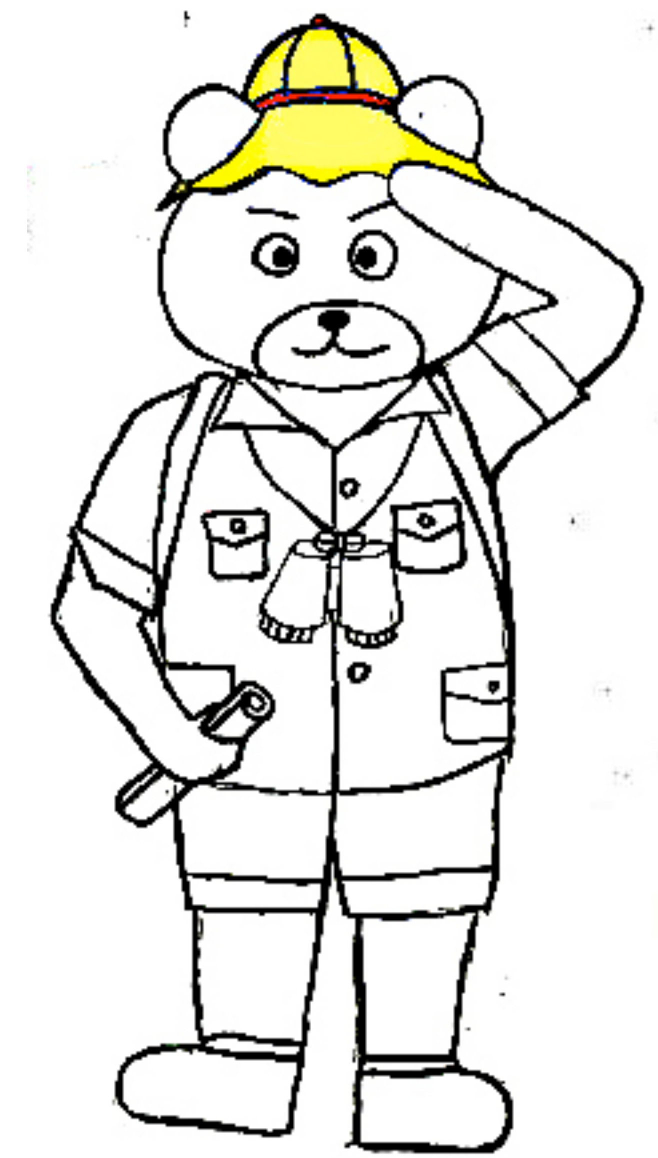
Pursuing QCD Phase Transition with Lattice QCD and Experimental Data

Keitaro Nagata and Atsushi Nakamura

Lattice 2013, Mainz

Details of the Talk can be found in

A.Nakamura K.Nagata
arXiv:1305.0760



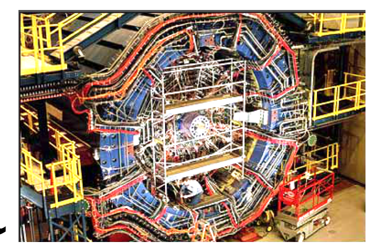
Strano.
Non capito

Strange Paper.
I could not
understand.

Komisches papier
Ich verstehe nicht

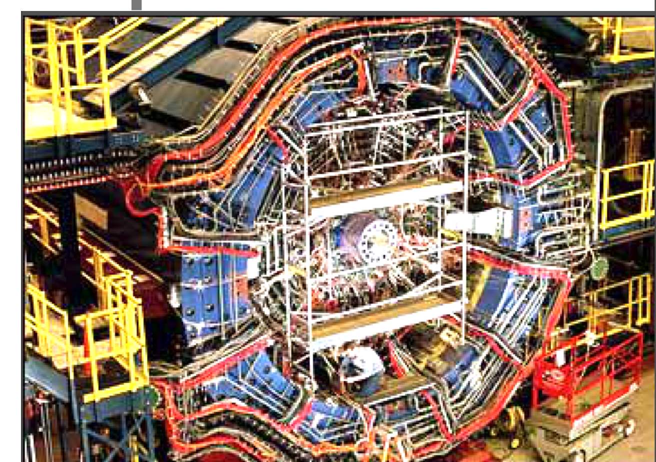
これ格子の論文？
よく分からない

It includes

- Experimental data  and Lattice data 
- Undergraduate Physics (Quantum Mechanics)
- Undergraduate Math. (Cauchy's Integral)

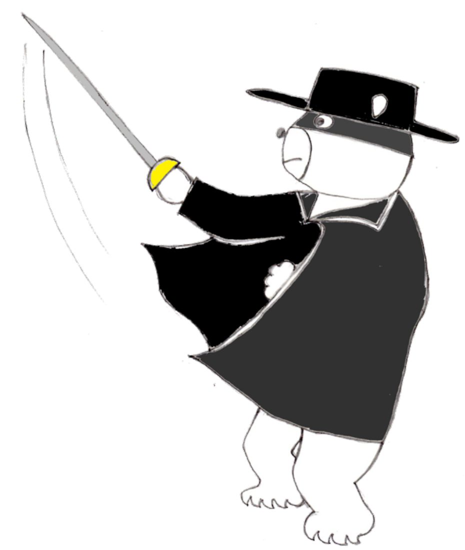
Canonical Partition Functions

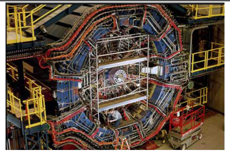
Experiments



Lattice QCD
Simulaitions

Z_N



 and 

Knowledge of whole Z_n $\Longleftrightarrow$ $Z(\mu, T)$ for $\forall \mu$, and T $\Longleftrightarrow$ QCD Phase

$$Z(\mu, T) = \mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$$

$$= \sum_n \langle n| e^{-(H-\mu\hat{N})/T} |n\rangle$$

$$= \sum_n \boxed{\langle n| e^{-H/T} |n\rangle}\, e^{\mu n/T}$$

$$= \sum_n Z_n(T) \xi^n \qquad \left(\xi \equiv e^{\mu/T}\right)$$

Fugacity

If $[H, \hat{N}] = 0$

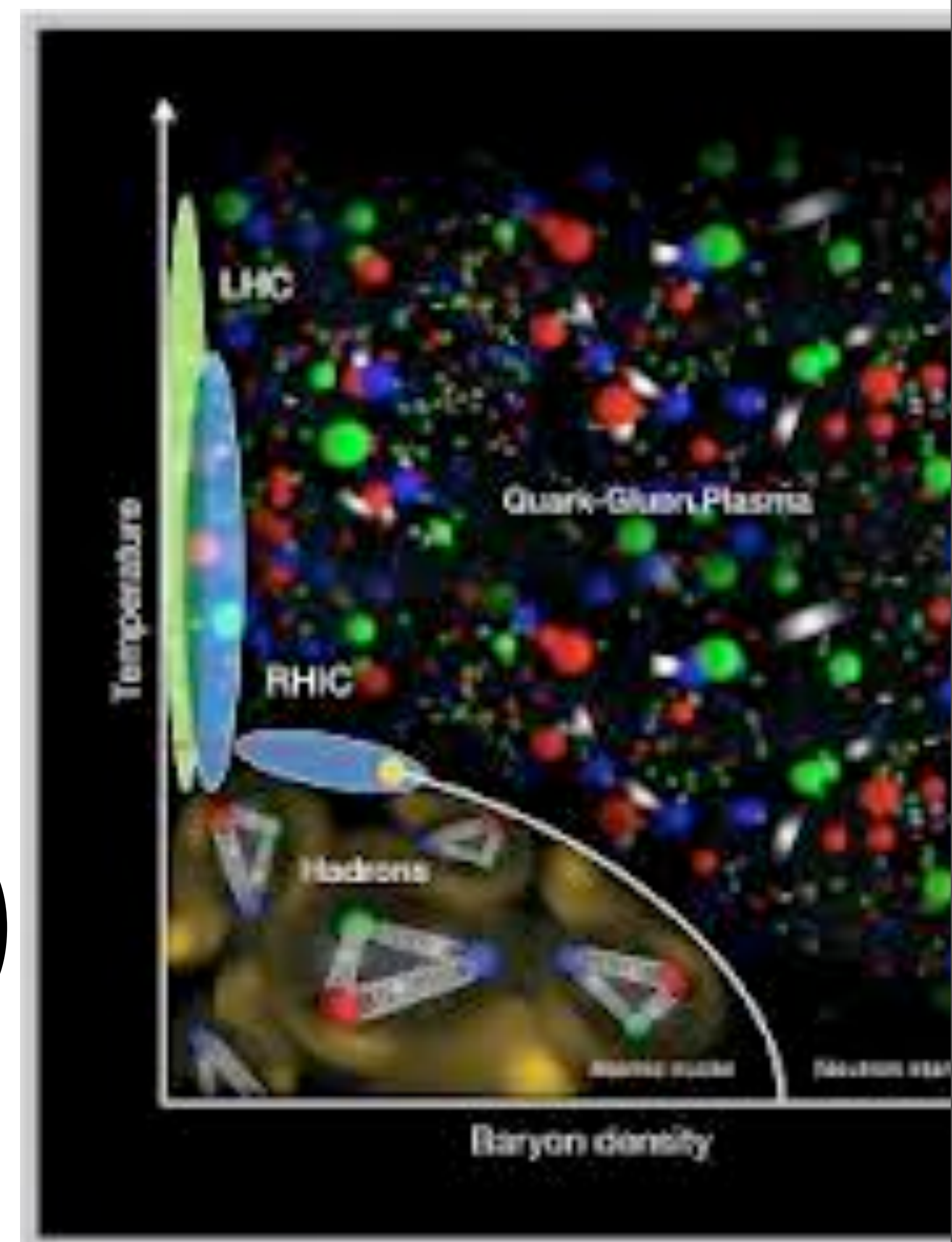
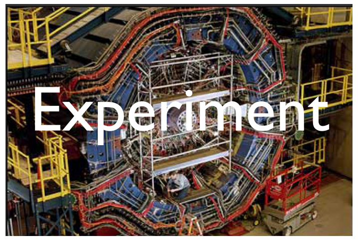
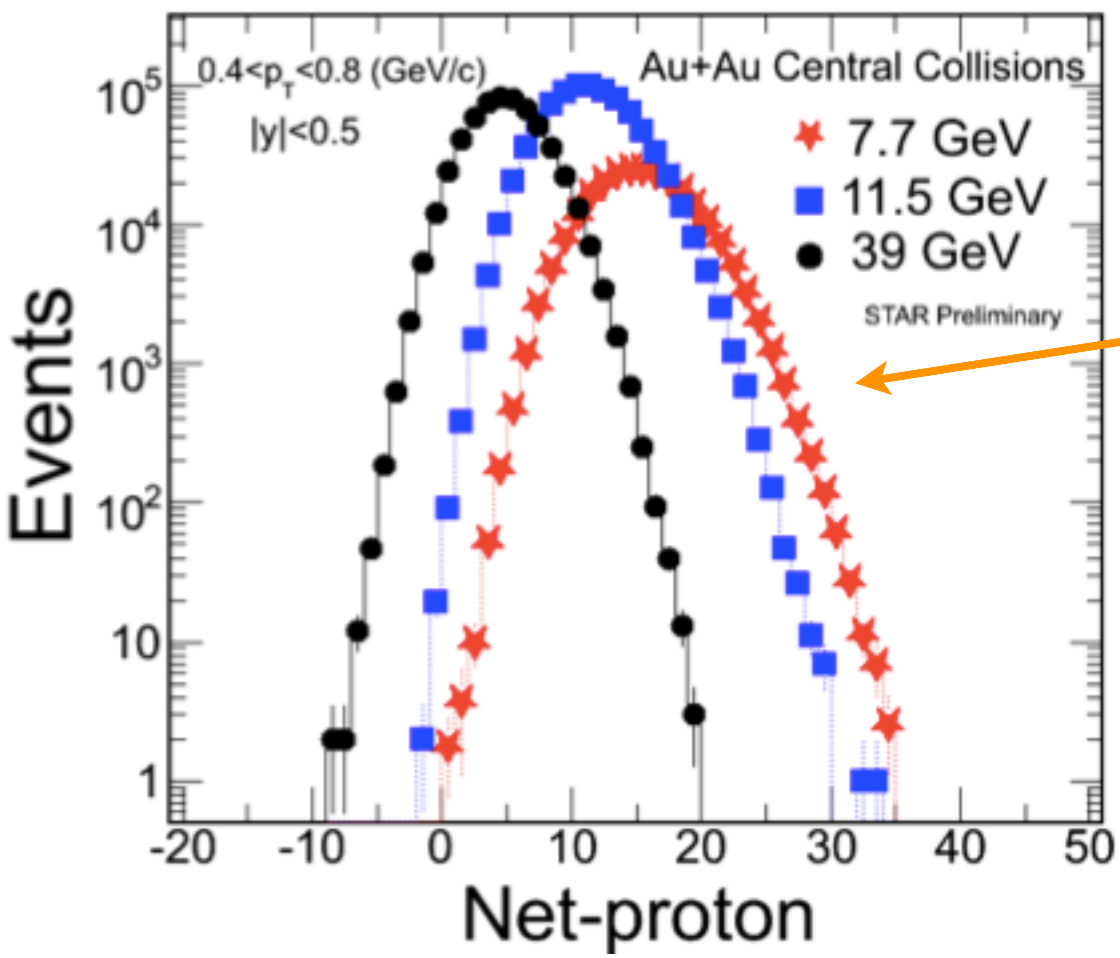


