D and D_s decay constants from a chiral analysis on HISQ ensembles

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*Ph.D thesis research

Lattice 2013, Mainz, Germany

MILC HISQ 2+1+1 Ensembles

- ✦ For higher precision than available with the asqtad action, we have moved to the HISQ action [Follana et al. [HPQCD], PRD 75 (2007) 054502].
  - Reduced $O(\alpha_S\, a^2)$ and $O(\alpha_S^2\, a^2)$ [taste-violation] errors with respect to asqtad.
  - $\alpha_S\, (am_c)^2$, $(am_c)^4$ errors also reduced.
    - Non-relativistic expansion says that m_c errors further reduced in heavy-light physics by powers of charm quark velocity.
    - $\Rightarrow$ treat charm with same relativistic action as light quarks.
  - Ensembles include charm sea quarks:
    - Although error of quenching charm is probably quite small in most cases, at today's level of precision it is safer to include charm in the sea; doesn't cost much.

D meson decay constants with HISQ

- Advantage of HISQ is that charm may be treated with same action as light quarks.
  - avoid renormalization errors and many tuning issues.
  - share to some degree the small statistical errors of staggered light pseudoscalars.
  - how large a value of $m_c a$ may reasonably be used is not obvious *a priori.*
    - HPQCD has included HISQ valence on asqtad lattices as coarse as a=0.15 fm, for which $m_c a$ = 0.85.
    - We consider HISQ on HISQ up to a=0.15 fm also.
      - Power counting estimate is that errors are ~5%, with further reduction by dimensionless factors possible.
      - We currently are keeping a=0.15 fm data in central values, but compare with fits dropping it in systematic error estimate.

Ensembles Used

Ensembles with physical strange quark mass:

$\approx a$ (fm)	m_l/m_s	$N_s^3 \times N_t$	$M_\pi L$	M_π (MeV)	N_{lats}
0.15	1/5	$16^3 \times 48$	3.78	306.9(5)	1020
0.15	1/10	$24^3 \times 48$	3.99	214.5(2)	1000
0.15	1/27	$32^3 \times 48$	3.30	131.0(1)	1000
0.12	1/5	$24^3 \times 64$	4.54	305.3(4)	1040
0.12	1/10	$24^3 \times 64$	3.22	218.1(4)	1020
0.12	1/10	$32^3 \times 64$	4.29	216.9(2)	1000
0.12	1/10	$40^3 \times 64$	5.36	217.0(2)	1032
0.12	1/27	$48^3 \times 64$	3.88	131.7(1)	1000
0.09	1/5	$32^3 \times 96$	4.50	312.7(6)	1012
0.09	1/10	$48^3 \times 96$	4.71	220.3(2)	1000
0.09	1/27	$64^3 \times 96$	3.66	128.2(1)	872
0.06	1/5	$48^3 \times 144$	4.51	319.3(5)	1016
0.06	1/10	$64^3 \times 144$	4.25	229.2(4)	837
0.06	1/27	$96^3 \times 192$	3.95	135.5(2)	586

Red = ensemble generation still in progress

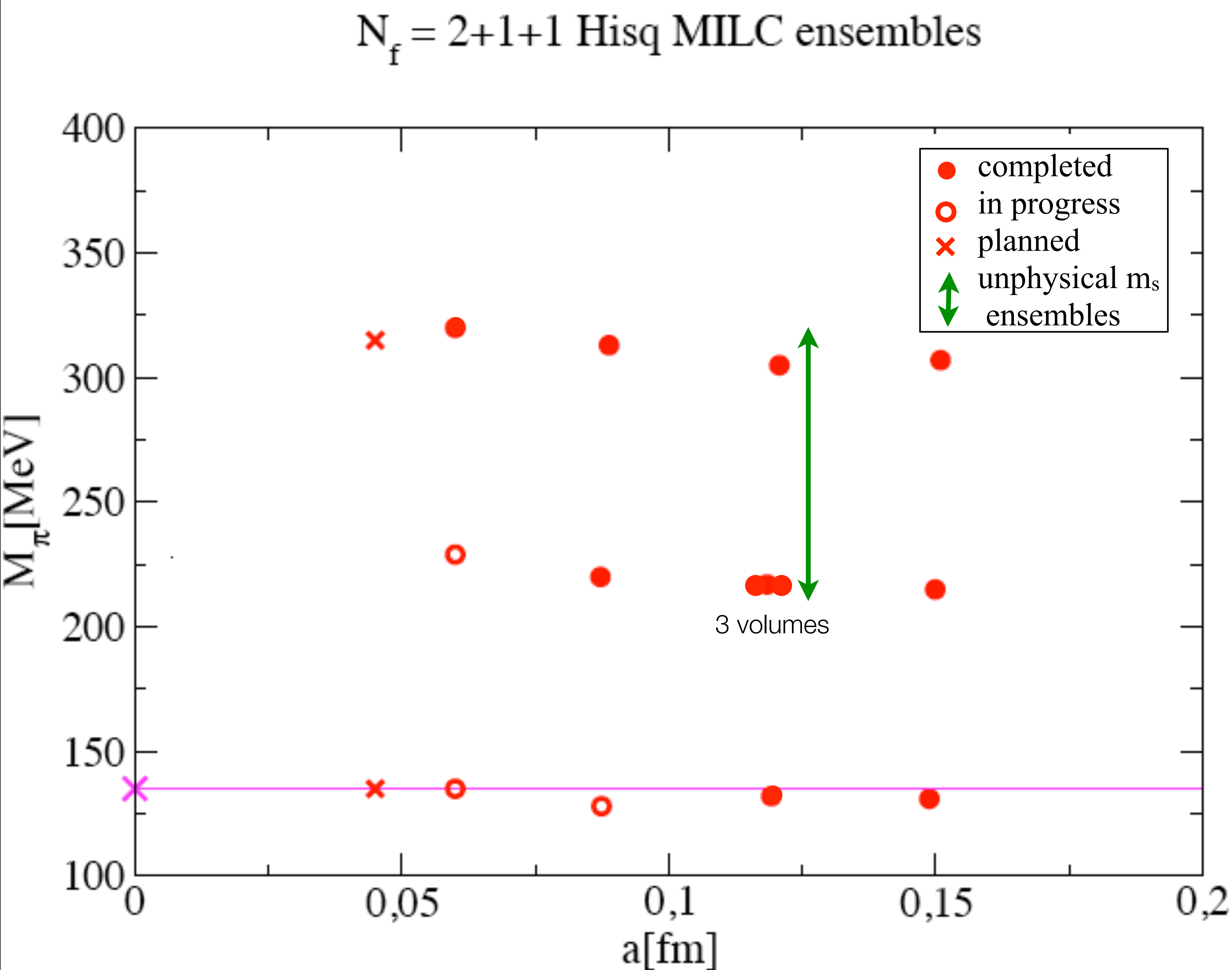
Ensembles Used

Ensembles with strange quark mass lighter than physical:

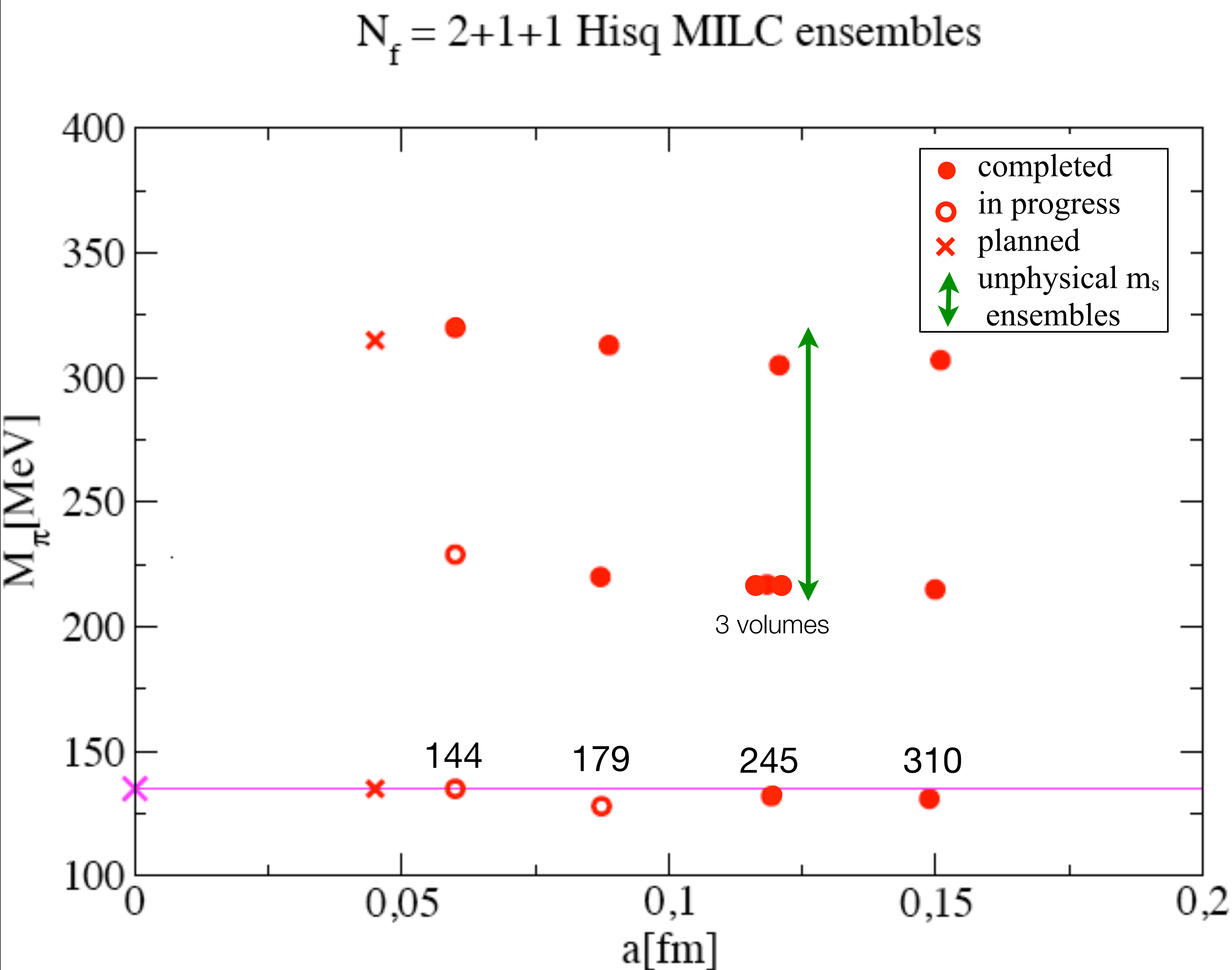
$\approx a$ (fm)	m_l/m_s	m'_s/m_s	$N_s^3 \times N_t$	$N_{\rm lats}$
0.12	0.10	0.10	$32^3 \times 64$	1020
0.12	0.10	0.25	$32^3 \times 64$	1020
0.12	0.10	0.45	$32^3 \times 64$	1020
0.12	0.10	0.60	$32^3 \times 64$	1020
0.12	0.25	0.25	$24^3 \times 64$	1020
0.12	0.20	0.60	$24^3 \times 64$	1020
0.12	0.175	0.45	$32^3 \times 64$	1020

(These ensembles are not crucial to D decay project, but are useful for adjusting for mistunings in strange (and light) masses.)

MILC HISQ Ensembles



MILC HISQ Ensembles



sea masses:

$m_\pi^{\rm RMS}$ [MeV]

Valence Masses Used

β	am_l	am_s	am_c	light masses m_x (m_x/m_s)	heavy mass m_y (m_y/m_c)
5.80	0.013	0.065	0.838	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
5.80	0.0064	0.064	0.828	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
5.80	0.00235	0.0647	0.831	0.036,0.07,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.00	0.0102	0.0509	0.635	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.00	0.00507	0.0507	0.628	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
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6.00	0.00507	0.0304	0.628	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.00	0.00507	0.00507	0.628	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.00	0.00184	0.0507	0.628	0.036,0.073,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.30	0.0074	0.037	0.440	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.30	0.00363	0.0363	0.430	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.30	0.0012	0.0363	0.432	0.033,0.066,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	0.0048	0.024	0.286	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	0.0024	0.024	0.286	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	0.0008	0.022	0.260	0.036,0.068,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0

Valence Masses Used

β	am_l	am_s	am_c	light masses m_x (m_x/m_s)	heavy mass m_y (m_y/m_c)
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5.80	0.0064	0.064	0.828	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
5.80	0.00235	0.0647	0.831	0.036,0.07,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
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6.00	0.00507	0.00507	0.628	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
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6.30	0.0074	0.037	0.440	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.30	0.00363	0.0363	0.430	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.30	0.0012	0.0363	0.432	0.033,0.066,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	0.0048	0.024	0.286	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	0.0024	0.024	0.286	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	0.0008	0.022	0.260	0.036,0.068,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0

Physical sea mass ensembles have physical valence masses. (Volumes chosen appropriately.)

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6.72	0.0048	0.024	0.286	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	0.0024	0.024	0.286	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	0.0008	0.022	0.260	0.036,0.068,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0

Physical sea mass ensembles have physical valence masses. (Volumes chosen appropriately.)

Allows correction for mistuning in m_c.

Chiral Perturbation Theory

- ✦ J. Komijani and CB have worked out appropriate heavy-meson chiral perturbation theory when both light and heavy quarks are staggered: “Heavy-meson, rooted, all-staggered chiral perturbation theory” (HMrASχPT) [arXiv:1211.0785, and to appear].
  - Used here to fit all heavy-light data and interpolate/extrapolate to physical quark masses and to continuum.
  - Useful also for understanding the pattern of taste-symmetry breaking in the heavy-light meson masses [MILC: A. Bazavov et al., PRD 87 054505 (2013)].

