Anomalous Dimensions of Four-Fermion Operators from Conformal EWSB Dynamics

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in collaboration with:
L Del Debbio
L Keegan

Lattice 2013, Mainz

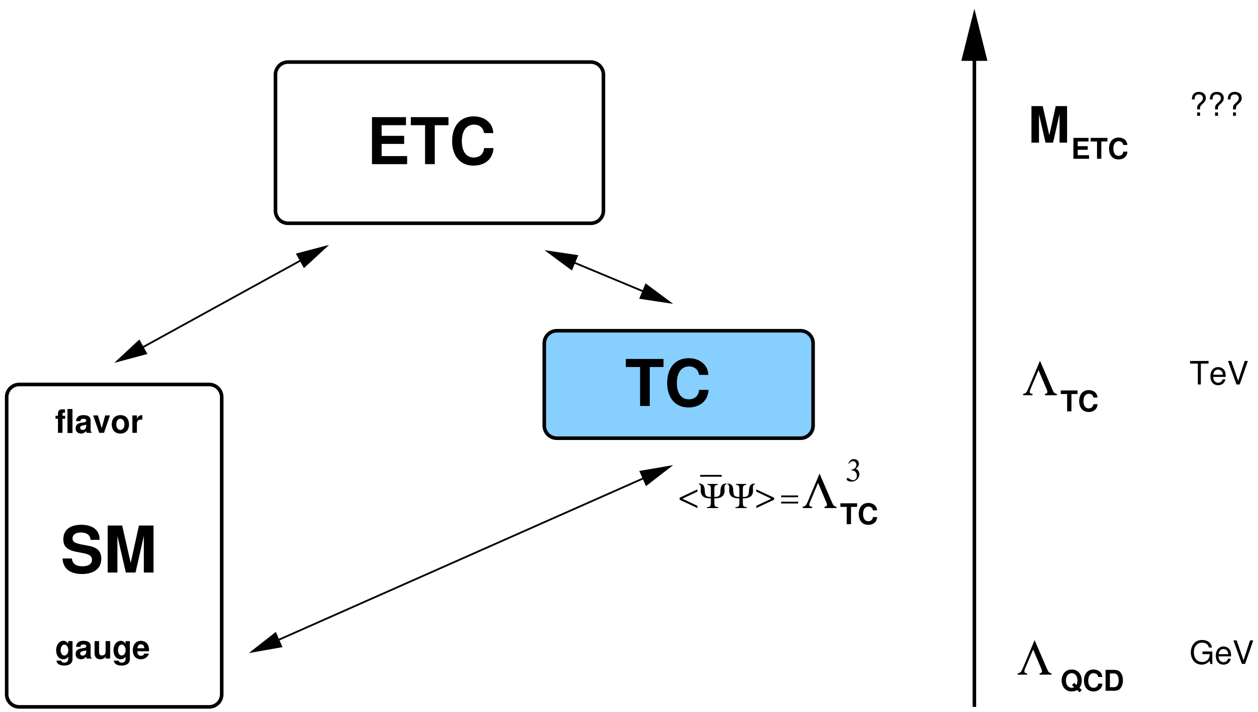
strong conformal EWSB + flavour

- strong interactions break $SU(2)_L$, generate small W mass ⇒ break EW symmetry with some UV QCD-like dynamics: technicolour

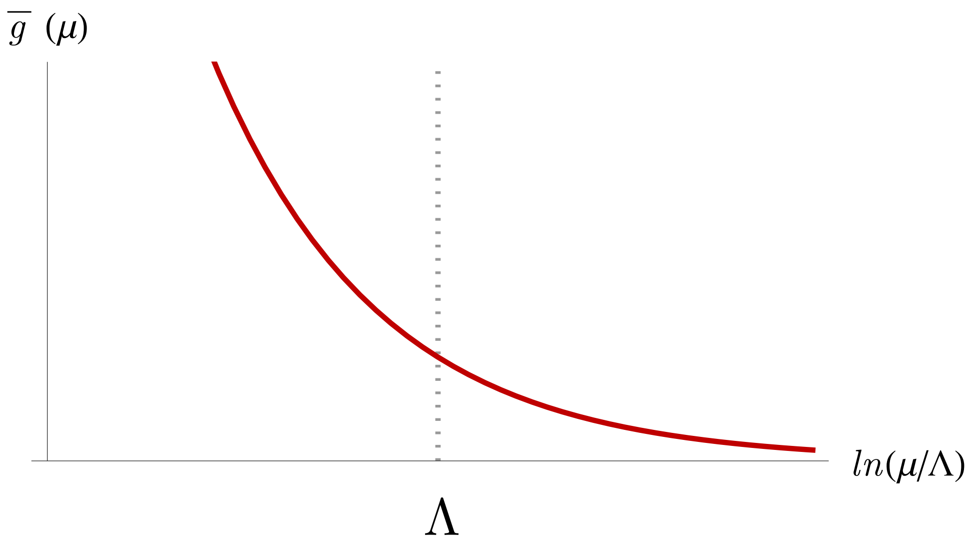
[Weinberg, Susskind 78]

- potential to generate fermion masses and mixings, but realistic models require more elaborate dynamics (extended technicolour)

[Dimopoulos, Susskind 79; Eichten, Lane 80]

