On the phase diagram of Yang-Mills theories in the presence of a θ term

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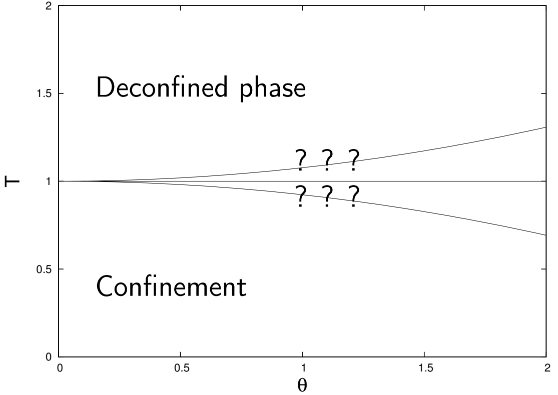
and work in progress

Outline

- 1) Topological θ-term and sign problem.
- 2) $T_c(\theta)$ for $SU(3)$: analytic continuation & reweighting.
- 3) $T_c(\theta)$ for $SU(N_c)$: large N_c & preliminary $N_c = 2, 4$.
- 4) Some issues regarding fixing topology.
- 5) Conclusions.

1) Topological θ-term and sign problem.

Aim: SU(3) gauge theory phase diagram in the $T - \theta$ plane.



Does T_c depend on θ? Is it growing or decreasing?

- PNJL model [Mizher, Fraga, Sakai, Kouno et al.]
- semiclassical approximations [Anber, Unsal, Poppitz and Schaefer]

1) Topological θ-term and sign problem.

We consider the following continuum action in euclidean metric:

$$S = S_{YM} + S_\theta$$

The Yang-Mills term

$$S_{YM} = -\frac{1}{4}\int d^4x \; F^a_{\mu\nu}(x) F^a_{\mu\nu}(x)$$

and the topological θ-term

$$S_\theta = -i\theta \frac{g_0^2}{64\pi^2}\int d^4x \; \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x) \equiv -i\theta \int d^4x \; q(x) \equiv -i\theta Q[A]$$

But it is complex! Bad news... sign problem!

1) Topological θ-term and sign problem.

Analytic continuation

Via an imaginary $\theta = i\theta_I$ term we can "solve" the sign problem.

[Azcoiti et al., PRL 2002; Alles and Papa, PRD 2008; Horsley et al., arxiv:0808.1428 [hep-lat]; Panagopoulos and Vicari, JHEP 2011]

Analyticity is supported by the current knowledge of the vacuum free energy derivatives with respect to θ evaluated at $\theta = 0$.

[Alles, D'Elia and Di Giacomo, PRD 2005; Vicari and Panagopoulos, Physics Reports 2008]

Reweighting

From simulation at $\theta = 0$, if we measure the topological charge Q of each configuration, we can reweight towards nonzero θ:

$$\langle O \rangle_\theta = \frac{\langle O \; e^{-i\theta Q} \rangle_0}{\langle e^{-i\theta Q} \rangle_0}$$

1) Topological θ-term and sign problem.

The lattice partition function is:

$$\boxed{Z(T,\theta) = \int D[U] e^{-S^L_{YM}[U] - \theta_L Q_L[U]}}$$

$$S^L_{YM} = \text{Standard Wilson Plaquette Action}$$

$$Q_L[U] = \frac{-1}{2^9\pi^2} \sum_n^{\text{Lattice}} \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \tilde{\epsilon}_{\mu\nu\rho\sigma} \mathrm{Tr}\left(\Pi_{\mu\nu}(n)\Pi_{\rho\sigma}(n)\right)$$

Due to a finite multiplicative renormalization Q_L is related to the integer valued Q by :

$$Q_L = Z(\beta) Q + O(a^2)$$

[Campostrini, Di Giacomo and Panagopoulos, Phys Lett B 1988]

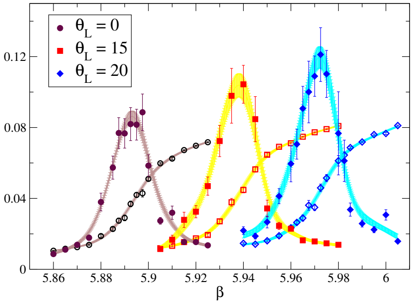
So the θ-term is also

$$S_\theta \equiv -\theta_L Q_L = -\theta_L Z(\beta) Q = -\theta_I Q$$

Polyakov Loop → order parameter for deconfinement even at $\theta \neq 0$.

$$\chi_L(\beta, \theta_L) = V_s \left(\langle |L|^2 \rangle_{\beta,\theta_L} - \langle |L| \rangle^2_{\beta,\theta_L} \right)$$

Determination of β_c on $24^3 \times 6$ lattice.



