D_s to ϕ and other transitions from lattice QCD

Gordon Donald, Christine Davies, Jonna Koponen
(HPQCD collaboration)

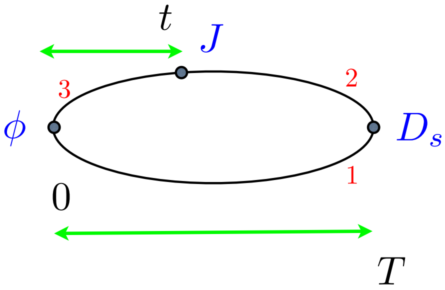
31 July 2013

Outline

- I will discuss our calculations of the vector and axial vector form factors which appear in transitions between pseudoscalar and vector mesons.
- Start with $D_s \to \phi\ell\nu$, which is a weak transition with a $c \to s$ quark decay.
- Same methods used for other transitions, such as charmonium radiative decay $J/\psi \to \eta_c\gamma$.

How we do $D_s \to \phi$

- We use HISQ (staggered) action for s and c valence quarks.
- Our ϕ meson is $s\bar{s}$ vector.
- We want to calculate 3-point correlation function, which depends on t and T.



Gauge configurations

Set	a/fm	$au_0 m_l^{asq}$ / $au_0 m_s^{asq}$	$L_s/a \times L_t/a$	n_{cfg}	T
1	0.12	0.005/0.05	24×64	2088	12, 15, 18
2	0.12	0.01/0.05	20×64	2259	12, 15, 18
3	0.09	0.0062/0.031	28×96	1911	16, 19, 20, 23

- Calculated on MILC configurations with 2+1 flavours of asqtad sea quark.
- Vary the lattice spacing and sea quark masses.
- Also calculate the 3-point correlator for different time extents, T.

Fitting

- Fit the 2pt and 3pt correlators simultaneously.
- Use staggered quarks, so include oscillations in the fit.

$$C_{2pt}^{(P)}(t) = \sum_i \{a_i^{(P)}\}^2 [e^{-Et} + e^{-E(L_t - t)}] + \text{oscillations}$$

$$C_{3pt}^{P \to Q}(t, T) = \sum_{i,j} a_i^{(P)} [e^{-Et} + e^{-E(L_t - t)}] J_{i,j} a_j^{(Q)} \times \\ [e^{-E(T-t)} + e^{-E(L_t - T + t)}] + \text{oscillations}$$

- a_i are shared parameters in the 2pt and 3pt fits.
- $J_{i,j}$ related to matrix element $\langle P|\Gamma|Q\rangle$.

General form of $D_s \to \phi$ matrix element

$$\begin{aligned}\langle \phi(p', \varepsilon) | V^\mu - A^\mu | D_s(p) \rangle = & \frac{2i\epsilon^{\mu\nu\alpha\beta}}{m_{D_s} + m_\phi} \varepsilon_\nu p_\alpha p'_\beta V(q^2) \\ & - (m_{D_s} + m_\phi) \varepsilon^\mu A_1(q^2) \\ & + \frac{\varepsilon \cdot q}{m_{D_s} + m_\phi} (p + p')^\mu A_2(q^2) \\ & + 2m_\phi \frac{\varepsilon \cdot q}{q^2} q^\mu A_3(q^2) \\ & - 2m_\phi \frac{\varepsilon \cdot q}{q^2} q^\mu A_0(q^2)\end{aligned}$$

$$A_3(q^2) = \frac{m_{D_s} + m_\phi}{2m_\phi} A_1(q^2) - \frac{m_{D_s} - m_\phi}{2m_\phi} A_2(q^2)$$

$$A_3(0) = A_0(0)$$

How we get form factors: $A_1(q^2)$

- Simulate with the D_s at rest.
- Get range of q^2 with twisted boundary condition to tune p_ϕ.
- We can get $A_1(q^2)$ with $\varepsilon \cdot q = 0$. Matrix element reduces to

$$\langle \phi(p', \varepsilon) | A^\mu | D_s(p) \rangle = (m_{D_s} + m_\phi) \varepsilon^\mu A_1(q^2).$$

- Can use local or point-split A_μ and ϕ.
- The vector and axial vector operators we use are nonperturbatively normalised.