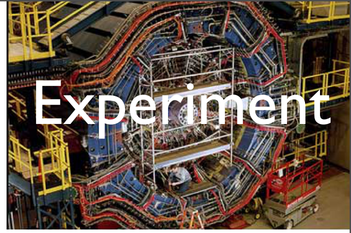
Image:BNL



$$Z(\xi, T) = \sum_n \boxed{Z_n(T)\, \xi^n}$$

Partition Function is
Sum of the Probability ...
If I know ξ, then I have Zn.



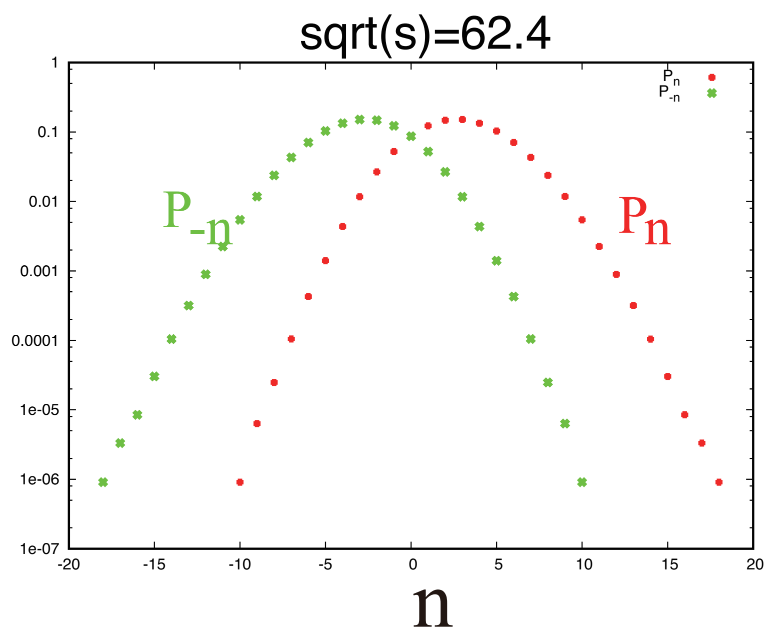


How can we extract Z_n from $P_n = Z_n \xi^n$?