Chiral Perturbation Theory

- An alternative, simpler analysis that avoids χPT and focuses on the physical-mass ensembles will be presented by Doug Toussaint tomorrow (session 8C).
  - That analysis also includes light-light mesons and provides the absolute scale setting (through f_π) and physical quark masses needed here.
  - At the cost of significant complications, the χPT analysis allows us to use all our data (not just physical-mass ensembles), thereby reducing statistical errors, and to help control the continuum extrapolation.
  - Ultimately will extend the χPT analysis to the light-light sector to have a completely self-contained version of the analysis.

Chiral Perturbation Theory

- NLO form in HMrASχPT, including hyperfine and flavor splittings in heavy-light masses:

$$
\begin{aligned}
\frac{f_{D_{x\Xi}}\sqrt{M_{D_{x\Xi}}}}{\kappa} = \; & 1 + \frac{1}{16\pi^2 f^2}\frac{1}{2}\Bigg\{ -\frac{1}{16}\sum_{\mathcal{S},\Xi'} \ell(m^2_{\mathcal{S}x,\Xi'}) \\
& - \frac{1}{3}\sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})\ell(m_j^2)\right] \\
& - \left(a^2\delta'_V \sum_{j\in\mathcal{M}_V^{(4,x)}} \frac{\partial}{\partial m^2_{X,V}}\left[R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})\ell(m_j^2)\right] + [V\to A]\right) \\
& - 3g_\pi^2 \frac{1}{16}\sum_{\mathcal{S},\Xi'} J(m_{\mathcal{S}x,\Xi'},\Delta^* + \delta_{\mathcal{S}x}) \\
& - g_\pi^2 \sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})J(m_j,\Delta^*)\right] \\
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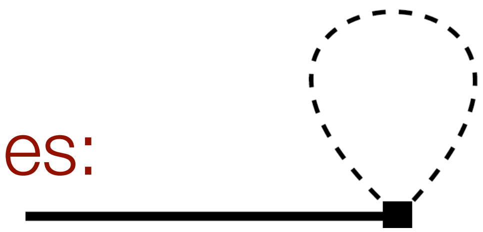
& + c_s(m_u + m_d + m_s) + c_v m_x + c_{a,\Xi} a^2 \; .
\end{aligned}
$$

Chiral Perturbation Theory

- NLO form in HMrASχPT, including hyperfine and flavor splittings in heavy-light masses:

chiral logs
from tadpoles:



$$
\begin{aligned}
\frac{f_{D_{x\Xi}}\sqrt{M_{D_{x\Xi}}}}{\kappa} \quad = \quad & 1 + \frac{1}{16\pi^2 f^2}\frac{1}{2}\Bigg\{ -\frac{1}{16}\sum_{\mathcal{S},\Xi'} \ell(m^2_{\mathcal{S}x,\Xi'}) \\
& -\frac{1}{3}\sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})\ell(m_j^2)\right] \\
& -\left(a^2\delta'_V \sum_{j\in\mathcal{M}_V^{(4,x)}} \frac{\partial}{\partial m^2_{X,V}}\left[R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})\ell(m_j^2)\right] + [V\to A]\right) \\
& -3g_\pi^2\frac{1}{16}\sum_{\mathcal{S},\Xi'} J(m_{\mathcal{S}x,\Xi'},\Delta^* + \delta_{\mathcal{S}x}) \\
& -g_\pi^2 \sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})J(m_j,\Delta^*)\right] \\
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& + c_s(m_u+m_d+m_s) + c_v m_x + c_{a,\Xi}a^2 \ .
\end{aligned}
$$

Chiral Perturbation Theory

✦ NLO form in HMrASχPT, including hyperfine and flavor splittings in heavy-light masses:

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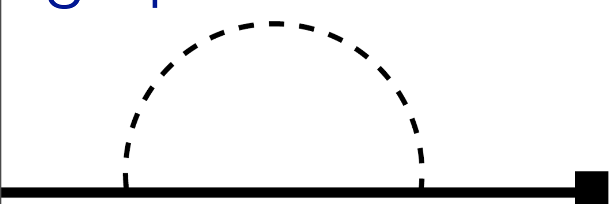
& + c_s(m_u+m_d+m_s) + c_v m_x + c_{a,\Xi}a^2 \;.
\end{aligned}
$$

chiral logs from sunset graphs



Chiral Perturbation Theory

✦ NLO form in HMrASχPT, including hyperfine and flavor splittings in heavy-light masses:

$$
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&\quad - \frac{1}{3}\sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})\ell(m_j^2)\right] \\
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&\quad + c_s(m_u+m_d+m_s) + c_v m_x + c_{a,\Xi}a^2 \ .
\end{aligned}
$$

heavy-light
hyperfine
splitting

Chiral Perturbation Theory

- NLO form in HMrASχPT, including hyperfine and flavor splittings in heavy-light masses:

residue functions for neutral light propagators

$$
\begin{aligned}
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& - 3g_\pi^2\frac{1}{16}\sum_{\mathcal{S},\Xi'} J(m_{\mathcal{S}x,\Xi'},\Delta^* + \delta_{\mathcal{S}x}) \\
& - g_\pi^2 \sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[\boxed{R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})}\,J(m_j,\Delta^*)\right] \\
& - 3g_\pi^2\left(a^2\delta'_V \sum_{j\in\mathcal{M}_V^{(4,x)}} \frac{\partial}{\partial m^2_{X,V}}\left[\boxed{R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})}\,J(m_j,\Delta^*)\right] + [V\to A]\right)\Bigg\} \\
& + c_s(m_u+m_d+m_s) + c_v m_x + c_{a,\Xi}a^2 \;.
\end{aligned}
$$

Chiral Perturbation Theory

- NLO form in HMrASχPT, including hyperfine and flavor splittings in heavy-light masses:

$$
\begin{aligned}
\frac{f_{D_{x\Xi}}\sqrt{M_{D_{x\Xi}}}}{\kappa} = \; & 1 + \frac{1}{16\pi^2 f^2}\frac{1}{2}\Bigg\{ -\frac{1}{16}\sum_{\mathcal{S},\Xi'} \ell(m^2_{\mathcal{S}x,\Xi'}) \\
& - \frac{1}{3}\sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})\ell(m_j^2)\right] \\
& - \left(a^2\delta'_V \sum_{j\in\mathcal{M}_V^{(4,x)}} \frac{\partial}{\partial m^2_{X,V}}\left[R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})\ell(m_j^2)\right] + [V\to A]\right) \\
& - 3g_\pi^2 \frac{1}{16}\sum_{\mathcal{S},\Xi'} J(m_{\mathcal{S}x,\Xi'}, \Delta^* + \delta_{\mathcal{S}x}) \\
& - g_\pi^2 \sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})J(m_j,\Delta^*)\right] \\
& - 3g_\pi^2 \left(a^2\delta'_V \sum_{j\in\mathcal{M}_V^{(4,x)}} \frac{\partial}{\partial m^2_{X,V}}\left[R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})J(m_j,\Delta^*)\right] + [V\to A]\right)\Bigg\} \\
& + c_s(m_u+m_d+m_s) + c_v m_x + c_{a,\Xi}a^2 \; .
\end{aligned}
$$

only taste-dependence at this order

Chiral Perturbation Theory

- Convenient to redefine LECs in terms of natural dimensionless factors of χPT:

$$c_s(m_u + m_d + m_s) + c_v m_x + c_a a^2$$
$$\to L_s\,(x_u + x_d + x_s) + L_v(x_x) + L_a \frac{x_{\bar\Delta}}{2} \quad .$$

$$x_{u,d,s,x} \quad \equiv \quad \frac{4B}{16\pi^2 f_\pi^2} m_{u,d,s,x} \quad , \qquad\qquad x_{\bar\Delta} \quad \equiv \quad \frac{2}{16\pi^2 f_\pi^2}\bar\Delta \quad ,$$

  - where B is the LEC that gives pion mass: $m_\pi^2 = B(m_u + m_d)$, and $\bar\Delta$ is the mean-squared pion taste splitting.

- With these definitions, LECs are expected to be $O(1)$.

Chiral Perturbation Theory

- Have very precise data (~0.2% stat errors), with ~200--366 points, depending on cuts.
- Need to add higher-order analytic terms to the fit function:
  - "Generic" *a*-dependence of NLO LECs (a NNLO effect).
    - e.g., $L_s\,(x_u + x_d + x_s) \to (L_s + L_{s\delta}\;\alpha_S a^2)\,(x_u + x_d + x_s)$.
    - So add parameters $\kappa_\delta,\ L_{s\delta},\ L_{v\delta},\ \delta'_{A\delta},\ \delta'_{V\delta},\ g^2_{\pi\delta},\ L_{a\delta}$.
  - NNLO and NNNLO terms in quark masses needed to fit masses ~m$_s$:

$q_1(x_x^2)$	$c_1(x_x)^3$
$q_2(2x_l + x_s)^2$	$c_2(x_x)(2x_l + x_s)^2$
$q_3(2x_l + x_s)(x_v)$	$c_3(x_x)(2x_l^2 + x_s^2)$
$q_4(2x_l^2 + x_s^2)$	$c_4(x_x)^2(2x_l + x_s)$
	$c_5(2x_l + x_s)^3$
	$c_6(2x_l + x_s)(2x_l^2 + x_s^2)$
	$c_7(2x_l^3 + x_s^3)$

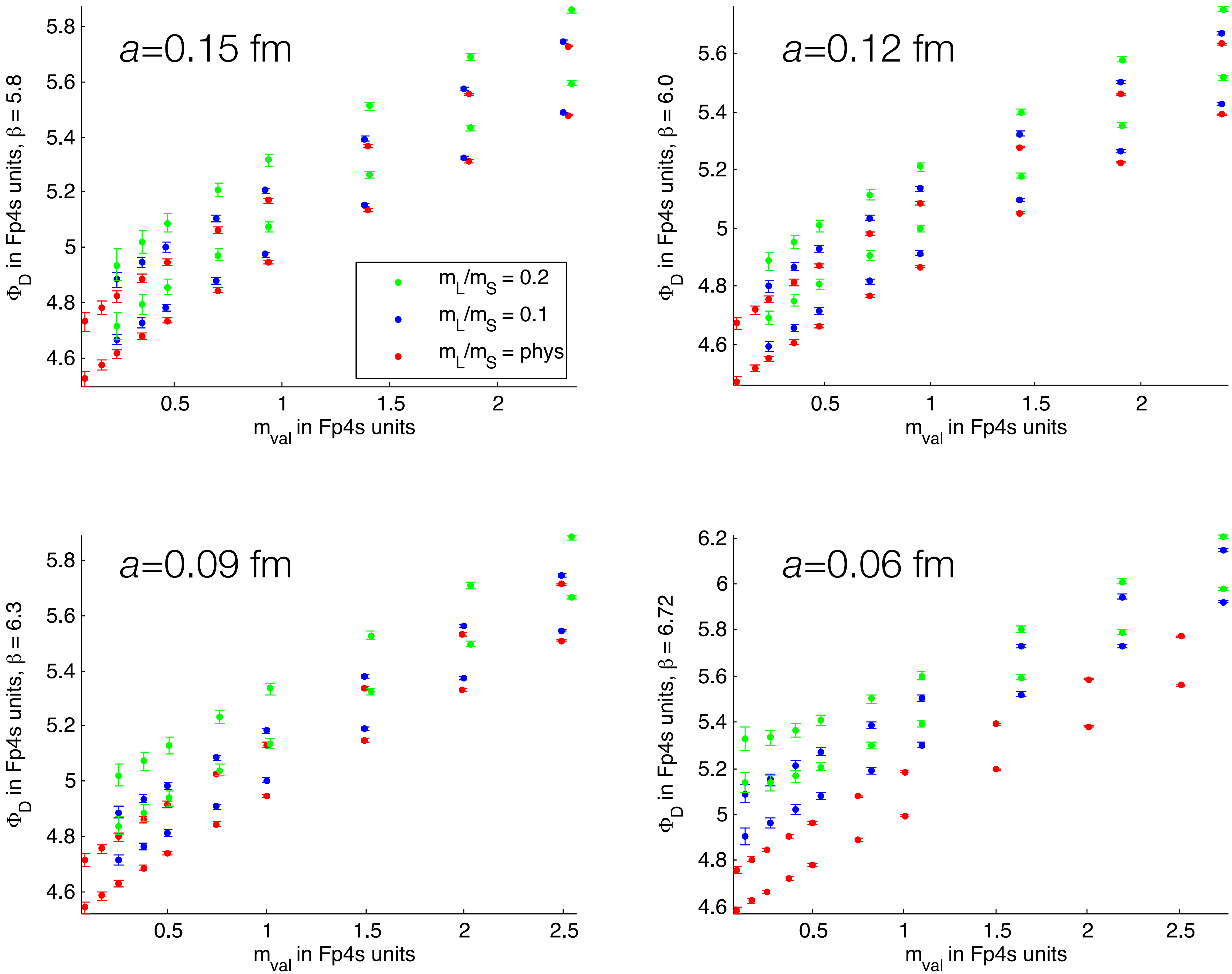
Chiral Perturbation Theory

- ✦ Because of mistunings in m_c, and especially if $m_c = 0.9\, m_c^{\rm phys}$ valence masses are included, need higher order HQET terms.
  - multiply by $1 + c_{m_c1}(\Lambda_{\rm QCD}/m_c) + c_{m_c2}(\Lambda_{\rm QCD}/m_c)^2$, where we take $\Lambda_{\rm QCD} \sim 350\ {\rm MeV}$.
- ✦ If β = 5.8 (a = 0.15 fm) ensembles are included, additional discretization correction for large am_c needed, although difficult to distinguish $\alpha_s(am_c)^2$ from $(am_c)^4$.

