The $D_s$, $D^+$, $B_s$ and $B^+$ decay constants from
2 + 1 flavor lattice QCD

Fermilab Lattice and MILC collaborations
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31 July 2013
The $D$ and $B$ decay constants

- Among simplest Weak matrix elements to compute.
- Test of lattice technology: e.g., heavy quark mass sensitivity, HQ formalisms, chiral extrapolations.
- The $D$ and $B$ decay constants provide tests of the CKM picture e.g., $B^0 \rightarrow \mu^+\mu^-$ in SM and $|V_{ub}|$ from $B^+ \rightarrow \tau^+\nu$.
- This talk: Decay constants from three-flavor MILC asqtad lattices using asqtad light and clover (Fermilab interpretation) heavy valence quarks. PRELIMINARY.
- $D$ decay constants with HISQ charm on MILC four-flavor HISQ lattices see talks by:
  - C. Bernard  Wed 11:00 session: 6C
  - D. Toussaint  Thu 17:50 session: 8C

We have published results from our previous asqtad study...
Conclusions from our previous study

PRD.85.114506, arXiv:1112.3051

Our current study addresses many of these sources of error...
Conclusions from our previous study

**PRD.85.114506, arXiv:1112.3051**

Improvements in current study:

- No u⁰ adjustment.
- HQ discretization, LQ discr., chiral extrap.

and statistical errors reduced by including finer

\( a \approx 0.058 \) and 0.043 fm

lattices and higher 2-pt statistics.

- Nearer to physical

\( m_l/m_h = 1/20 \) helps in chiral extrap.
Conclusions from our previous study

PRD.85.114506, arXiv:1112.3051

Contributions in quadrature to percent error

\[ \text{Contributions in quadrature to percent error} \]

\[ f_{D^-}, f_{D^0}, f_{B^-}, f_{B^0} \]

\[ \begin{array}{c}
\text{source} \\
2\text{-pt fits} \\
\text{chiral extrapolation} \\
\text{finite volume} \\
\text{heavy quark mass} \\
\text{higher-order rho}_A \text{A4} \\
\text{HQ discretization} \\
\text{light quark mass} \\
\text{LQ discretization} \\
\text{match. ZvQQ, Zvqq} \\
\text{scale (r1)} \\
\text{statistics} \\
\text{u0 adjustment}
\end{array} \]

\[ \begin{array}{c}
5.2 \\
4.2 \\
4.5 \\
3.9
\end{array} \]

\[ \text{and more improvements:} \]

- Better kappa tuning to reduce HQ mass errors.
- Better runs to compute \( Z_{V4}^{QQ} \) and \( Z_{V4}^{qq} \).
- New 2-pt fit technology to control 2-pt fit errors.
MILC asqtad $N_f = 2 + 1$

Current study includes

<table>
<thead>
<tr>
<th>id</th>
<th>a [fm]</th>
<th>beta</th>
<th>$m_l/m_h$</th>
<th>$am_h$</th>
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<th>$r_1/a$</th>
<th>$N_{\text{config}}$</th>
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</table>

*Cf.* our previous study:

- Two finer lattice spacings: 0.043 and 0.058 fm (id=A-E).
- Sea quarks nearer to physical: $m_l/m_h = 1/20$ (F).
- Better statistics: around $3.6 \times$ more $N_{\text{config}} \cdot N_{\text{tsrc}}$. 
Charm and Bottom 2-pt functions

On each ensemble, for $H = \text{charm, bottom}$ and a range of $m_q$ compute six 2-pt functions

$$C^{(j,k)}(t) = \langle O^{(j)\dagger}_{Hq}(t) \ O^{(k)}_{Hq}(0) \rangle$$

$$C^{(k)}_{A_4}(t) = \langle A^\dagger_{4}(t) \ O^{(k)}_{Hq}(0) \rangle$$

with smearings $j, k \in \{\text{point, smeared}\}$.

The quantity $\phi_{Hq} = f_{Hq} \sqrt{M_{Hq}}$ is found from the overlap

$$\frac{\langle 0 \mid A_\mu \mid H_q(p) \rangle}{\sqrt{m_{Hq}}} = (p_\mu/m_{Hq}) \phi_{Hq}$$

$O(a)$-improved $A_4$ is matched to continuum by factor

$$\sqrt{Z^{QQ}_{4}Z^{qq}_{4}} \rho_{A_4}$$

with nonperturbative $Z^{QQ}_{4}$ and small one-loop $\rho_{A_4}$ correction.
2-pt fitting details

- Fit four (five) 2-pt functions.