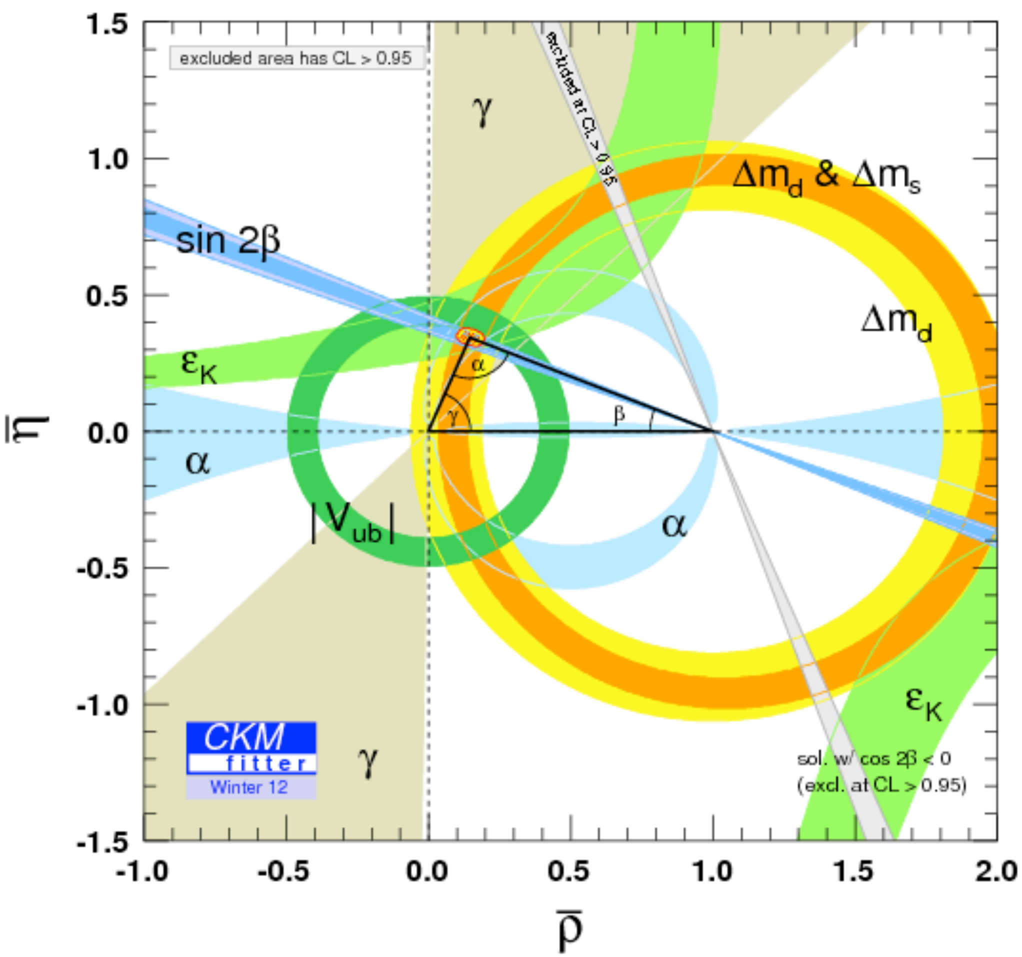


strong conformal EWSB + flavour

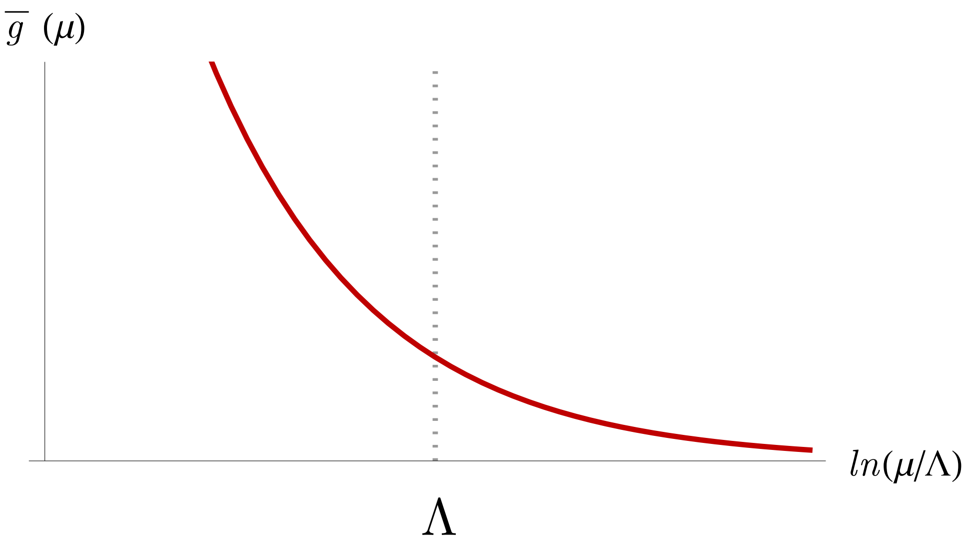


[Weinberg, Susskind 78]
[Dimopoulos, Susskind 79; Eichten, Lane 80]

- EWSB from strong dynamics
- Higgs?
- well-defined scale, log running
  ⇒ flavour hierarchies unnatural
- suppression mechanism for FCNC needed



strong conformal EWSB + flavour



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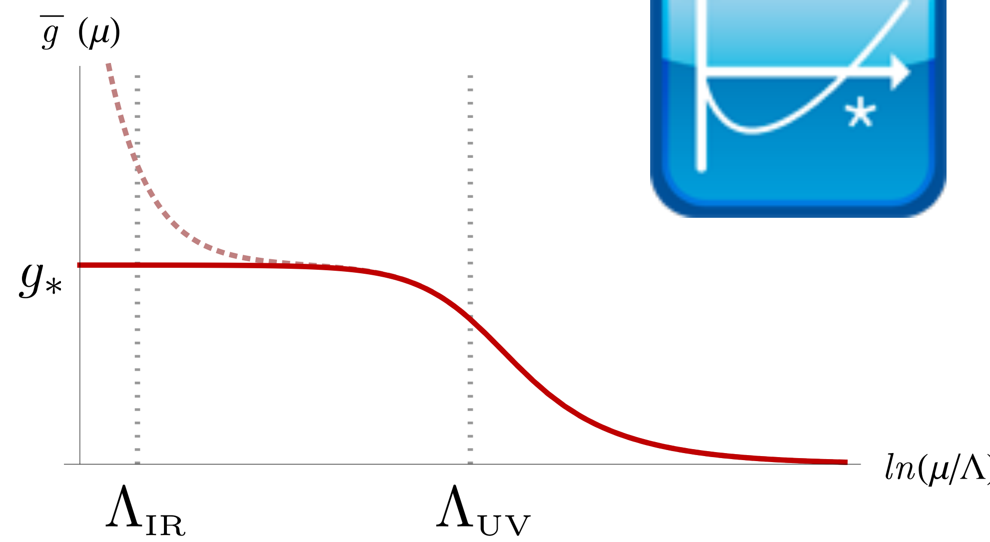


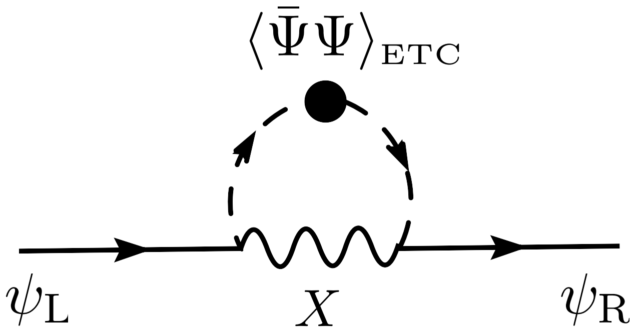
[Holdom 81, 85]
[Yamawaki et al. 86; Akiba, Yanagida 86]
[Appelquist, Wijewardhana 87]
[Appelquist, Sannino 99]

- EWSB from strong dynamics
- Higgs?
- well-defined scale, log running $\Rightarrow$ flavour hierarchies unnatural
- suppression mechanism for FCNC needed

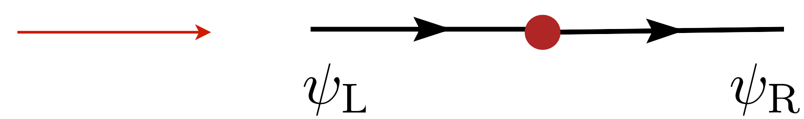
- EWSB from strong dynamics
- composite Higgs: dilaton?
- scale-invariant window $\Rightarrow$ flavour hierarchies
- flavour dynamics?

fermion masses

- TC condensates ⇒ masses



$$m_q \sim c(\mu/M_X)\,\frac{\Sigma_{\mathrm{ETC}}(\mu)}{M_X^2}$$



- (moderately large) anomalous dimensions + conformality ⇒ hierarchies

$$\frac{\Sigma(\Lambda_{\mathrm{UV}})}{\Sigma(\Lambda_{\mathrm{IR}})} \sim \log\left(\frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{IR}}}\right)^{\gamma} \qquad \text{vs.} \qquad \frac{\Sigma(\Lambda_{\mathrm{UV}})}{\Sigma(\Lambda_{\mathrm{IR}})} \sim \left(\frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{IR}}}\right)^{\gamma}$$

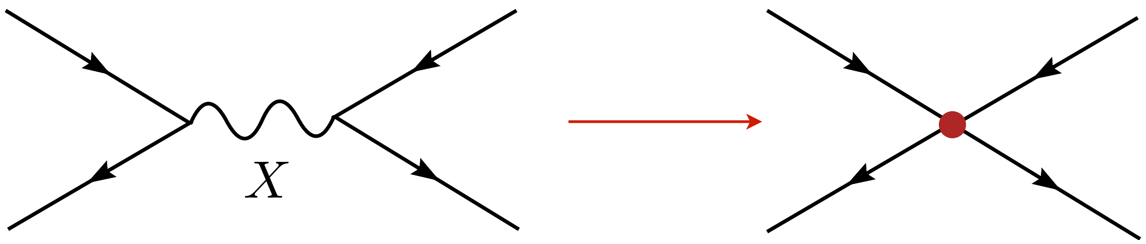
model building blocks

fermion couplings

- matrix elements ⇒ modify CKM (resp. PMNS), Higgs couplings, ...

  respect experimental constraints, suppress FCNC

$$\mathcal{L}_{\rm SM} + \frac{c(\mu/M_X)}{M_X^2}\,[\bar\psi\Gamma\psi][\bar\psi\Gamma'\psi](\mu)$$



- anomalous dimensions ⇒ hierarchies

$$\frac{\mathcal{O}(\Lambda_{\rm UV})}{\mathcal{O}(\Lambda_{\rm IR})} \sim \log\left(\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}}\right)^{\gamma} \qquad \text{vs.} \qquad \frac{\mathcal{O}(\Lambda_{\rm UV})}{\mathcal{O}(\Lambda_{\rm IR})} \sim \left(\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}}\right)^{\gamma}$$

non-flavour example: Higgs-Yukawa sector

[cf. Rattazzi, Rychkov, Tonni, Vichi 08]

leading correction to CFT action in top-Higgs sector [SM + strongly coupled CFT with (composite $H \sim \bar{\Psi}\Psi$) Higgs doublet]:

$$\mathcal{L}_{tH} = \frac{1}{16\pi^2}\lambda_t H\bar{Q}_L t_R + \text{h.c.} + \underbrace{\left(\frac{1}{16\pi^2}\right)^2 \lambda_t^2 \int \mathrm{d}^4x\mathrm{d}^4y\, H(x)^\dagger H(y)\bar{Q}_L t_R(x)\bar{t}_R Q_L(y)}$$

$$\approx \frac{1}{16\pi^2}\lambda_t^2 \Lambda_{\text{UV}}^{2+2d-\Delta_S} \int \mathrm{d}^4x\, H(x)^\dagger H(x)$$

