Lattice 2013

Determining the anomalous dimension through the eigenmodes of Dirac operator

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Outline

- Goal/results:

  Determining energy-dependent mass anomalous dimension through Dirac eigenmodes

- Method

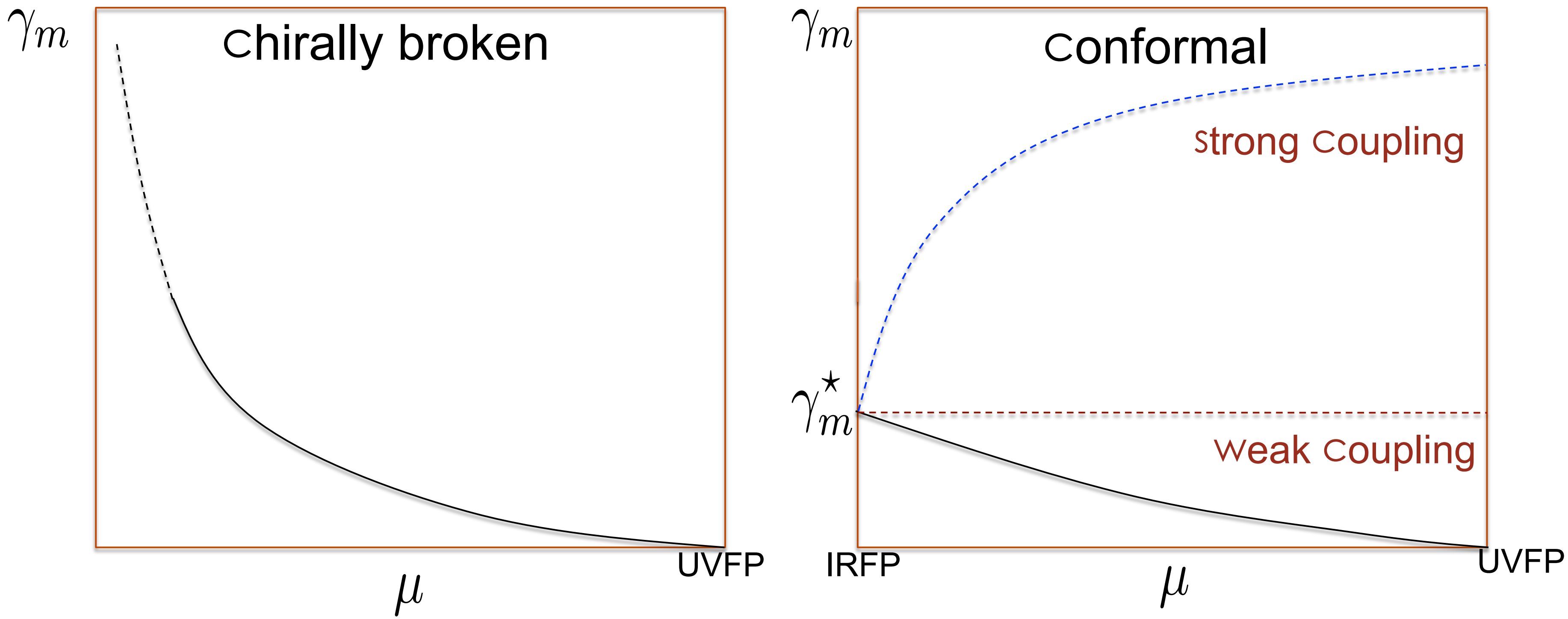
- Applications (nHYP smeared staggered fermion):

  1) SU(3) 4-flavor: QCD-like, test case

  2) SU(3) 12-flavor: conformal or χSB ? new developments

Goal and Results

Dirac eigenmodes can predict the scale dependent mass anomalous dimension in both chirally broken and IR conformal systems



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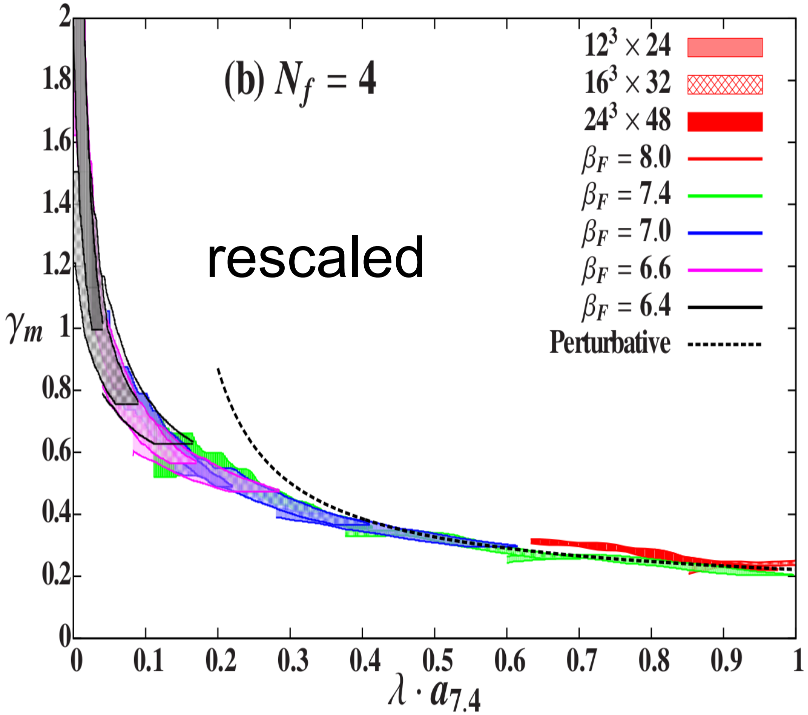
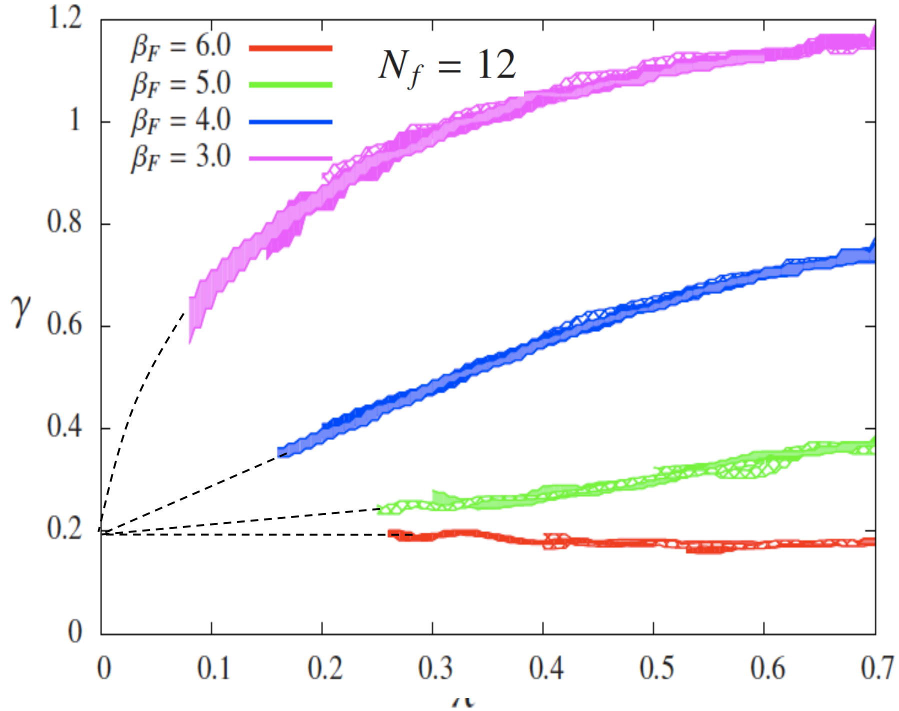




Dirac eigenvalue density and mode number

$$S_F[\psi, \bar{\psi}] = \int d^4x \bar{\psi}(x)(\not{D} + m)\psi(x)$$

$$D\psi_k = \lambda_k \psi_k$$

Dirac eigenvalue density: $\rho(\lambda)$

Dirac eigenmode number: $\nu(\lambda) = V \int_{-\lambda}^{\lambda} \rho(\omega) d\omega$

Dirac eigenvalue density and mode number

In chirally broken system we know :

