Excited Spectroscopy of Mesons Containing Charm Quarks From Lattice QCD

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(For the Hadron Spectrum Collaboration)
1. Experimental motivation
2. Ensemble details
3. HadSpec recipe for spectroscopy
4. Results
   - Charmonium spectrum
   - $D$ and $D_s$ spectra
   - Hybrid mesons
   - $D\pi$ scattering ($l = 3/2$) - Preliminary
Pre 2003 - charm spectroscopy well explained via quark models - $^2S+1L_J$

New narrow charmonium-like structures are observed by BABAR and Belle above the open charm threshold ("X,Y,Z’s")

Too many states for the $^2S+1L_J$ pattern to explain $\Rightarrow$ renewed theoretical interest . . . what could the states be?

- $X(3872)$: close to the $D\bar{D}^*$ threshold $\Rightarrow$ a molecular meson?
- $X(4260)$: a $1^{--}$ hybrid meson?
- $X(4430)\pm$: a charged entity $\Rightarrow$ can’t be $c\bar{c}$, maybe a tetra-quark?

Still no clear picture has emerged
BABAR observes the $D_{s0}^*(2317)^\pm$ state


CLEO confirms the BABAR discovery and observes a further resonance $D_{s1}(2460)^\pm$


Significantly Lighter and narrower than quark model predictions

Ensemble Details

Calculations performed on lattices generated by the Hadron Spectrum Collaboration

- Dynamical - $N_f = 2 + 1$
- Anisotropic - $\xi = a_s/a_t \sim 3.5$
- Scale set via $M_\Omega$: $a_s = 0.1227(8)$ fm, $a_t^{-1} = 5.67(4)$ GeV
- Two volumes: $16^3 \times 128$ and $24^3 \times 128$
- Clover fermions: On-shell $O(a)$ improvement
- Spatial links are stout smeared
- Quark fields are distilled
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*Caveat: Pion mass \( \sim 391 \text{ MeV} \)*
Recipe for the calculation of **extensive** spectra:

1. Use basis of local and non-local operators from distilled fields

\[ \bar{\Psi}(x) \Gamma D_i D_j \ldots \Psi(x) \]

We include:
- All combinations of $\gamma$-matrices and derivatives up to three derivatives
- Operators $\sim F_{\mu\nu} \Rightarrow$ access gluonic degrees of freedom
- Operators that let us explore all $J^P(C)$ up to $J = 4$

2. Build a correlation matrix from two-point correlation functions

\[
C_{ij} = \langle 0 | O_i O_j^\dagger | 0 \rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} \ e^{-E_n t}
\]
Use a variational method - solve the generalised eigenvalue equation

$$C_{ij}(t)v^{(n)}_j = \lambda^{(n)}(t)C_{ij}(t_0)v^{(n)}_j$$

This gives:

- **Eigenvalues**: $\lambda^{(n)}(t) \sim e^{-E_n t} \left[1 + O(e^{-\Delta E t})\right]$ - principle correlator

- **Eigenvectors**: Relate to overlaps $Z^{(n)}_i = \sqrt{2E_n} e^{E_n t_0/2} v^{(n)\dagger}_j C_{ji}(t_0)$

Use overlaps to assign each extracted state a continuum spin

- Operators of definite $J^{PC}$ were constructed in step 1 and subduced into the relevant irrep

- A subduced operator carries a memory of the continuum spin $J$, from which it was subduced - it overlaps predominantly with states of this $J$
Results - Hidden Charm Sector
- Large overlap with operators $\mathcal{O} \sim F_{\mu\nu}$

- **Lightest hybrid supermultiplet:**
  
  $(c\bar{c} \text{ in } S\text{-wave}) \otimes (J_g^{PC} = 1^{+-}) \Rightarrow [(0, 1, 2)^{-+}, 1^{--}]$

- **Excited hybrid supermultiplet:**
  
  $(c\bar{c} \text{ in } P\text{-wave}) \otimes (J_g^{PC} = 1^{+-}) \Rightarrow [0^{+-}, (1^{+-})^3, (2^{+-})^2, 3^{+-}, (0, 1, 2)^{++}]$
Results - Open Charm Sector
Pattern of Hybrid Mesons

- Large overlap with operators $\mathcal{O} \sim F_{\mu\nu}$

- Lightest hybrid supermultiplet - same pattern and scale as in Charmonium and Light meson sectors [J. Dudek, arXiv:1106.5515]

[Diagram showing mass spectrum of D and Ds mesons with excited states labeled]
Results - $D\pi$ Scattering
$D_{\pi}$ Multi-particle Spectra - Preliminary

$A_1^+$

$P = (0,0,0)$

$A_1$

$P = (1,1,0)$

$A_1$

$P = (1,0,0)$

$A_1$

$P = (1,1,1)$
$D_{π}$ Scattering Phase Shift for $l = 0$ - Preliminary

\[ \alpha_t E_{cm} \]

\[ \delta_0(\text{deg}) \]

- $P = (0,0,0)$
- $P = (1,0,0)$
- $P = (1,1,0)$
- $P = (1,1,1)$

PRELIMINARY
Conclusions

- Computed charmonium spectrum - observe exotic states
- Computed $D$ and $D_s$ spectra - multi-hadron effects may be important to understand the $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$ states
- Spectra generally well explained by quark model
- Observe extra hybrid states
- Early stages of $D\pi$ Scattering ($I = 3/2$)