L and χ_L data and β-reweighting.

Several lattice spacings in order to approach the continuum limit:

$a \simeq 1/(4T_c(0))$, $1/(6T_c(0))$, $1/(8T_c(0))$ and $1/(10T_c(0))$.

Lattices aspect ratio $= 4$.

2) $T_c(\theta)$ for $SU(3)$: analytic continuation & reweighting.

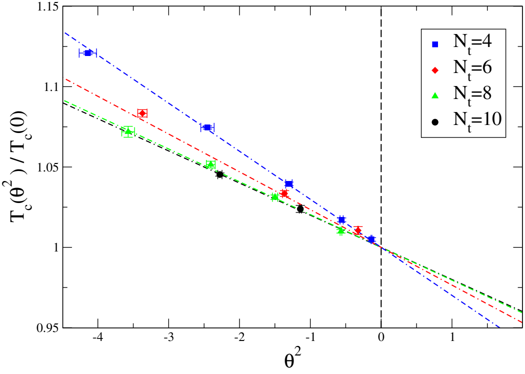
$T_c(\theta)/T_c(0) \simeq 1 + R_\theta \theta^2$

$R_\theta^{N_t=4} = 0.0299(7)$
$\chi^2/d.o.f. \sim 0.3$

$R_\theta^{N_t=6} = 0.0235(5)$
$\chi^2/d.o.f. \sim 1.6$

$R_\theta^{N_t=8} = 0.0204(5)$
$\chi^2/d.o.f. \sim 0.7$

$$\boxed{R_\theta^{N_t=10} = 0.0200(5) \quad \chi^2/d.o.f. \sim 1.0}$$



T_c increases for imaginary coupling then,
by analytic continuation, it decreases for real θ.

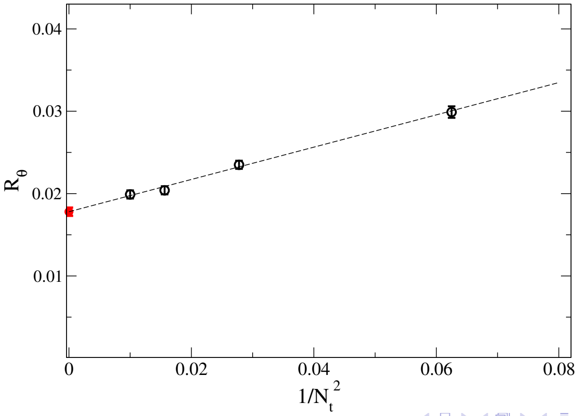
2) $T_c(\theta)$ for $SU(3)$: analytic continuation & reweighting.

Assuming quadratic finite lattice spacing corrections to R_θ:

$$R_\theta^{N_t} = R_\theta^{\text{cont}} + c/N_t^2$$

we can extrapolate to the continuum limit to get

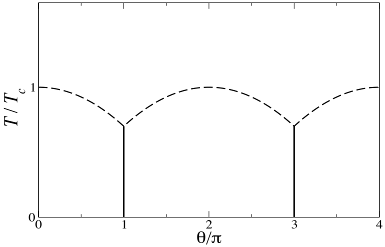
$$R_\theta^{\text{cont}} = 0.0178(5) \text{ with } \chi^2/d.o.f. \sim 0.6$$

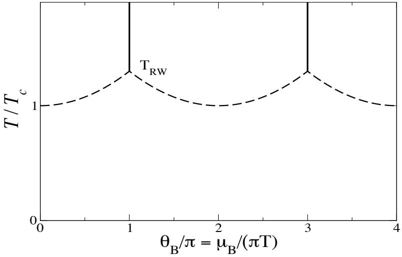


2) $T_c(\theta)$ for $SU(3)$: analytic continuation & reweighting.

How does the phase diagram looks like?