Banks-Casher relation: $\rho(0) = \pi\langle\bar{\psi}\psi\rangle \neq 0$

In conformal system we expect :

$$\rho(\lambda) \sim \lambda^{\alpha}$$

$$\nu(\lambda) = cV\lambda^{\alpha+1} = c(L\lambda^{(\alpha+1)/4})^4$$

RG invariance of $\nu(\lambda)$ implies:

$$y_m = 1 + \gamma_m = \frac{4}{\alpha+1}$$

γ_m anomalous dimension

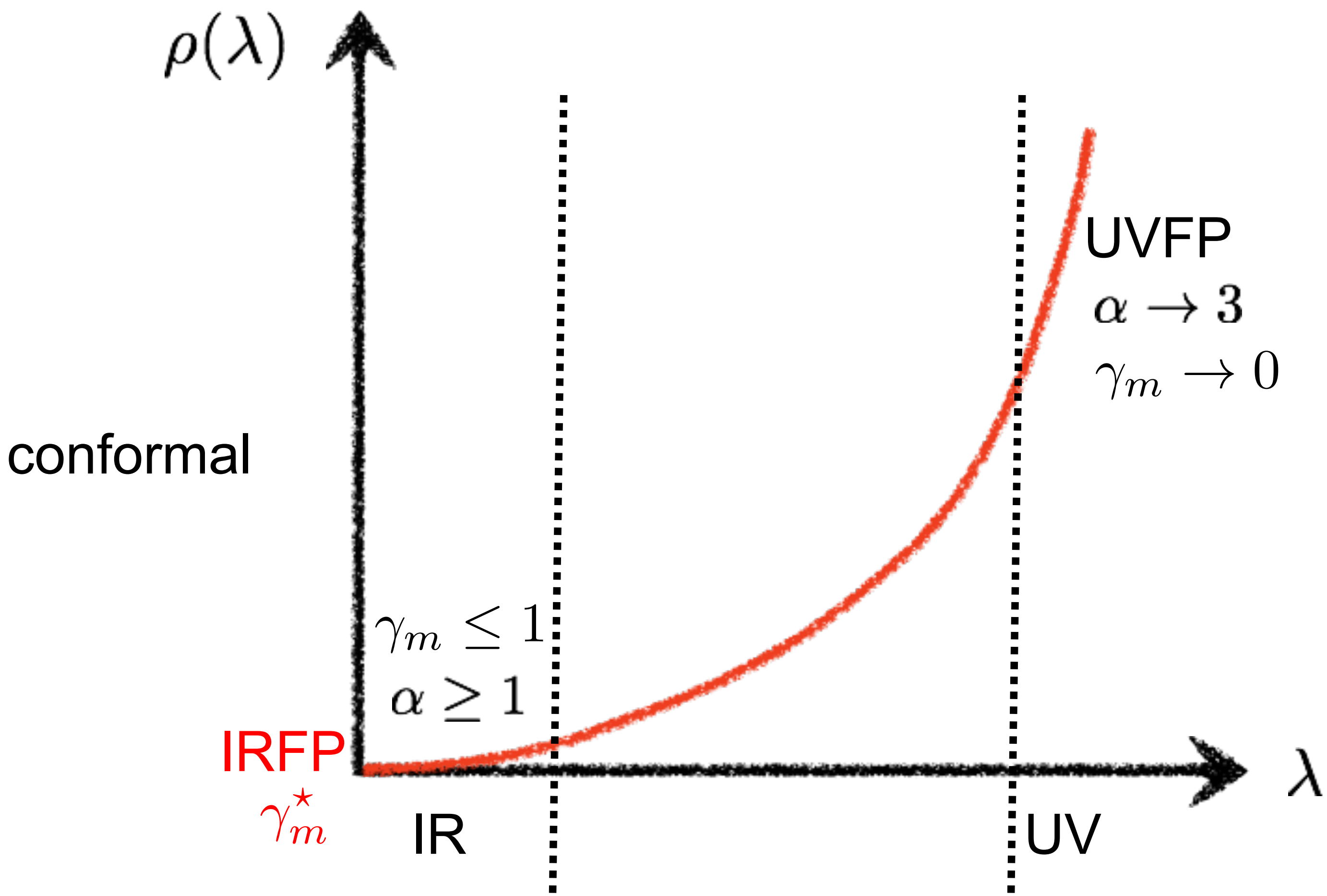
(Del Debbio and Zwicky,
Giusti and Luscher)

New: $\gamma_m,\ \alpha$ energy dependent

$\lambda \Longleftrightarrow$ energy

$\Rightarrow$ $\boxed{\gamma_m,\ \alpha \text{ depends on } \lambda}$

Scaling of eigenvalue density in IR conformal systems

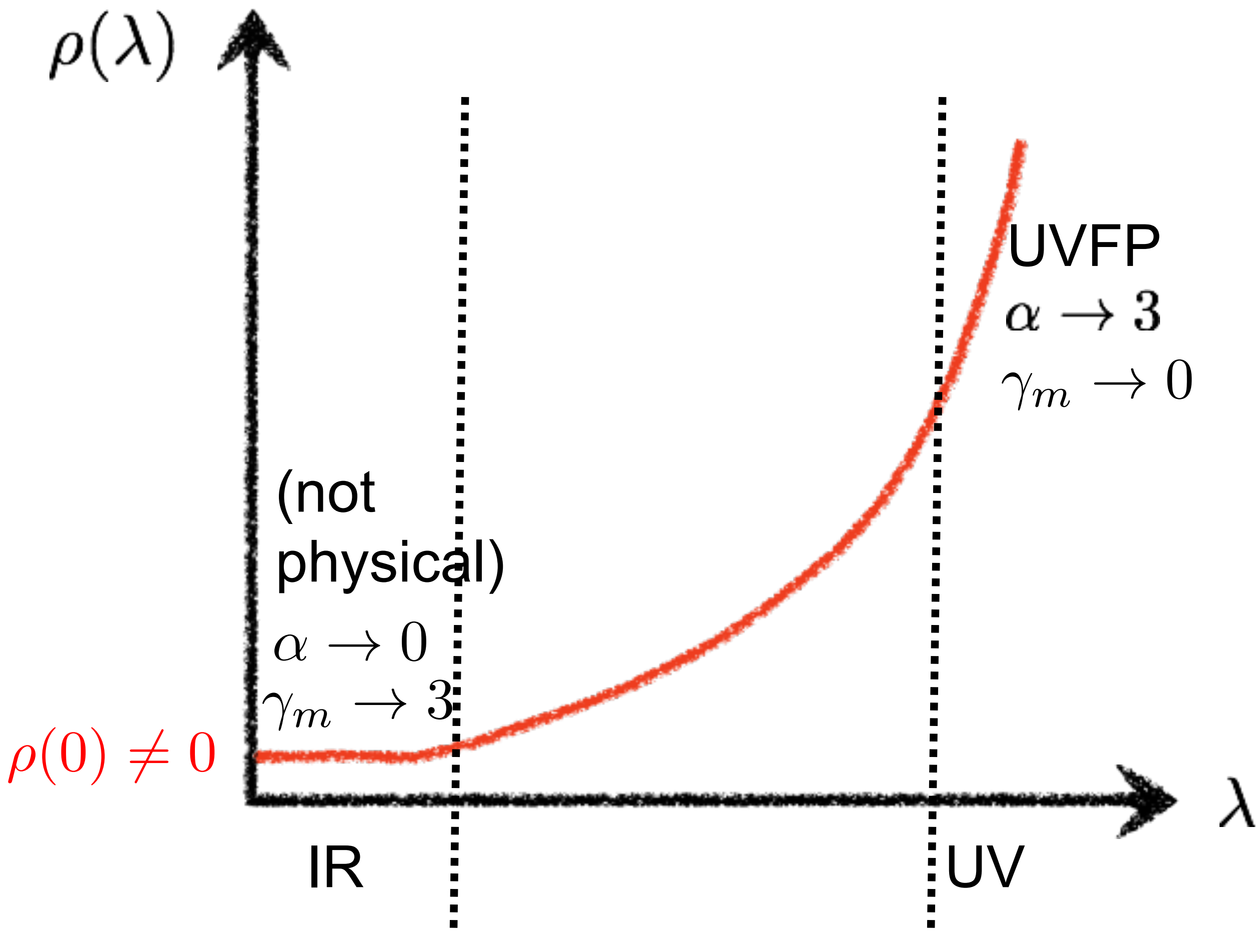


Fit:

$$\nu(\lambda) \propto \lambda^{1+\alpha(\lambda)}$$

$$1 + \gamma_m = \frac{4}{\alpha + 1}$$

Scaling of eigenvalue density in chirally broken systems



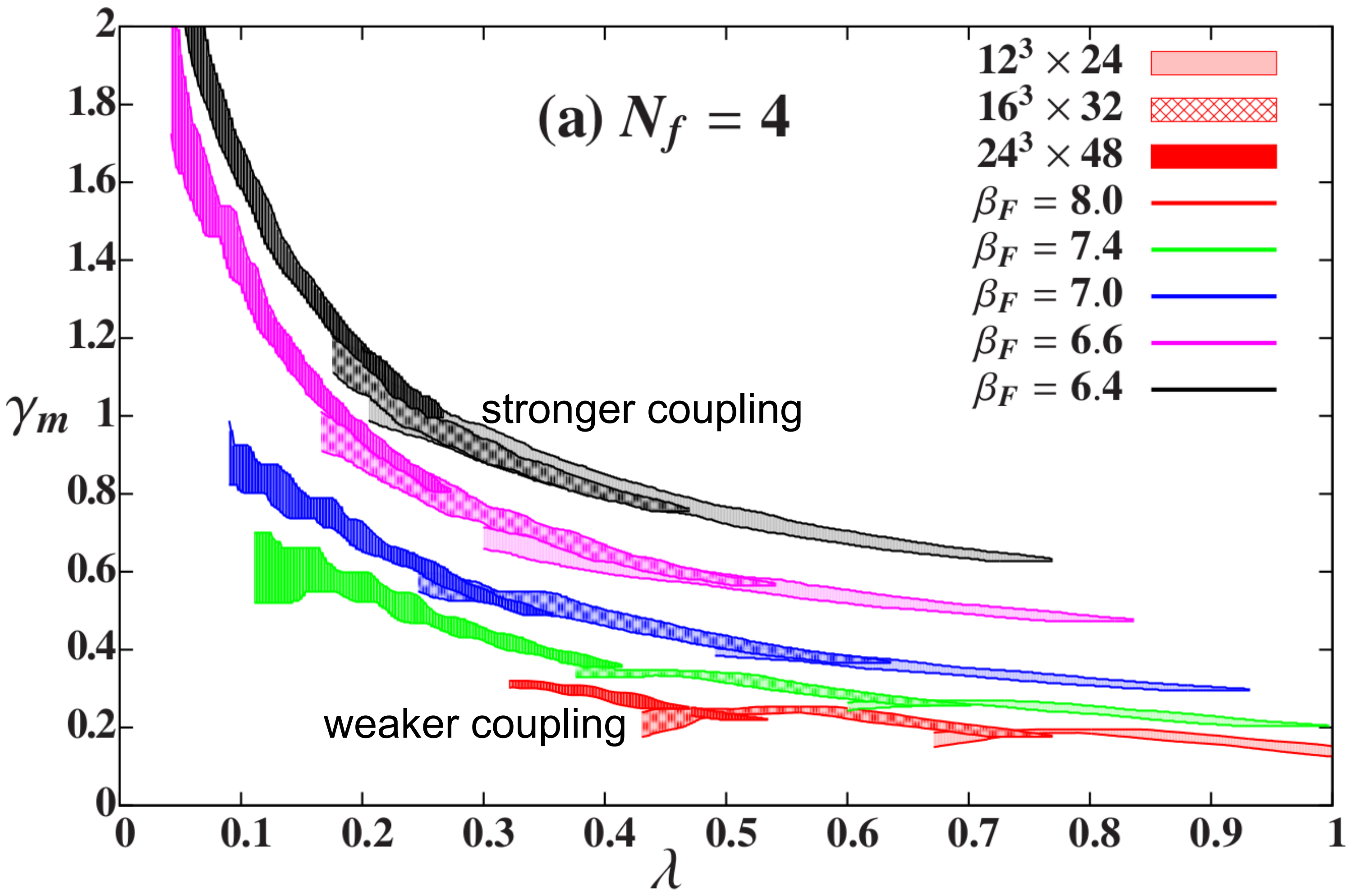
Fit:

$\nu(\lambda) \propto \lambda^{1+\alpha(\lambda)}$

(in limited range)

$$1 + \gamma_m = \frac{4}{\alpha + 1}$$

Scaling of mode number for Nf=4



$$\nu(\lambda) = c\lambda^{\alpha+1}$$

$$\beta_F \sim 12/g^2$$

QCD like!

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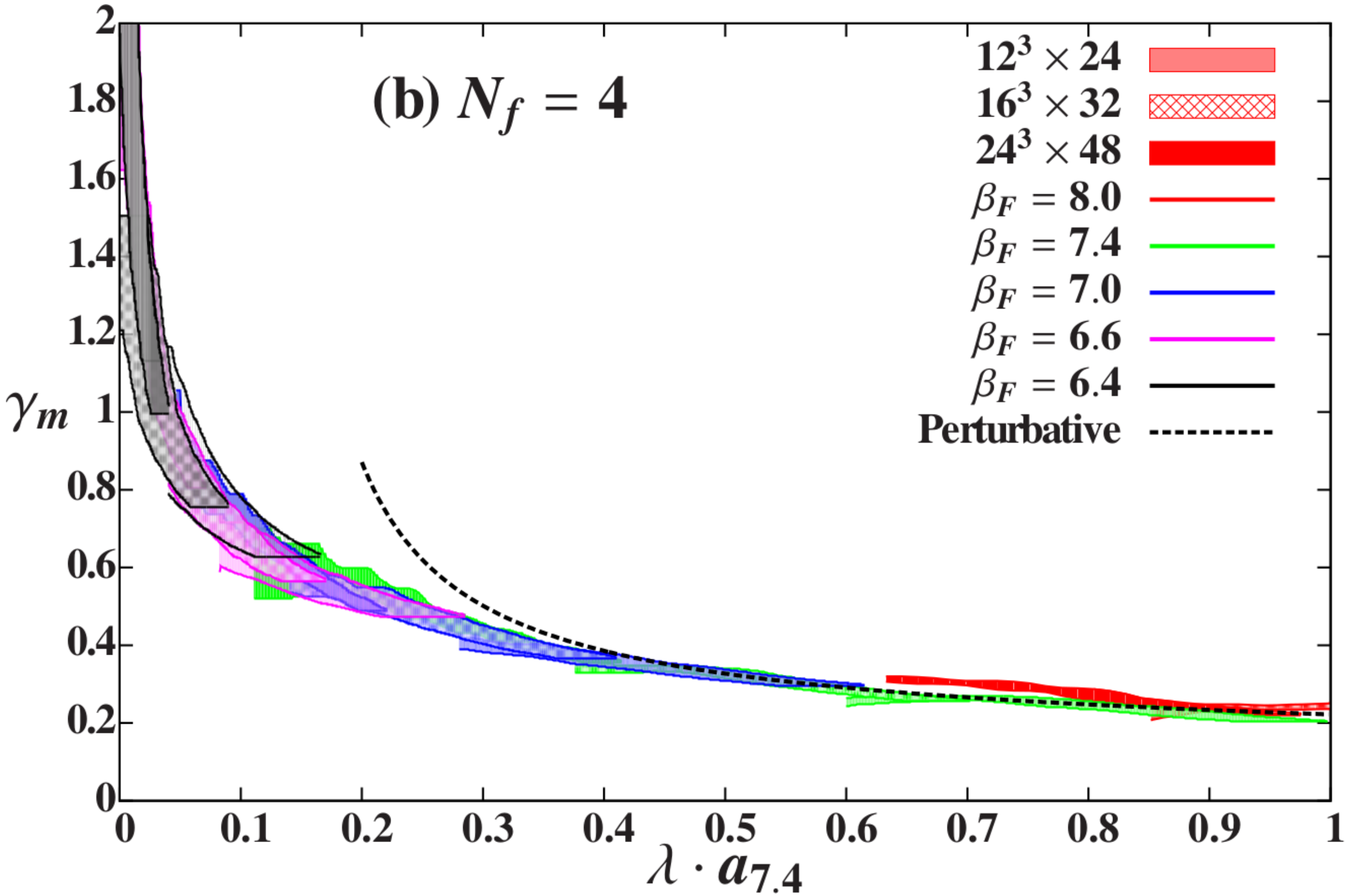
Scaling of mode number for Nf=4



Rescaled to a common lattice spacing

$$\lambda_\beta \to \lambda_\beta\left(\frac{a_{7.4}}{a_\beta}\right)^{1+\gamma_m}$$

Match to 1-loop pertb. theory at $\lambda \cdot a_{7.4} = 0.8$

QCD like

WORKS!

