Local Polyakov loop domains and their fractality

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Center clusters

- In pure gluodynamics: The **Polyakov loop**

$$L(x) = \mathrm{Tr} \prod_{t=1}^{N_t} U_4(x,t)$$

  is an order parameter for the deconfinement transition.

- $L(x)$ transforms non-trivially under center transformations.

- **Center domains:** Clusters of spatial points x where the phase of $L(x)$ is near the same center element.

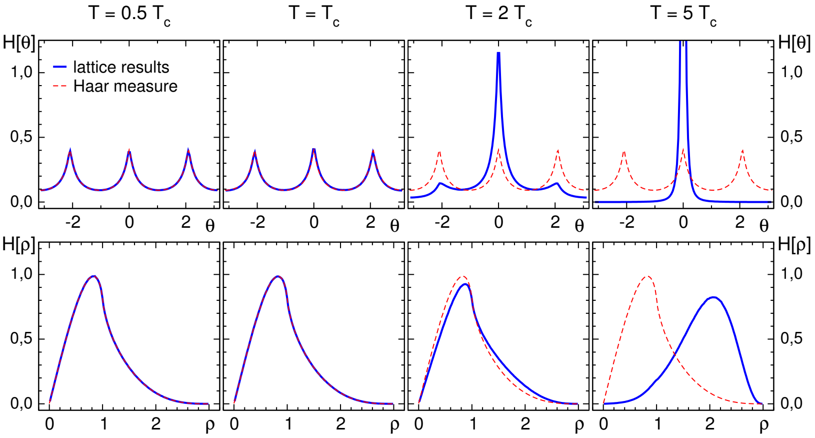
- Center domains are reminiscent of Weiss domains.

- The center domains may play a role in QCD phenomenology.

Technical details

- **Pure gluodynamics** with Wilson gauge action.
- **Lattices:** $30^3 \times N_t$, $40^3 \times N_t$, $48^3 \times N_t$.
- **Fixed scale approach**, i.e., we use N_t to drive the temperature.
- **Temperature:** $0.3 \leq T/T_c \leq 7$.
- **Inverse coupling:** $\beta = 5.90, 6.20, 6.45$.
- **Lattice spacing:** $a = 0.112$ fm, 0.068 fm, 0.048 fm.

Histogram of phase and modulus of $L(x)$



$$L(x) = \rho(x)\exp(i\theta(x))$$

Definition of center domains - 1

We assign to a spatial point x the **center sector number** $n(x)$:

$$n(x) = \begin{cases} -1 & \text{for } \theta(x) \in [-\pi+\delta\,,\,-\pi/3-\delta] \\ 0 & \text{for } \theta(x) \in [-\pi/3+\delta\,,\,\pi/3-\delta] \\ +1 & \text{for } \theta(x) \in [\pi/3+\delta\,,\,\pi-\delta] \end{cases}$$

with the real and non-negative parameter

$$\delta = f\frac{\pi}{3}\,, \quad f \in [0,1)\,.$$

Cut parameter f:

- $f > 0$: Sites far from the center elements are removed
- $f = 0$: No sites removed
- $f \to 1$: All sites removed

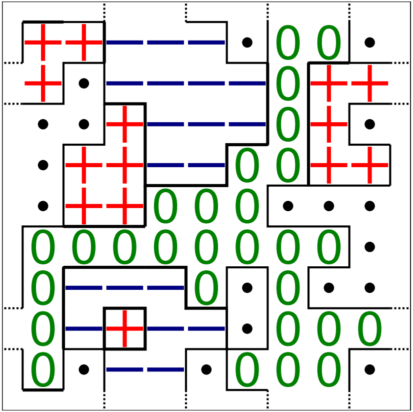
Definition of center domains - 2

Two spatial points

$$x \, , \, y = x \pm \hat{\mu}$$

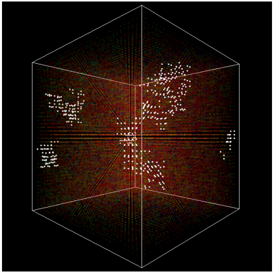
belong to the same cluster if

$$n(x) = n(y) \, .$$

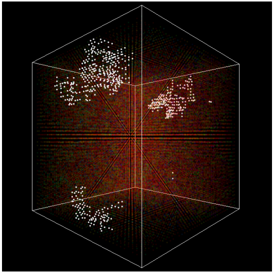


- Do we observe cluster formation?
- How do these clusters change with temperature?
- What is their fractal dimension?