Assumption for the sketched phase diagram:
- critical line depending on θ^2
- periodicity in $\theta \to$ cusps
- critical point at $\theta = \pi$ and $T = 0$ connected with the cusps

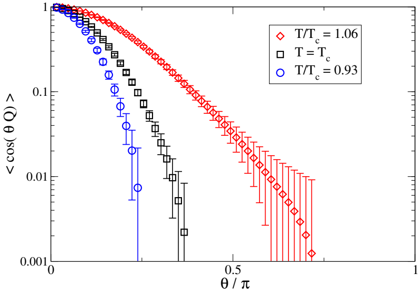


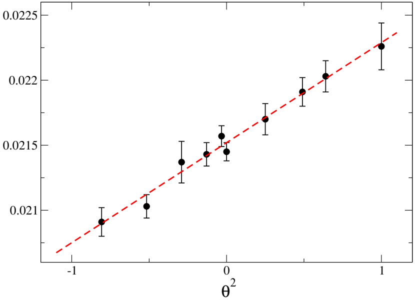


Similarity with the $T - \mu_B^I$ phase diagram!!

2) $T_c(\theta)$ for $SU(3)$: analytic continuation & reweighting.

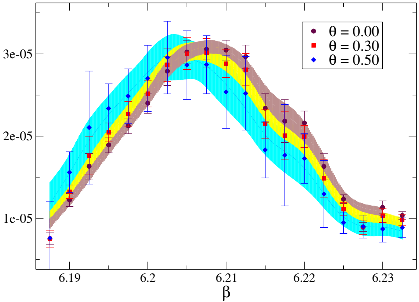
The reweighting factor
$\cos(\theta Q)$
at different temperatures.





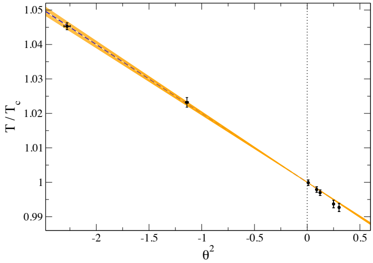
Polyakov Loop both as a function of both real and imaginary θ at $T = 1.055\, T_c$.

2) $T_c(\theta)$ for $SU(3)$: analytic continuation & reweighting.



Real θ-reweighted Polyakov Loop Susceptibility.

Critical temperature ratios for both real and imaginary θ on the $40^3 \times 10$ lattice.



3) $T_c(\theta)$ for $SU(N_c)$: large N_c & preliminary $N_c = 2, 4$.

Critical line curvature R_θ in the Large N_c limit.

Ingredients:
1) the transition is first order (Latent heat $\Delta\epsilon$).
2) the free energies of the confined-deconfined (c-d) phase are:

$$\frac{f_c(t)}{T} = \frac{\chi\theta^2}{2T} + A_c t + O(t^2) \qquad \frac{f_d(t)}{T} = A_d t + O(t^2).$$

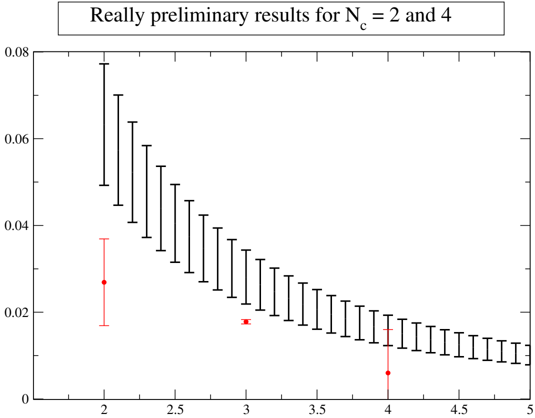
Then one gets [M. D'Elia and FN, PRL 109 (2012) 072001]:

$$\frac{T_c(\theta)}{T_c(0)} = 1 - \frac{\chi}{2\Delta\epsilon}\theta^2 = 1 - R_\theta^{large\ N_c}\theta^2$$

Using the estimates of these quantities from [Lucini, Teper and Wenger, JHEP 2005] we have:

$$R_\theta^{large\ N_c} = \frac{\chi}{2\Delta\epsilon} = \frac{0.253(56)}{N_c^2} + O(\frac{1}{N_c^4})$$

3) $T_c(\theta)$ for $SU(N_c)$: large N_c & preliminary $N_c = 2, 4$.