Lattice Scale

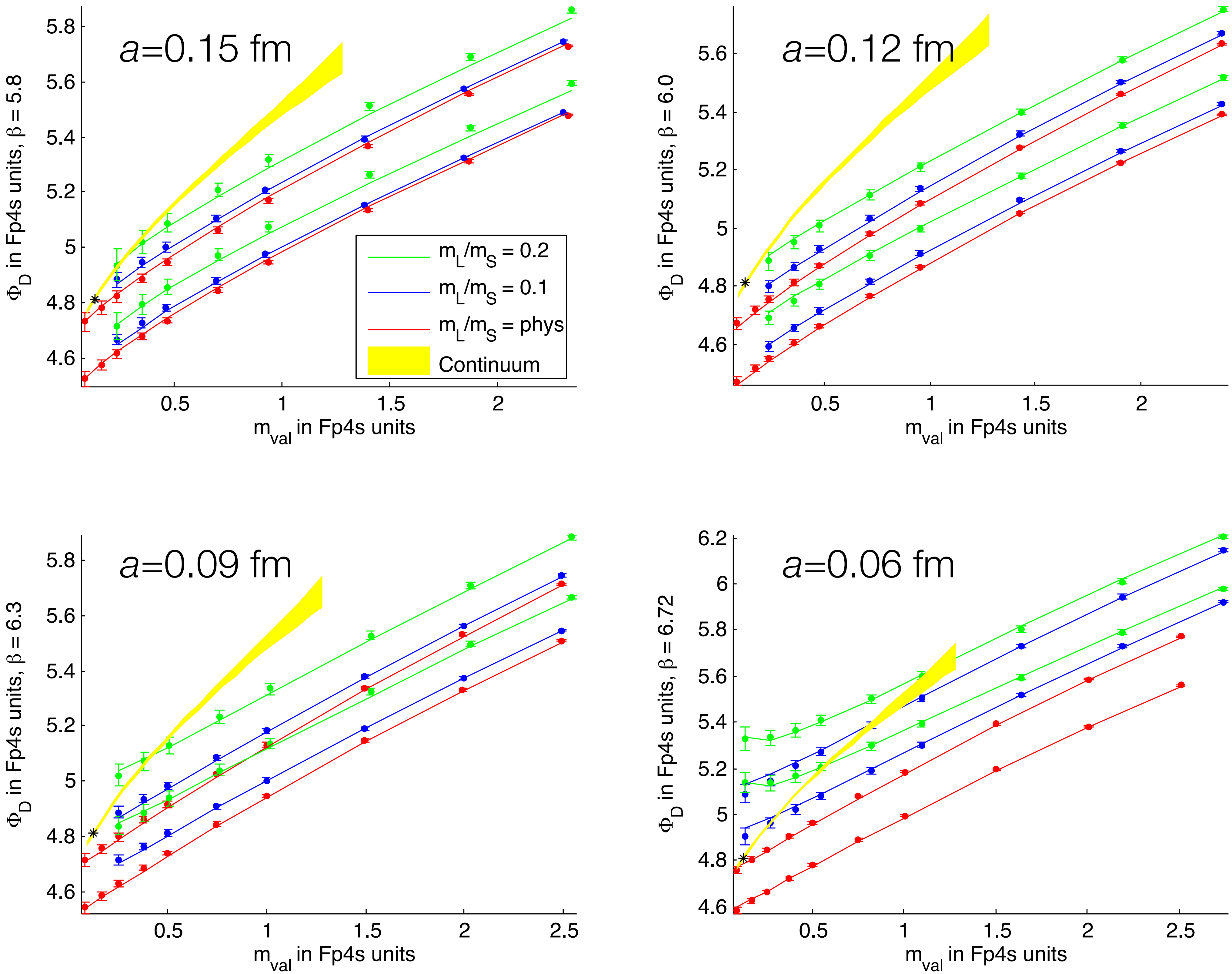
- ✦ Relative lattice scales are determined by f_{p4s} , the decay constant when valence masses are 0.4 $m_s^{\rm phys}$.
  - (sea masses are physical).
- ✦ Has very small statistical errors (comparable to that of Symanzik or Wilson flow w_0 when computed on the same numbers of configurations).
- ✦ Absolute scale is set by computing f_{p4s}/f_π, extrapolated to the continuum.

Lattice Data



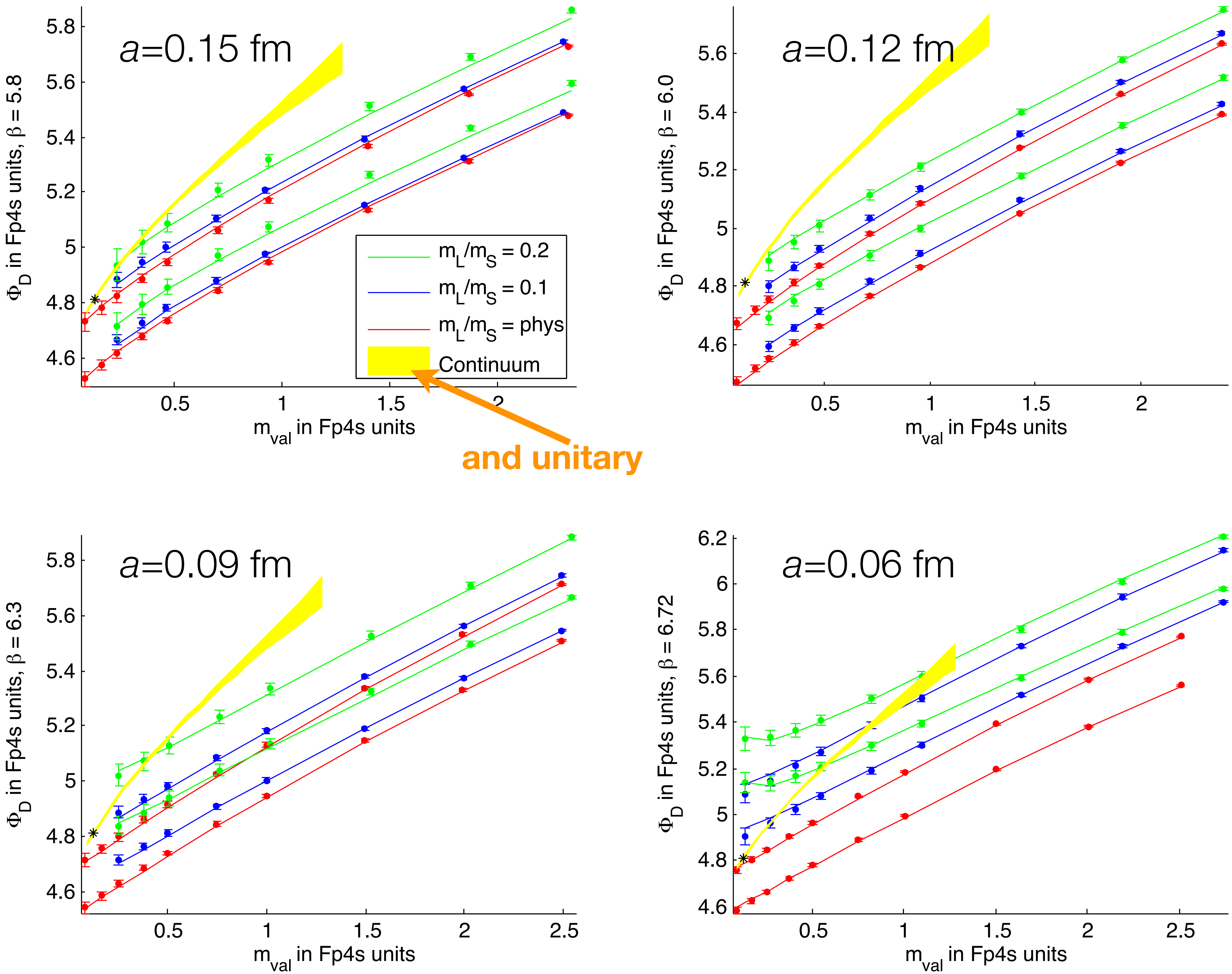
- For each color, higher points have $m_c = m_c^{phys}$; lower points have $m_c = 0.9\ m_c^{phys}$.
- Data for unphysical m_s ensembles (as well as multiple a=0.12 fm volumes) not shown, but included in fits.
- 366 data points, total.

Central Chiral Fit



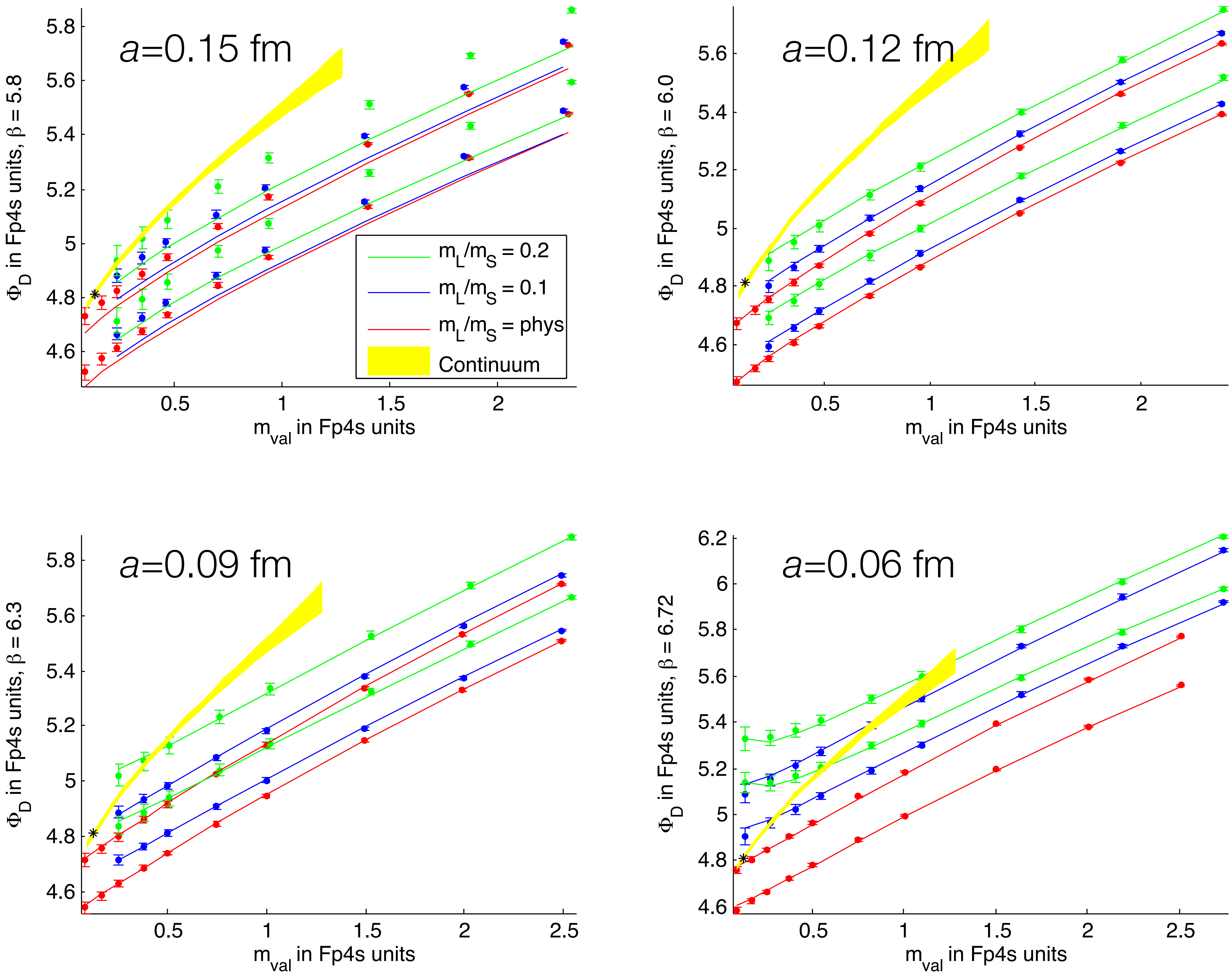
- Fit is to all 366 data points; 31 parameters.
- correlated χ^2/dof=325/335; p=0.64.
- Black burst shows continuum-extrapolated physical result.
- Statistical errors from jackknife (including all inputs) are tiny.