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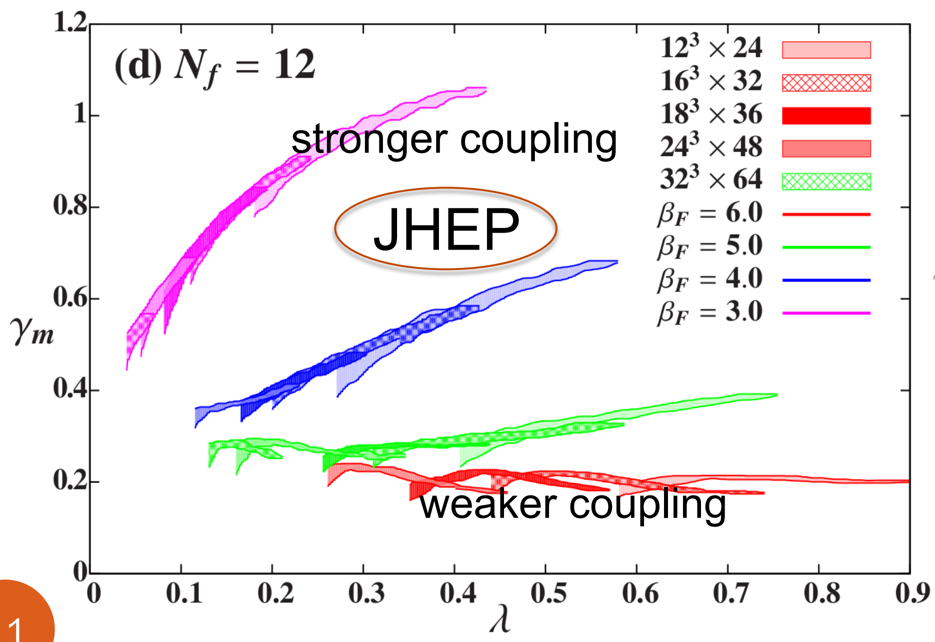
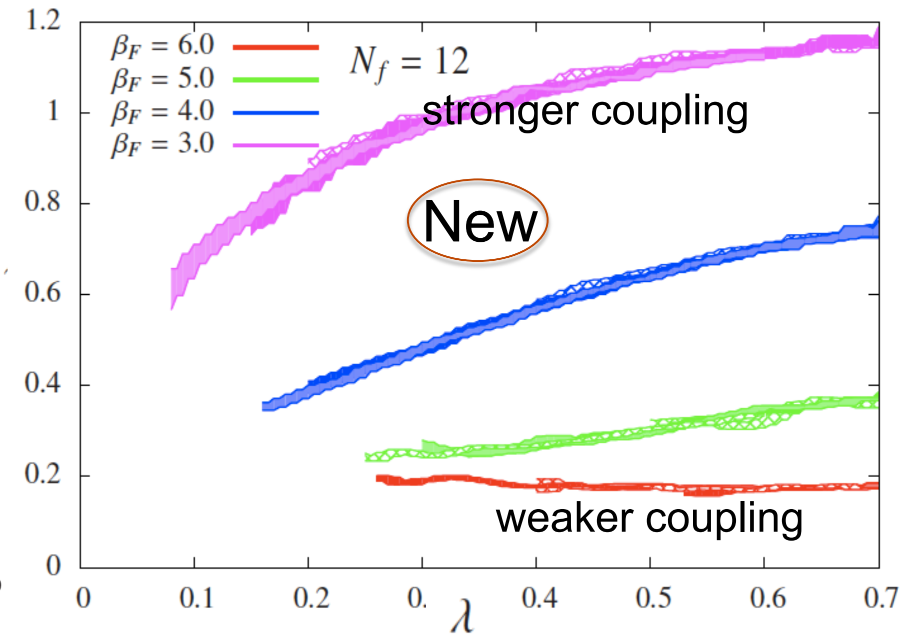
New developments for Nf=12 system

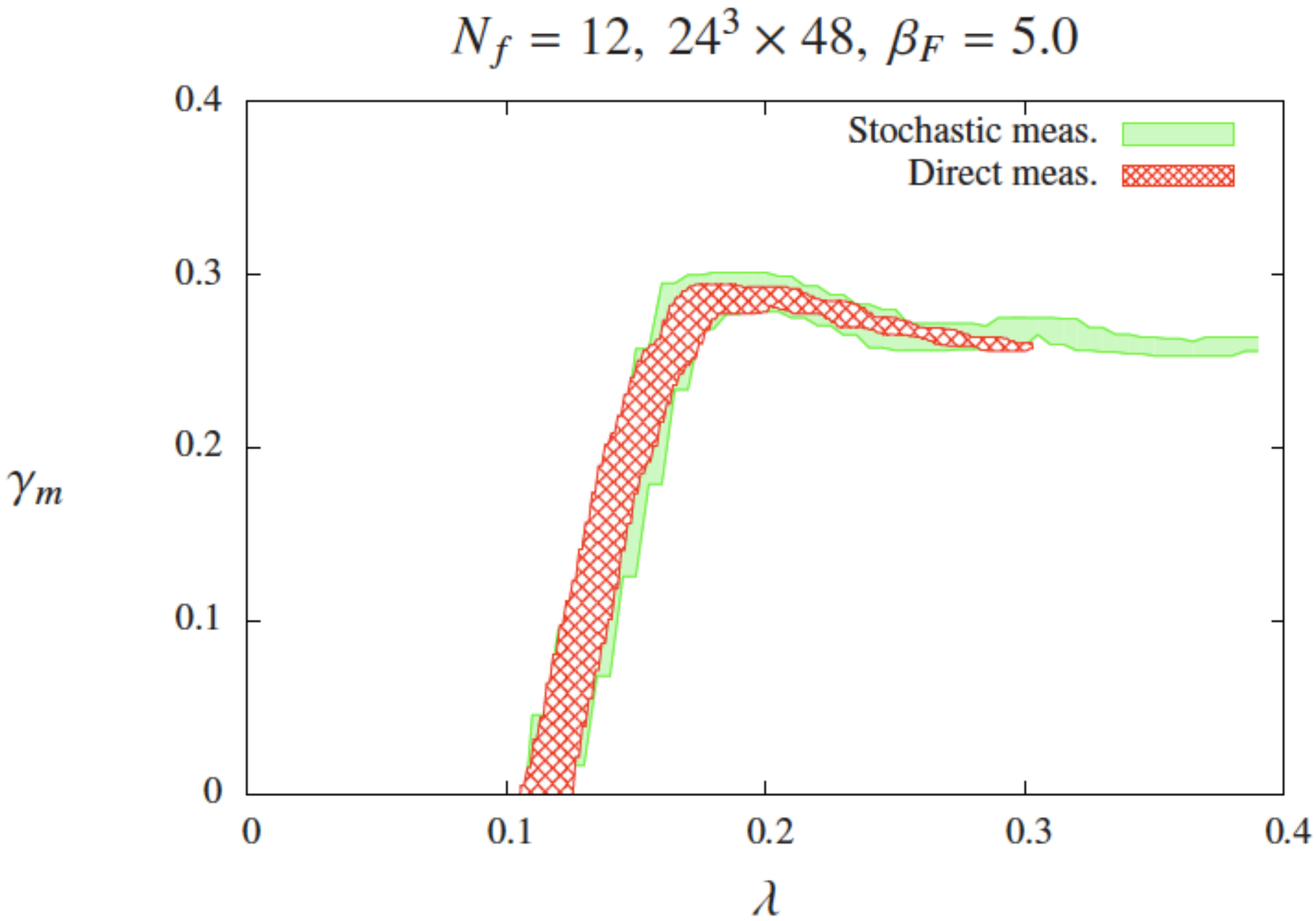
	Published	New
Measurement Method	Direct (eigenvalue)	Stochastic (mode number) L. Giusti, M. Luscher, A. Patella, et al.
Fermion mass	<= 0.0025	0.0
Boundary Con. (spatial)	Periodic	Anti-periodic
Volumes	$L^3 \times 2L, L \leq 32$	$L^4, L \leq 48$