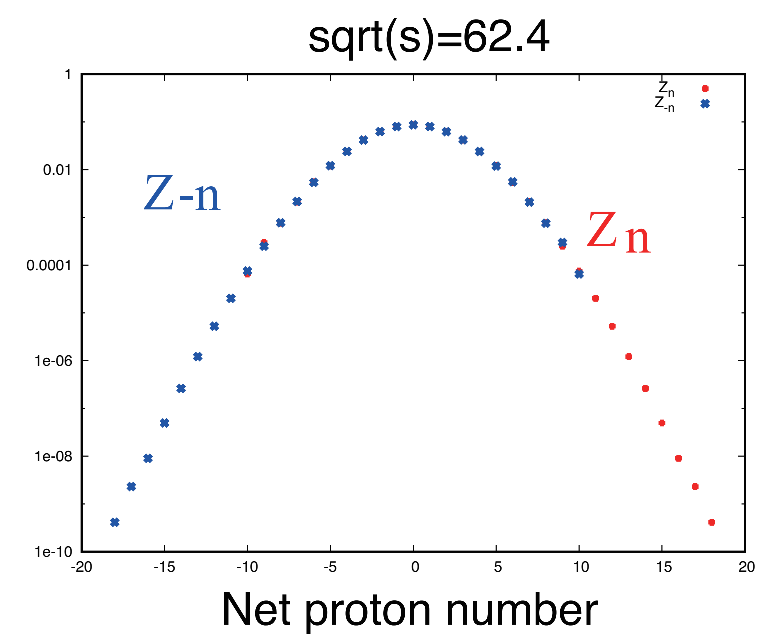
$\boxed{\xi}$ unknown

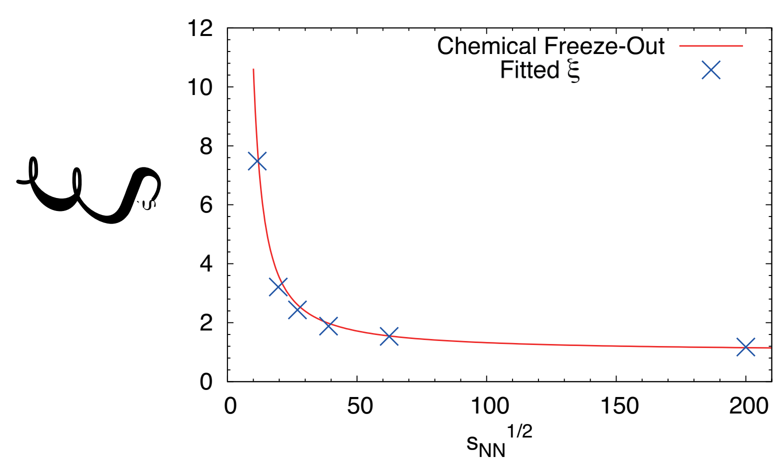
$$Z_n = P_n / \xi^n$$

We require $\boxed{Z_{+n} = Z_{-n}}$



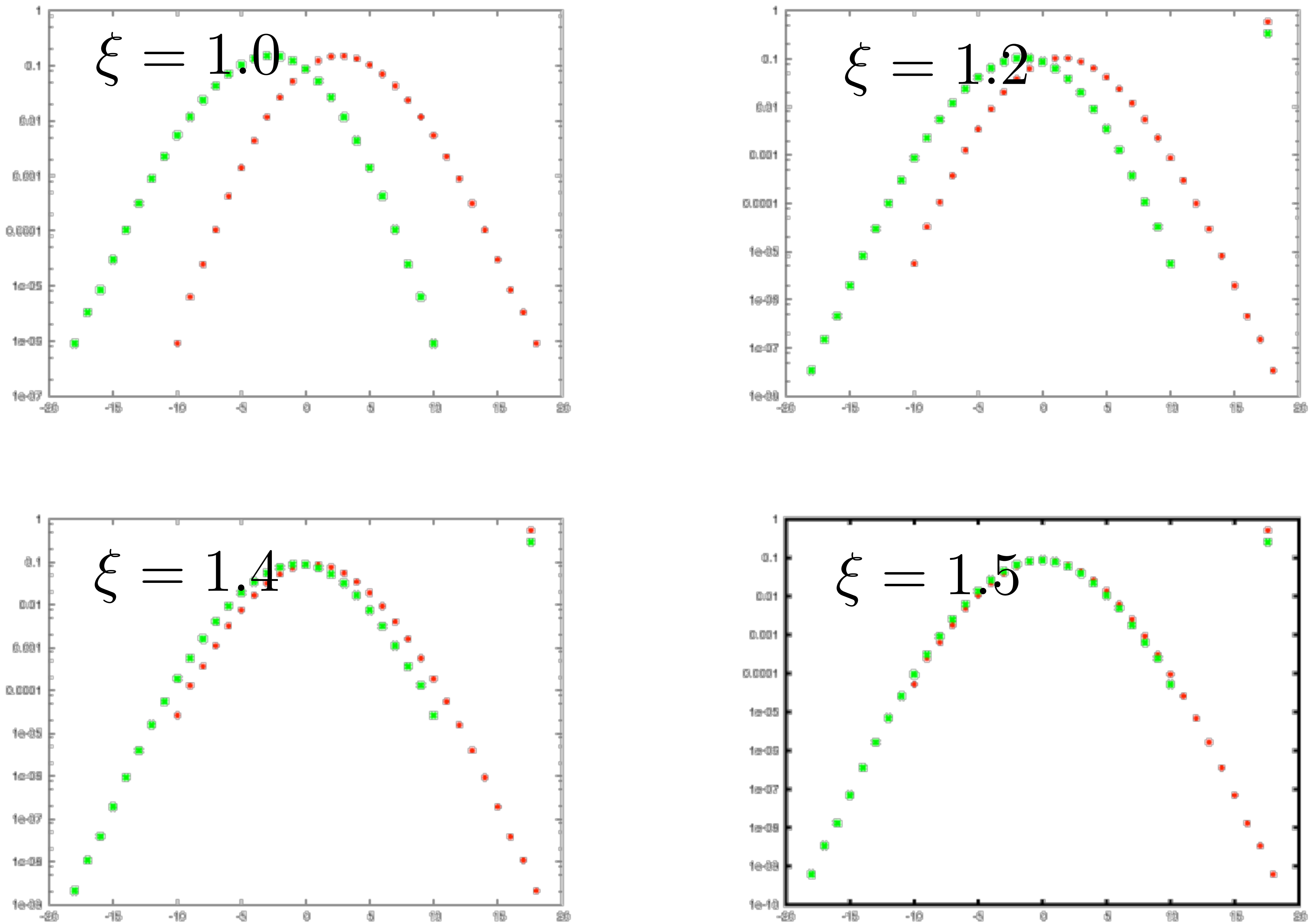
Fit ξ



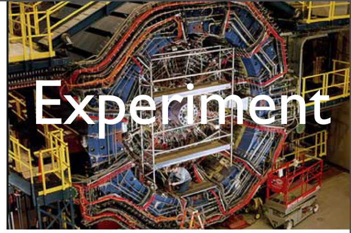


Obtained ξ are consistent with those by the phenomenological analysis, ``Freeze-out''