Central Chiral Fit



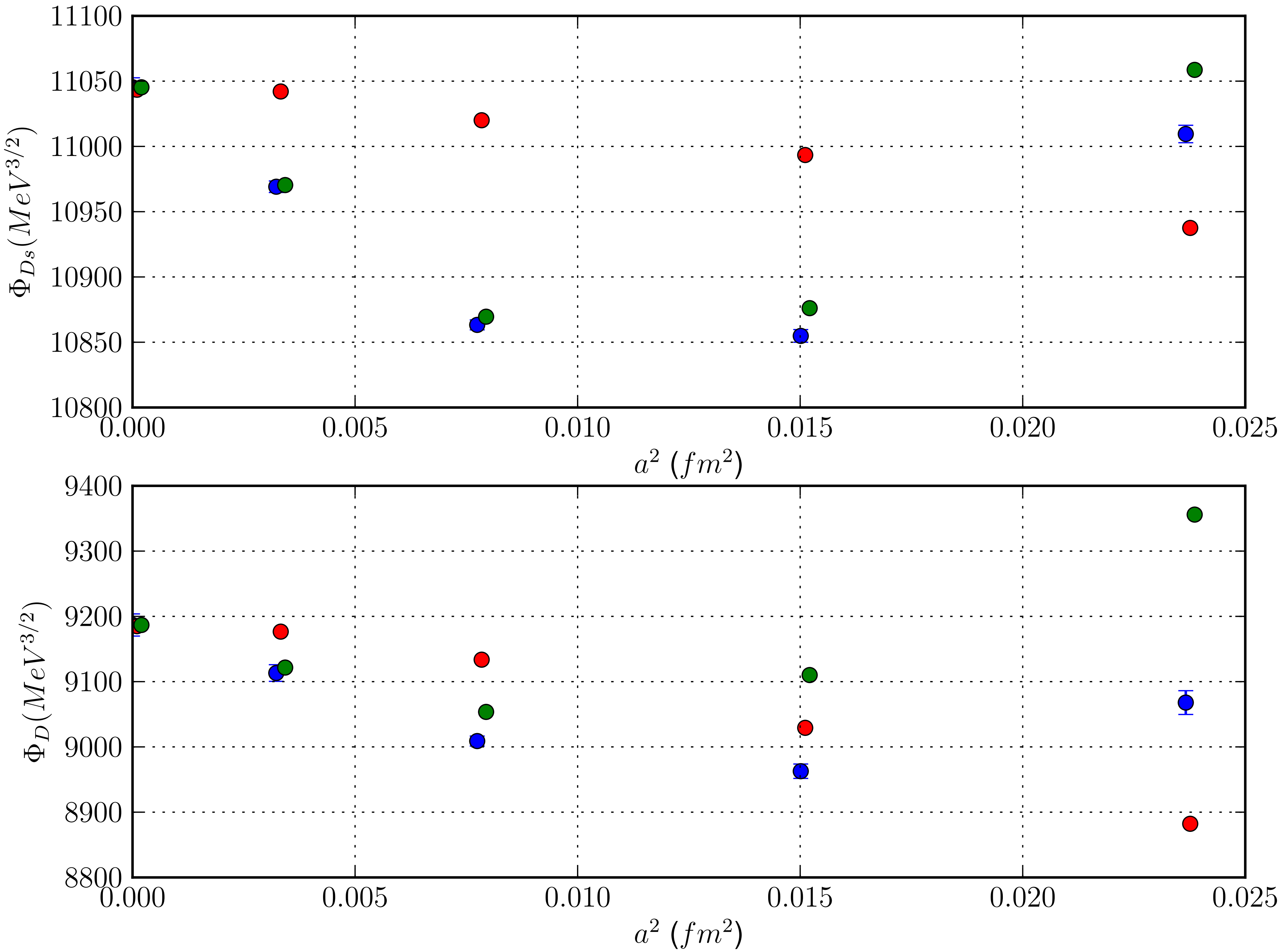
- Fit is to all 366 data points; 31 parameters.
- correlated χ^2/dof=325/335; p=0.64.
- Black burst shows continuum-extrapolated physical result.
- Statistical errors from jackknife (including all inputs) are tiny.

An Alternative Chiral Fit



- Fit does not include a=0.15 fm data.
- Fit is to 314 data points; 29 parameters.
- Does not include $\alpha_s(am_c)^2,\ (am_c)^4$ terms.
- correlated χ^2/dof=298/285; p=0.28.

Dependence on a^2



- data (adjusted for mistunings to physical masses)
- staggered chiral log contribution
- contribution from fit a-dependence (αa^2, $\alpha^2 a^2$, a^4)

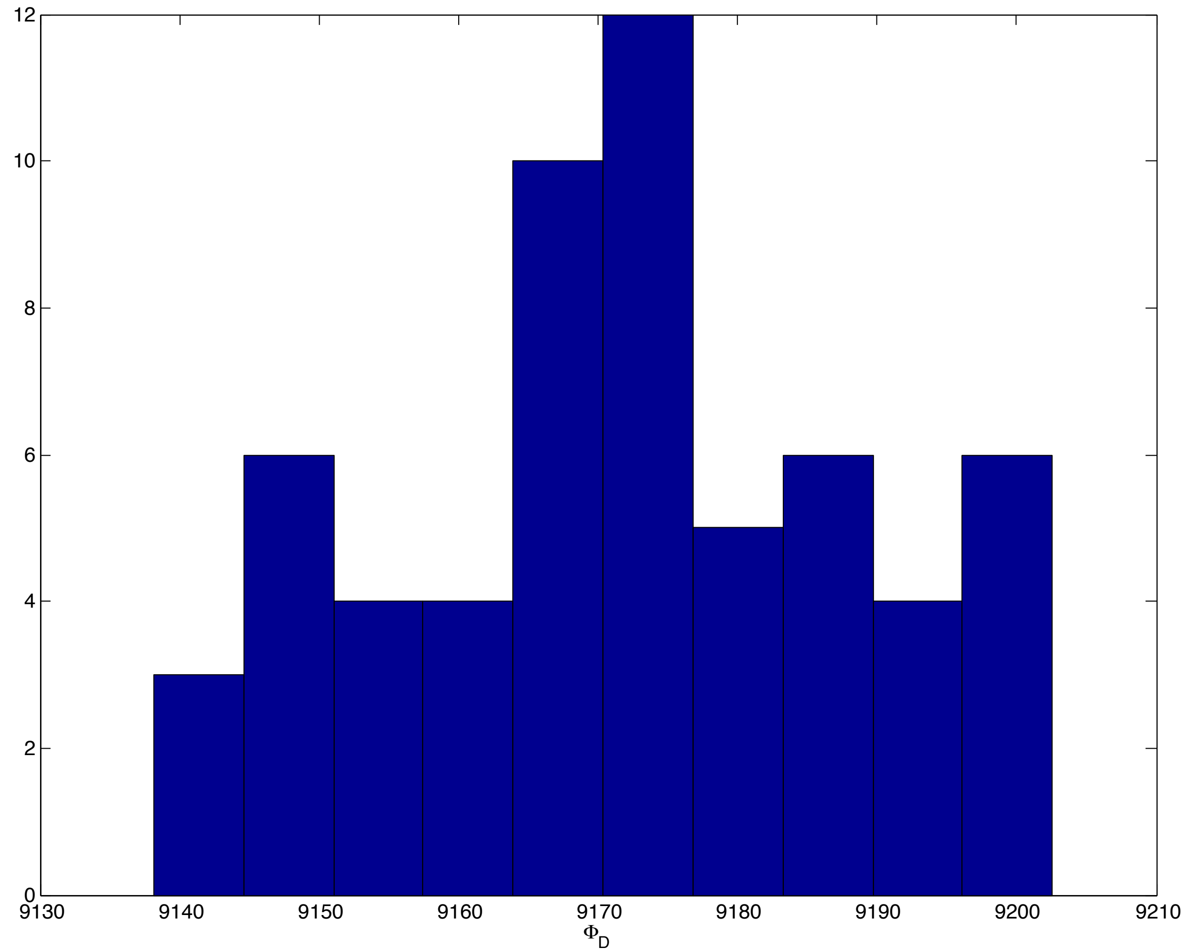
- clear competition of a^2 and a^4 fit terms, needed for parabolic shape of data.
- total variation with a, as well as individual contributions to a-dependence, are ~ 2--3%.
- Scale dependence is different in physical-point analysis (Doug Toussaint's talk), which uses f_π for relative scale setting, but rough shape is similar.

Discussion of Systematic Errors

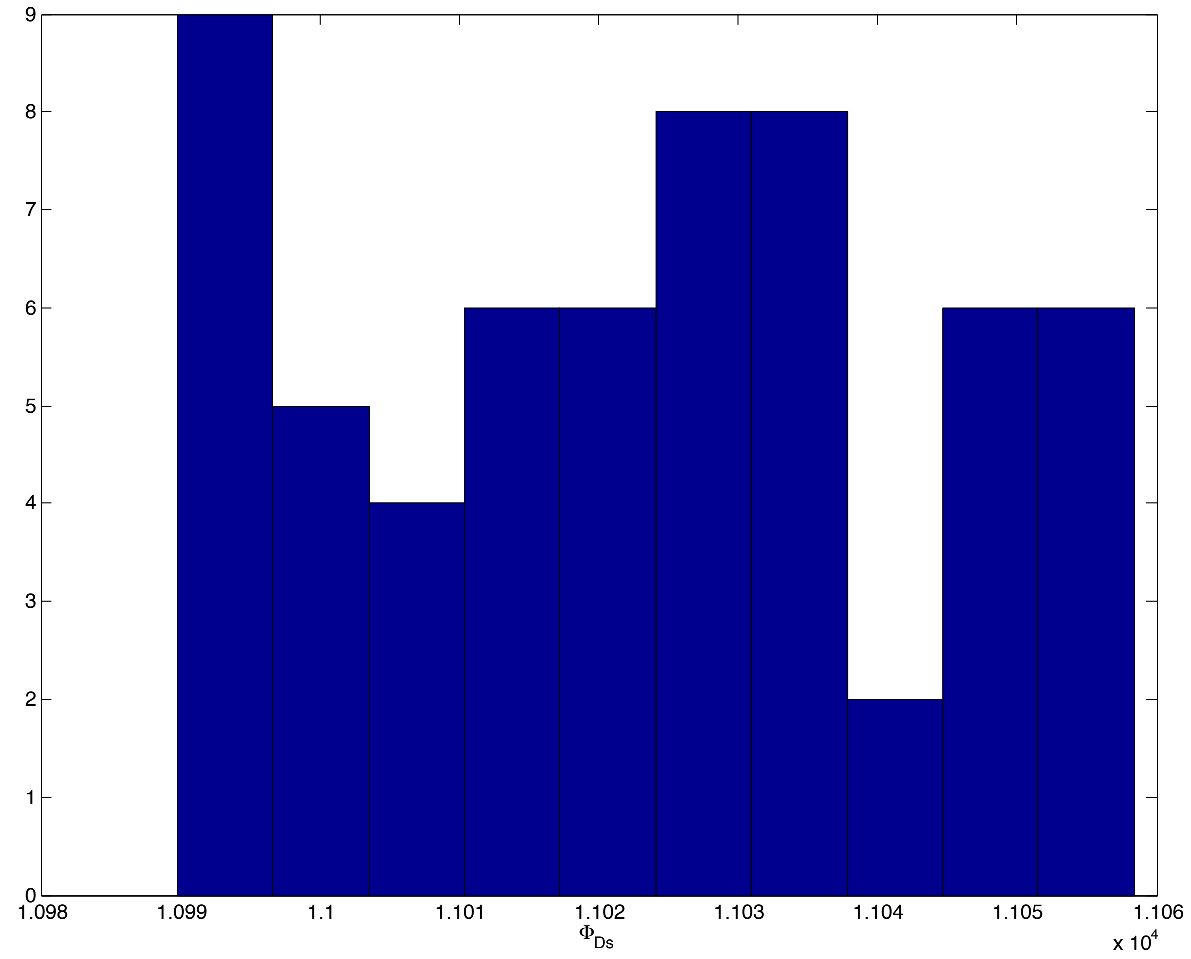
- ✦ For continuum extrapolation/chiral interpolation errors, use two methods:
  - By straightforward comparison with various continuum extrapolations of physical-mass ensemble results (Doug Toussaint's talk).
  - "Self-contained" error analysis:
    - Have 10 acceptable chiral fits ($p>0.05$), which:
      - keep or drop a=0.15 fm ensembles.
      - keep or drop $(a\, m_c)^4$ and $\alpha_s(a\, m_c)^2$ terms.
      - constrain higher order chiral terms and/or discretization terms with priors, or leave them unconstrained.
    - Have 6 versions of inputs (quark masses, f_{p4s} in physical units from f_π) from physical-mass ensemble results.
    - Histogram results of 60 composite analyses.