How we get form factors: $A_0(q^2)$

- $A_0(q^2)$ is not experimentally accessible as it is suppressed by the lepton mass.
- However, it is quite easy to calculate with HISQ.
- Using PCAC relation, we have

$$
\begin{aligned}
q_\mu \langle \phi(p', \varepsilon) | A^\mu | D_s(p) \rangle &= 2 m_\phi \varepsilon \cdot q A_0(q^2) \\
&= (m_c + m_s) \langle \phi(p', \varepsilon) | \gamma_5 | D_s(p) \rangle .
\end{aligned}
$$

- Use local (absolutely normalised) pseudoscalar current.

How we get form factors: $A_2(q^2)$

- $A_2(q^2)$ is important and we need to have it to reconstruct the decay.
- Only enters the matrix element when $\varepsilon \cdot q \neq 0$, along with everything else.
- We calculate the whole matrix and extract $A_2(q^2)$ because we already know $A_1(q^2)$ and $A_0(q^2)$.
- Use same axial current as for $A_1(q^2)$, but set up different kinematics.
- Easiest at $q^2 = 0$ because we have relation $A_0(0) = A_3(0)$.

How we get form factors: $V(q^2)$

- We also need $V(q^2)$ to build the whole decay rate.
- To make a taste-singlet correlator, we use $\gamma_\mu \otimes \gamma_\mu \gamma_\nu$ vector operator for our ϕ and use $\gamma_t \gamma_5$ for the D_s.
- This is fine as the differences are only in taste; discretisation effect and very small for HISQ.

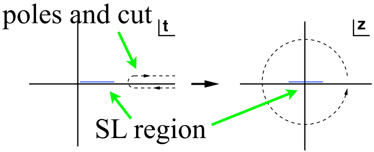
$$\frac{2i\epsilon_{\mu\nu\alpha t}}{m_{D_s} + m_\phi} \varepsilon^\mu E_{D_s} p_\phi^\nu V(q^2).$$

- Same vector form factor as appears in radiative decays ($J/\psi \to \eta_c \gamma$ appears later).

Extrapolation

- Extrapolated using z-space fit.

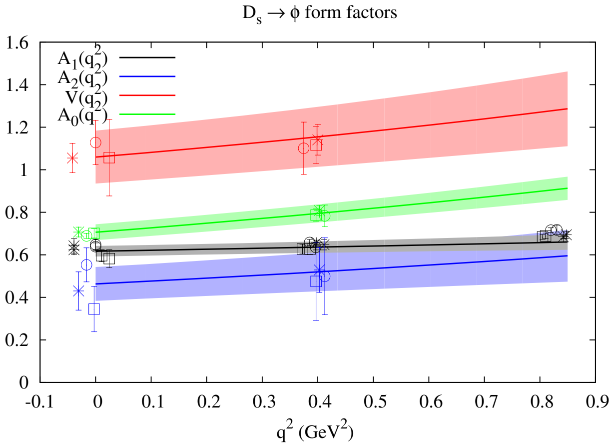
$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}.$$



- In z-space, the semileptonic region maps to $|z| < 1$.
- $t_+ = (M_{D_s} + M_\phi)^2$ and $t_0 = 0$.
- For each form factor (A_1, A_2, V, A_0):

$$\tilde{F}(z) = \sum_{n=0}^{3} A_n \left\{1 + B_n a^2 + C_n a^4 + D_n \delta_l\right\} z^n.$$

Form factors



- Lattice: $r_2 = A_2(0)/A_1(0) = 0.74(12)$, $r_V = V(0)/A_1(0) = 1.72(21)$
- BaBar[PhysRevD.78.051101 (2008)]: $r_2 = 0.763(71)(65)$, $r_V = 1.849(60)(95)$