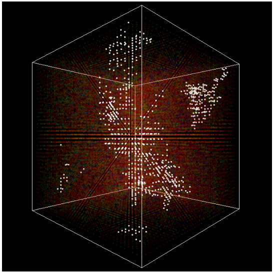
Visualization of center domains



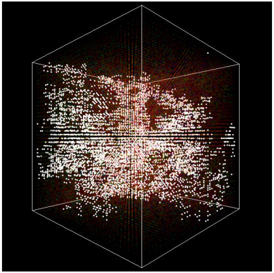
$T = 0.62\,T_c$



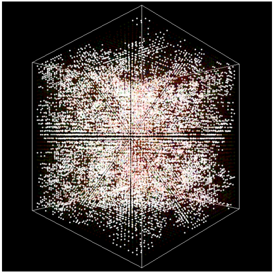
$T = 0.82\,T_c$



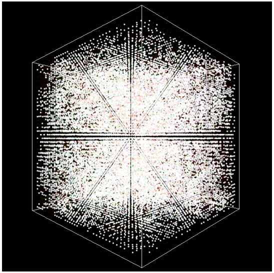
$T = 0.90\,T_c$



$T = 0.98\,T_c$

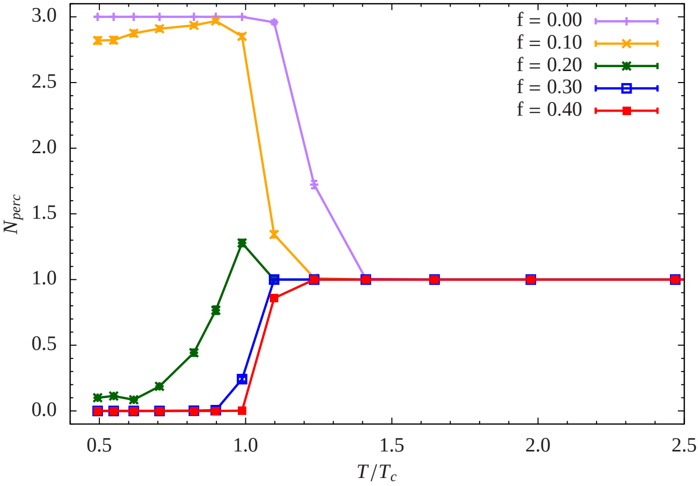


$T = 1.10\,T_c$



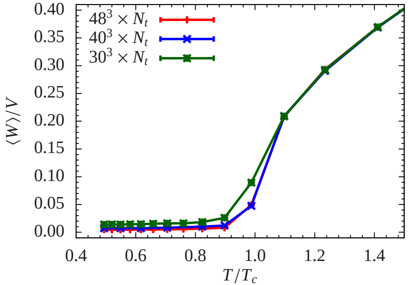
$T = 1.64\,T_c$

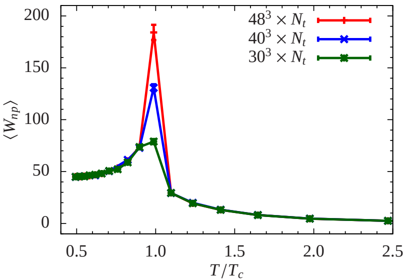
Percolation probability ($40^3 \times N_t$, $\beta = 6.2$)



Number of percolating clusters N_{perc}.

Number of sites in a cluster / weight ($\beta = 6.2, f = 0.30$)

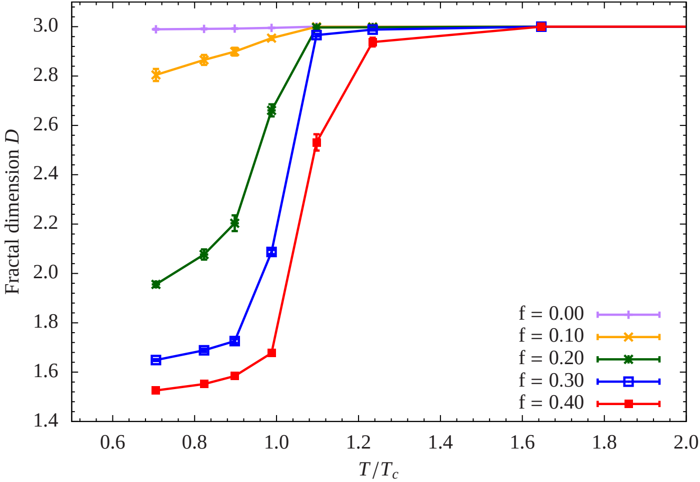




$\langle W \rangle / V$ ... **Weight of the largest cluster** normalized by the volume.