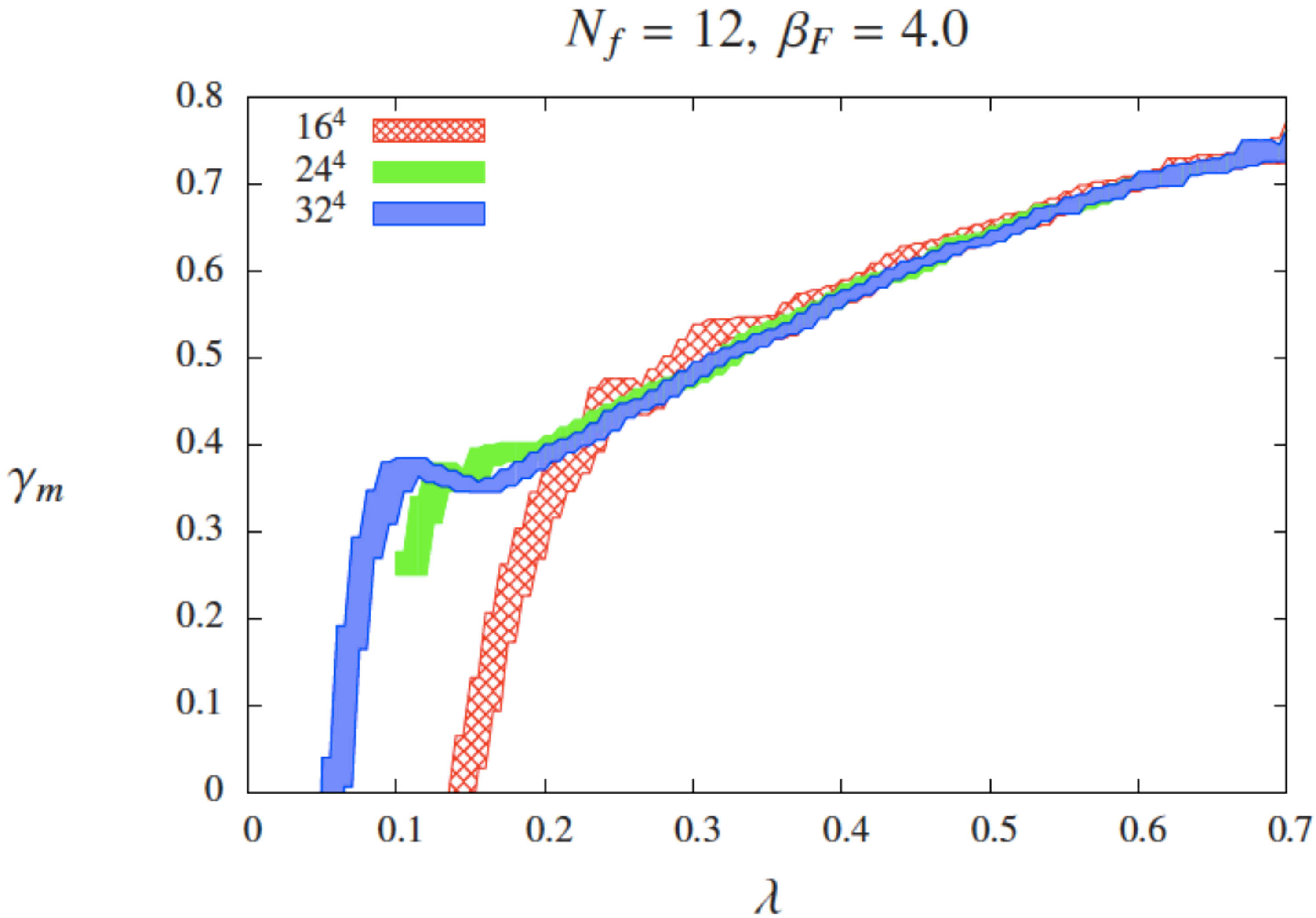
Test 1: Direct vs. stochastic meas.

Two methods: Direct measurement / Stochastic estimator are consistent



Test 2: Finite volume effects

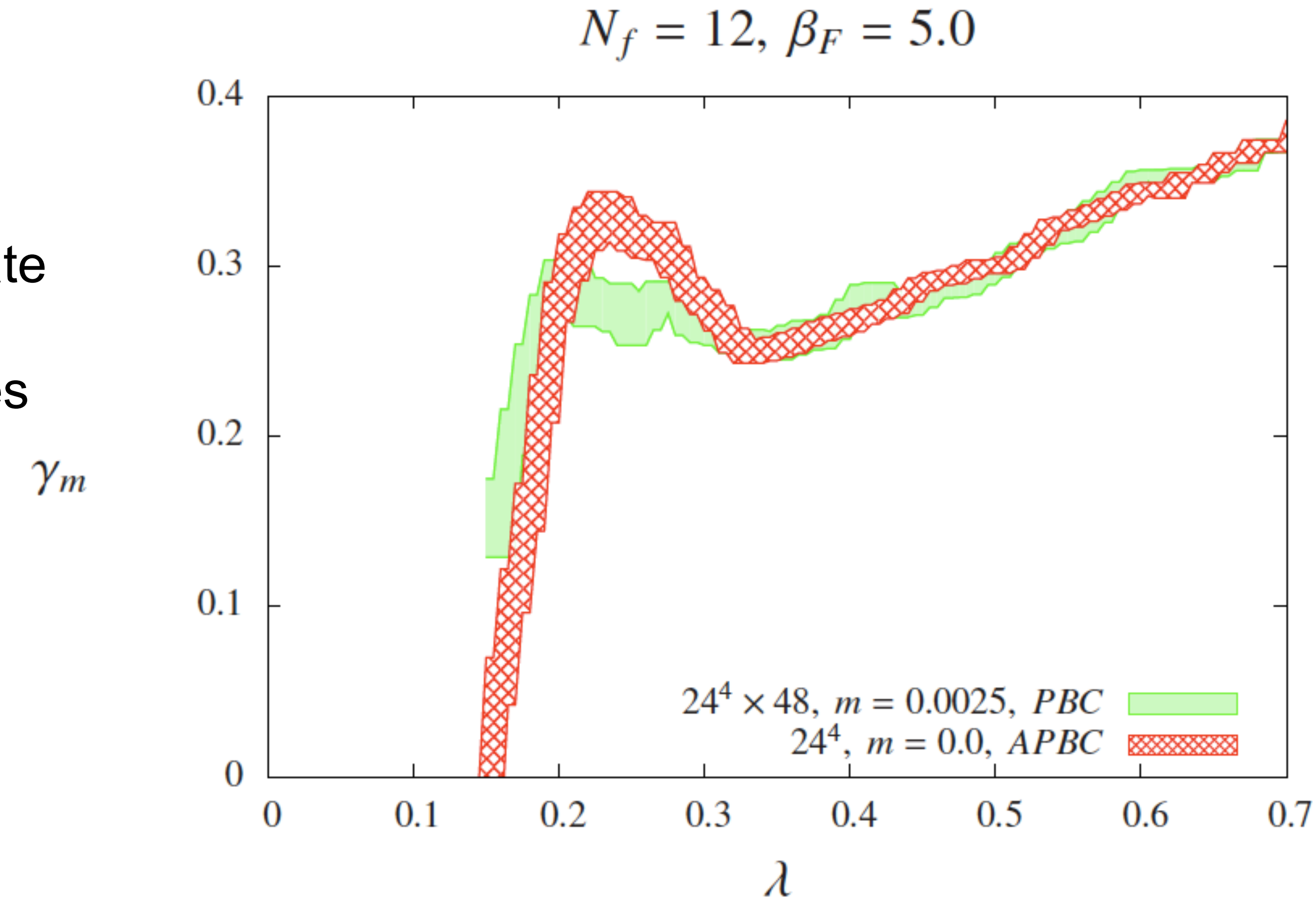
Finite volumes effects go away as lambda increases:



Test 3: B.C. effects

2 ensembles: $24^3 \times 48$, m=0.0025, periodic b.c.
24^4 , m=0.0, anti-periodic b.c are consistent