$3 < \Delta_S < 4$ leads to relevant deformation of the CFT, strongly self-coupled Higgs at IR (EW) scale; scale dependence of V_{eff} determined by the value of Δ_S

renormalisation of four-fermion operators

4f operators (engineering dimension *d*=6) mix under renormalisation with all other *d*≤6 operators with same transformation properties under all symmetries

$$\mathcal{O} = (\bar{\psi}\Gamma T\psi)(\bar{\psi}\Gamma' T')$$

spin flavour / colour

$\{\mathcal{O}_i\}$ operator basis in given symmetry sector

$$\bar{\mathcal{O}}_i(\mu) = \sum_j Z_{ij}(\mu)\mathcal{O}_j$$

rules of the game (mass-independent schemes):

- symmetries dictate mixing pattern
- mixing due to symmetry breaking by regulator involves coefficients that depend on bare couplings only, not on renormalisation scale [Testa 98]
- spurionic symmetries allow to constrain mass dependence of mixing with lower-dimensional operators

anomalous dimensions: QCD

scale dependence (anomalous dimensions) can be obtained by considering operators without subtractions $\longleftrightarrow$ four distinct flavours in chiral limit

[Donini, Giménez, Martinelli, Talevi, Vladikas 99]

full basis (after Fierzing):

$$
\begin{aligned}
Q_1^{\pm} &= (\bar{\psi}_1\gamma_\mu(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3\gamma_\mu(\mathbf{1}-\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)\\
Q_2^{\pm} &= (\bar{\psi}_1(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3(\mathbf{1}+\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)\\
Q_3^{\pm} &= (\bar{\psi}_1\gamma_\mu(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3\gamma_\mu(\mathbf{1}+\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)\\
Q_4^{\pm} &= (\bar{\psi}_1(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3(\mathbf{1}-\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)\\
Q_5^{\pm} &= (\bar{\psi}_1\sigma_{\mu\nu}(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3\sigma_{\mu\nu}(\mathbf{1}-\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)
\end{aligned}
$$

anomalous dimensions: QCD

scale dependence (anomalous dimensions) can be obtained by considering operators without subtractions $\longleftrightarrow$ four distinct flavours in chiral limit

[Donini, Giménez, Martinelli, Talevi, Vladikas 99]

full basis (after Fierzing):

$$
\begin{aligned}
Q_1^{\pm} &= Q_{\mathrm{VV+AA}}^{\pm} - Q_{\mathrm{VA+AV}}^{\pm} \\
Q_2^{\pm} &= Q_{\mathrm{VV-AA}}^{\pm} + Q_{\mathrm{VA-AV}}^{\pm} \\
Q_3^{\pm} &= Q_{\mathrm{SS-PP}}^{\pm} + Q_{\mathrm{SP-PS}}^{\pm} \\
Q_4^{\pm} &= Q_{\mathrm{SS+PP}}^{\pm} - Q_{\mathrm{SP+PS}}^{\pm} \\
Q_5^{\pm} &= Q_{\mathrm{TT}}^{\pm} + Q_{\mathrm{T\tilde{T}}}^{\pm}
\end{aligned}
$$

parity-even parity-odd

anomalous dimensions: QCD

scale dependence (anomalous dimensions) can be obtained by considering operators without subtractions $\longleftrightarrow$ four distinct flavours in chiral limit

[Donini, Giménez, Martinelli, Talevi, Vladikas 99]

renormalisation pattern with Wilson fermions:

$$\bar{Q}_i^{\pm}(\mu) = \sum_{j,k=1}^{5} Z_{ij}^{\pm}(\mu)[\delta_{jk} + \Delta_{jk}(g_0^2)]\, Q_k^{\pm}(g_0^2)$$

$$\begin{pmatrix} Z_{11}^{\pm} & 0 & 0 & 0 & 0 \\ 0 & Z_{22}^{\pm} & Z_{32}^{\pm} & 0 & 0 \\ 0 & Z_{23}^{\pm} & Z_{33}^{\pm} & 0 & 0 \\ 0 & 0 & 0 & Z_{44}^{\pm} & Z_{45}^{\pm} \\ 0 & 0 & 0 & Z_{54}^{\pm} & Z_{55}^{\pm} \end{pmatrix} \qquad \begin{pmatrix} 0 & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} \\ \Delta_{21} & 0 & 0 & \Delta_{24} & \Delta_{25} \\ \Delta_{31} & 0 & 0 & \Delta_{34} & \Delta_{35} \\ \Delta_{41} & \Delta_{42} & \Delta_{43} & 0 & 0 \\ \Delta_{51} & \Delta_{52} & \Delta_{53} & 0 & 0 \end{pmatrix}$$

$\neq 0$ in parity-even sector only!

anomalous dimensions: beyond QCD

scale dependence (anomalous dimensions) can be obtained by considering operators without subtractions $\longleftrightarrow$ four distinct flavours in chiral limit

[Donini, Giménez, Martinelli, Talevi, Vladikas 99]

results can be extended to e.g. fermions in adjoint representation of SU(2):

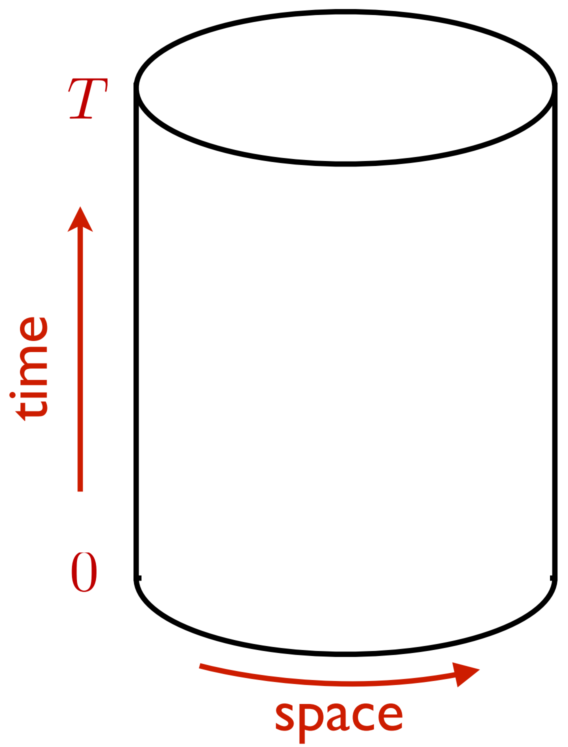
- Fierzing changes: real representation gives rise to Fierzing in both particle-antiparticle and particle-particle channels
- can show that no extra correlation functions are needed for SF renormalisation conditions
- can check explicitly that anomalous dimensions of unflavoured operators (e.g. $(\bar{\psi}\psi)^2$) are retrieved with the same formalism

[Del Debbio, Keegan, CP in progress]

non-perturbative RG running

non-perturbative renormalisation: Schrödinger Functional in Euclidean space

[Lüscher, Jansen, Narayanan, Sint, Sommer, Weisz, Wolff 91-96]
[ALPHA, 96-]



$$e^{-\Gamma} = \int D[A,\bar\psi,\psi]\,\exp\{-S[A,\bar\psi,\psi]\}$$

Dirichlet boundary conditions in time; abelian background gauge field controlled by parameter η