$$Z_n = P_n / \xi^n$$

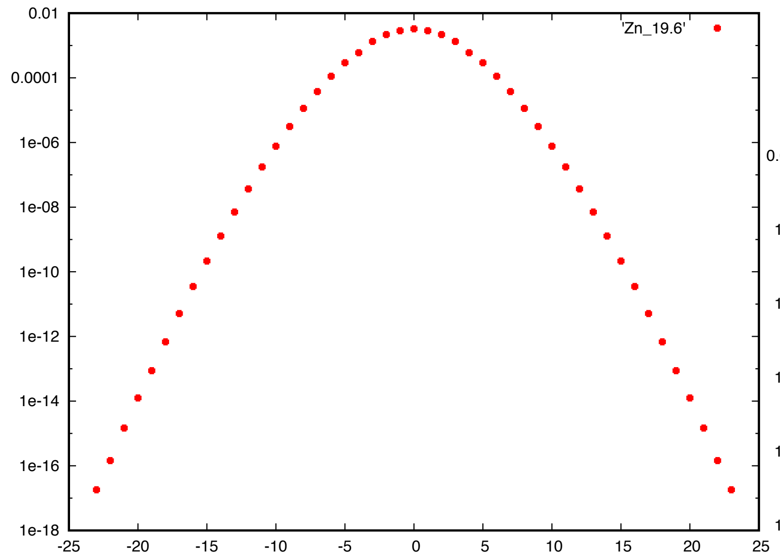


Final Value $\xi = 1.534$

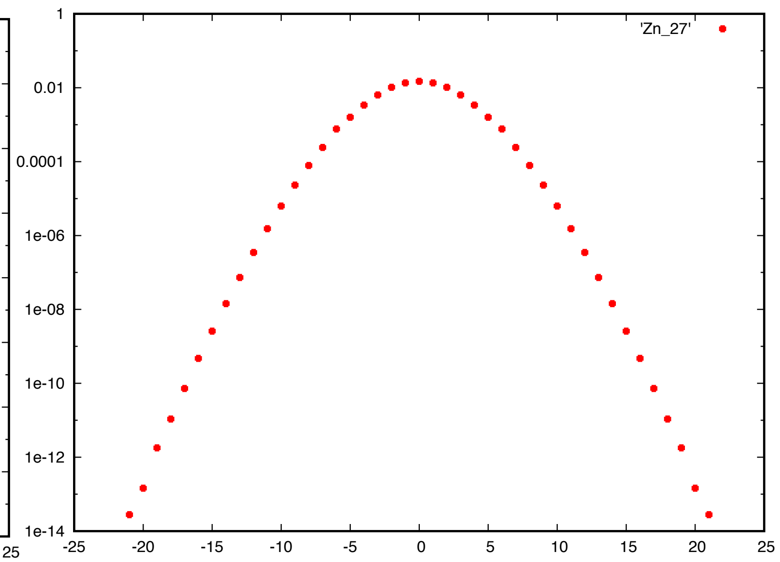


Z_n from RHIC data

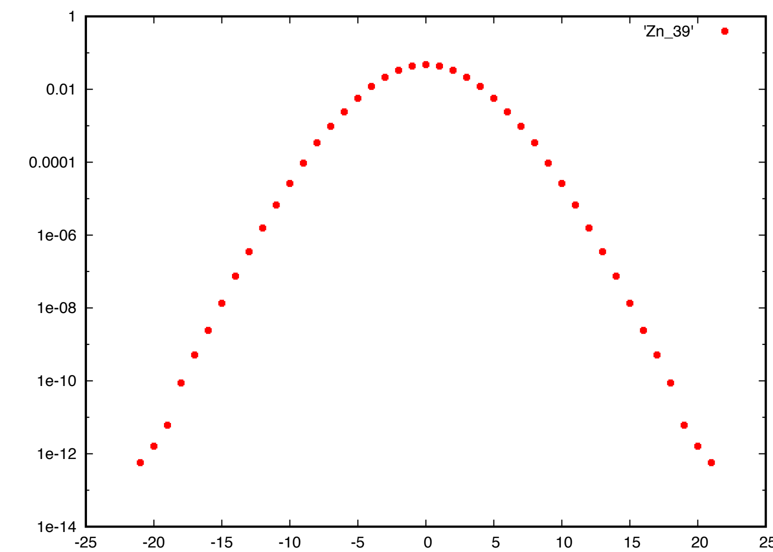
$\sqrt{s} = 19.6\mathrm{GeV}$



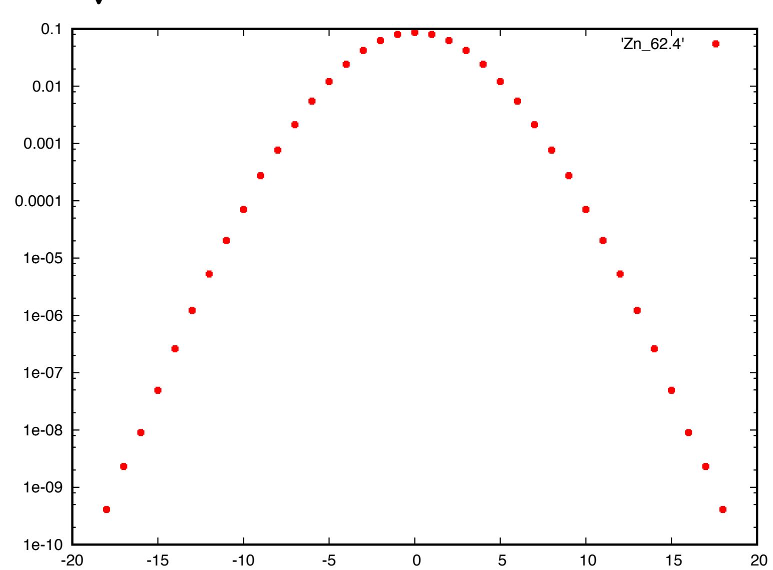
$\sqrt{s} = 27\mathrm{GeV}$



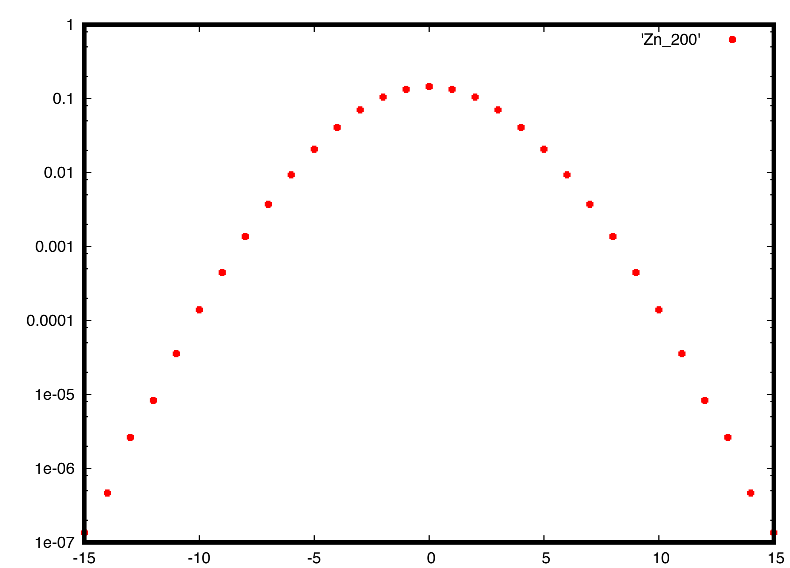
$\sqrt{s} = 39\mathrm{GeV}$

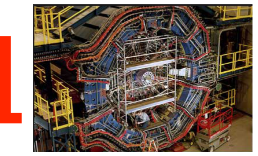


$\sqrt{s} = 62.4\mathrm{GeV}$



$\sqrt{s} = 200\mathrm{GeV}$



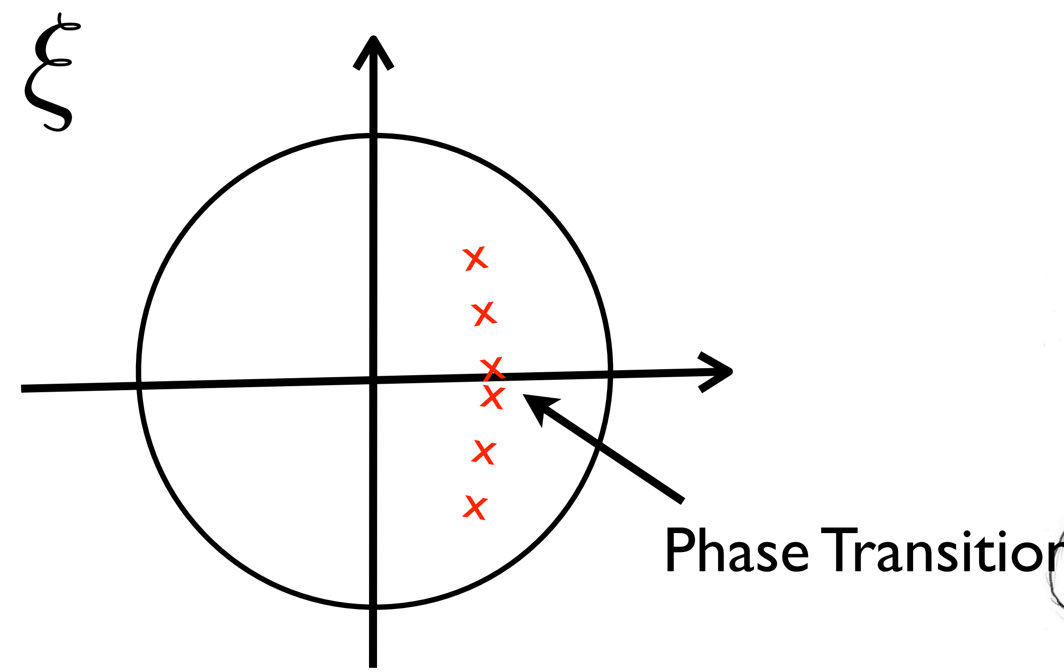
 and 

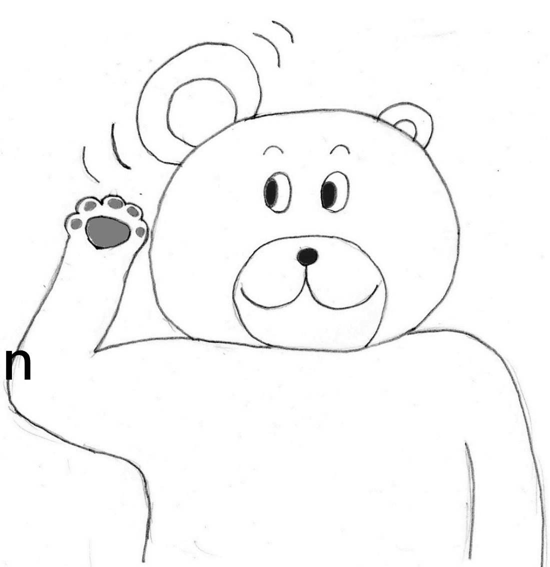
Lee-Yang Zeros

Zeros of $Z(\xi)$ in Complex Fugacity Plane.

$$Z(\alpha_k) = 0$$

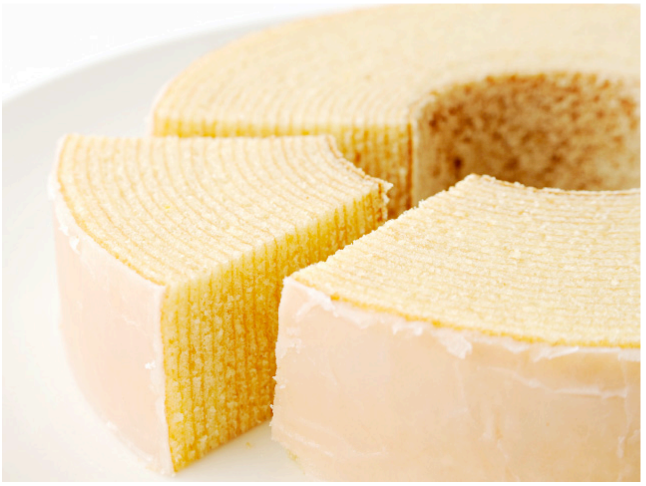
Great Idea to investigate a Statistical System

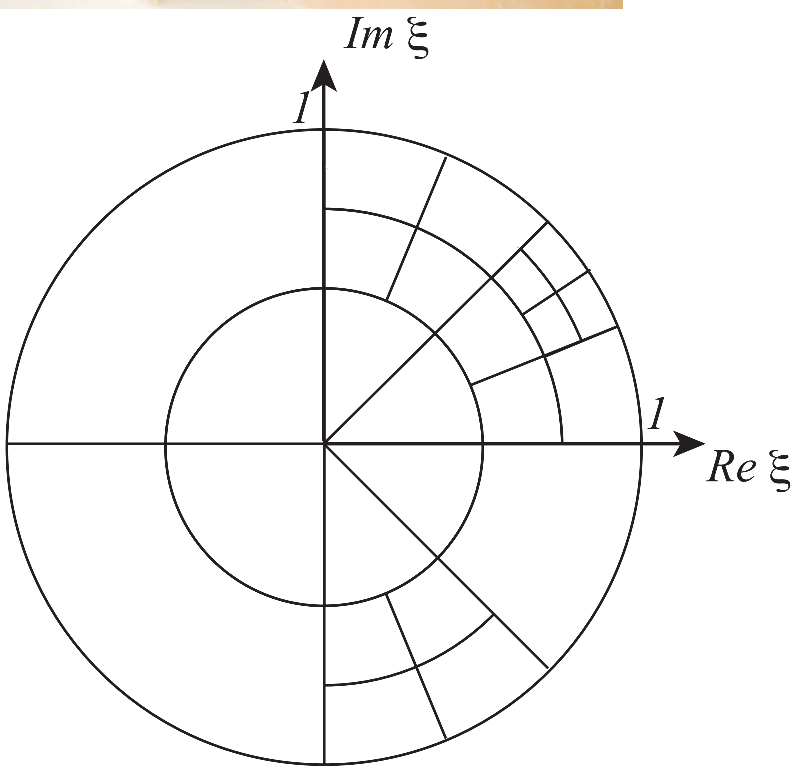





and


cut Baum-Kuchen (cBK) Algorithm

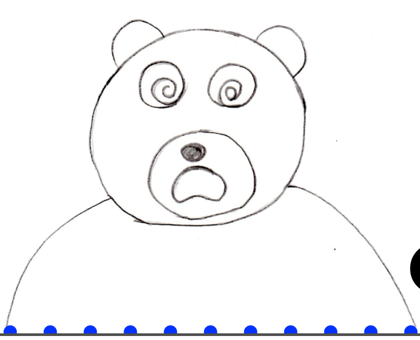




$$f(\xi) = \prod_k (\xi - \alpha_k)$$

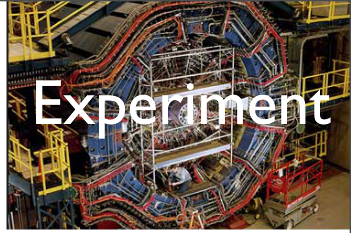
$$\frac{f'}{f} = \sum_k \frac{1}{\xi - \alpha_k}$$

$$\frac{1}{2\pi i} \oint_C \frac{f'}{f} d\xi = \left(\begin{array}{c} \text{Number of} \\ \text{Zeros in} \\ \text{Contour } C \end{array} \right)$$

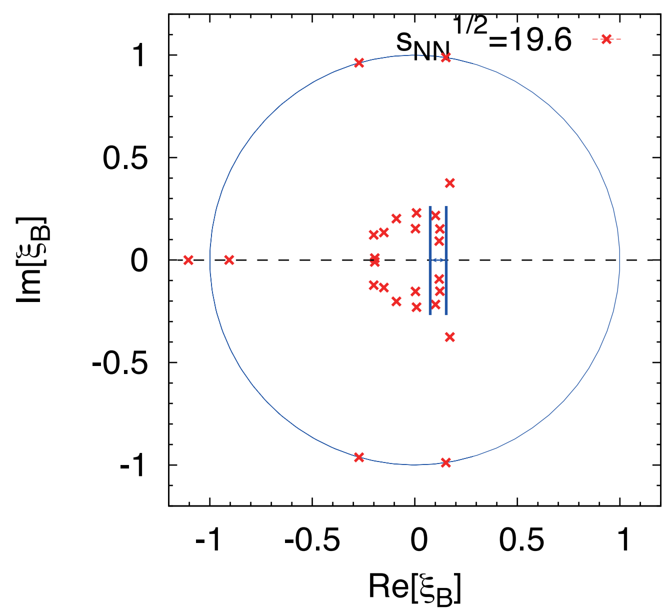


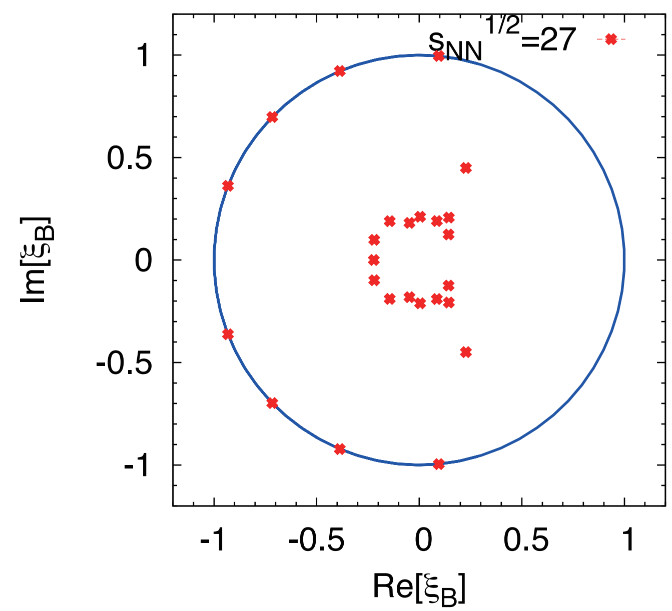
50 - 100 number of significant digits

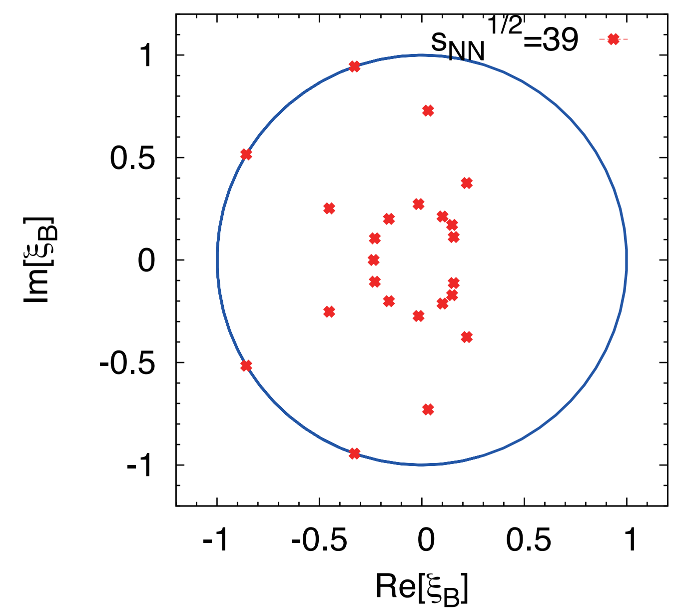
A Coutour is cut into four pieces if there are zeros inside.

