is tuned by setting pseudoscalar $ss$ mass at 685 MeV
  
  $m_s a = 0.048$ (a = 0.0888 fm)
  $= 0.028$ (a = 0.0582 fm)

- Charm mass is tuned by
  
  $m_c a = 0.425-0.43$ (a = 0.0888fm),
  $= 0.29$ (a = 0.0582 fm)

  Considering kinetic masses of mesons (a la Fermilab formulation)
Lattice spacings and tuning of charm and strange masses

Lattice spacings are calculated by Omega(sss) mass = 1672 GeV

48^3 \times 144 : 0.0582(5) \text{ fm}
32^3 \times 96 : 0.0877(10) \text{ fm} \quad \text{which are quite consistent with lattice spacings determined by MILC}

- Strange mass is tuned by setting pseudoscalar ss mass at 685 MeV

  \begin{align*}
  m_s a &= 0.048 \text{ (a = 0.0888 fm)} \\
  &= 0.028 \text{ (a = 0.0582 fm)}
  \end{align*}

Taking m_s = 100 \text{ MeV}

  \begin{align*}
  m_s a &= 0.0450 \text{ (a = 0.0888fm)}, \\
  &= 0.0295 \text{ (a = 0.0582fm)}
  \end{align*}

- Charm mass is tuned by

  \[ \frac{1}{4} \left( m_{\eta_c} + 3 m_{J/\psi} \right) \]

  \begin{align*}
  m_c a &= 0.425—0.43 \text{ (a = 0.0888fm)}, \\
  &= 0.29 \text{ (a = 0.0582 fm)}
  \end{align*}

Considering kinetic masses of mesons (a la Fermilab formulation)
Pseudo-scalar eff. masses
Pseudoscalar meson mass

Graphs showing the relationship between $m_{\pi}$ (MeV) and $ma$ (MeV) for two different sets of data.
Pseudoscalar meson mass

\[(m_\pi a)^2\]

\[(m_\rho a)^2\]

\[(m_\omega a)^2\]

\[(m_\phi a)^2\]
Effective mass for HFS \((48^3 \times 144, a = 0.0582\text{fm})\)
Meson mass splittings

- $h_c - \eta_c$
- $\chi_{c1} - \eta_c$
- $D_{s1} - D_s$
- $D_s - \eta_c/2$
- $\chi_{c0} - \eta_c$
- $D_{s0} - D_s$
- $D_{s*} - D_s$
- $\psi - \eta_c$

Mass Splitting (MeV) vs. Lattice Spacing (fermi)
Singly and doubly-charmed $\Omega$ baryons

(48$^3 \times 144$, a = 0.0582fm)
Triply-charmed $\Omega_{ccc}^{(3/2^+)}$ baryon

\begin{center}
\begin{tikzpicture}
\begin{axis}[
    xlabel={HSC-2013, ILGTI-2013, PACS-CS-2012},
    ylabel={$m_{\Omega_{ccc,g}} - 3/2 m_{J/\psi}$ (GeV)},
    xmin=0.65, xmax=0.21,
    ymin=0.06, ymax=0.21,
    xtick={0.65, 0.75, 0.85},
    ytick={0.06, 0.09, 0.12, 0.15, 0.18, 0.21},
    legend pos=north west,
]
\addplot[fill=blue!30] coordinates {(0.66, 0.18) (0.74, 0.15)} node [pos=0.5] {$c_{SW}=1.35$};
\addplot[fill=red!30] coordinates {(0.74, 0.15) (0.82, 0.12)} node [pos=0.5] {$a=0.0582\text{fm}$};
\addplot[fill=green!30] coordinates {(0.82, 0.12) (0.9, 0.09)} node [pos=0.5] {$a=0.09\text{fm}$};
\end{axis}
\end{tikzpicture}
\end{center}
Decay constants

\[ \langle 0 | \bar{c}(0) \gamma_\mu \gamma_5 q(0) | D_q(p) \rangle = f_{D_q} p_\mu \]

\[ \langle 0 | \bar{c}(0) \gamma_\mu q(0) | D_q^*(p, \lambda) \rangle = f_{D_q^*} m_{D_q^*} e^\lambda \]

From PCAC:

\[ M_{PS}^2 f_{PS} = (\mu_1 + \mu_2) |\langle 0 | P^1(0) | PS \rangle| \]

\[ \mu_{1,2} \text{ are the bare quark masses} \]

\[ Z_m Z_P = 1 \]

\[ f_{D_s} = \frac{(m_c + m_s)}{m_{D_s}^2} \sqrt{2A m_{D_s}} \]

\[ x \equiv |\langle 0 | P | D_s \rangle|, \quad 2A = x^2/m_{D_s} \]

Both i) point-point propagators and ii) wall-point with wall-wall propagators were utilized.
The ratio \( \frac{f_{D^*s}}{f_{Ds}} \)

\[ \langle 0 | \bar{c}(0) \gamma_\mu \gamma_5 q(0) | D_q(p) \rangle = f_{D_q} p_\mu \]

\[ \langle 0 | \bar{c}(0) \gamma_\mu q(0) | D^*_q(p, \lambda) \rangle = f_{D^*_q} m_{D^*_q} e_\mu^\lambda \]

\[ m_{Ds} f_{Ds} = Z_A |\langle 0 | A4 | D_s \rangle| \]

\[ Z_A = \frac{m_{Ds} f_{Ds}}{\sqrt{2A m_{Ds}}} \]

\[ f_{D^*_s} = \frac{Z_A}{m_{D^*_s}} |\langle 0 | V | D^*_s \rangle| \]

It is better to calculate the ratio \( \frac{f_{D^*s}}{f_{Ds}} \) where the effect of various normalization factors and mixed action effect will be smaller.
Summary and outlook

✓ Overlap valence on 2+1+1 flavour HISQ configurations is a promising approach to do lattice QCD simulation with light, strange and charm quark together in same lattice formulation.

✓ However, we found that the dispersion relation with overlap fermions, at charm mass, is not better than that of clover fermions found in literature.

✓ Kinetic masses of mesons are used instead of pole masses to tune charm quark mass. Dispersion relation improved at kinetic masses.

✓ Preliminary results are encouraging, particularly, the hyperfine splitting for charmonium. We are studying meson and baryon spectra in details.

✓ We are also studying heavy-light decay constants. Necessary renormalization constant calculations are ongoing.

✓ We also need to calculate the mixing parameter for this mix action approach.

Acknowledgement:

➢ Computations : ILGTI-TIFR BG/P
➢ Thanks to MILC collaboration (particularly, S. Gottlieb) for giving access to HISQ configurations
➢ Overlap issues : χQCD collaboration
Mixed action effects

Mixed action

For chirally symmetric valence, it is like partial quenching with one extra parameter in valence-sea mass (Chen, O’Connell, Walker-Loud, hep-lat/0611003, arXiv:0706.0035)

\[
\begin{align*}
    m^2_{v_1 v_2} &= B_0 (m_{v_1} + m_{v_2}), \\
    \tilde{m}^2_{us} &= B_0 (m_u + m_s) + a^2 \Delta_{Mix}, \\
    \tilde{m}^2_{s_1 s_2} &= B_0 (m_{s_1} + m_{s_2}) + a^2 \Delta_{sea},
\end{align*}
\]

Mixed action effect for overlap on domain wall gauge configurations was found to be small...  M. Lujan et. al., arXiv:1204.6256v1