Continuum Extrapolation Error

$$\Phi_D \equiv f_{D^+}\sqrt{M_{D^+}}$$

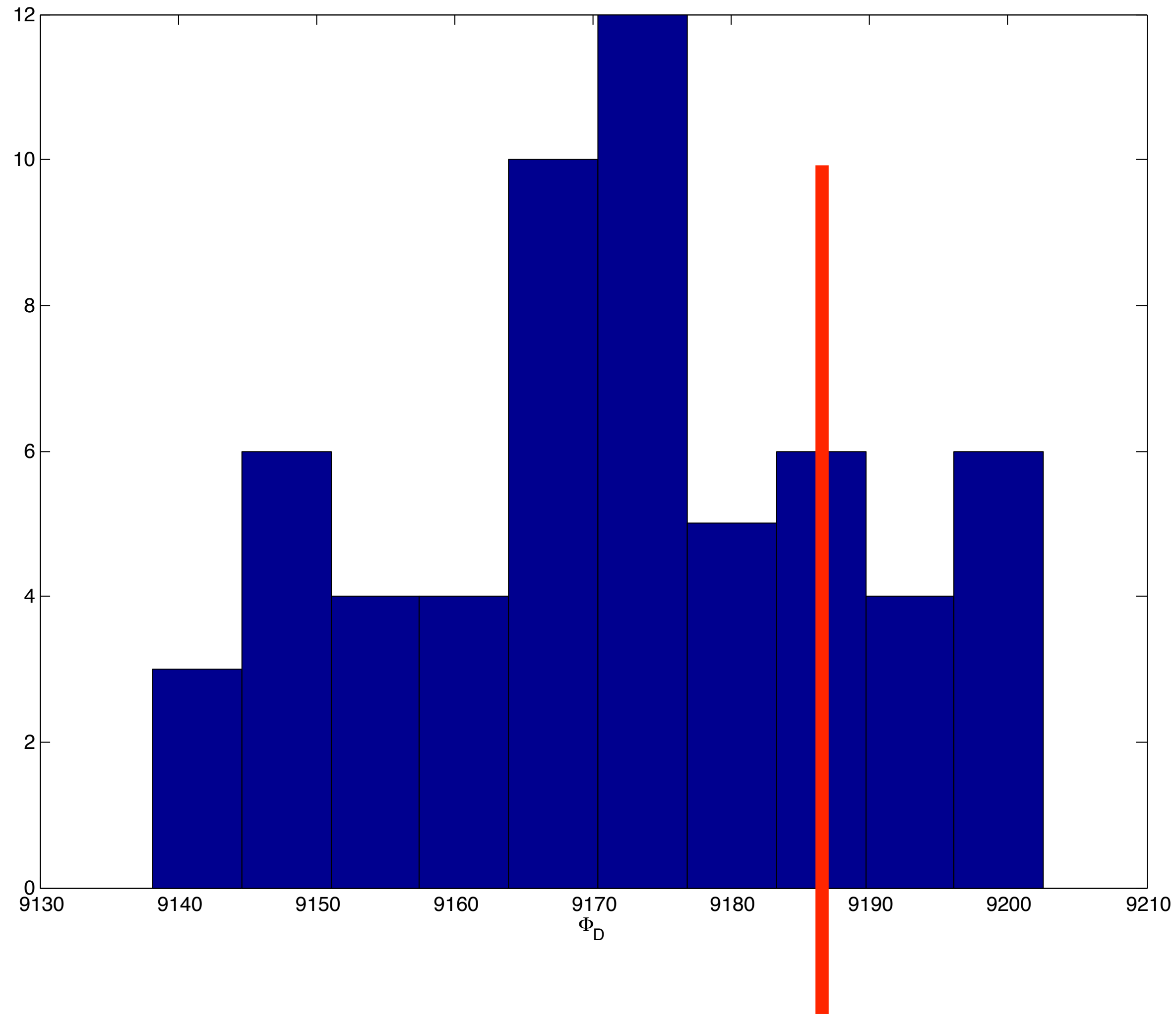


$$\Phi_{D_s} \equiv f_{D_s}\sqrt{M_{D_s}}$$



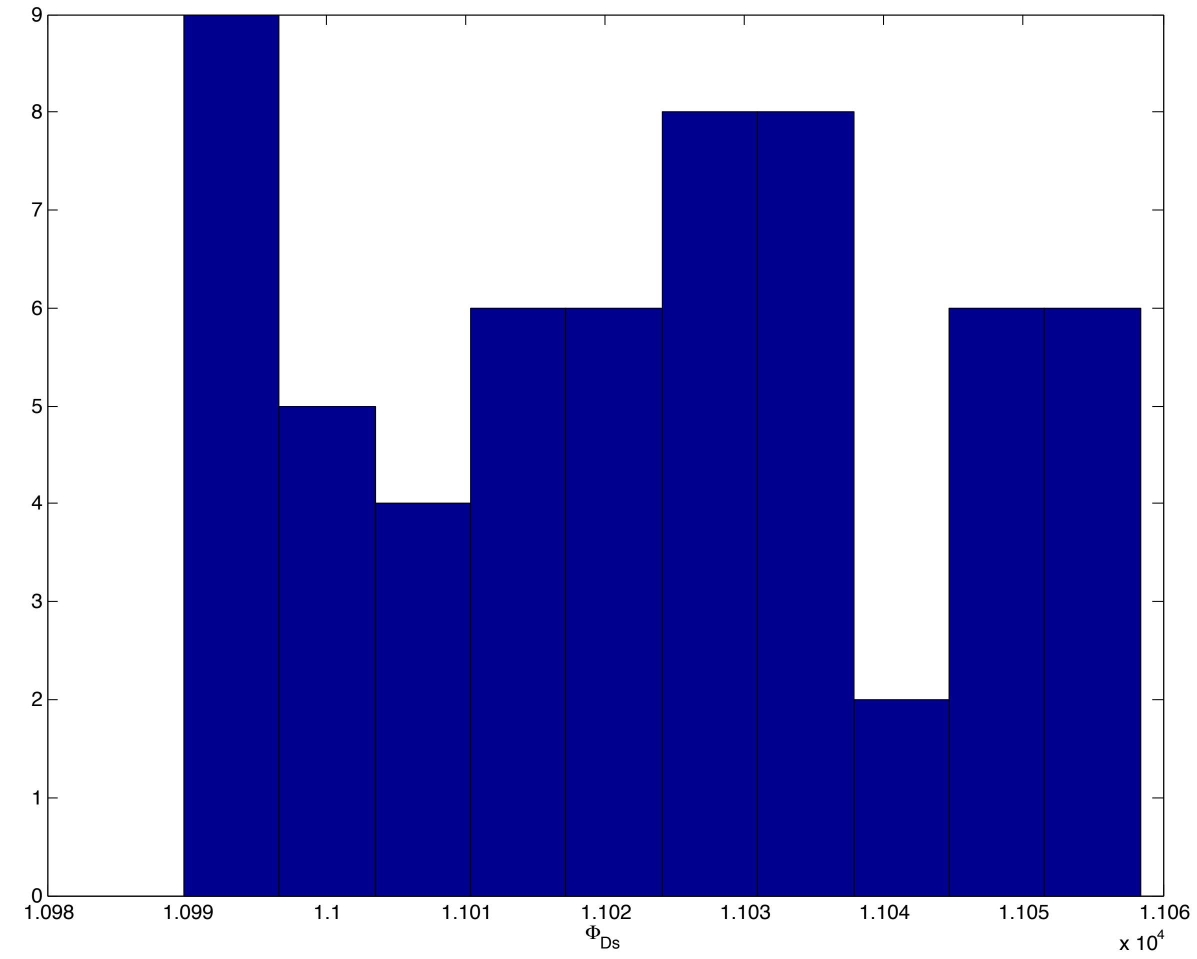
Continuum Extrapolation Error

$$\Phi_D \equiv f_{D^+}\sqrt{M_{D^+}}$$



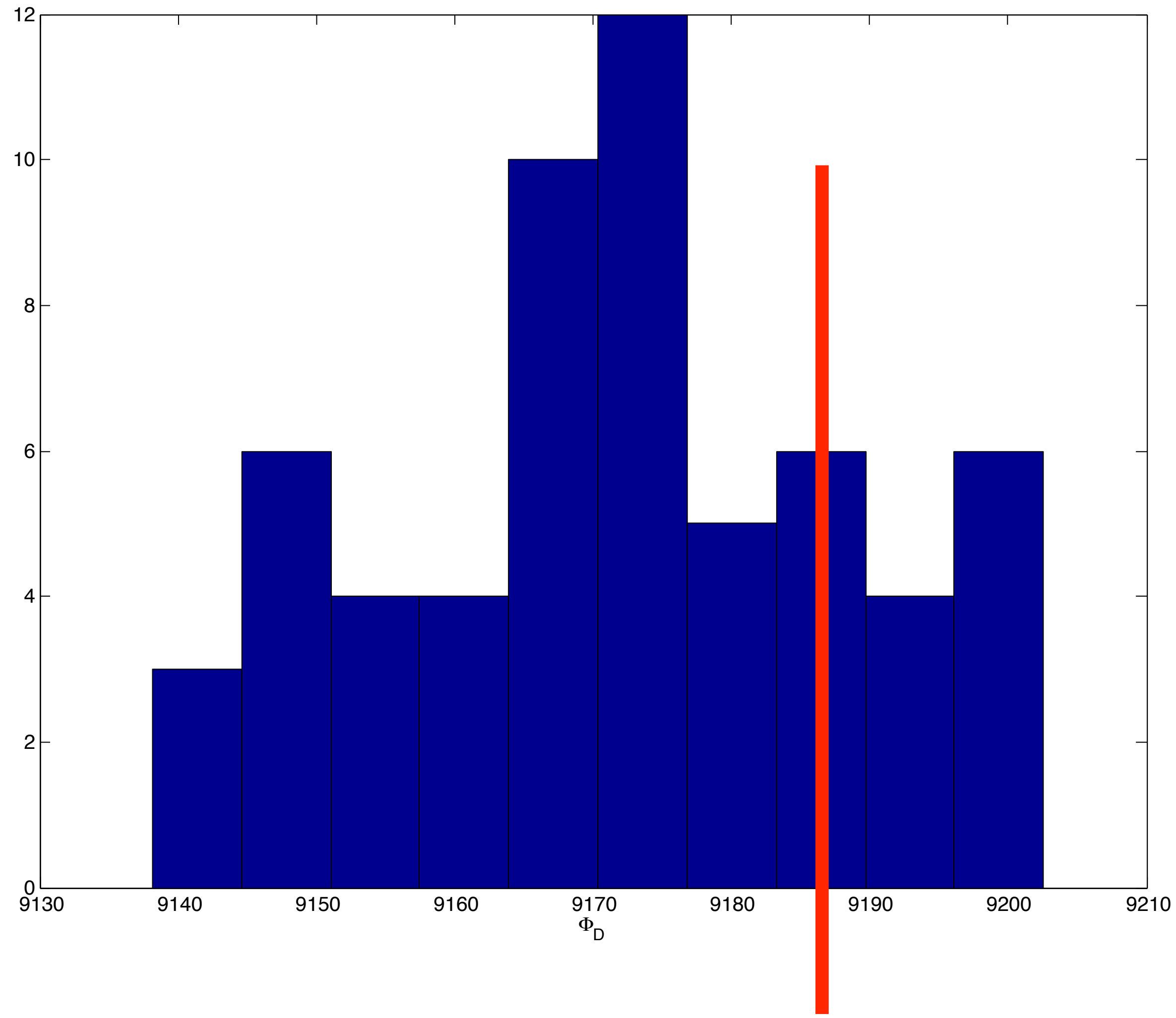
“Central” fit
→ 9187 MeV$^{3/2}$

$$\Phi_{D_s} \equiv f_{D_s}\sqrt{M_{D_s}}$$

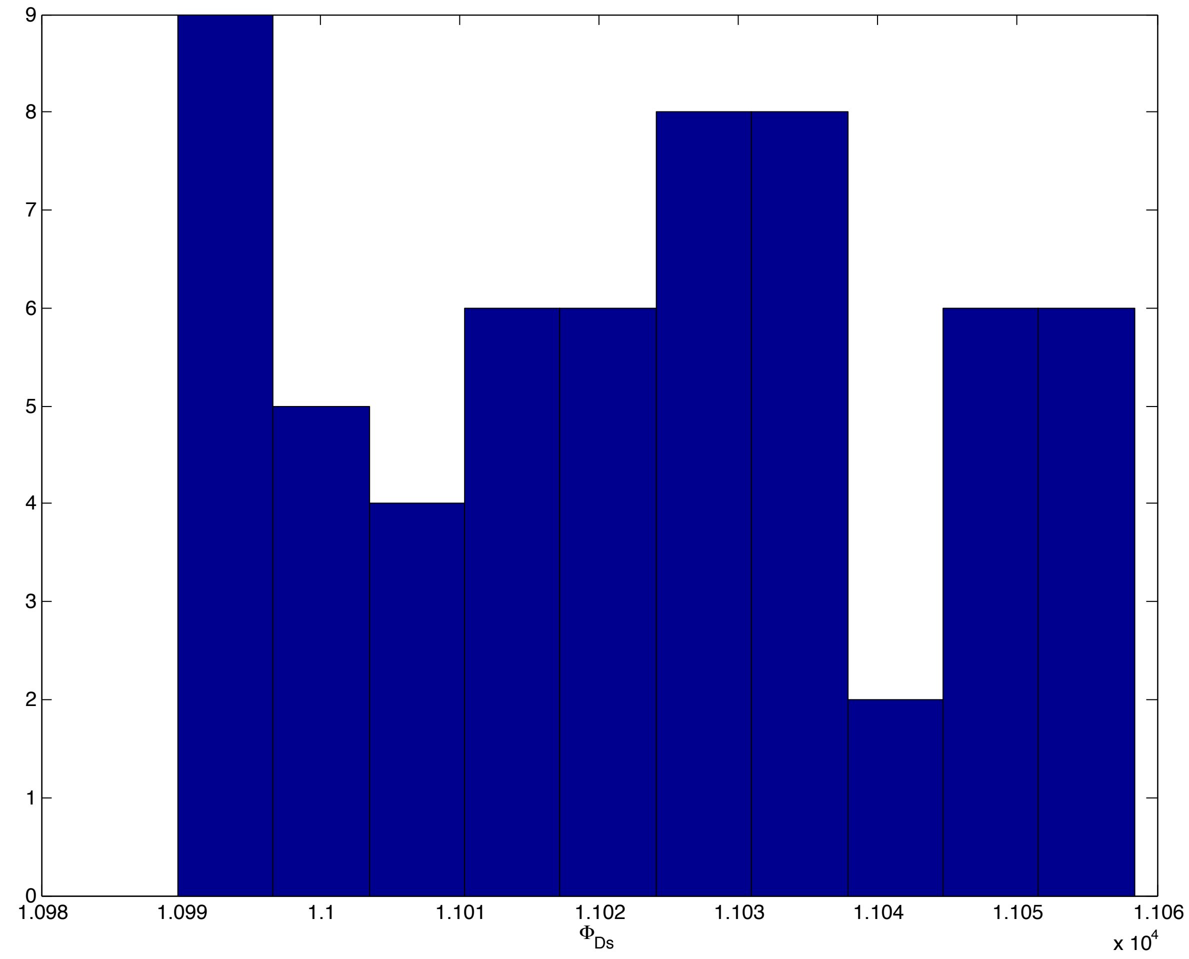


Continuum Extrapolation Error

$$\Phi_D \equiv f_{D+}\sqrt{M_{D+}}$$



$$\Phi_{D_s} \equiv f_{D_s}\sqrt{M_{D_s}}$$



"Central" fit
→ 9187 MeV$^{3/2}$

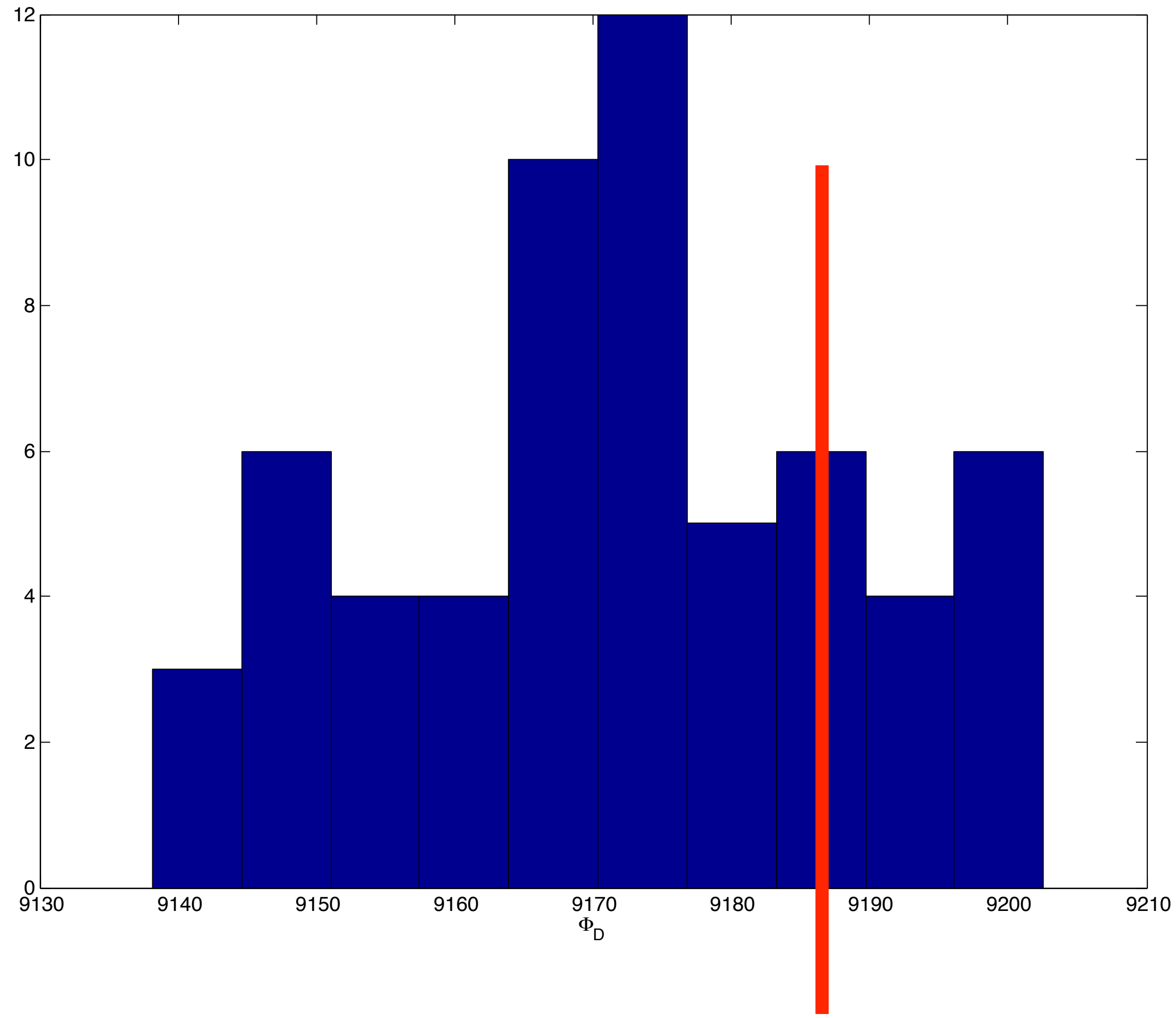