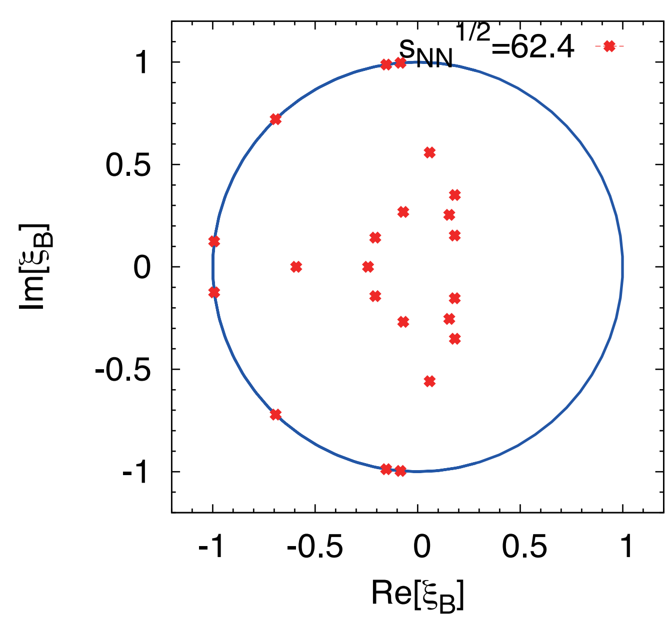


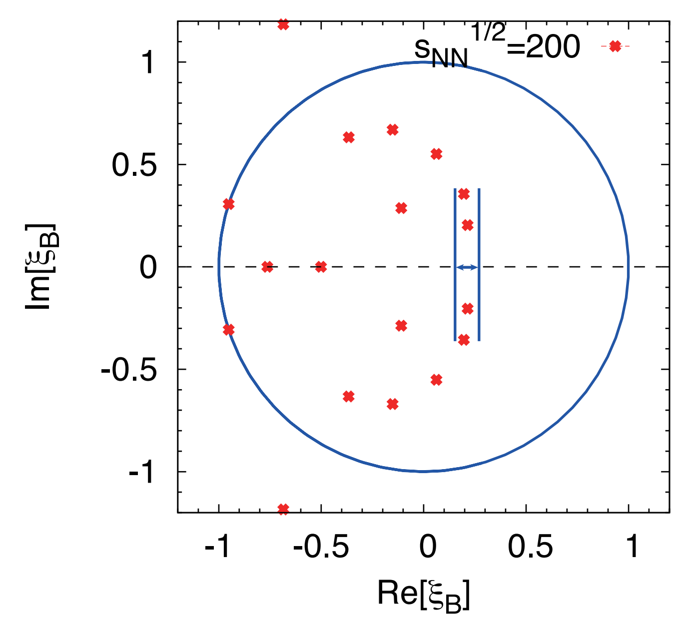
Lee-Yang Zeros: RHIC Experiments











Lattice QCD
Canonical Approach

- Miller and Redlich
  - Phys. Rev. D35 (1987) 2524
- A.Hasenfratz and Toussaint
  - Nucl. Phys.B371 (1992) 539
- Barbour and Bell
  - Nucl. Phys. B372 (1992) 385
- Engels, Kaczmarek, Karsch and Laermann
  - Nucl.Phys. B558 (1999) 307
- deForcrand and Kratochvila
  - Nucl. Phys. B (P.S.) 153 (2006) 62 (hep-lat/0602024)
- A.Li, Meng, Alexandru, K-F. Liu
  - PoS LAT2008:032 and 178
  - Phys.Rev. D82(2010) 054502, D84 (2011) 071503
- Danzer and Gattringer
  - arXiv:1204-1020 Europian Journal

Lattice: How to Calculate

$$Z(\mu, T) = \int \mathcal{D}U \ \ (\det \Delta(\mu))^{\mathrm{N_f}} \exp(-S_G)$$

$$\det \Delta(\mu) \to \left(\frac{\det \Delta(\mu)}{\det \Delta(0)} \right) \det \Delta(0)$$

$$\det \Delta(\mu) \propto \xi^{-N_{\mathrm{red}}/2} \prod_{n=1}^{N_{\mathrm{red}}} (\lambda_n + \xi)$$

$$\propto \sum_{n=-N_{\mathrm{red}}/2}^{N_{\mathrm{red}/2}} c_n \xi^n$$

Fugacity Expansion
Nagata and A. Nakamura,
Phys. Rev. D82 (2010) 094027

Alexandru and Wenger,
Phys.Rev.D83 (2011) 034502

$$Z_{GC}(\mu) = \int DU \left[\frac{C_0 \sum c_n \xi^n}{\det \Delta(0)} \right]^{N_f} (\det \Delta(0))^{N_f} e^{-S_G}$$

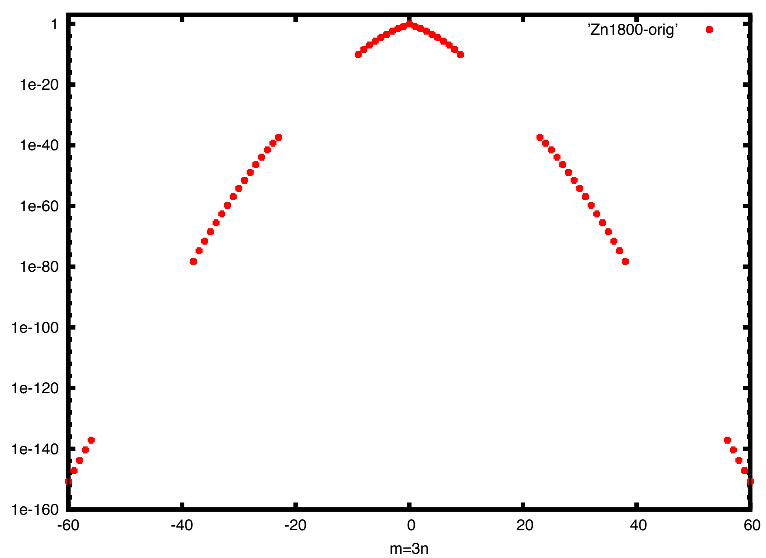
$$= \int DU (\sum a_n \xi^n)(\det \Delta(0))^{N_f} e^{-S_G}$$

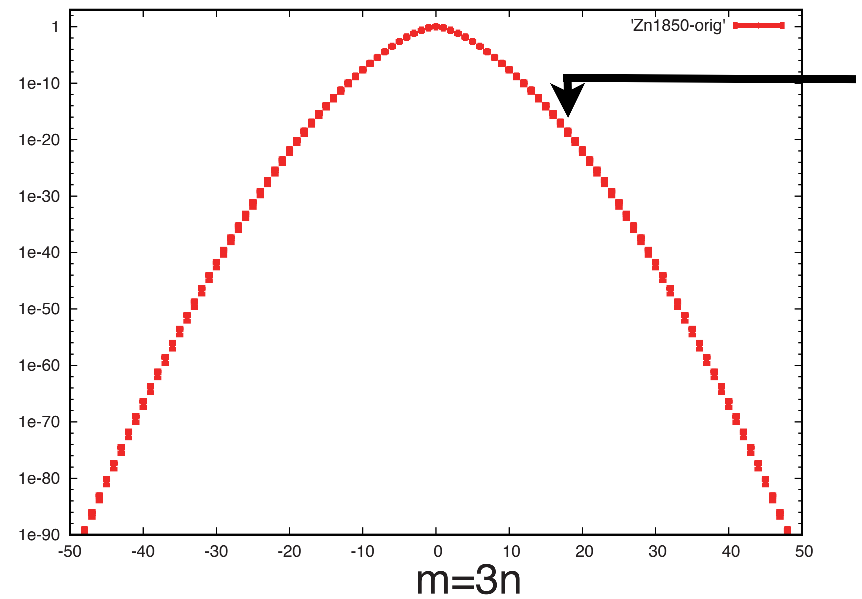
$$= \int DU (\sum a_n \xi^n)(\det \Delta(0))^{N_f} e^{-S_G}$$

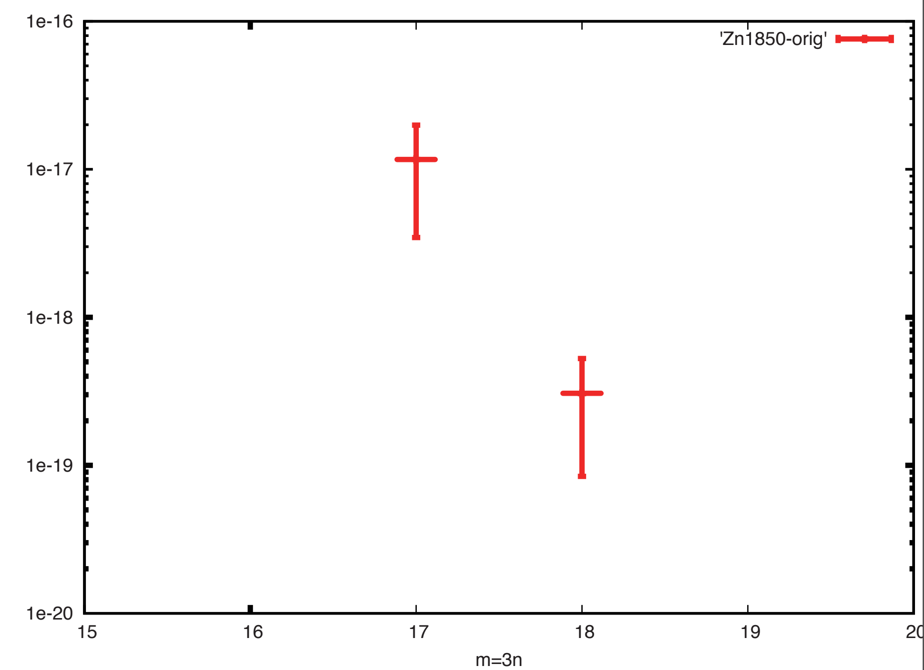
$$= \sum_n \xi^n \int DU a_n (\det \Delta(0))^{N_f} e^{-S_G}$$