Helicity amplitudes

- Decay rate written in terms of helicity amplitudes, which are:

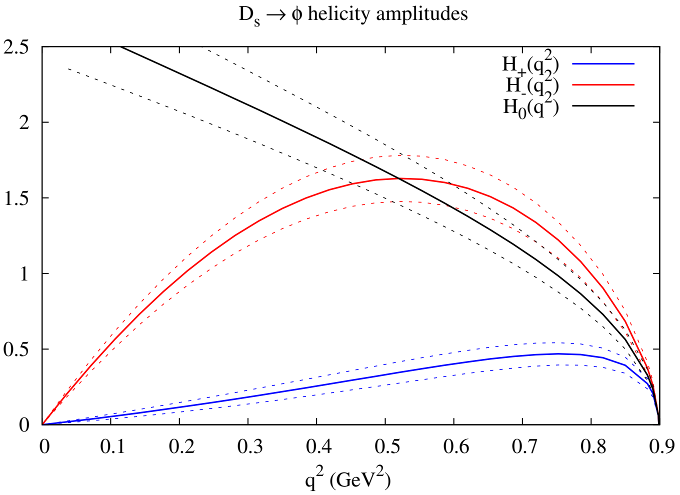
$$H_\pm(q^2) = (M_{D_s} + M_\phi)A_1(q^2) \mp \frac{2M_{D_s}p_\phi}{M_{D_s} + M_\phi}V(q^2).$$

and

$$\begin{aligned} H_0(q^2) &= \frac{1}{2M_\phi\sqrt{q^2}} \times \\ & [(M_{D_s}^2 - M_\phi^2 - q^2)(M_{D_s} + M_\phi)A_1(q^2) \\ &- 4\frac{M_{D_s}^2 p_\phi^2}{M_{D_s} + M_\phi}A_2(q^2)]. \end{aligned}$$

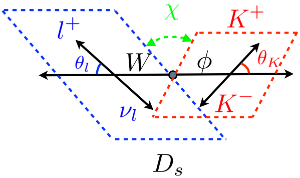
Helicity amplitudes

- Plot $p_\phi q^2 |H_i(q^2)|^2$ as they appear in decay rate.



Differential decay rate

- For massless leptons

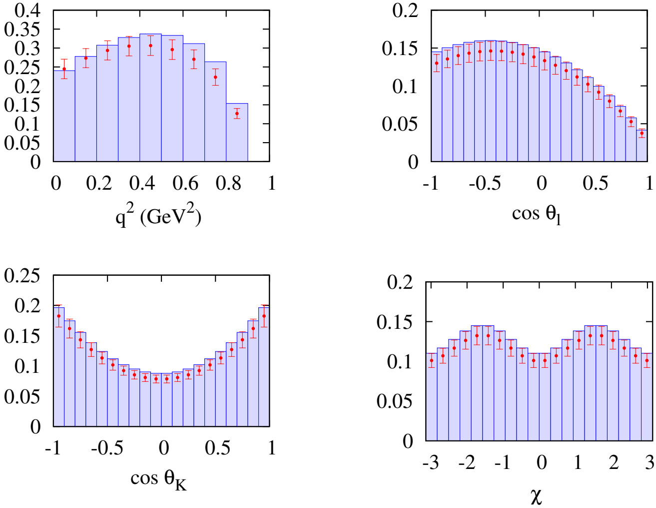


$$\frac{d\Gamma(D_s \to \phi\ell\nu, \phi \to K^+K^-)}{dq^2 d\cos\theta_K d\cos\theta_\ell d\chi} =$$

$$\frac{3}{8(4\pi)^4} G_F^2 |V_{cs}|^2 \frac{p_\phi q^2}{M_{D_s}^2} \mathcal{B}(\phi \to K^+K^-) \times$$