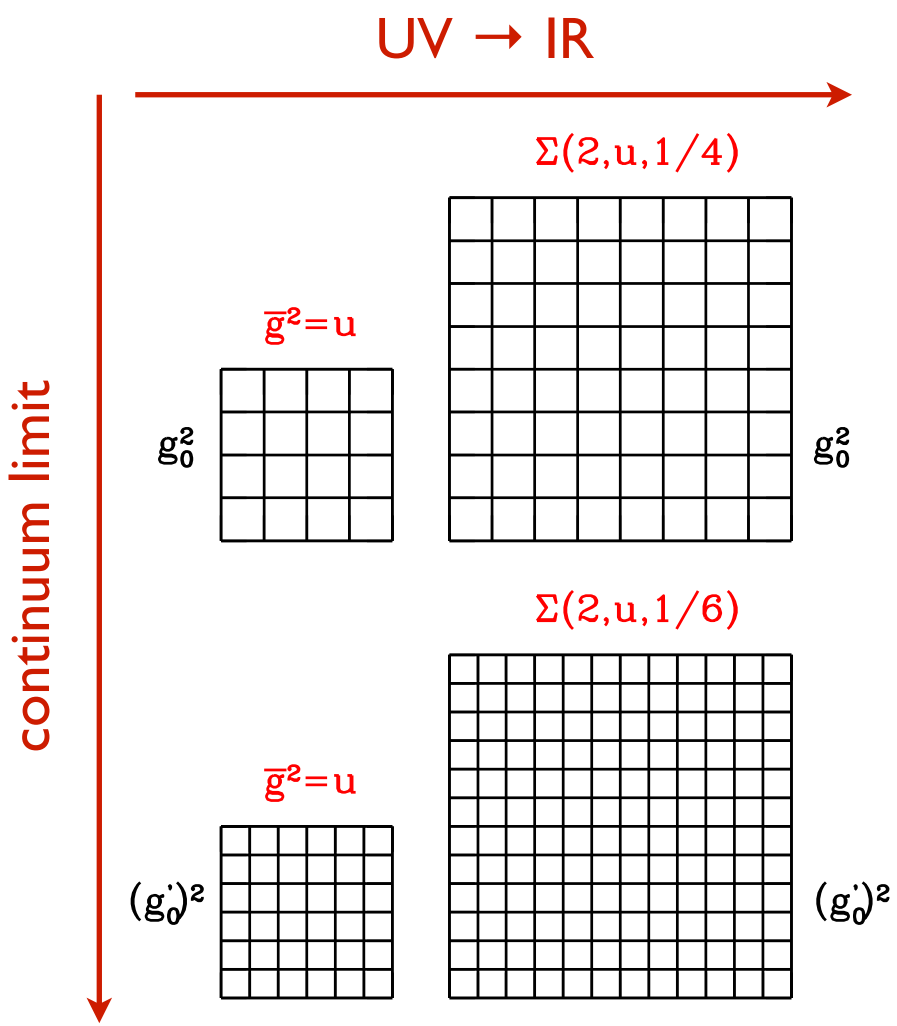
periodic boundary conditions in space (up to global phase)

renormalised coupling: response to change in bkg field

$$\bar g^2(\mu = L^{-1}) = k\left(\frac{\partial\Gamma}{\partial\eta}\right)^{-1}$$

running coupling

- compute at fixed value of the coupling ⇔ *L* for several lattice spacings, take continuum limit
- change coupling such that *L* changes in fixed steps of *s* and iterate



step-scaling function:

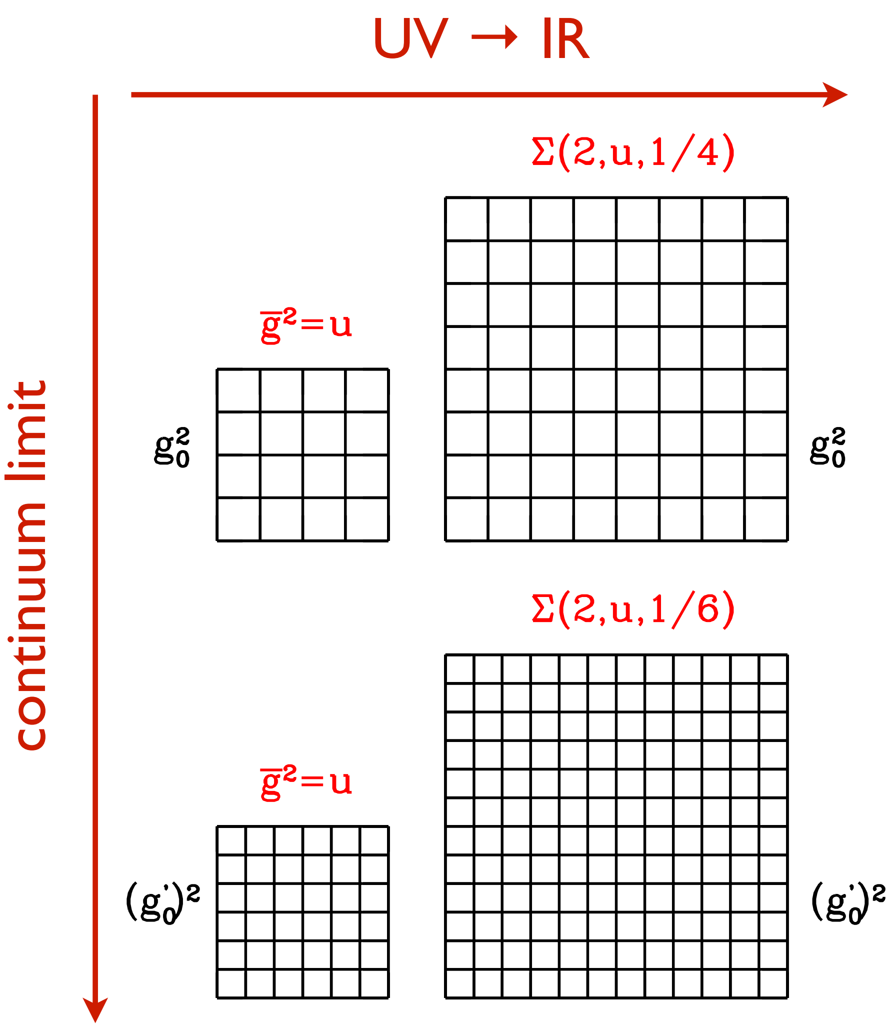
$$\sigma(s, \bar{g}^2(L^{-1})) = \bar{g}^2((sL)^{-1})$$

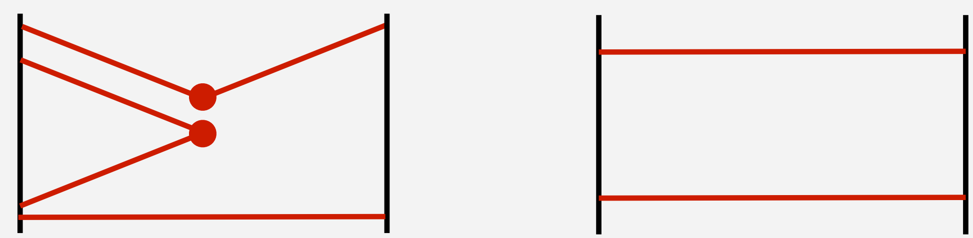
$$\Updownarrow$$

$$\log s = \int_{\bar{g}^2(L^{-1})}^{\sigma(s,\bar{g}^2(L^{-1}))} \mathrm{d}g^2 \, \frac{1}{\beta(g^2)}$$

RG evolution of composite operators

- compute at fixed value of the coupling ⇔ L for several lattice spacings, take continuum limit
- change coupling such that L changes in fixed steps of s and iterate





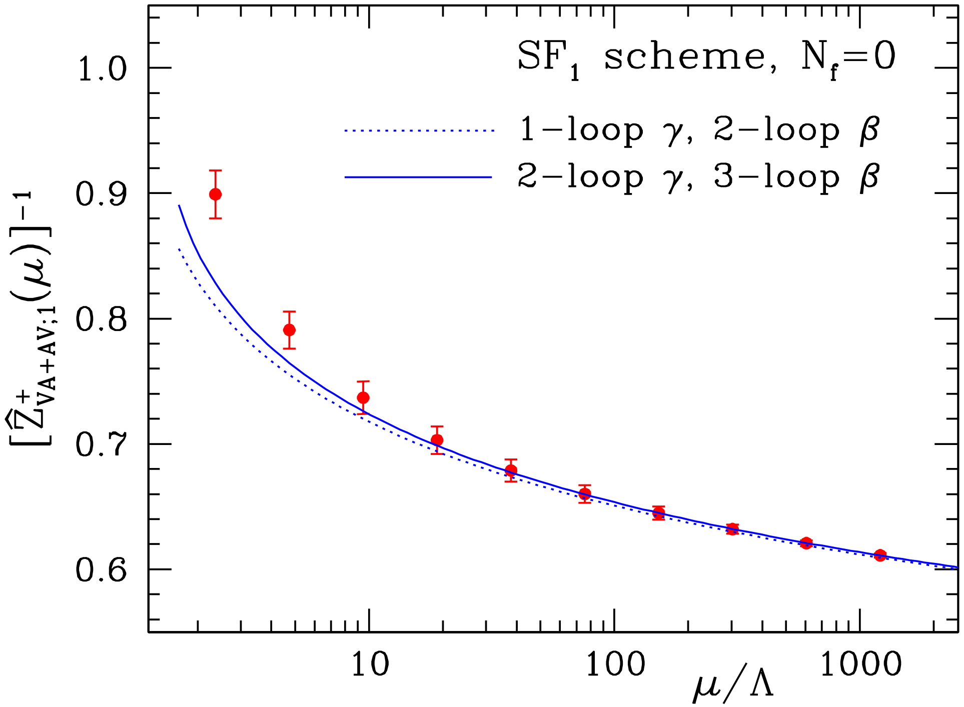
$$f_k^{\pm}(x_0) = \langle \mathcal{O}_{\rm b}\, \tilde{\mathcal{O}}_{\rm b}\, Q_k^{\pm}(x)\, \mathcal{O}'_{\rm b}\rangle \qquad f_1 = \langle \mathcal{O}_{\rm b} \mathcal{O}'_{\rm b}\rangle$$