$$Z_{GC}(\mu) = \sum Z_n \xi^n$$

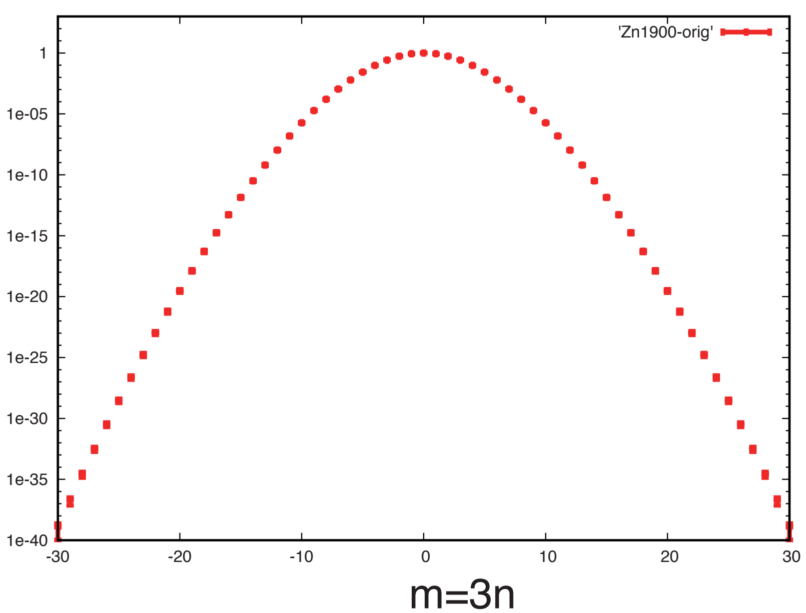
Z_n from lattice QCD

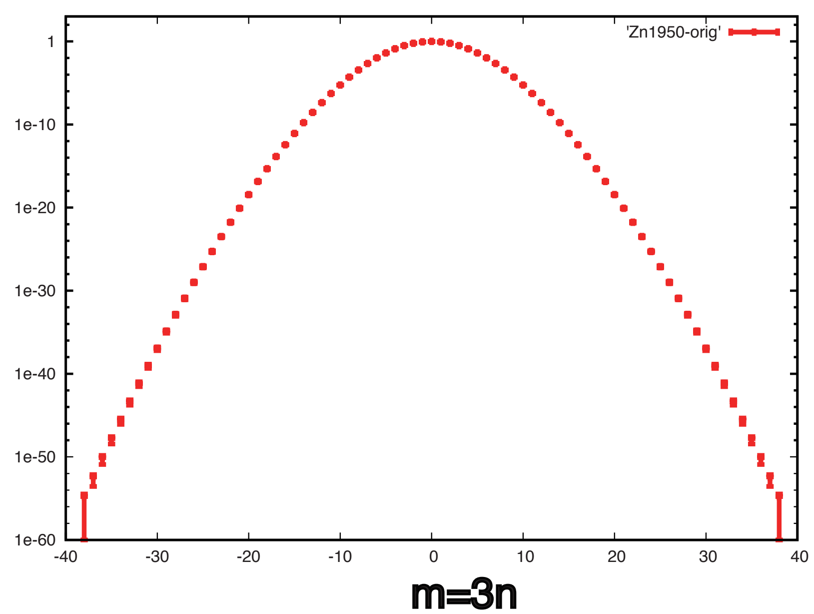


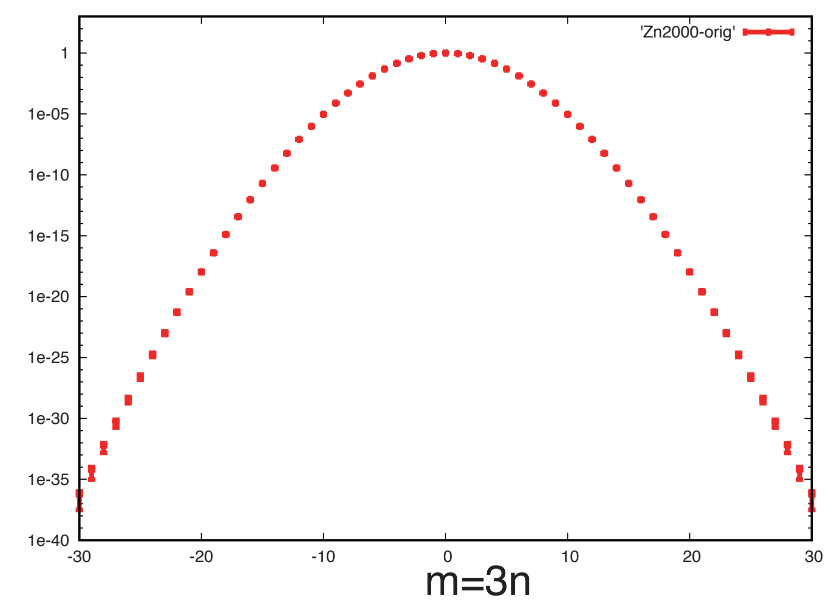




$Im(Z_n)$ are used as an error



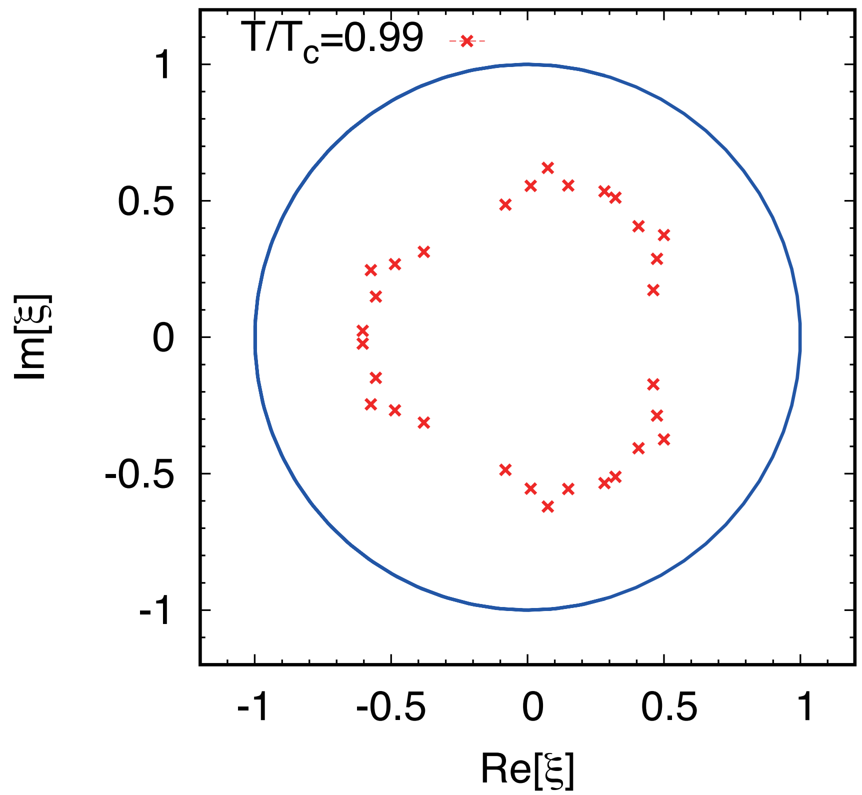




Lee-Yang Zeros
Lattice QCD

$$Z(\xi) = \sum_m Z_{3m} \times \xi^{3m}$$

$Z_n = 0$ **unless** $n \neq 3m$

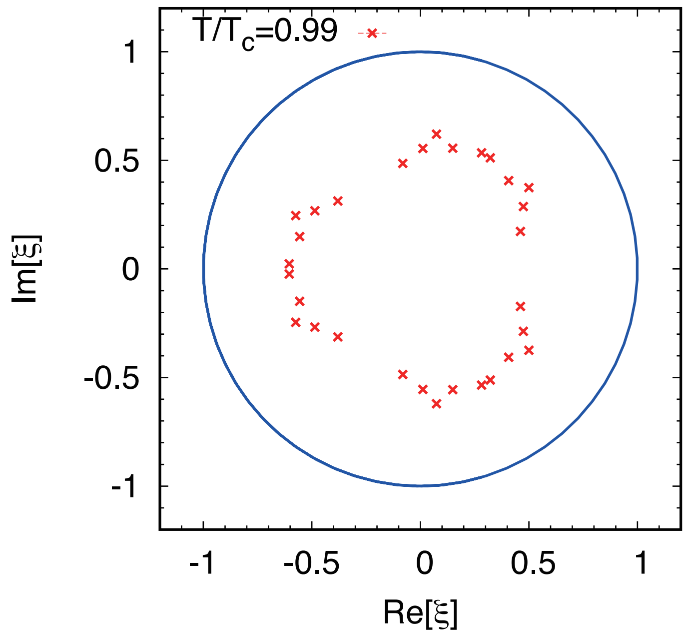


Periodicity $\theta = \dfrac{2\pi}{3}$

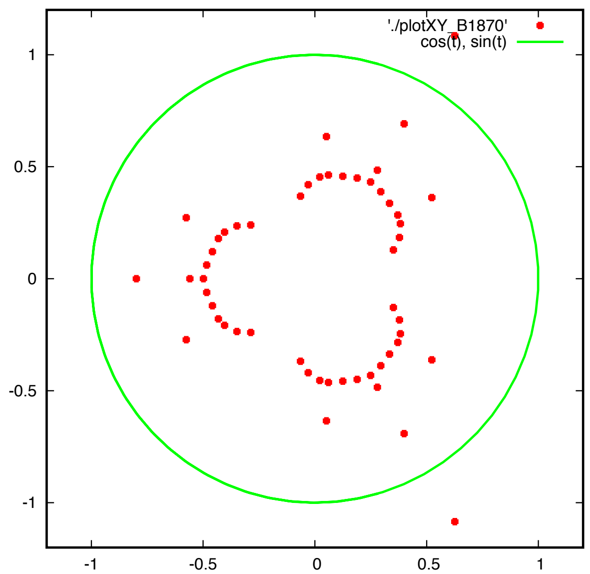
$(\xi = e^{i\theta})$