- For $SU(2)$ we have 2 lattice spacings: $N_t = 6$ and 8.
- For $SU(4)$ we have only 2 lattice spacings: $N_t = 5$ and 6.

4) Some issues regarding fixing topology.

Motivated by the general expression for a reweighted observable, that can be rewritten as

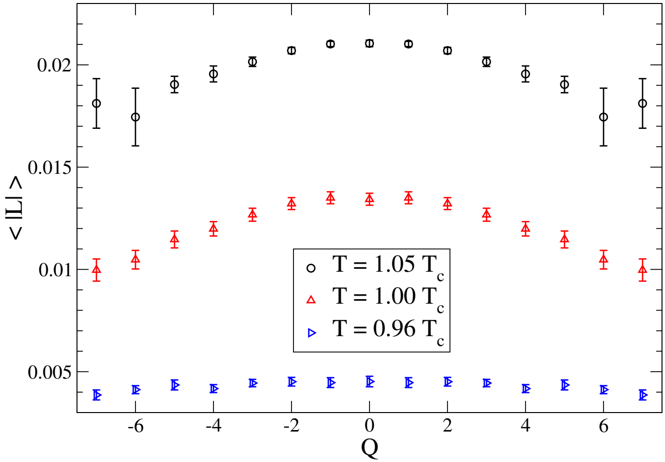
$$\langle O \rangle = \frac{1}{\langle \cos(\theta Q) \rangle} \sum_{Q=-\infty}^{+\infty} e^{i\theta Q} \, \mathcal{P}(Q) \, \langle O \rangle_Q \ ,$$

we observe that a non trivial dependence on θ is present iff observables restricted to different topological sectors have different expectation values:

$$\frac{\partial(\text{anything})}{\partial \theta} \neq 0 \iff (anything)_Q \neq (anything)_{Q'}$$

4) Some issues regarding fixing topology.

Polyakov loop vs topological charge at various temperatures.



Results from our finest lattice: $40^3 \times 10$.

4) Some issues regarding fixing topology.

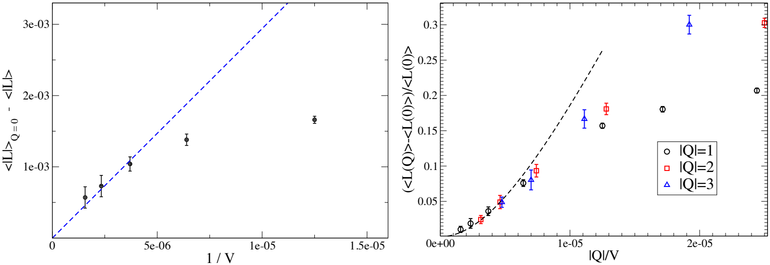
Instanton gas approximation sets up just above T_c

[Bonati, D'Elia, Panagopoulos and Vicari, PRL 2013]

We assume each (anti)-instanton to contribute in modifing $\langle |L| \rangle_Q$:

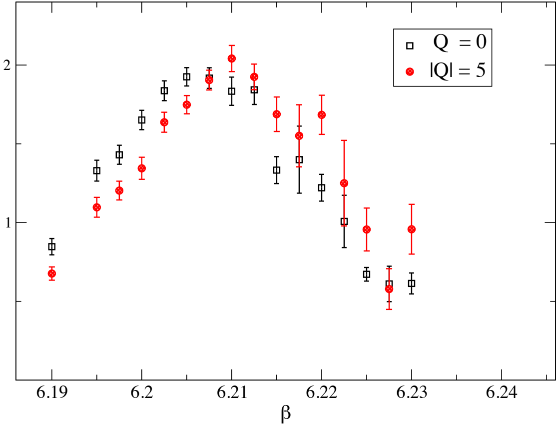
$$\langle |L| \rangle_Q \simeq \text{const} - \frac{\gamma}{V} \langle n + \overline{n} \rangle_Q \simeq \langle |L| \rangle + \frac{\gamma}{2V} \left(1 - \frac{Q^2}{V \chi_t} \right)$$

At $T = 1.018\ T_c$ we get:



4) Some issues regarding fixing topology.

Also the Polyakov Loop Susceptibility is senstitive to the topological sector, giving rise to a shift in the critical temperature.