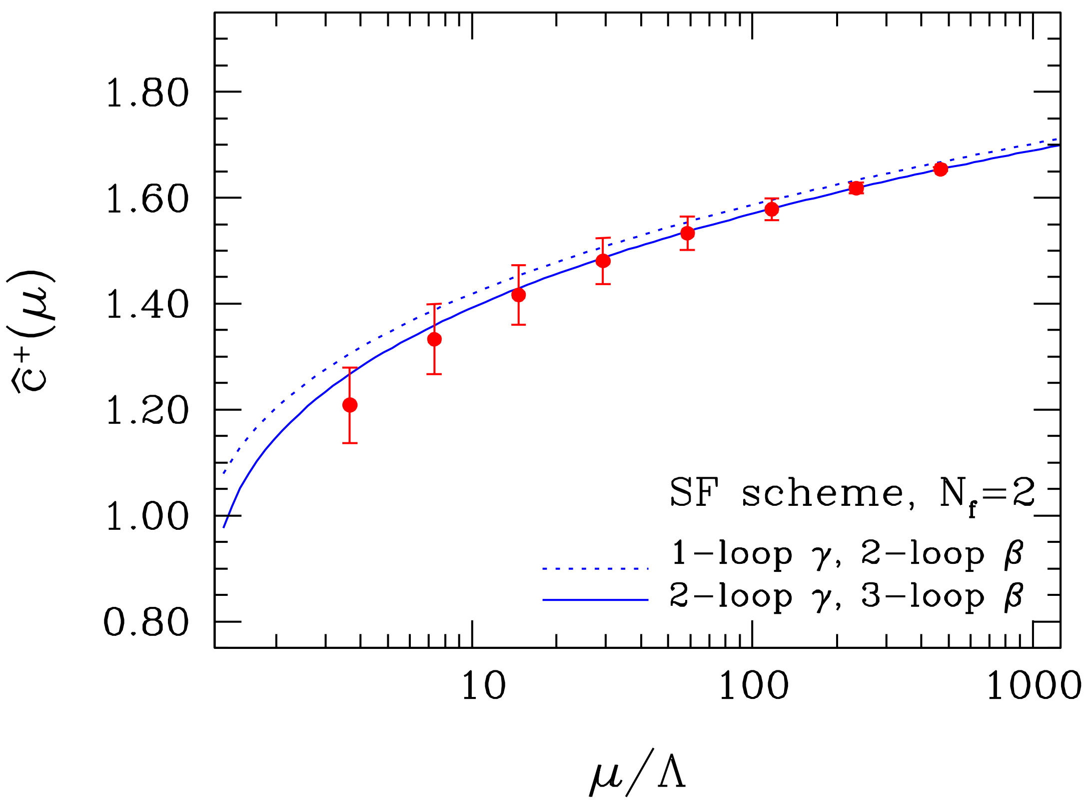
$$Z_k^{\pm}(L^{-1})\, \frac{f_k^{\pm}(T/2)}{f_1^{3/2}} = \left.\frac{f_k^{\pm}(T/2)}{f_1^{3/2}}\right|_{\text{tree level}}$$

$$\sigma_k^{\pm}(s, \bar{g}^2(L^{-1})) = \lim_{a\to 0} \frac{Z_k^{\pm}(g_0^2, a/(sL))}{Z_k^{\pm}(g_0^2, a/L)}$$

$$= \exp\left\{ \int_{\bar{g}^2(L^{-1})}^{\bar{g}^2((sL)^{-1})} \mathrm{d}g^2\, \frac{\gamma_k^{\pm}(g^2)}{\beta(g^2)} \right\}$$

RG evolution of composite operators





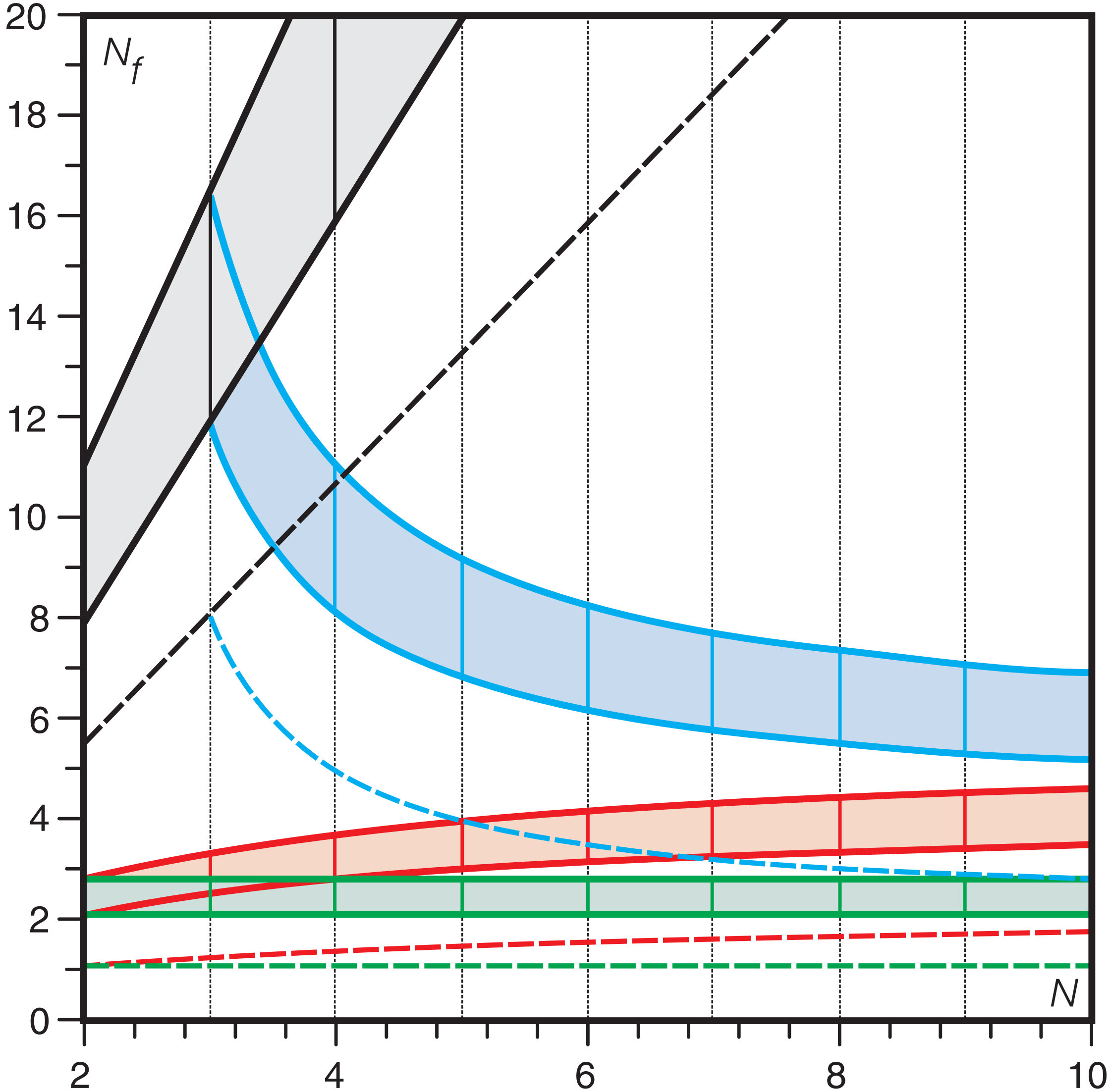
ALPHA Collaboration

[ALPHA 05-07]

proof of concept in minimal walking technicolour

perturbative parameter space for walking theories:

[sketch from Dietrich, Sannino 06]