$\beta = 1.85$

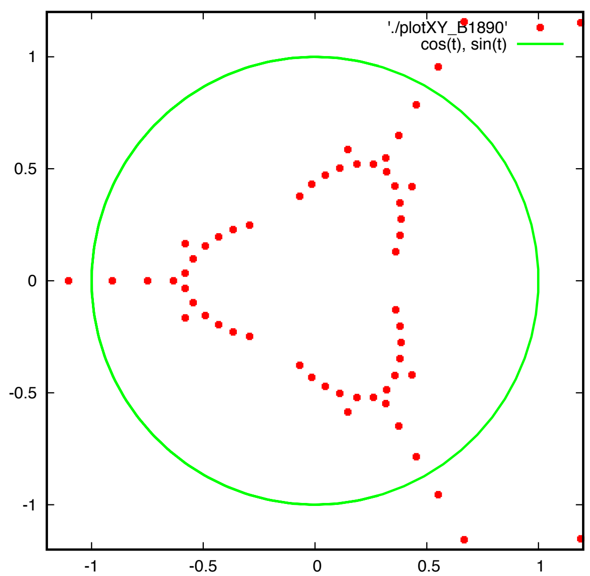
$T/T_c \sim 0.99$



$$\beta = 1.85 \qquad T/T_c \sim 0.99$$

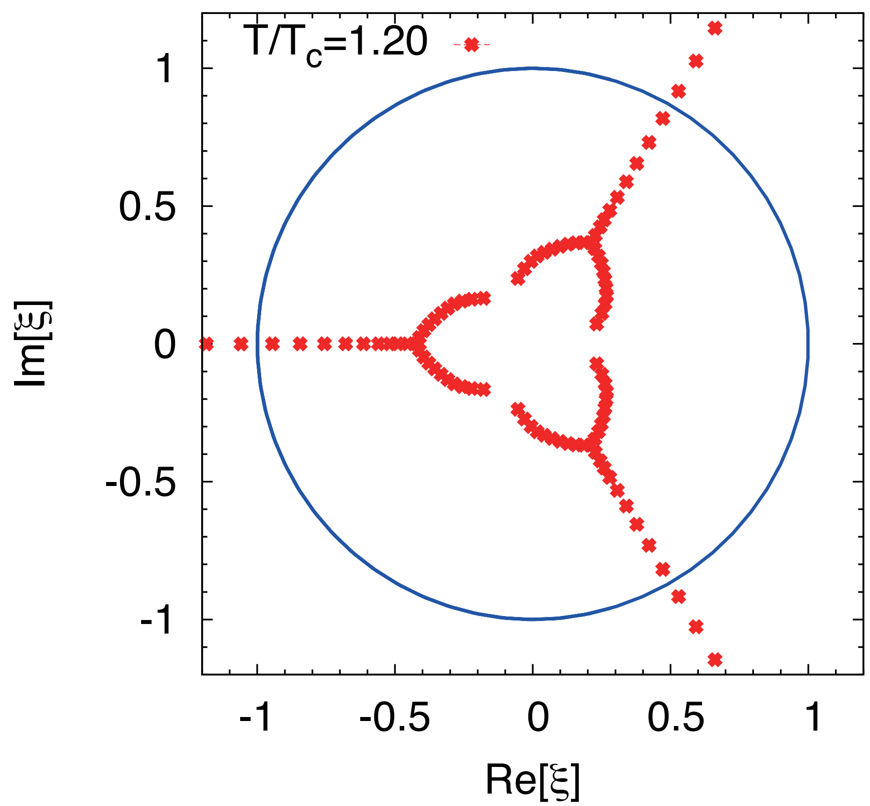


$$\beta = 1.87 \qquad T/T_c \sim 1.01$$

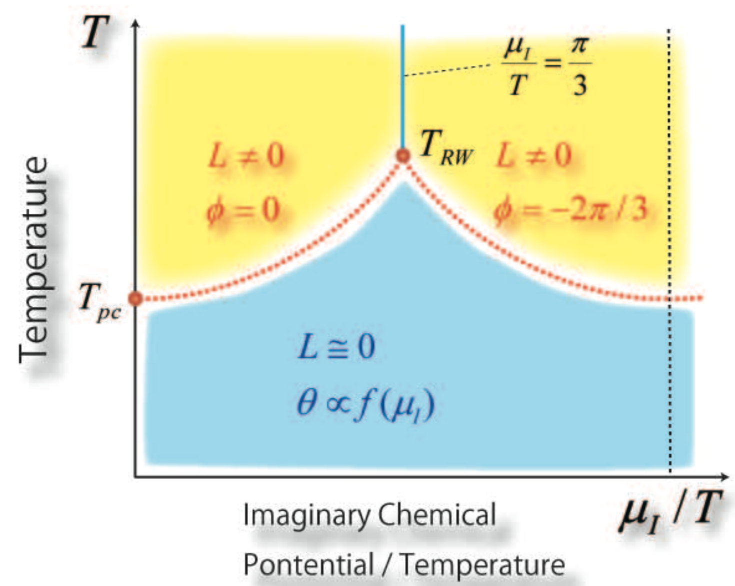


$$\beta = 1.89 \qquad T/T_c \sim 1.04$$

Lee-Yang Zeros
Lattice QCD



$$T/T_c \sim 1.20$$



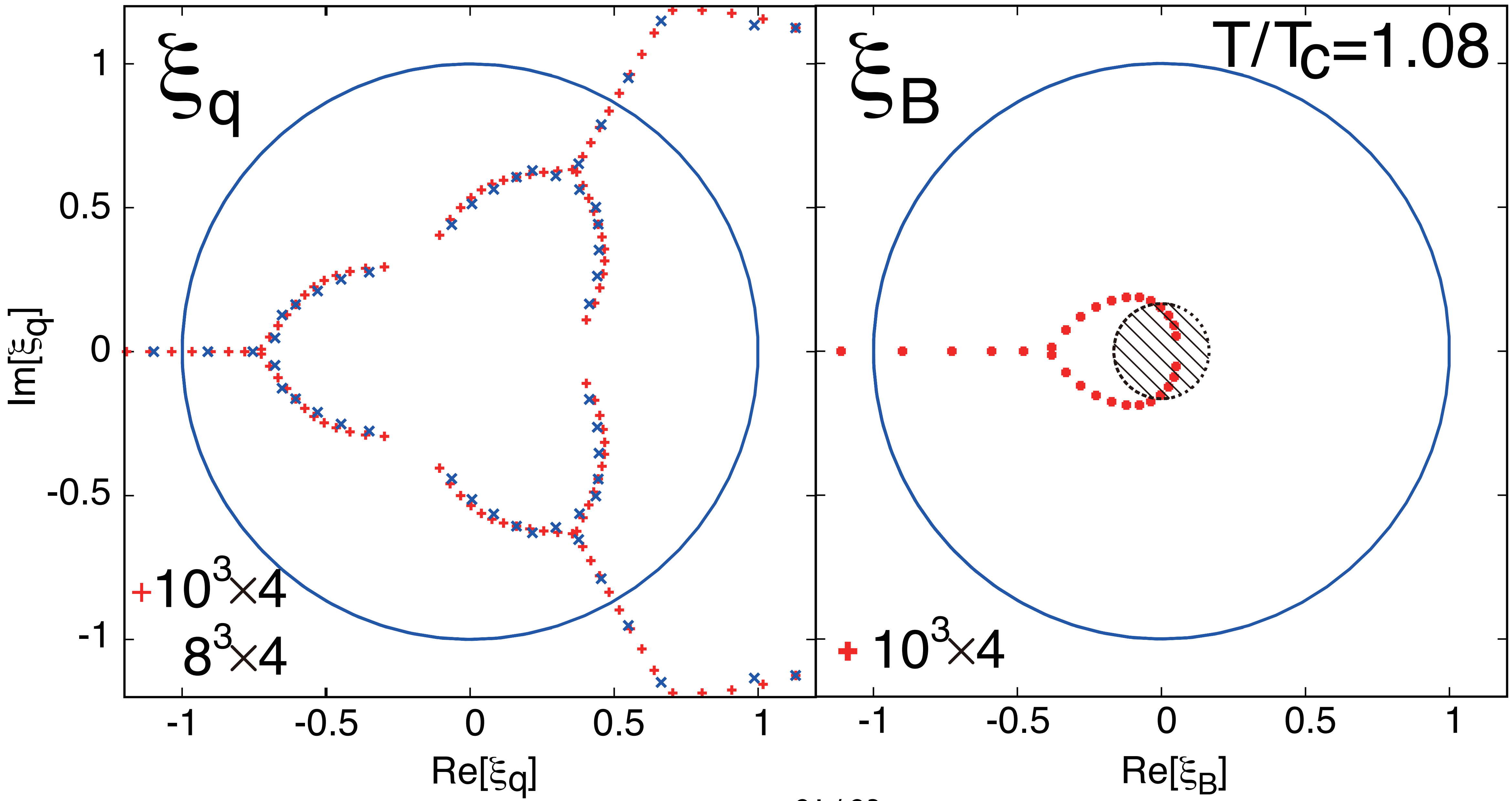
$$\xi = e^{\mu/T}$$

The Unit Cirle in ξ

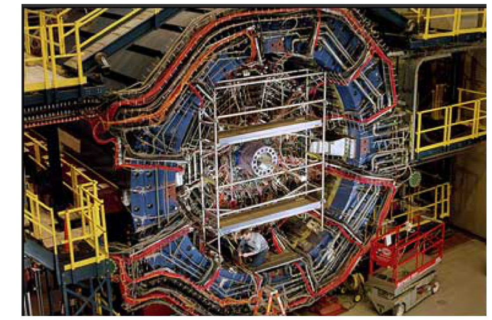
Imaginary μ

Roberge-Weise Transition !