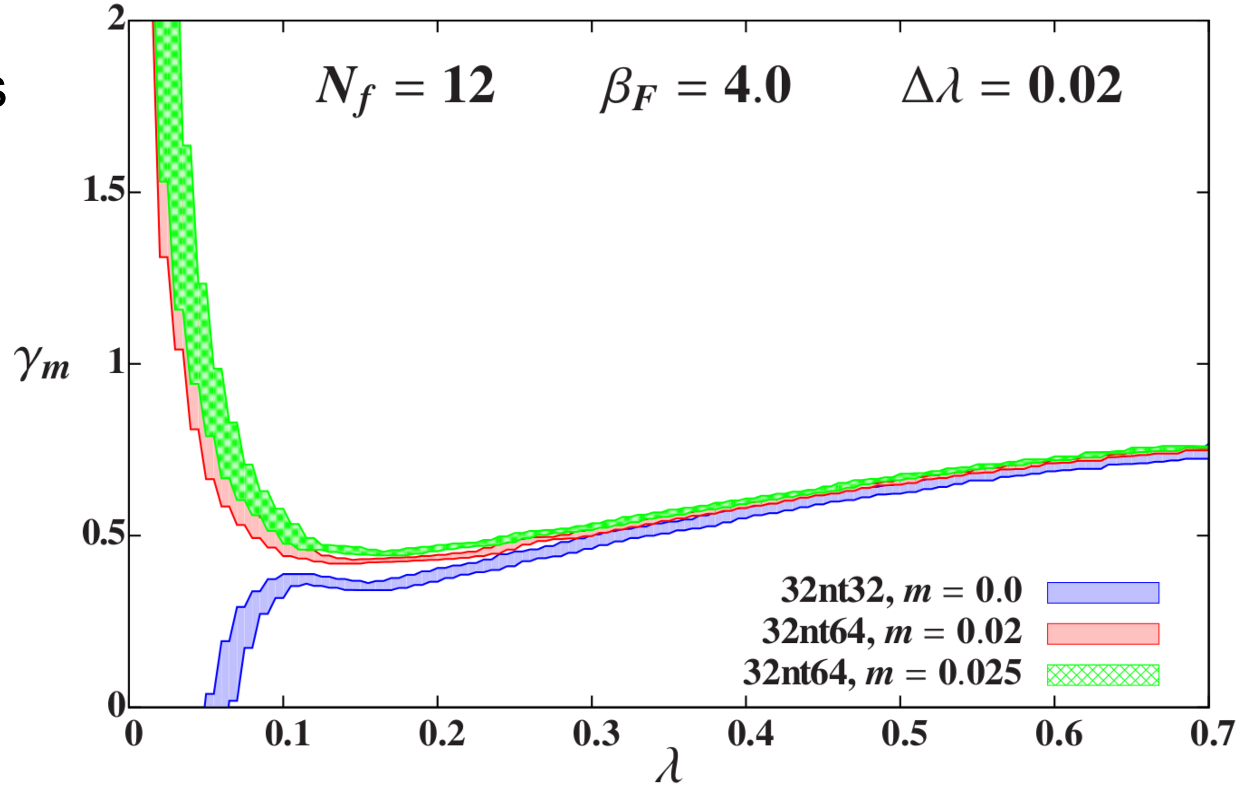
No chiral condensate for these ensembles



Test 3: Finite mass effects

3 ensembles:
32^4, m=0.0 (chirally symmetric)
$32^3 \times 64$, m=0.02 (chirally broken)
$32^3 \times 64$, m=0.025 (chirally broken)
are consistent

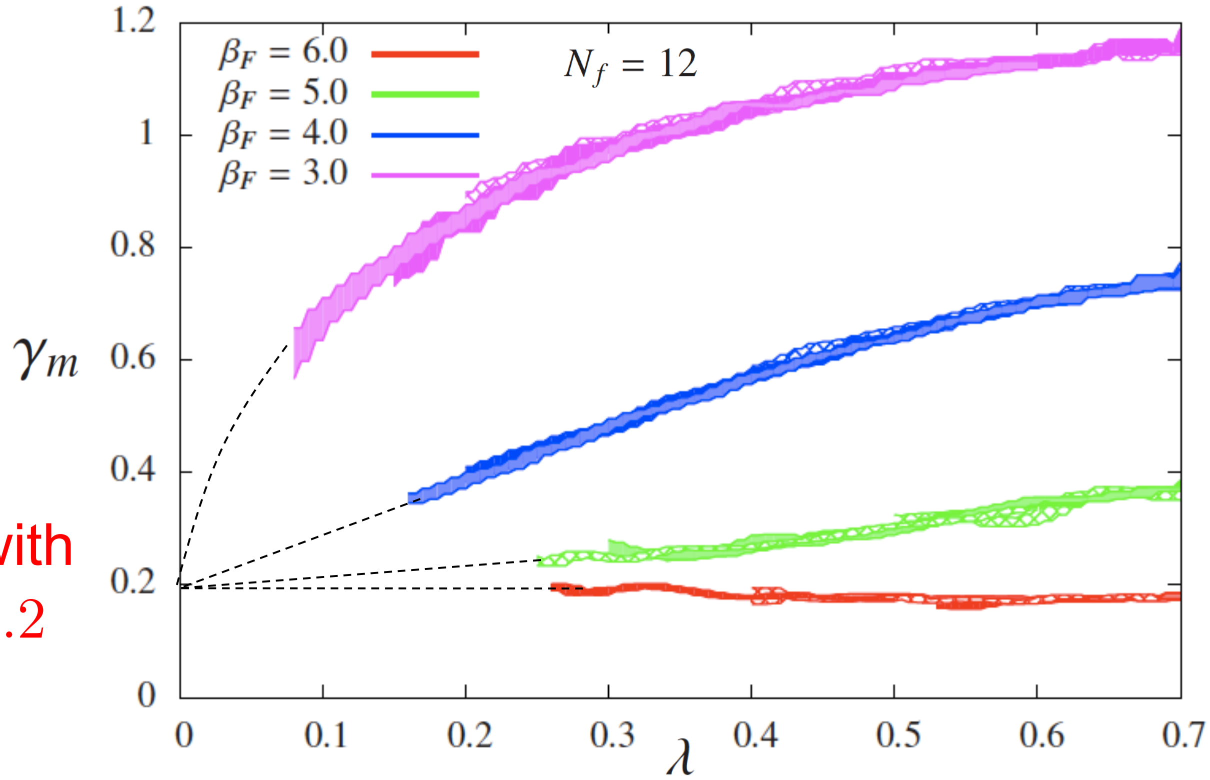
- Finite mass breaks chiral symmetry
- Finite mass effects disappear above $\lambda \approx 0.3$
- “Backward flow”



Again: SU(3) 12-flavor results

All finite volume, mass, b.c. effects only affect small lambda transient

IRFP with
$\gamma_m^\star \sim 0.2$



- Investigating ranges of couplings & energy scales required
- Extrapolation to IR limit required

Conclusions

- Goal/results: $\gamma_m(\lambda) \Longleftarrow \nu(\lambda)$

- Applications:
  1) SU(3) 4-flavor system: QCD-like
  2) SU(3) 12-flavor system: consistent IRFP with $\gamma_m \sim 0.2$

- Unique probe to study systems from IR to UV

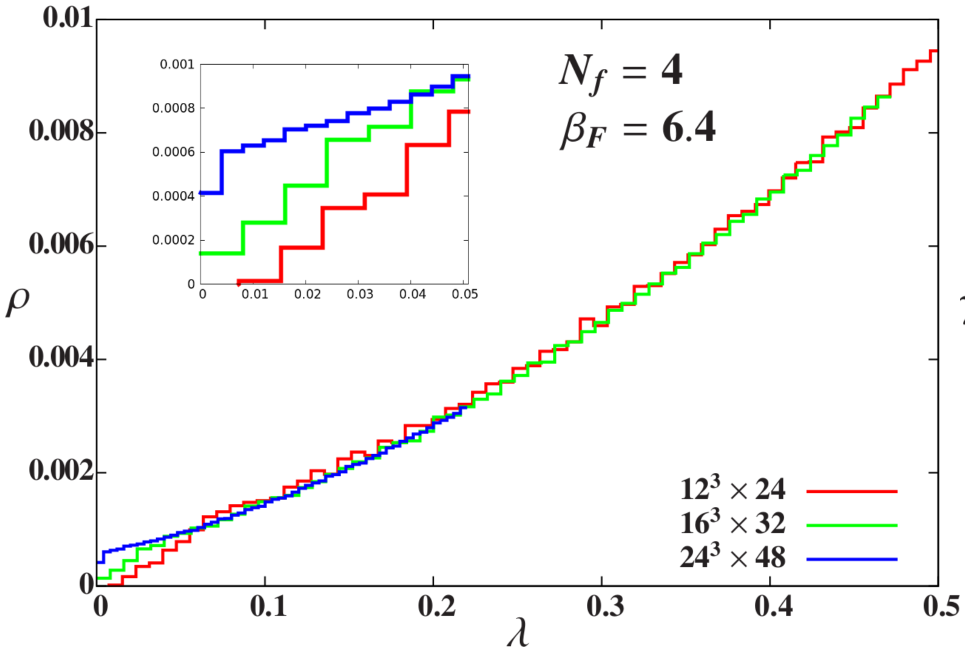
- Universal and applicable to any lattice model of interest, including both conformal and chirally broken systems.

