

$$\begin{aligned}
e^{\Delta S_P} &= \frac{\int DU_k D\phi \, e^{S'_L}}{\int DU_k D\phi \, e^{S''_L}} \\
&= \frac{\int DU_k D\phi \, \exp[S'_L - S''_L] e^{S''_L}}{\int DU_k D\phi \, e^{S''_L}} \\
&= \left\langle \exp[S'_L - S''_L] \right\rangle''
\end{aligned}$$