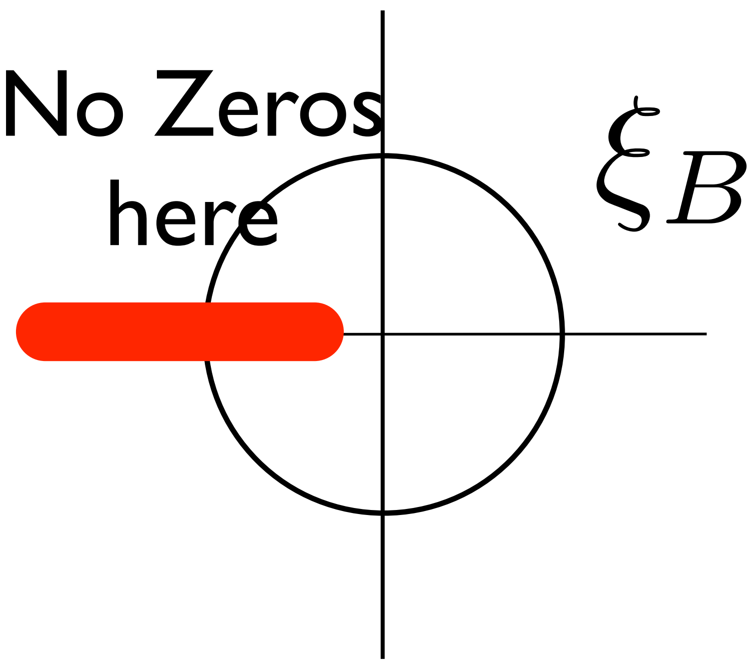
$$\xi_q \quad \Rightarrow \quad \xi_B = {\xi_q}^3$$

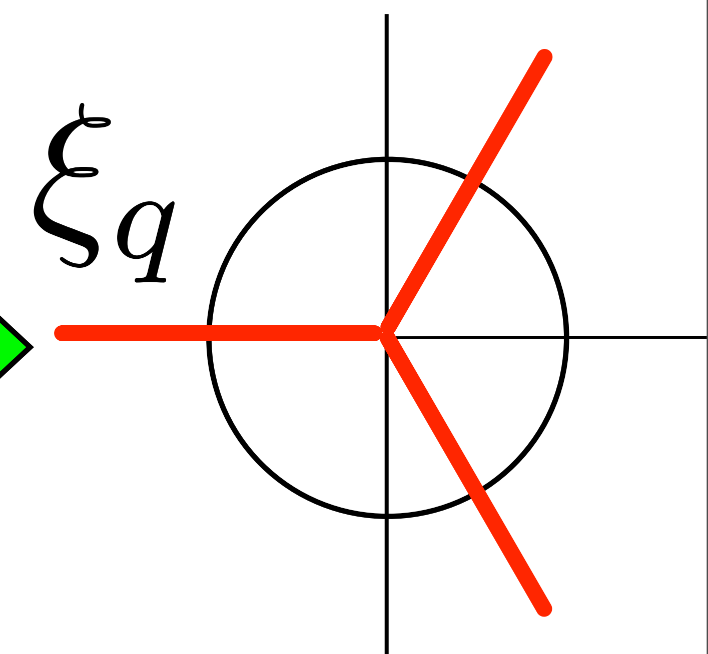


13年7月31日水曜日

A Message from  to 

In the Lee-Yang Diagram constructed from your multiplicity,



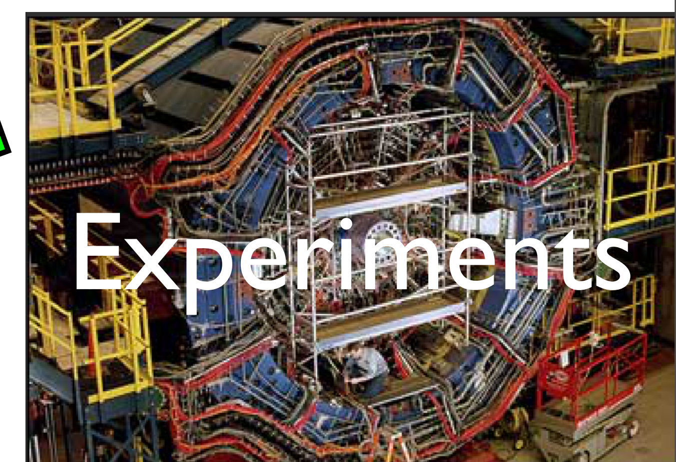


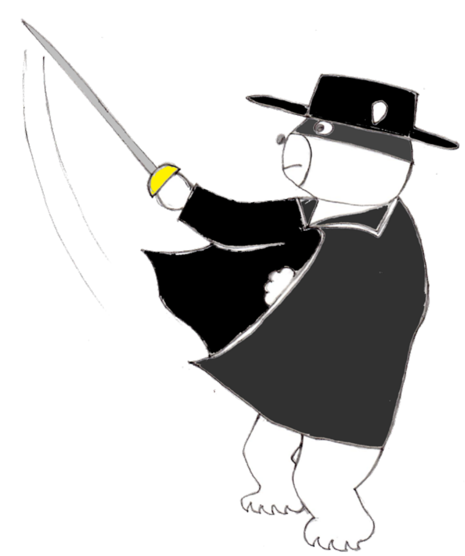
No Roberge-Weise Transition

Your Temperature $T \leq T_{RW} \sim 1.2 T_c$

Canonical Partition Functions is a Bridge between Two Approaches to Study QCD Phase

Z_N



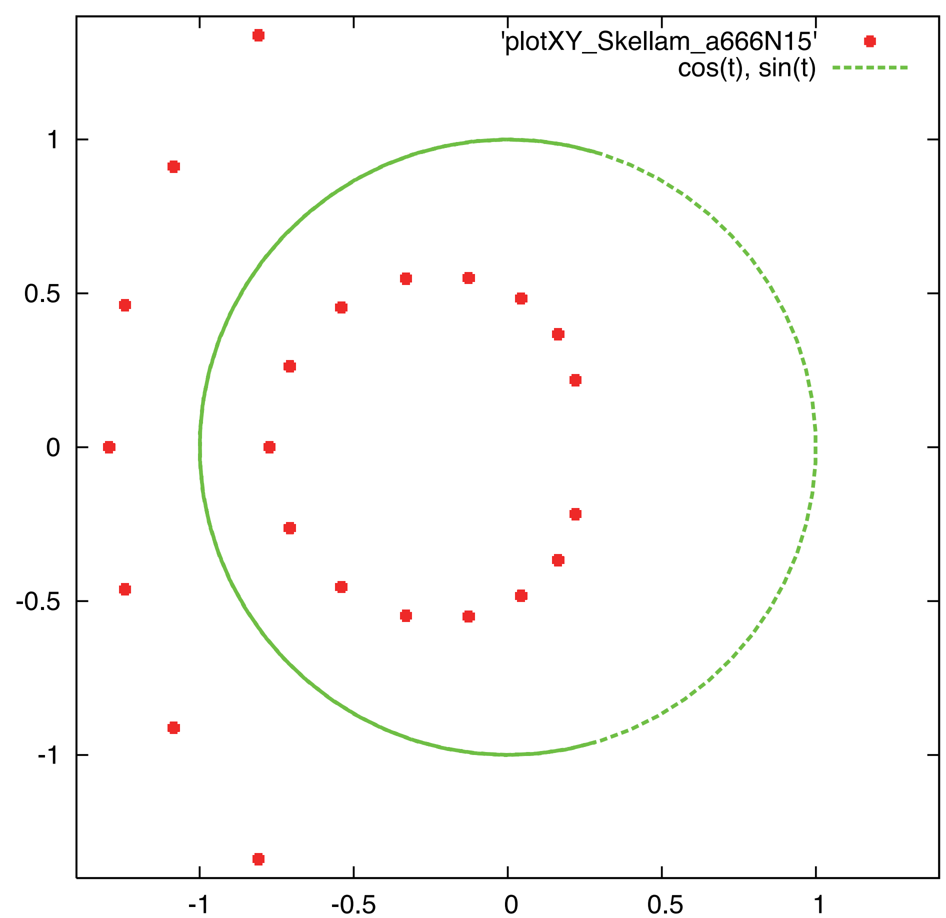


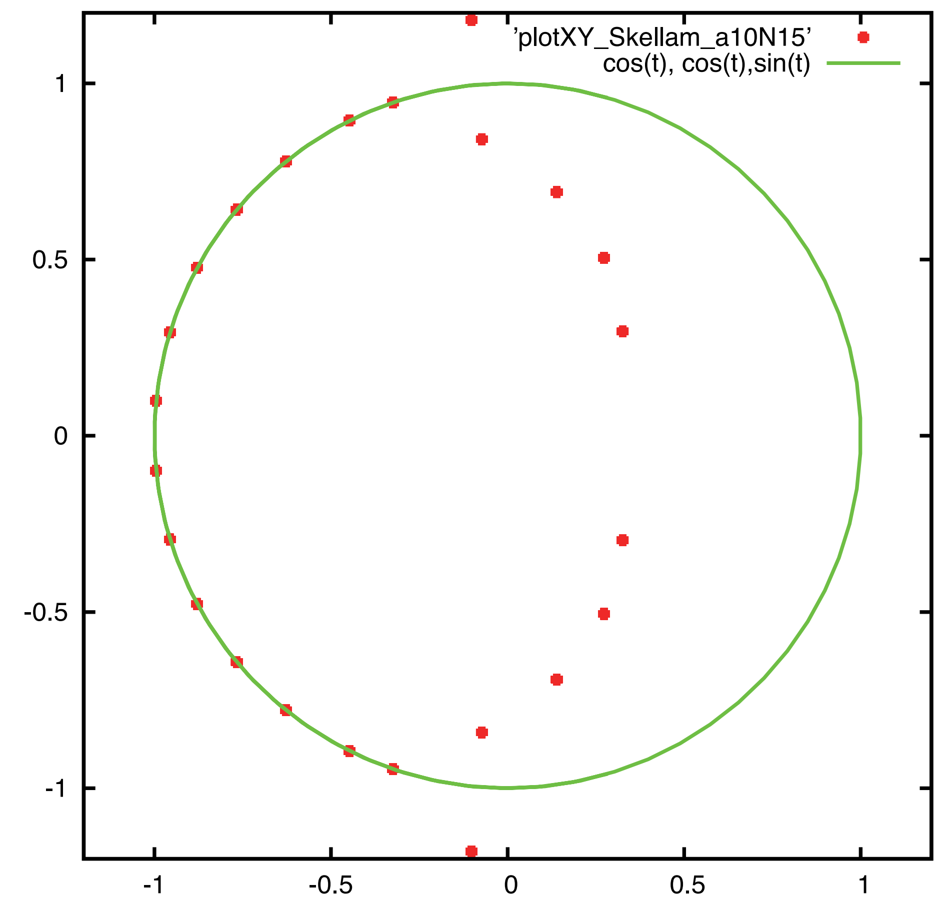
Backup Slides

Skellam

- Difference of two Poisson distributed variables.
  In our case proton and anti-proton

LYZ : Skellam


'plotXY_Skellam_a666N15'
cos(t), sin(t)
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1


'plotXY_Skellam_a10N15'
cos(t), cos(t),sin(t)
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1