- MWTC: 2 Dirac fermions in adjoint of gauge group
- extensively studied for SU(2) gauge group
- extensive (conclusive?) evidence of conformal behaviour
- mass anomalous dimension likely on the small side for successful phenomenology

[Bursa et al., Del Debbio et al. 09-10]
[Hietanen, Rummukainen, Tuominen 09]
[Catterall, Del Debbio, Giedt, Keegan 10-11]
[DeGrand, Shamir, Svetitsky 11]
[Giedt, Weinberg 11-12]
[Patella 12]
[Karavirta et al. 12]

preliminary results MWTC

- determine four-fermion operator anomalous dimensions via SF non-perturbative RG running
- Wilson fermion regularisation ⇒ explicit breaking of chiral symmetry, work in parity-odd sector
- define several renormalisation schemes — scheme independence of anomalous dimensions at fixed point provides strong constraint

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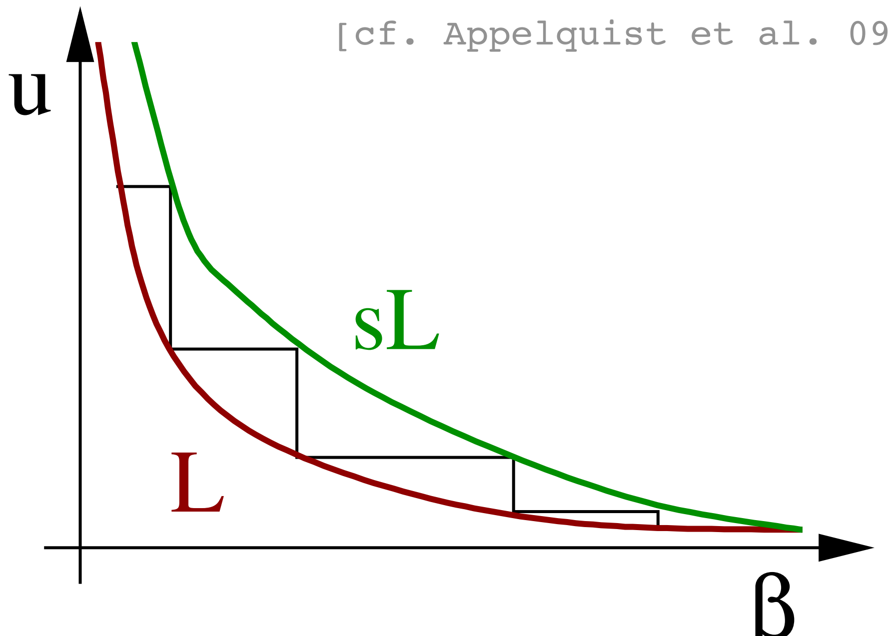
β	$L=8$	$L=10$	$L=12$	$L=16$
2.05	$20k$ (88%)	$20k$ (87%)	$20k$ (85%)	$20k$ (86%)
2.20	$20k$ (88%)	$20k$ (86%)	$20k$ (86%)	$20k$ (85%)
2.50	$20k$ (90%)	$20k$ (88%)	$20k$ (89%)	$20k$ (83%)
3.00	$20k$ (95%)	$20k$ (89%)	$20k$ (88%)	$20k$ (86%)
3.50	$20k$ (95%)	$20k$ (89%)	$20k$ (86%)	$20k$ (87%)
4.50	$20k$ (96%)	$20k$ (91%)	$20k$ (88%)	$20k$ (85%)
8.00	$20k$ (96%)	$20k$ (92%)	$20k$ (90%)	$20k$ (87%)
16.00	$20k$ (96%)	$20k$ (90%)	$20k$ (87%)	$20k$ (83%)

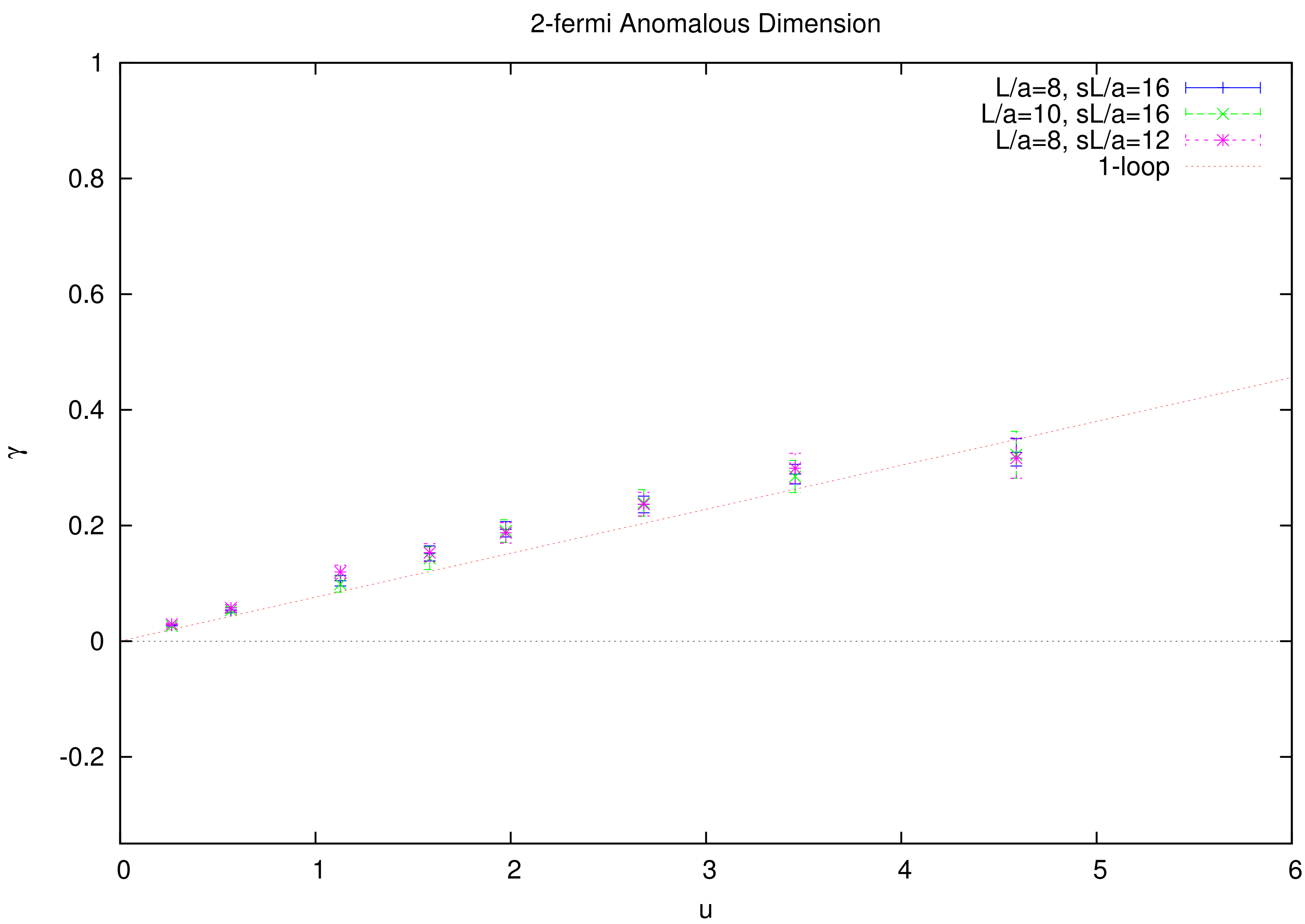
$$\gamma_X(\bar{g}^2(L^{-1})) \approx \frac{\log[\sigma_X(s,\bar{g}^2(L^{-1}))]}{\log s}$$

alleviate numerical cost by using mild scale dependence to perform interpolations

[cf. Appelquist et al. 09]