$\langle W_{np} \rangle$ ... **Mean cluster size** of non-percolating clusters.

Fractality ($40^3 \times N_t$, $\beta = 6.2$)



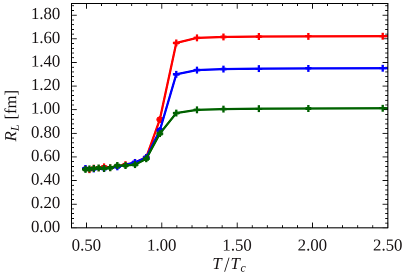
Fractal dimension D of largest cluster via **box counting method**:

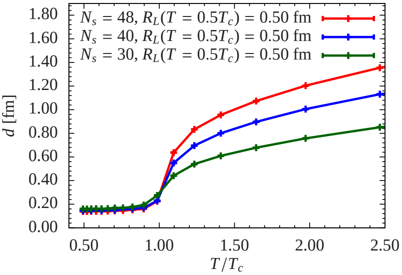
$$N(s) \propto s^{-D} ,$$

with N(s) number of boxes of size s needed to cover the whole cluster.

Linear cluster extents ($\beta = 6.2$)

(Lattice extent $N_s a$: $N_s = 30$: 2×1.02, $N_s = 40$: 2×1.35, $N_s = 48$: 2×1.63 fm)





Radius of a cluster of size s:

$$R_s^2 = \sum_{i=1}^{s} \frac{|\mathbf{r}_i - \mathbf{r}_0|^2}{s} ,$$

with "center of mass"

$$\mathbf{r}_0 = \sum_{i=1}^{s} \frac{\mathbf{r}_i}{s} .$$

Avg. distance traveled d:

$$d^2 = \sum_s n_s s \overline{R}_s^2 .$$

$n_s s$: Probability for a site to belong to a cluster of size s.

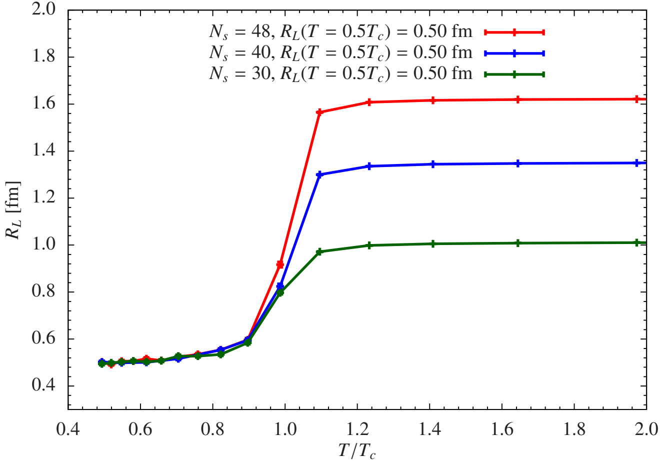
$\overline{R}_s$: avg. over all R_s for a given s.

Summary:

- We study the formation and properties of center domains.
- Drastic change of properties at the phase transition.
- Many small clusters below the phase transition.
- At $T \geq T_c$ one cluster wins $\Rightarrow$ percolation over the whole lattice.
- Fractality: Below T_c the clusters are highly complex objects; above T_c the fractal dimension becomes $D \rightarrow 3$.
- Results on properties of center clusters may be relevant for phenomenological description of heavy ion collisions.

Linear cluster extents ($\beta = 6.2$)

(Lattice extent $N_s a$: $N_s = 30 : 2 \times 1.02$, $N_s = 40 : 2 \times 1.35$, $N_s = 48 : 2 \times 1.63$ fm)



Radius of a cluster of size s:

$$R_s^2 = \sum_{i=1}^{s} \frac{|\mathbf{r}_i - \mathbf{r}_0|^2}{s} \quad , \quad \text{with "center of mass"} \quad \mathbf{r}_0 = \sum_{i=1}^{s} \frac{\mathbf{r}_i}{s} \, .$$