

# DK scattering and the $D_s$ spectrum

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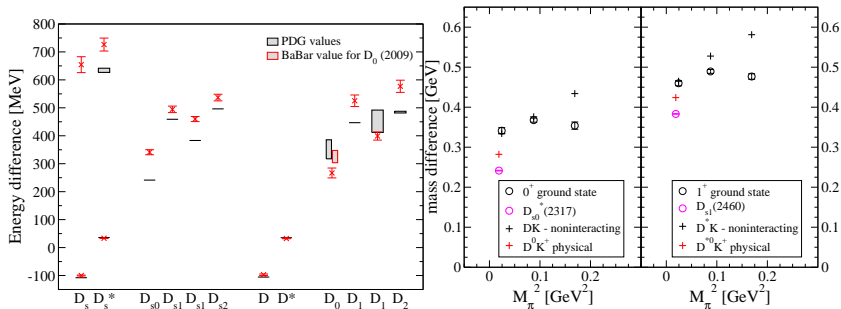


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# Motivation: Experimental $D_s$ spectrum

- Established states:
  - $D_s$  ( $J^P = 0^-$ ) and  $D_s^*$  ( $1^-$ )
  - $D_{s0}^*(2317)$  ( $0^+$ ),  $D_{s1}^*(2460)$  ( $1^+$ ),  $D_{s1}(2536)$  ( $1^+$ ),  $D_{s2}^*(2573)$  ( $2^+$ )
- More recent discoveries:
  - $D_{s1}^*(2710)$  seen by BaBar, Belle ( $1^-$ )
  - $D_{sJ}^*(2860)$  seen by BaBar ( $3^-?, 0^+?$ )
  - $D_{sJ}^*(3040)$  seen by BaBar ( $1^+?, 2^-?$ )
  - $D_{sJ}^*(2632)$  seen by SELEX ( $1^-?$ )
- There is a zoo of phenomenological models and lattice results are getting dated
- Some models suggest a tetraquark/molecular interpretations for controversial states

# Our previous attempt...

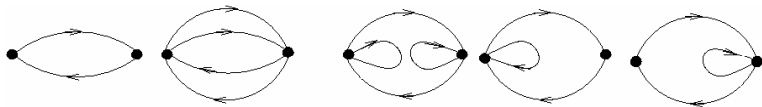


Mohler and Woloshyn, PRD 84 054503, 2011

- $DK$  threshold turned out to be unphysical
- Even with light sea-quark masses the lowest states with  $J^P = 0^+, 1^+$  remained unphysical
- Including the  $DK$  threshold explicitly might be vital

# For our current efforts

- Work with a partially quenched strange quark
  - Use  $\phi$  meson and  $\eta_s$  to set strange quark mass
  - We obtain  $\kappa_s = 0.13666$
- Improve charm quark tuning used for Fermilab charm
  - Use Landau link for  $c_{sw,c} = \frac{1}{u_0^3}$
  - Empirically this reduces discretization effects
- Explicitly include DK interpolators into the basis



# Technicalities: The “Distillation” method

Peardon et al. PRD 80, 054506 (2009); Morningstar et al. PRD 83, 114505 (2011)

- Idea: Construct separable quark smearing operator using low modes of the 3D lattice Laplacian  
Spectral decomposition for an  $N \times N$  matrix:

$$f(A) = \sum_{k=1}^N f(\lambda^{(k)}) v^{(k)} v^{(k)\dagger}.$$

With  $f(\nabla^2) = \Theta(\sigma_s^2 + \nabla^2)$  (Laplacian-Heaviside (LapH) smearing):

$$q_s \equiv \sum_{k=1}^N \Theta(\sigma_s^2 + \lambda^{(k)}) v^{(k)} v^{(k)\dagger} q = \sum_{k=1}^{N_v} v^{(k)} v^{(k)\dagger} q.$$

- Advantages: momentum projection at source; large interpolator freedom, small storage
- Disadvantages: expensive; unfavorable volume scaling
- Stochastic approach improves bad volume scaling

# Technicalities II: Lattices used

ID	$N_L^3 \times N_T$	$N_f$	$a[\text{fm}]$	$L[\text{fm}]$	#configs	$m_\pi[\text{MeV}]$	$m_K[\text{MeV}]$
(1)	$16^3 \times 32$	2	0.1239(13)	1.98	280/279	266(3)(3)	552(2)(6)
(2)	$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

- Ensemble (1) has 2 flavors of nHYP-smearred quarks

Gauge ensemble from Hasenfratz et al. PRD 78 054511 (2008)

Hasenfratz et al. PRD 78 014515 (2008)

- Ensemble (2) has 2+1 flavors of Wilson-Clover quarks

PACS-CS, Aoki et al. PRD 79 034503 (2009)

- On the larger volume we use stochastic distillation

Morningstar et al. PRD 83, 114505 (2011)

# Technicalities III: Charm quark treatment

- We use the *Fermilab method* for the heavy (charm) quark

El-Khadra et al., PRD 55, 3933 (1997)