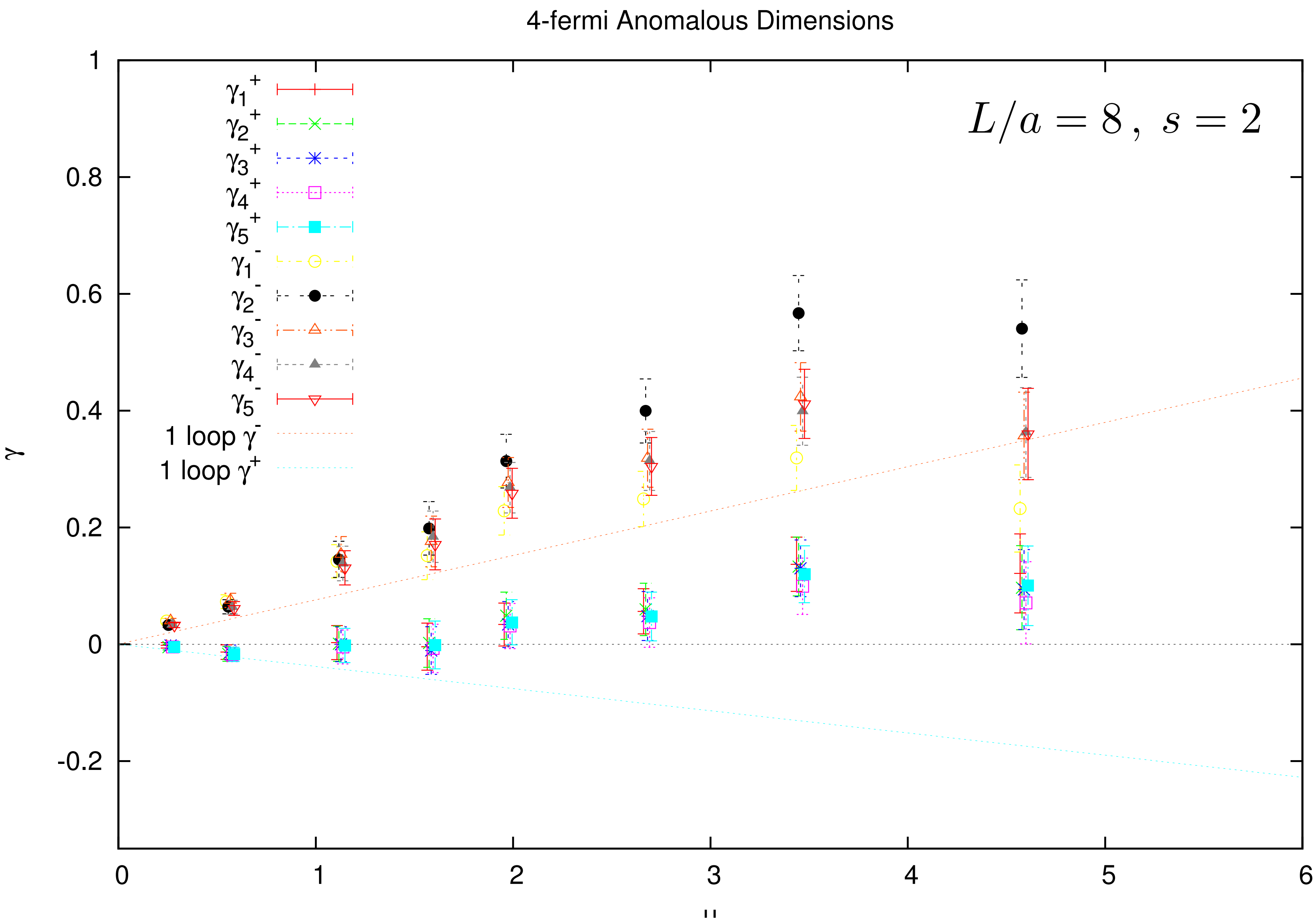


mass anomalous dimension redux



preliminary results MWTC

left current-left current operators, five different renormalisation schemes



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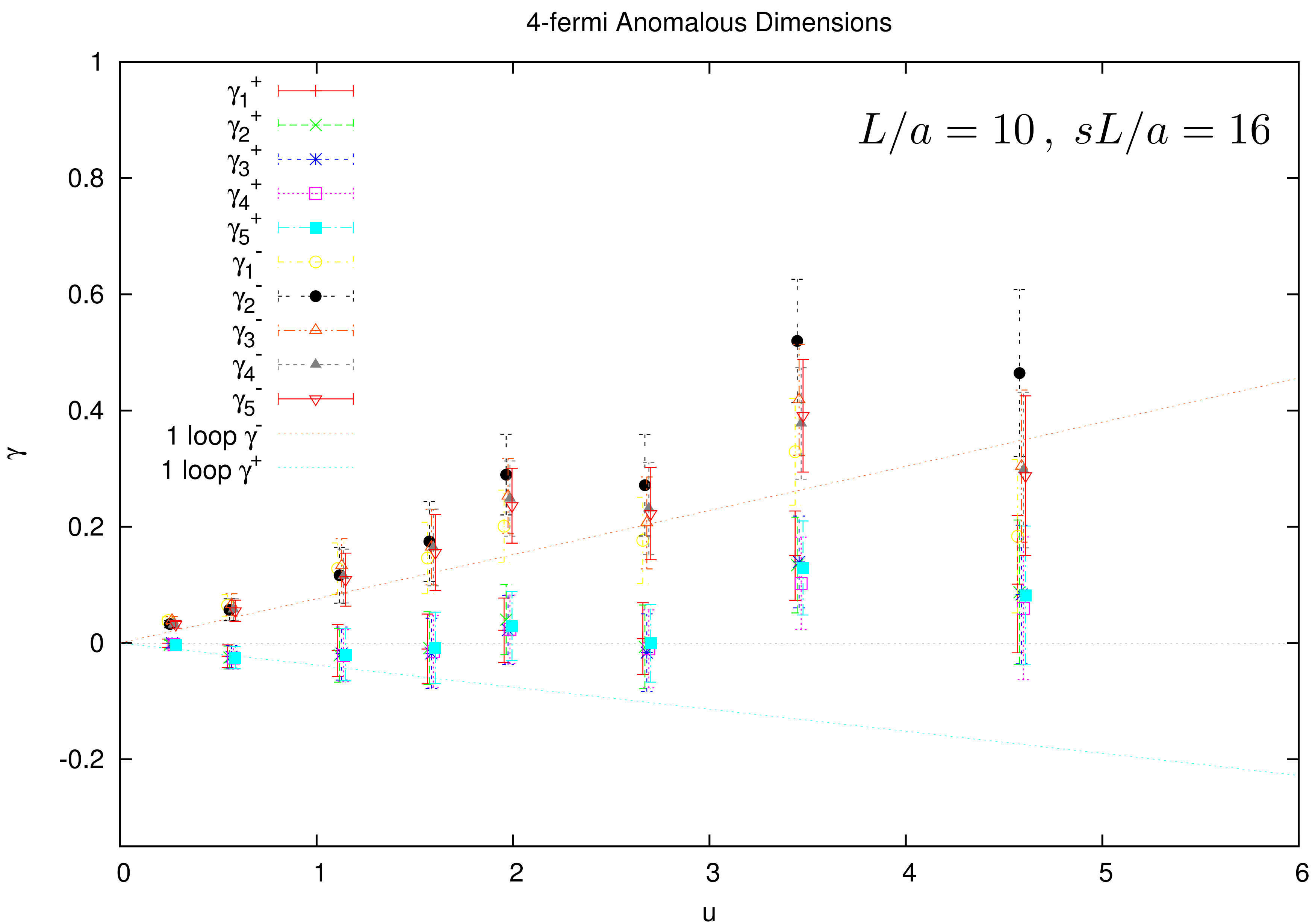