Thank you!

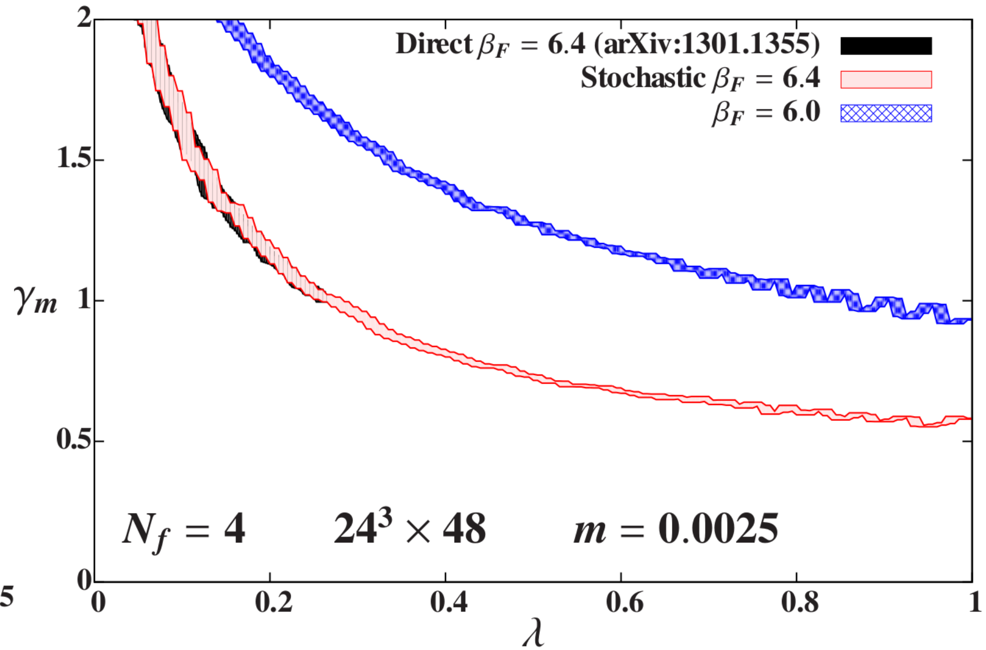
Funding and computing resources

4 flavor: QCD like

non-zero chiral condensate:

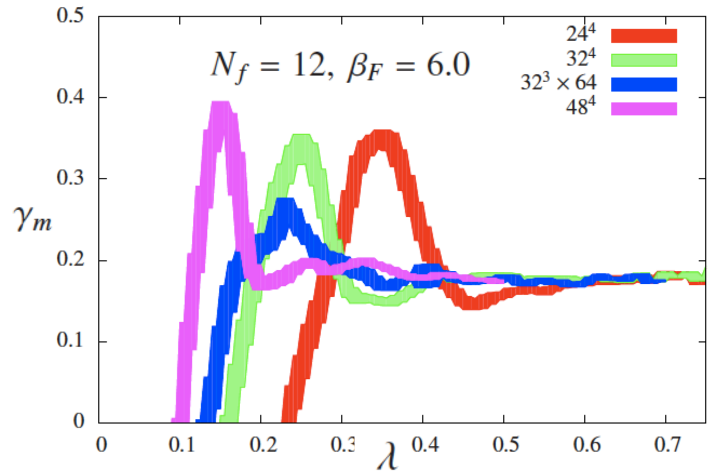


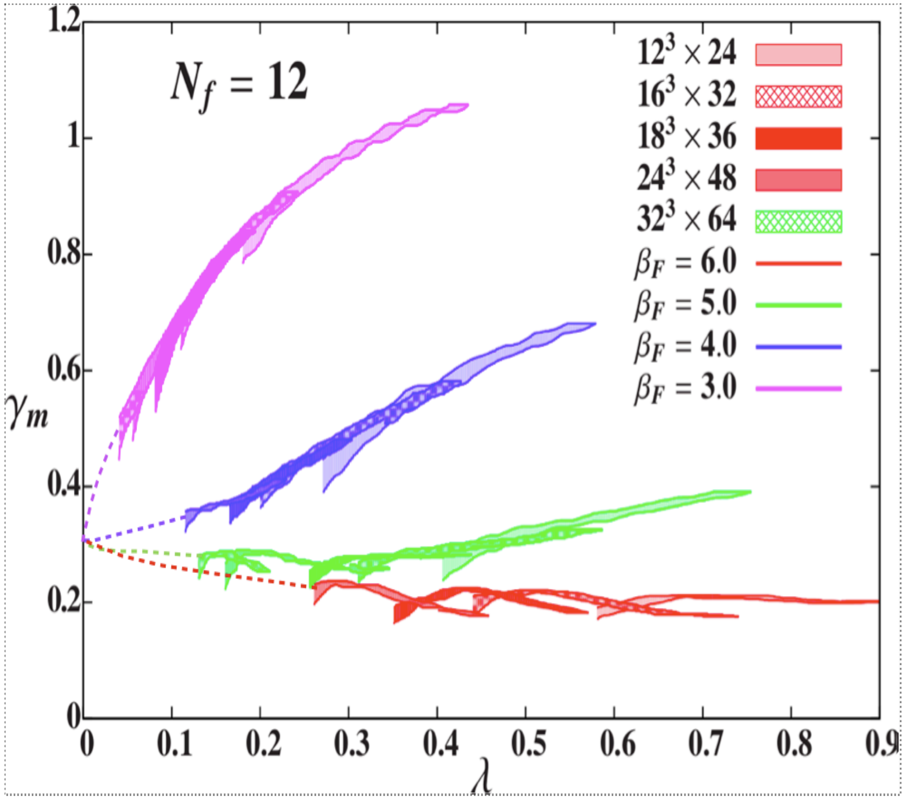
γ_m shoots up:

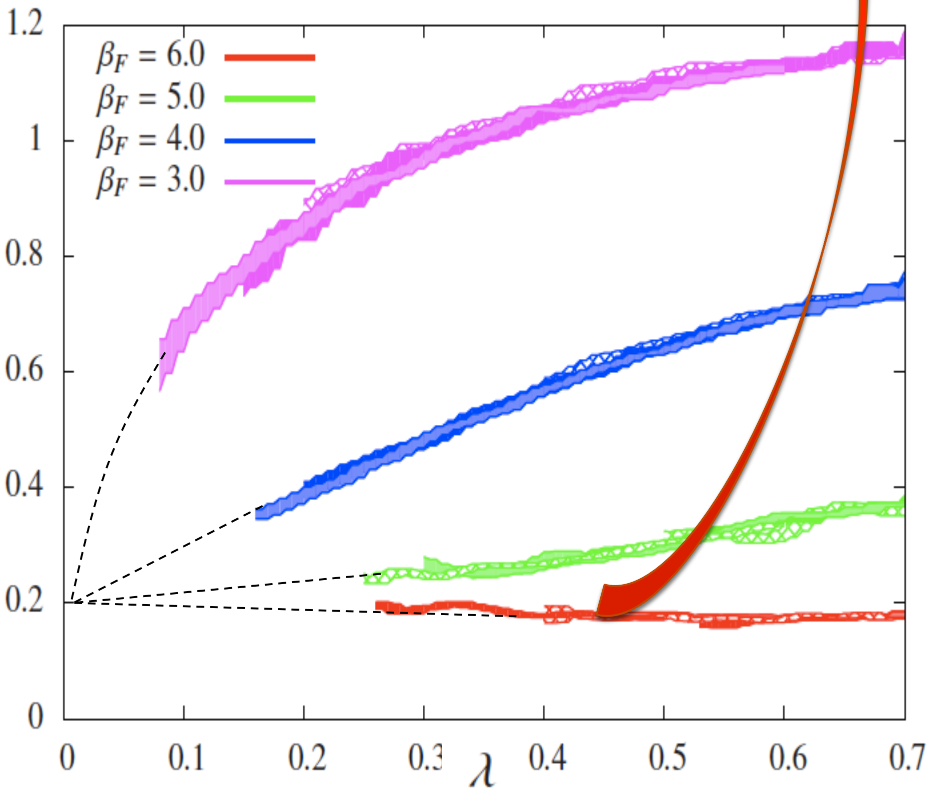


Old vs. New results

New results have better control over finite volume effects

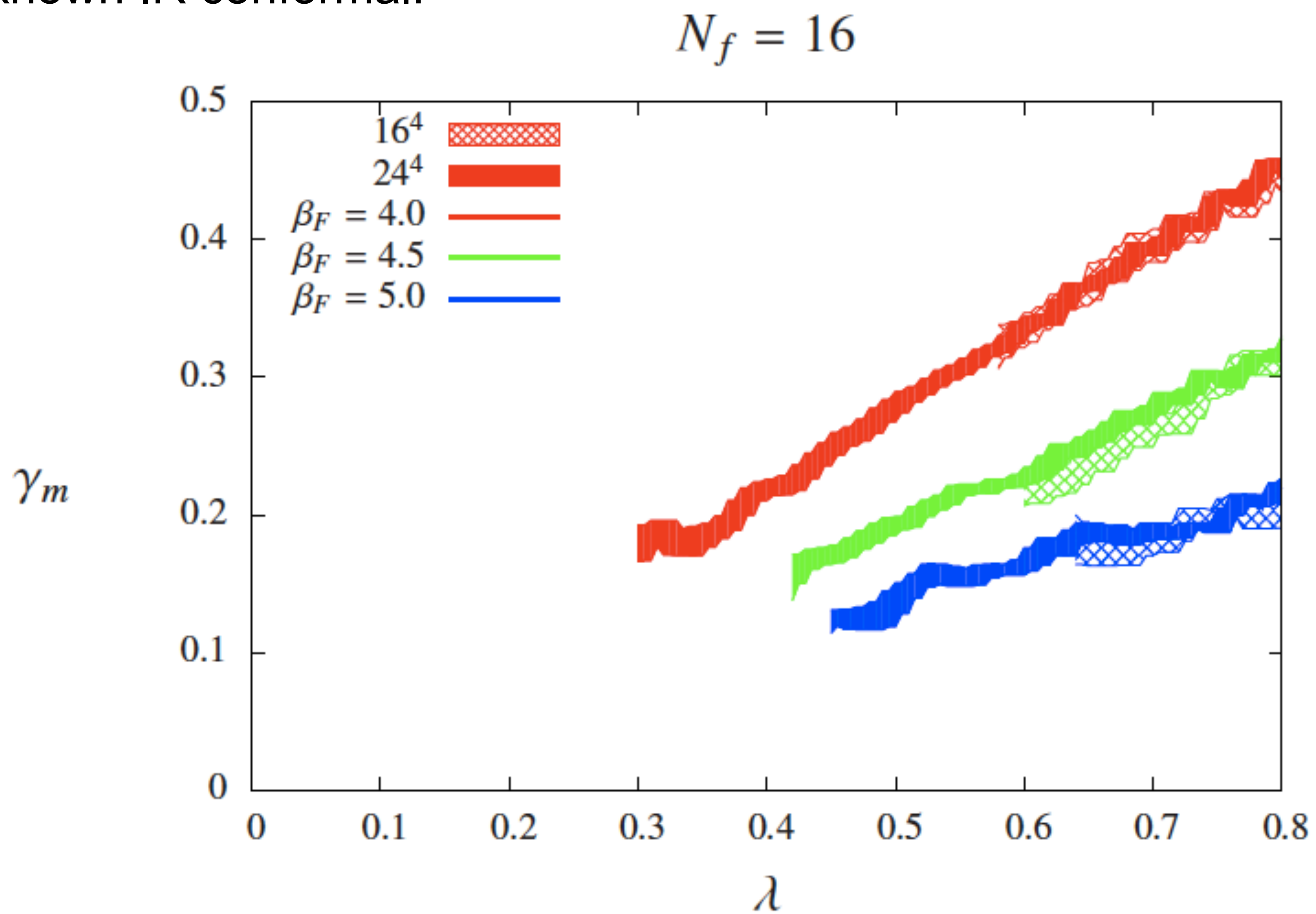




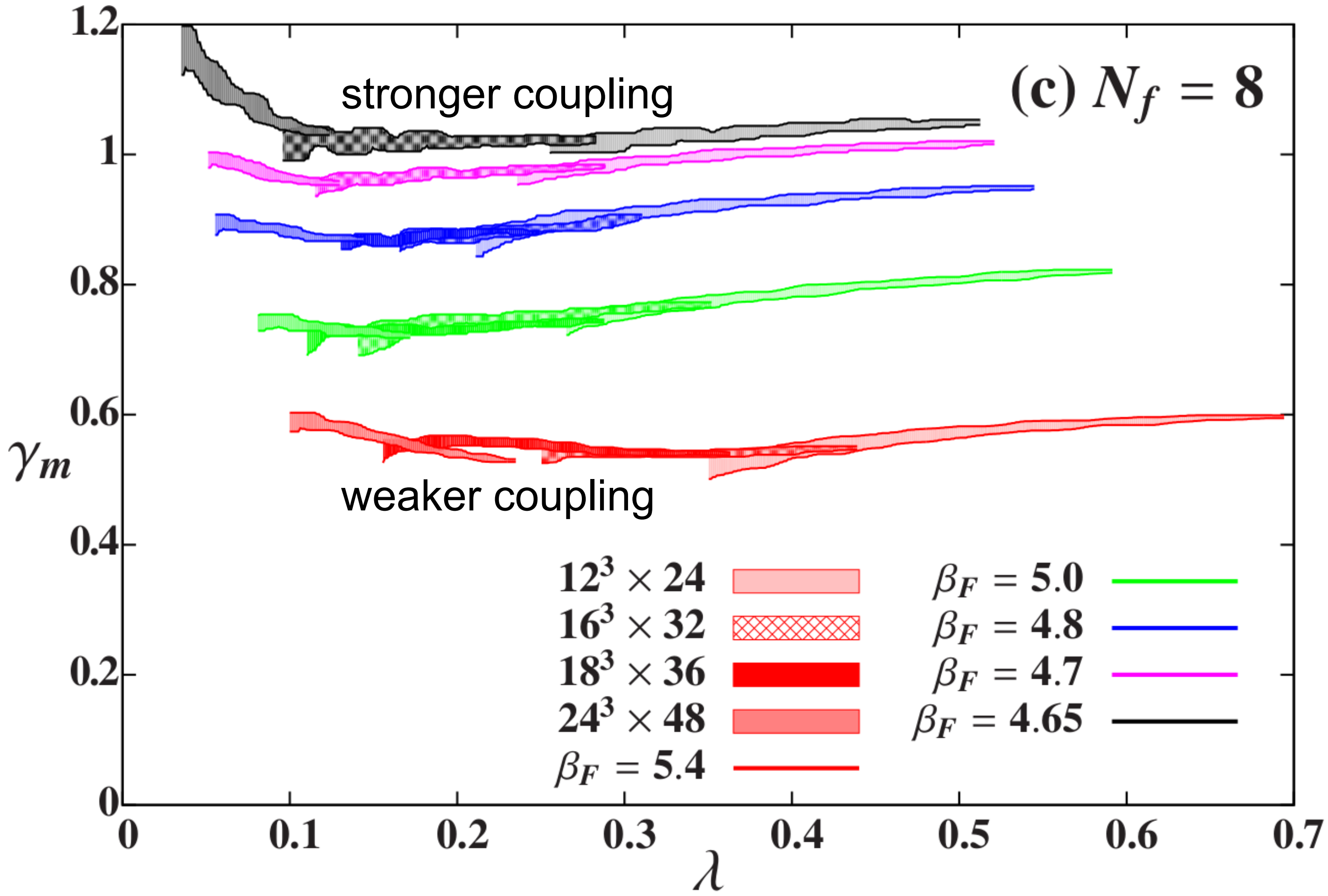


Scaling of mode number for Nf=16

known IR conformal:



Scaling of mode number for Nf=8



- little dependence on λ
- sensitive to gauge coupling
- large over large scales

"walking" chirally-broken or strongly-coupled IR conformal ?