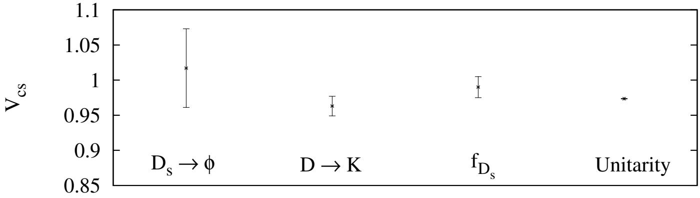
$$\{(1+\cos\theta_\ell)^2 \sin^2\theta_K |H_+(q^2)|^2$$
$$+(1-\cos\theta_\ell)^2 \sin^2\theta_K |H_-(q^2)|^2$$
$$+4\sin^2\theta_\ell \cos^2\theta_K |H_0(q^2)|^2$$
$$+4\sin\theta_\ell(1+\cos\theta_\ell)\sin\theta_K\cos\theta_K\cos\chi H_+(q^2)H_0(q^2)$$
$$-4\sin\theta_\ell(1-\cos\theta_\ell)\sin\theta_K\cos\theta_K\cos\chi H_-(q^2)H_0(q^2)$$
$$-2\sin^2\theta_\ell\sin^2\theta_K\cos 2\chi H_+(q^2)H_-(q^2)\}\,.$$

Decay distributions. Red lattice, blue experiment (BaBar)



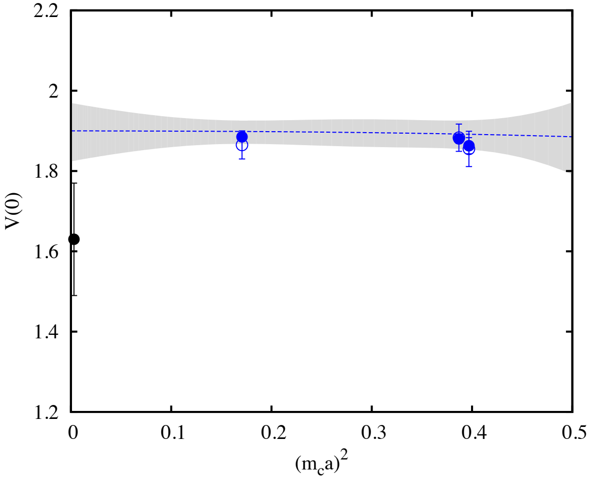
V_{cs}

- Integrating over q^2, $\cos\theta_\ell$, $\cos\theta_K$ and χ gives total decay rate.
- Comparison with BaBar gives $V_{cs} = 1.017(56)$
- In agreement with unitarity and with (semi)leptonic determinations.

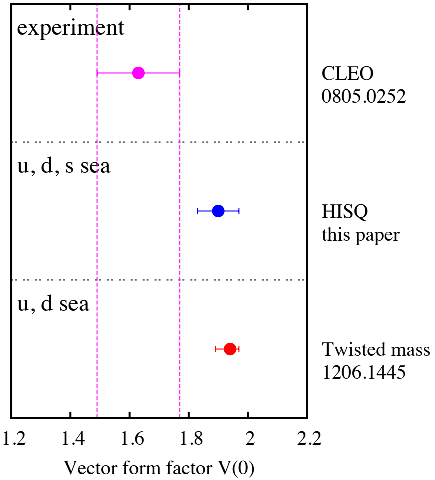


$V(0)$ for $J/\psi \to \gamma\eta_c$

- From PhysRevD.86.094501 (2012)



$V(0)$ for $J/\psi \to \gamma\eta_c$



Conclusion and outlook

- We have studied vector/pseudoscalar meson transitions with HISQ action.
- Calculated full q^2 range of all the vector and axial form factors for $D_s \to \phi$ and matched shape of experimental decay distributions.
- Nonperturbatively normalised currents used throughout.
- Same methods also allow us to study radiative decays, $J/\psi \to \eta_c \gamma$.
- The staggered correlation functions can also be used for pseudoscalar meson decays to 2 photons, $\eta_c/\pi^0 \to 2\gamma$ (in progress).