- New two stage fit process:
  1. Set empirical Bayesian priors for ground-state from $1+1$ state fits at large time.
  2. Use (broadened) priors in fit over a wide range of smaller non-overlapping times.

- Good isolation of ground-state using $4+4$ or $5+5$ states

- Bootstrap fits clearly show (expected) correlations.
Clover c- and b-quark kappa tuning

Checks: $D_s$ HFS

![Graph showing $M(D_s^* - D_s)$ vs $(a/l)^2$.]

Want $m_2(\kappa) = m(D_s)$ or $m(B_s)$

$$E(\bar{p})^2 = m_1^2 + \left( \frac{m_1}{m_2} \right)^2 \bar{p}^2 + O(p^4)$$

- High-statistics 2-pt tuning runs on $m_l = 0.2m_h$ ensembles.
- $E(\bar{p})$ vs $\bar{p}$ fits include priors for $O(p^4)$ effects.
- Tunings corrected for sea-quark effects.
- Predict tuned kappa for all ensembles.

...and $B_s$ HFS

![Graph showing $M(B_s^* - B_s)$ vs $(a/l)^2$.]

Expt. $\Delta M$ shown at zero
Chiral fits

- Correct simulation $\phi = f\sqrt{M}$ for any kappa mistuning.
- NLO expression for $\phi$ from partially-quenched staggered chiral perturbation theory [Aubin and Bernard, arXiv:hep-lat/0510088].
- Add NNLO analytic (quadratic in quark mass) terms.
- Model both light- and heavy-quark discretization effects in the fits.
- Distance scale $r_1$, quark masses $m_s$, $m_d$ and $m_u$ and $O(a^2)$ LECs from MILC light meson fits.
# D system fit

## Preliminary

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\mu_l/\mu_h$</th>
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<tbody>
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<td>0.043 fm</td>
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</tr>
</tbody>
</table>

![Graphs showing fits for different parameters](image-url)
Extrapolations for $D^+$ and $D_s$

**PRELIMINARY** analysis and data are still blinded.
B system fit

PRELIMINARY

\[
\begin{array}{ccc}
  a=0.043 \text{ fm}; \text{ml/mh}=0.2 & a=0.059 \text{ fm}; \text{ml/mh}=0.1 & a=0.058 \text{ fm}; \text{ml/mh}=0.14 \\
  a=0.058 \text{ fm}; \text{ml/mh}=0.2 & a=0.083 \text{ fm}; \text{ml/mh}=0.05 & a=0.083 \text{ fm}; \text{ml/mh}=0.15 \\
  a=0.083 \text{ fm}; \text{ml/mh}=0.1 & a=0.083 \text{ fm}; \text{ml/mh}=0.15 & a=0.082 \text{ fm}; \text{ml/mh}=0.2 \\
  a=0.081 \text{ fm}; \text{ml/mh}=0.4 & a=0.11 \text{ fm}; \text{ml/mh}=0.14 & a=0.082 \text{ fm}; \text{ml/mh}=0.2 \\
  a=0.11 \text{ fm}; \text{ml/mh}=0.1 & a=0.043 \text{ fm}; \text{ml/mh}=0.05 & a=0.11 \text{ fm}; \text{ml/mh}=0.14 \\
  a=0.11 \text{ fm}; \text{ml/mh}=0.2 & a=0.081 \text{ fm}; \text{ml/mh}=0.2 & a=0.14 \text{ fm}; \text{ml/mh}=0.2 \\
\end{array}
\]
Extrapolations for $B^+$ and $B_s$

**PRELIMINARY** analysis and data are still blinded.
Summary

Current study (predicted)

- Error budget is work in progress.
- Current projections shown on left.
- Anticipate reductions in systematic errors compared to our previous study.
- Underway: final cross-checks of this analysis and full error budget for this three-flavor asqtad study.
- Next: clover bottom quark calculations on MILC four-flavor HISQ lattices – including e.g., $f_{B^+}/f_{D^+}$ with HISQ charm.