From $Q = 0$ to $Q = 5$ the change in T_c is $\sim 6.5\%$

5) Conclusions

- Use of imaginary θ_I parameter to cure sign problem for LGT.
- Determination of the curvature R_θ of the critical line.
- Reweighting in θ to support analyticity.
- Large N_c estimate and comparison with $SU(2)$ and $SU(4)$.
- Dependence on the topological sector.

Perspectives:

- Explore the $(T-\theta,\ T-\mu_B)$ speculated duality.
- Complete the analysis to $SU(2)$ and $SU(4)$.
- Larger N_c.

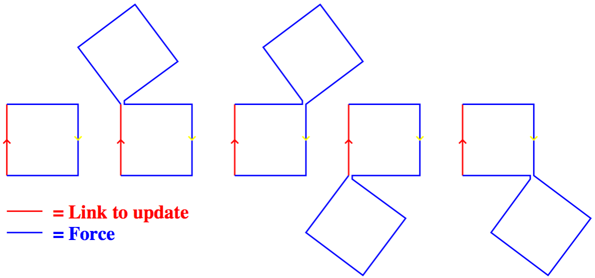
6) Backup: sampling algorithms.

Each link appears linearly in the simple action we employed.

$\Downarrow$

We can exploit standard Heatbath and Overrelaxation algorithms.

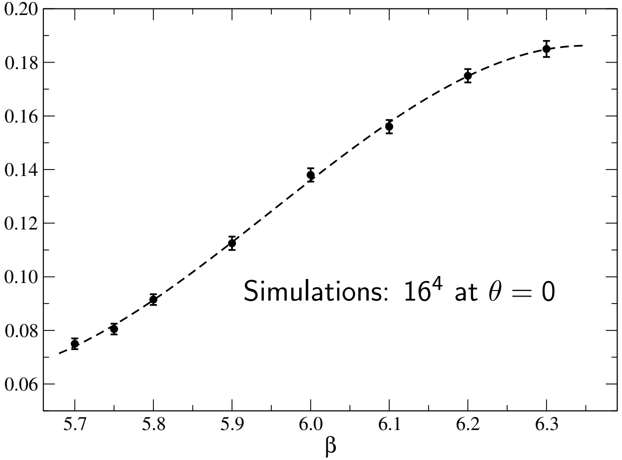
It is necessary to modify the staples definition. Pictorically:



With more complicated topological charge definitions on the lattice such standard algorithms wouldn't have been applicable.

6) Backup: renormalization factor $Z(\beta)$.

Renormalization factor determined as $Z(\beta) = \langle Q_L Q \rangle_0 / \langle Q^2 \rangle_0$, method proposed in [Panagopoulos and Vicari, JHEP 2011].
Needed to go from θ_L to $\theta_I \equiv Z(\beta)\theta_L$.



6) Backup: summary of the SU(3) simulations.

lattice	θ_L	β_c	θ_I	$T_c(\theta_I)/T_c(0)$
$16^3 \times 4$	0	5.6911(4)	0	1
$16^3 \times 4$	5	5.6934(6)	0.370(10)	1.0049(11)
$16^3 \times 4$	10	5.6990(7)	0.747(15)	1.0171(12)
$16^3 \times 4$	15	5.7092(7)	1.141(20)	1.0395(11)
$24^3 \times 6$	0	5.8929(8)	0	1
$24^3 \times 6$	5	5.8985(10)	0.5705(60)	1.0105(24)
$24^3 \times 6$	10	5.9105(5)	1.168(12)	1.0335(18)
$24^3 \times 6$	15	5.9364(8)	1.836(18)	1.0834(23)
$32^3 \times 8$	0	6.0622(6)	0	1
$32^3 \times 8$	5	6.0684(3)	0.753(8)	1.0100(11)
$32^3 \times 8$	8	6.0813(6)	1.224(15)	1.0312(14)
$32^3 \times 8$	10	6.0935(11)	1.551(20)	1.0515(21)
$40^3 \times 10$	0	6.2082(4)	0	1
$40^3 \times 10$	6.0	6.2241(13)	1.068(7)	1.0239(22)
$40^3 \times 10$	8.4	6.2381(5)	1.509(10)	1.0453(10)
$40^3 \times 10$	13.4	6.2821(9)	2.461(22)	1.1144(16)