Systematic error:

+14 MeV$^{3/2}$

-47 MeV$^{3/2}$

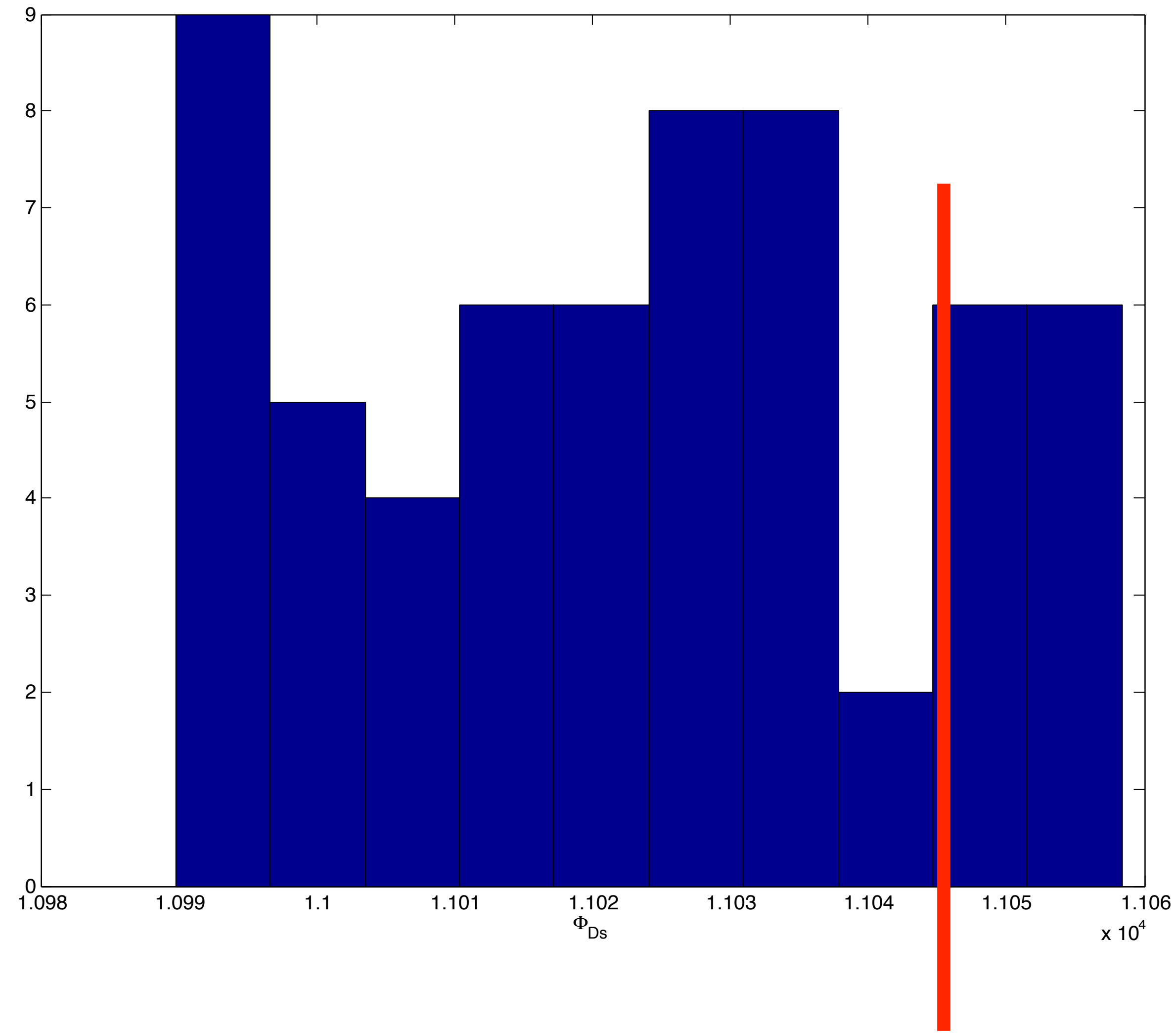
Continuum Extrapolation Error

$$\Phi_D \equiv f_{D^+}\sqrt{M_{D^+}}$$



“Central” fit
→ 9187 MeV$^{3/2}$

$$\Phi_{D_s} \equiv f_{D_s}\sqrt{M_{D_s}}$$



“Central” fit
→ 11045 MeV$^{3/2}$

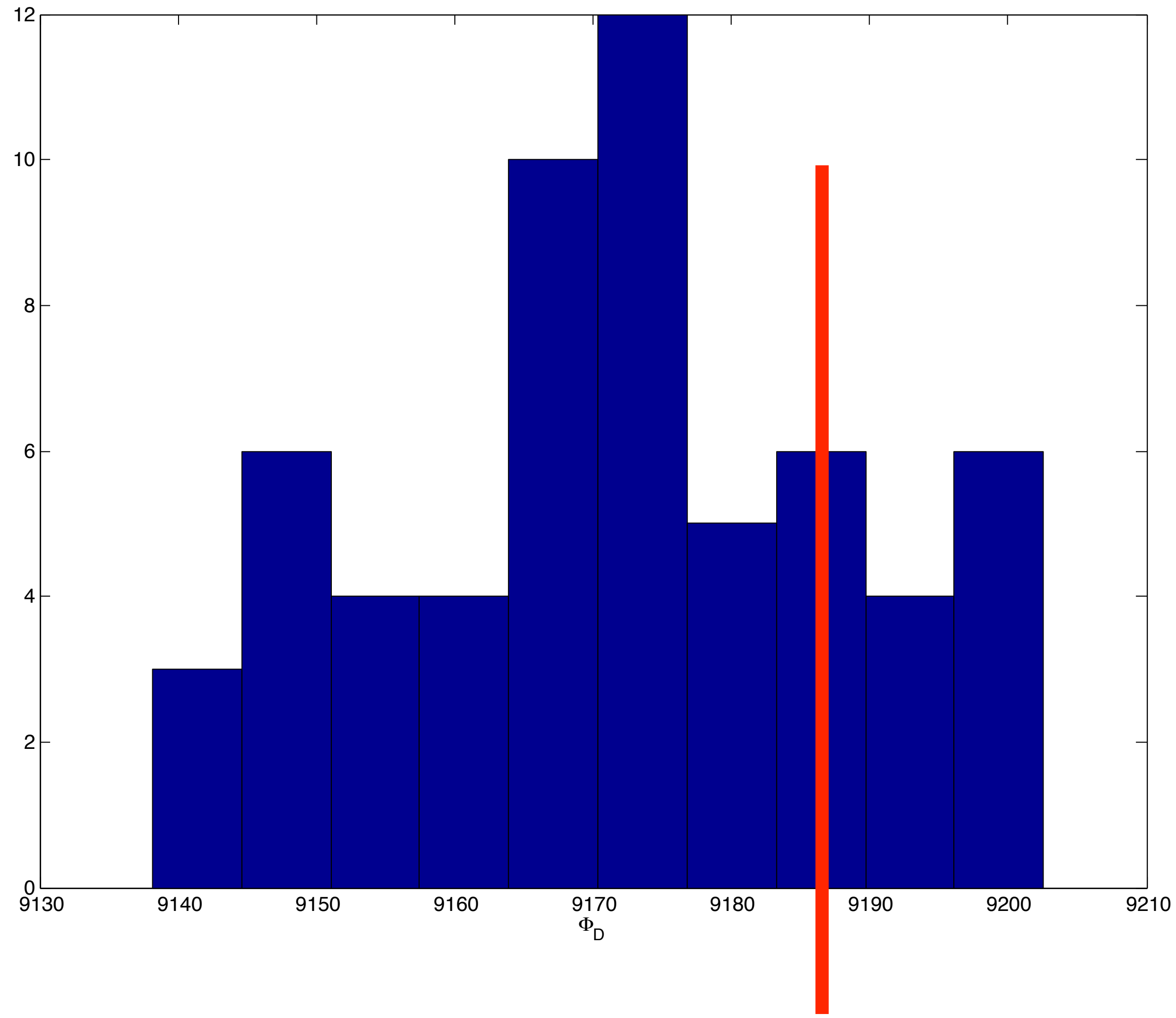
Systematic error:

+14 MeV$^{3/2}$

-47 MeV$^{3/2}$

Continuum Extrapolation Error

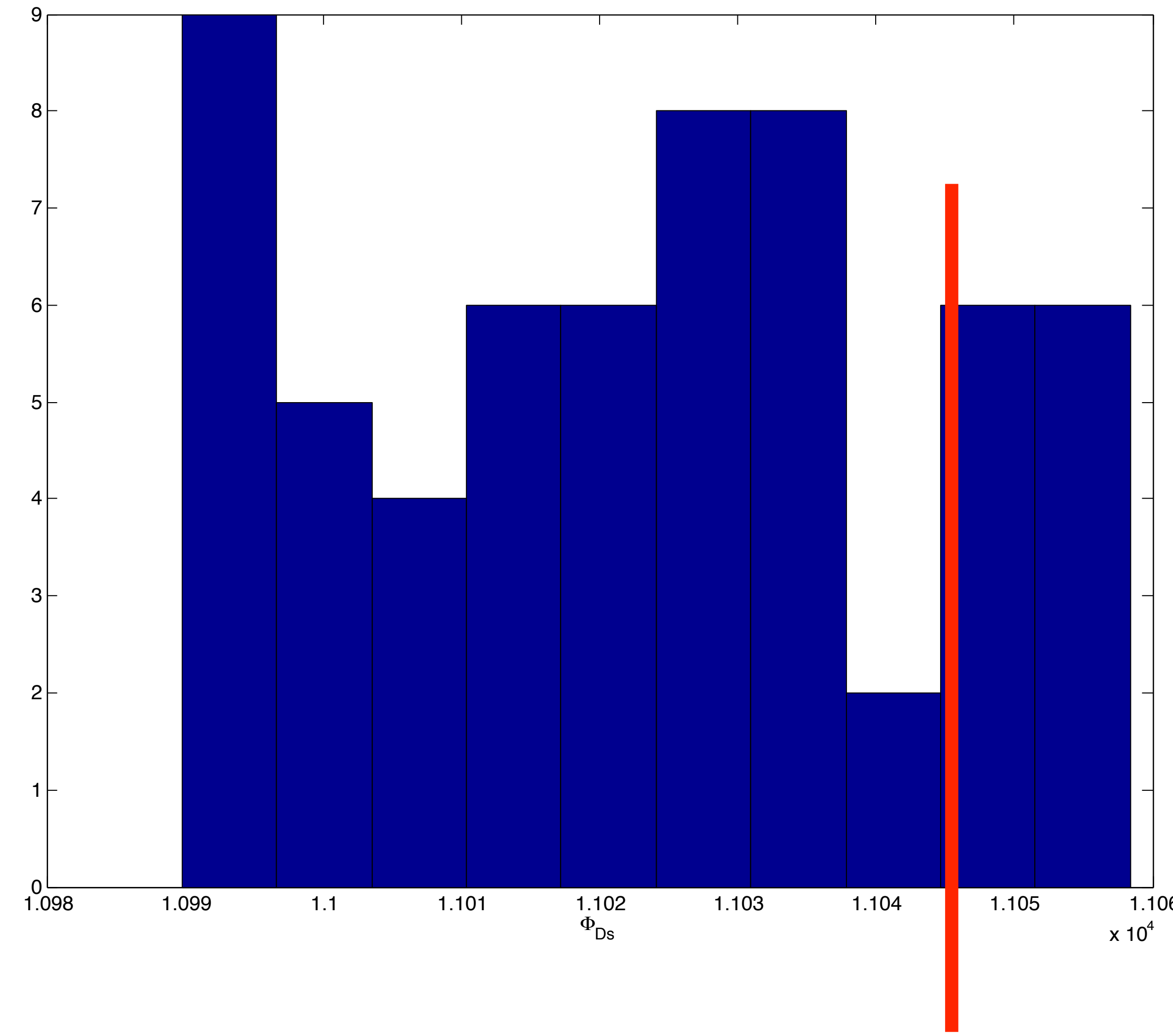
$$\Phi_D \equiv f_{D+}\sqrt{M_{D+}}$$



“Central” fit

→ 9187 MeV$^{3/2}$

$$\Phi_{D_s} \equiv f_{D_s}\sqrt{M_{D_s}}$$



“Central” fit

→ 11045 MeV$^{3/2}$

Systematic error:

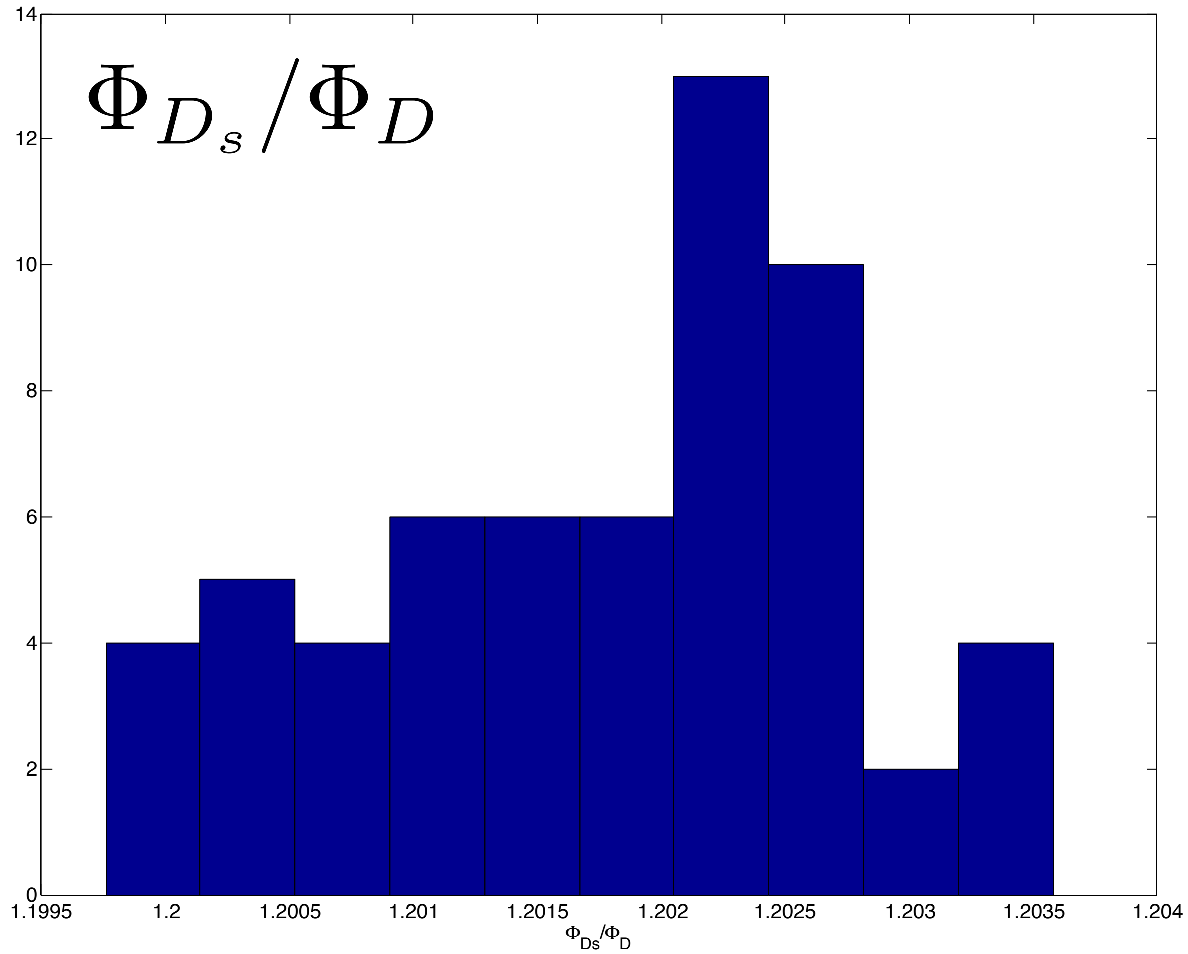
+14 MeV$^{3/2}$

-47 MeV$^{3/2}$

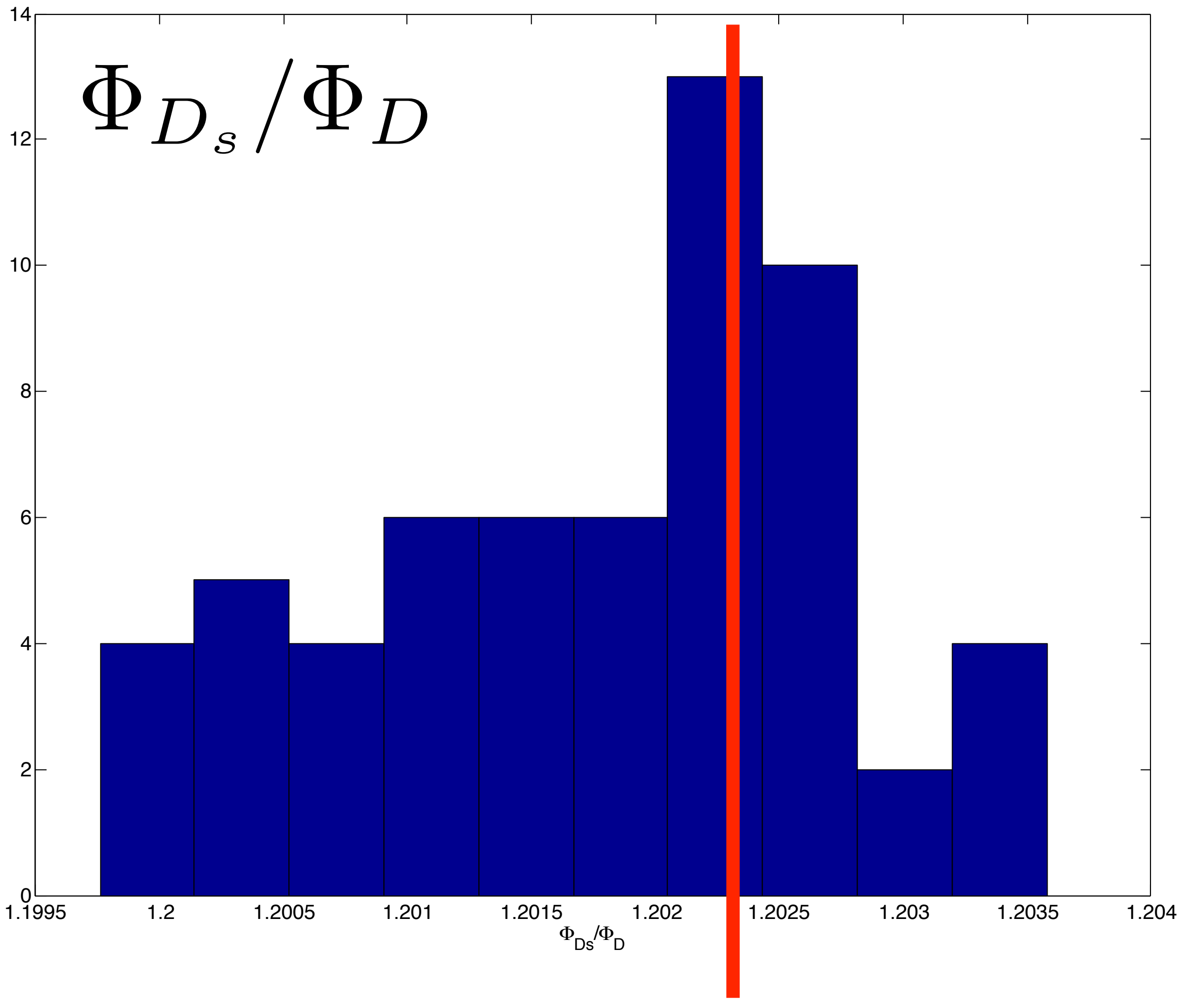
+13 MeV$^{3/2}$

-55 MeV$^{3/2}$

Continuum Extrapolation Error

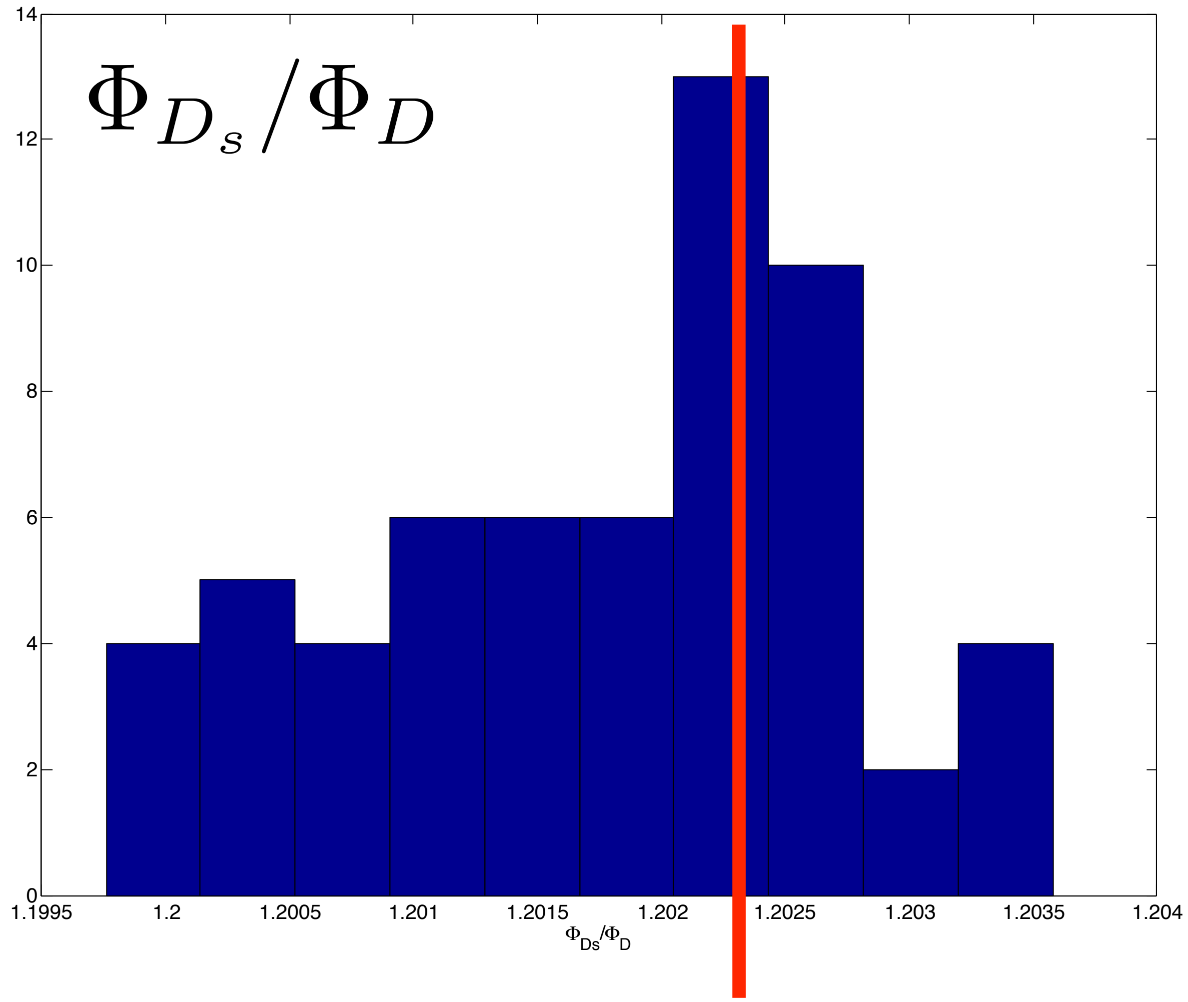


Continuum Extrapolation Error



"Central" fit
→ 1.2023

Continuum Extrapolation Error



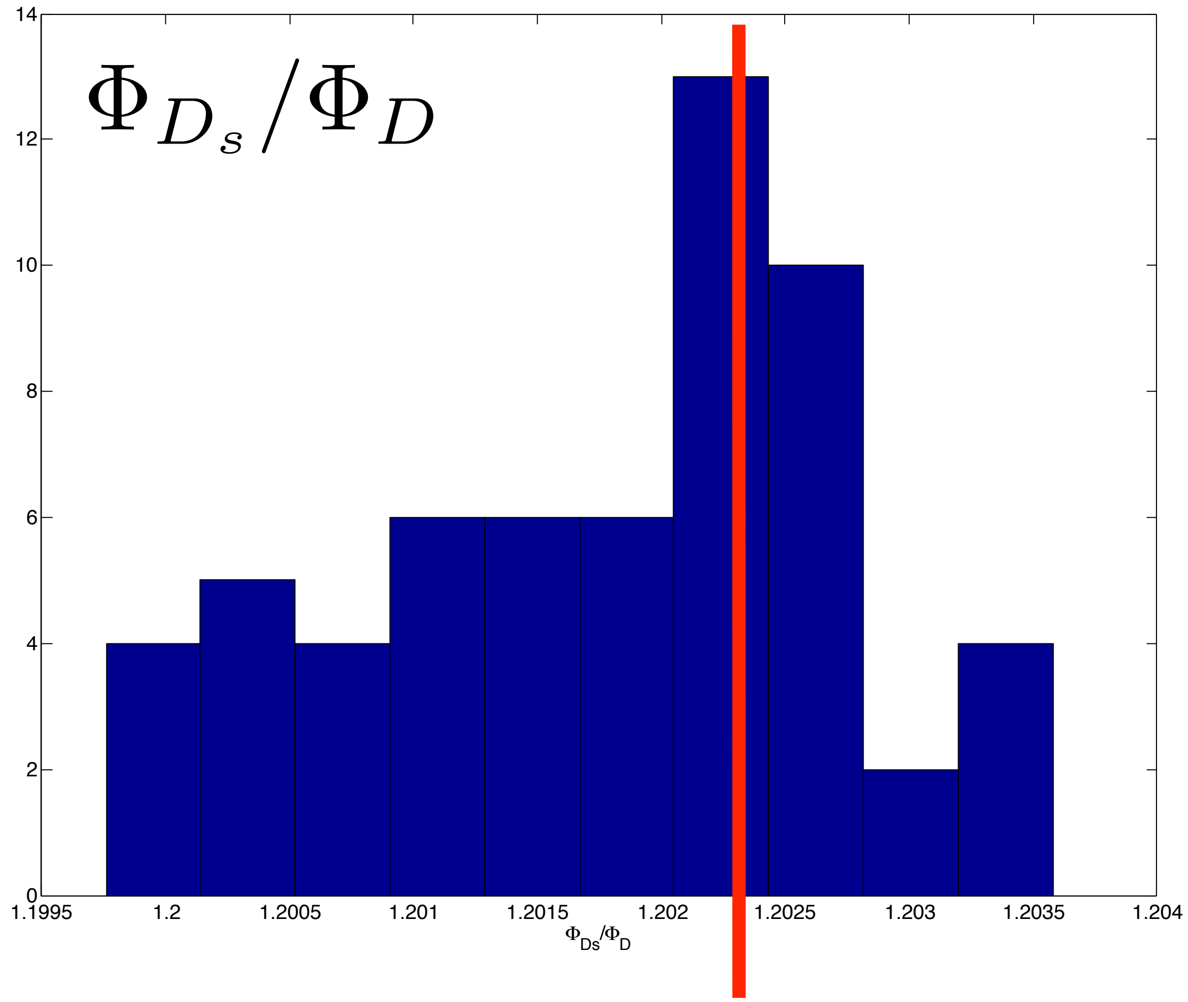
"Central" fit
→ 1.2023

Systematic error:

+0.0012

-0.0025

Continuum Extrapolation Error



"Central" fit
→ 1.2023

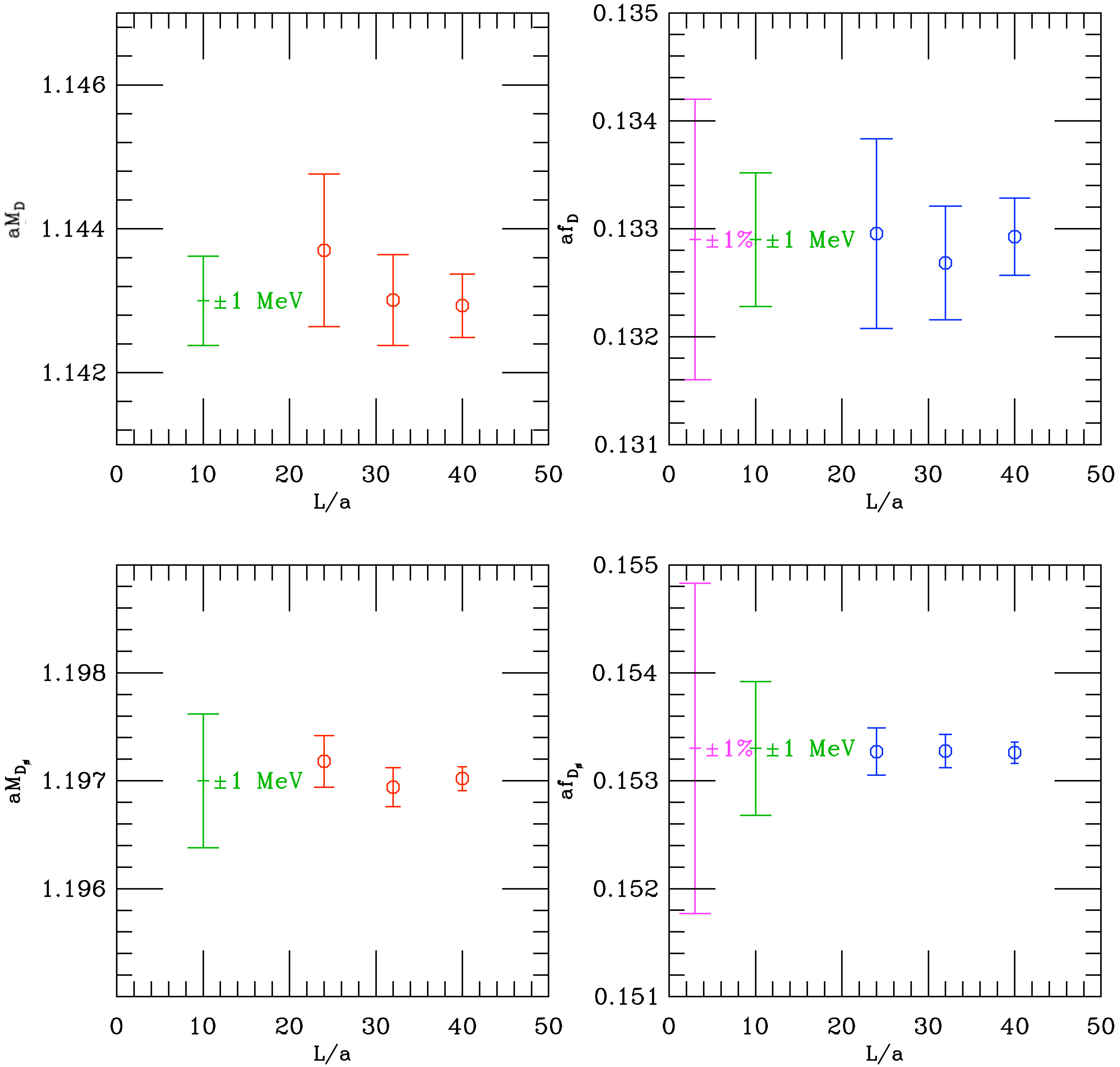
Systematic error:

-0.0025

+0.0012

- Roughly speaking, the chiral fits to account for ~2/3 of the variance, while the inputs of scale and quark masses account for ~1/3.

Finite size effects



- "Direct" finite size effects on heavy-light decay constants and masses are negligible.
- But note that results here are in lattice units.
- Small but non-negligible finite-size effects enter from scale setting through f_π.

EM errors

- EM effects included in error are only the ones coming from light quark masses.
- Do not include effects on m_c estimate (i.e. haven't taken out EM effects from the D_s mass, used to set m_c).
  - This appears to be a not-insignificant source of error (especially for f_{Ds}/f_D).
  - Can be relatively easily improved with an expansion of ongoing MILC EM project.
- In addition, errors associated with matching complete theory to pure QCD are not included (would be relevant for comparing to experiment).
    - Gläßle and Bali, arXiv:1111.3958 and Davies, et al., PRD 82 (2010) 114504 expect such errors are < 0.5%

Results

- Preliminary results, from "self-contained" chiral fit analysis only:

$$
\begin{aligned}
f_D &= 212.5 \pm 0.5_{\rm stat} \; {}^{+0.3}_{-1.1}|_{a^2\,{\rm extrap}} \pm 0.2_{\rm FV} \pm 0.0_{\rm EM} \pm 0.3_{f_\pi\,{\rm expt}} \;{\rm MeV} \\