- We tune  $\kappa$  so that the spin averaged **kinetic mass**  $(M_{\eta_c} + 3M_{J/\psi})/4$  assumes its physical value
- General form for the dispersion relation

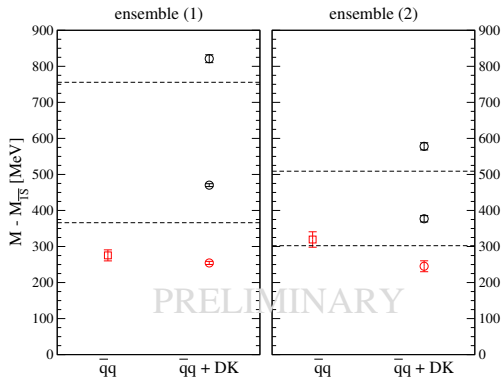
Bernard et al. PRD 83,034503 (2011)

$$E(p) = M_1 + \frac{p^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(p^2)^2}{8M_4^3} + \dots$$

- We tried different strategies and neglect the term with  $W_4$  for the final analysis
- For the DK we therefore use

$$E = \sqrt{m_K^2 + p^{*2}} + M_1 + \frac{p^{*2}}{2M_2} - \frac{p^{*4}}{8M_4^3}.$$

# Energy levels for $D_s$ with $J^P = 0^+$



- With the combined basis we obtain a much better quality of the ground state plateau
- The variational method yields two low-lying levels and fits are unambiguous



# Possible interpretations

## (1) A sub-threshold state stable under the strong interaction

- We call this “bound state scenario”
- This is irrespective of the nature of the state
- One expects a negative scattering length in this case

See Sasaki and Yamazaki, PRD 74 114507 (2006) for details.

See also NPLQCD, arXiv 1301.5790 for an example.

## (2) A resonance in a channel with attractive interaction

- The lowest state corresponds to the scattering level shifted below threshold in finite volume
- The additional level would indicate a QCD resonance
- One expects a positive scattering length in this case

This is the situation for the  $D_0^*(2400)$  DM, Prelovsek, Woloshyn PRD 87 034501 (2013).

# Using Lüscher's formula

- We can test the plausibility of these scenarios using Lüscher's formula and an effective range approximation

M. Lüscher Commun. Math. Phys. 105 (1986) 153;  
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi} L} Z_{00}(1, p^2),$$
$$\approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2,$$

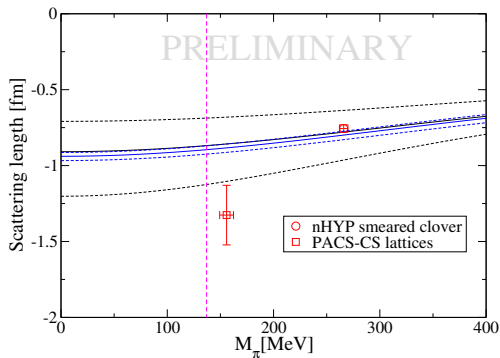
- **Preliminary** results for ensembles (1) and (2)

$$a_0 = -0.756 \pm 0.025 \text{ fm} \qquad r_0 = 0.056 \pm 0.031 \text{ fm} \qquad (1)$$

$$a_0 = -1.33 \pm 0.20 \text{ fm} \qquad r_0 = 0.27 \pm 0.17 \text{ fm} \qquad (2)$$

- We are still investigating the systematics

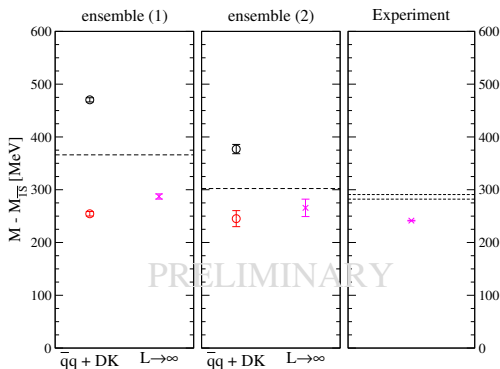
# Results for the scattering length $a_0$



- We compare to the predictions from an indirect calculation  
Liu *et al.* PRD 87 014508 (2013).
- Our determination robustly leads to negative values.

# Infinite volume bound states vs. experiment

- For a bound state we expect an S-matrix pole and  $\cot \delta = i$
- Using our  $a_0$  and  $r_0$  we can determine the binding momentum and calculate the corresponding energy level



# Conclusions

- We calculated energy levels in the  $D_s J^P = 0^+$  channel with a combined basis of  $\bar{q}q$  and DK interpolators
- We use partially quenched strange quarks, Fermilab c quarks and almost physical  $u/d$  quarks
- The DK interpolators are crucial to get reliable energy levels
- We observe an energy level compatible with the experimental  $D_{s0}^*(2317)$
- The situation is similar but more messy for the  $D_{s1}(2460)$ ...

For a similar situation see talk by S. Prelovsek Thursday 17:30 in 8G

# Thank you!

# Backup: Example energies