f_{D_s} &= 248.9 \pm 0.2_{\rm stat} \; {}^{+0.3}_{-1.2}|_{a^2\,{\rm extrap}} \pm 0.2_{\rm FV} \pm 0.1_{\rm EM} \pm 0.4_{f_\pi\,{\rm expt}} \;{\rm MeV} \\
f_{D_s}/f_D &= 1.1717(20)_{\rm stat}({}^{+12}_{-24})_{a^2\,{\rm extrap}}(3)_{\rm FV}(3)_{\rm EM}
\end{aligned}
$$

- Preliminary results, including comparison with continuum extrapolation of physical ensembles results only (Doug Toussaint talk):

$$
\begin{aligned}
f_D &= 212.5 \pm 0.5_{\rm stat} \; {}^{+0.3}_{-1.4}|_{a^2\,{\rm extrap}} \pm 0.2_{\rm FV} \pm 0.3_{\rm EM} \pm 0.3_{f_\pi\,{\rm expt}} \;{\rm MeV} \\
f_{D_s} &= 248.9 \pm 0.2_{\rm stat} \; {}^{+0.3}_{-1.5}|_{a^2\,{\rm extrap}} \pm 0.2_{\rm FV} \pm 0.1_{\rm EM} \pm 0.4_{f_\pi\,{\rm expt}} \;{\rm MeV} \\
f_{D_s}/f_D &= 1.1717(20)_{\rm stat}({}^{+52}_{-24})_{a^2\,{\rm extrap}}(4)_{\rm FV}(5)_{\rm EM}
\end{aligned}
$$

- For now, we've taken larger error values in each case to be conservative.

Results

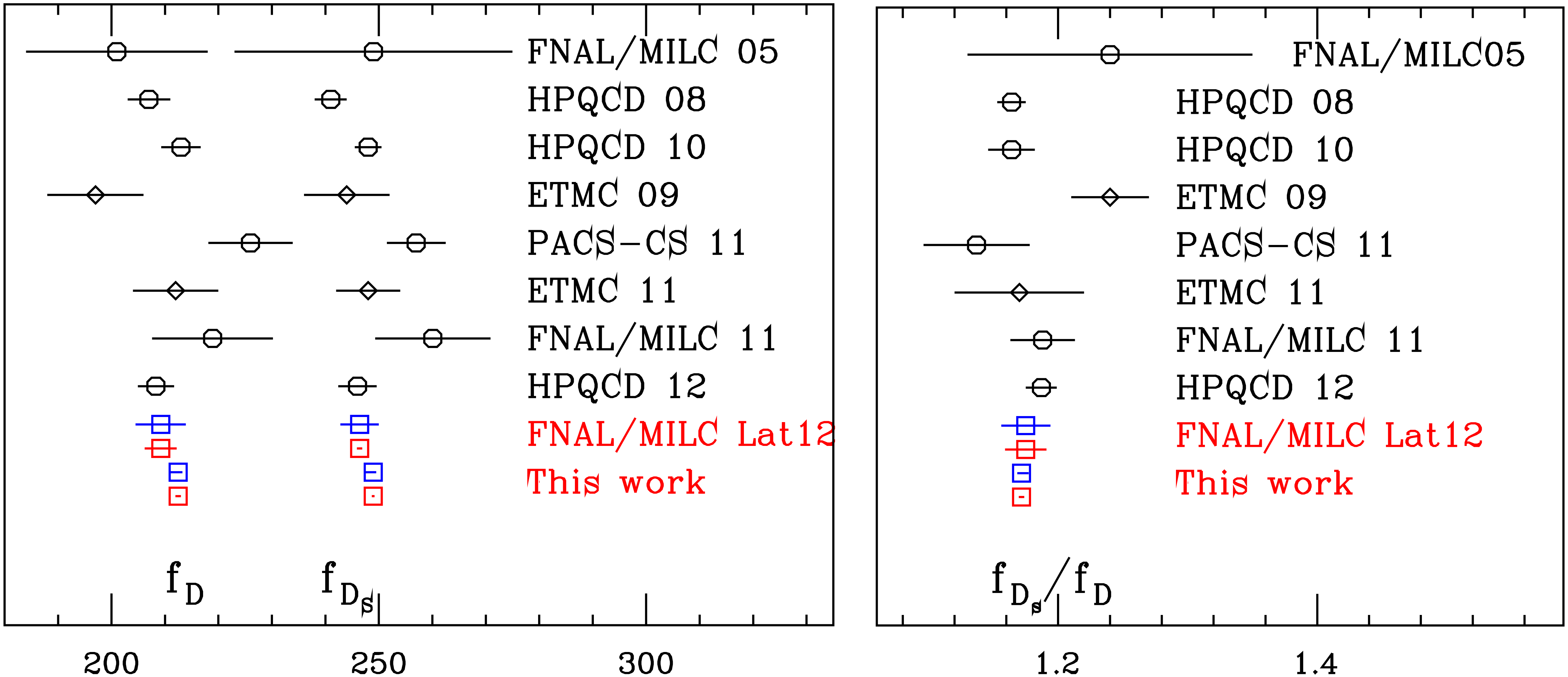
✦ Summary of our best current results (still preliminary):

$$\begin{aligned} f_D &= 212.5 \pm 0.5_{\rm stat} \;{}^{+0.6}_{-1.5}|_{\rm sys}\ \mathrm{MeV} \\ f_{D_s} &= 248.9 \pm 0.2_{\rm stat} \;{}^{+0.5}_{-1.6}|_{\rm sys}\ \mathrm{MeV} \\ f_{D_s}/f_D &= 1.1717(20)_{\rm stat}({}^{+52}_{-25})_{\rm sys} \end{aligned}$$

✦ In progress:

- finish 3 partial ensembles.
- chiral/continuum fits still need work:
  - Can we do with fewer parameters? Improve stability?
  - Understanding *a*-dependence better (esp. $\alpha_s(am_c)^2$, $(am_c)^4$ terms).
  - Choosing a more “central” central fit would be preferable, if one can be found that has comparable p value to current one and reasonable consistency with expectations of PT.
- Extending chiral-fit approach to light-light sector.

Compare to Previous Work



- Red points have statistical error only; blue include systematic errors.