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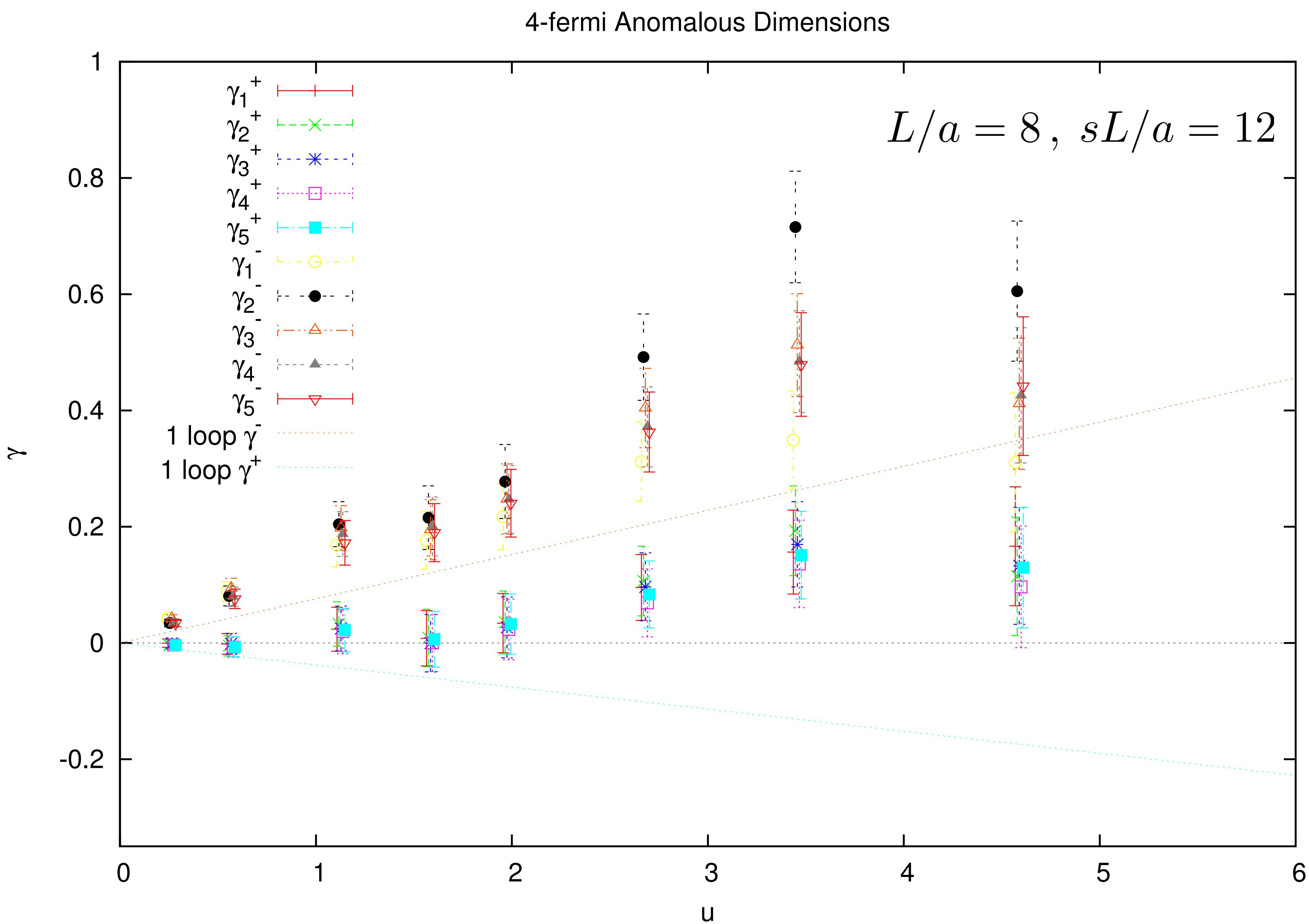


conclusions and outlook

- determine anomalous dimensions of four-fermion operators non-perturbatively in candidate conformal strong EWSB models for model building

- feasibility tested in MWTC — so far results for multiplicatively renormalisable left current-left current operator, remarkable scheme independence

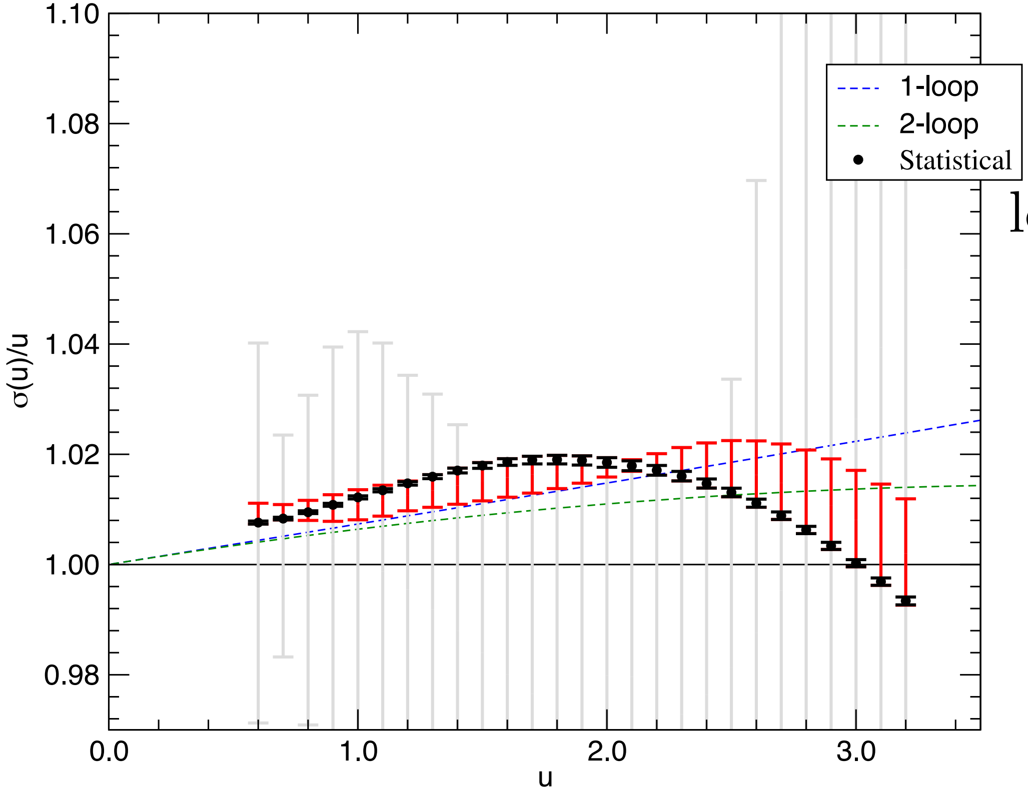
- anomalous dimensions for other operators being worked out, easy to extend to other models of interest (with vector couplings)

- many improvements possible for better precision:
  - more statistics, finer lattices
  - O(a) improvement for milder cutoff effects (chirally rotated SF?)
    [various parallel talks...]
  - gradient flow may simplify life enormously....
    [talk by M Lüscher]

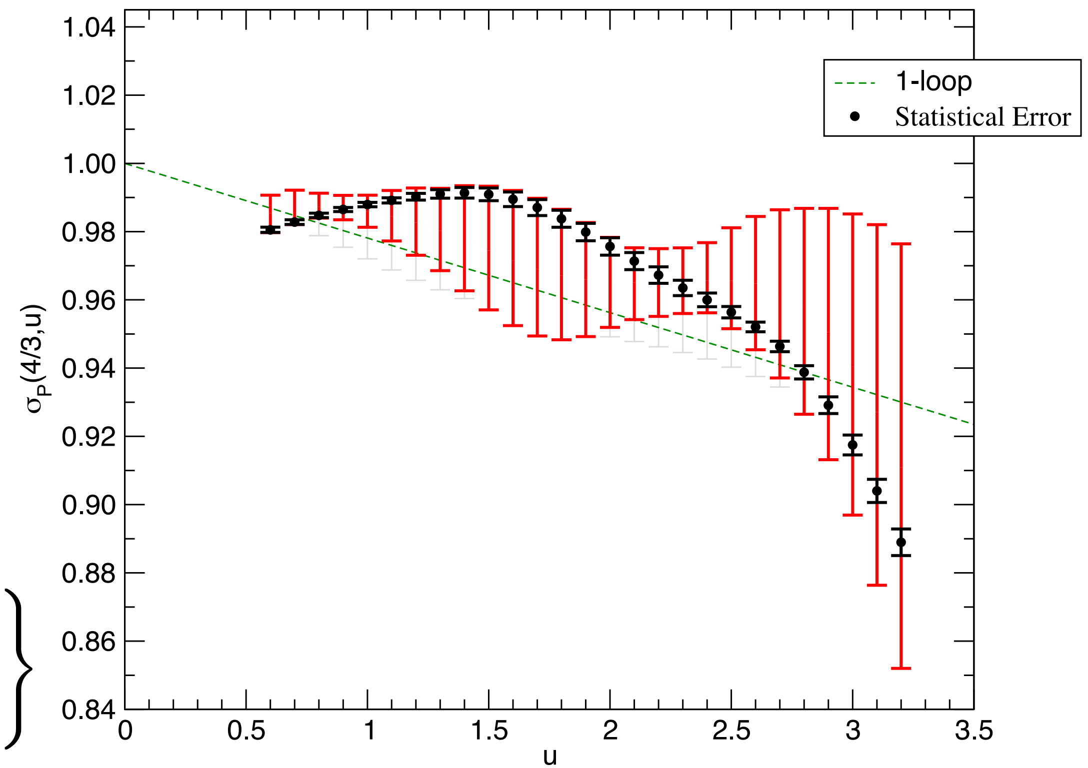
backup

minimal walking technicolour

[Bursa, Del Debbio, Keegan, Pica, Pickup 10]

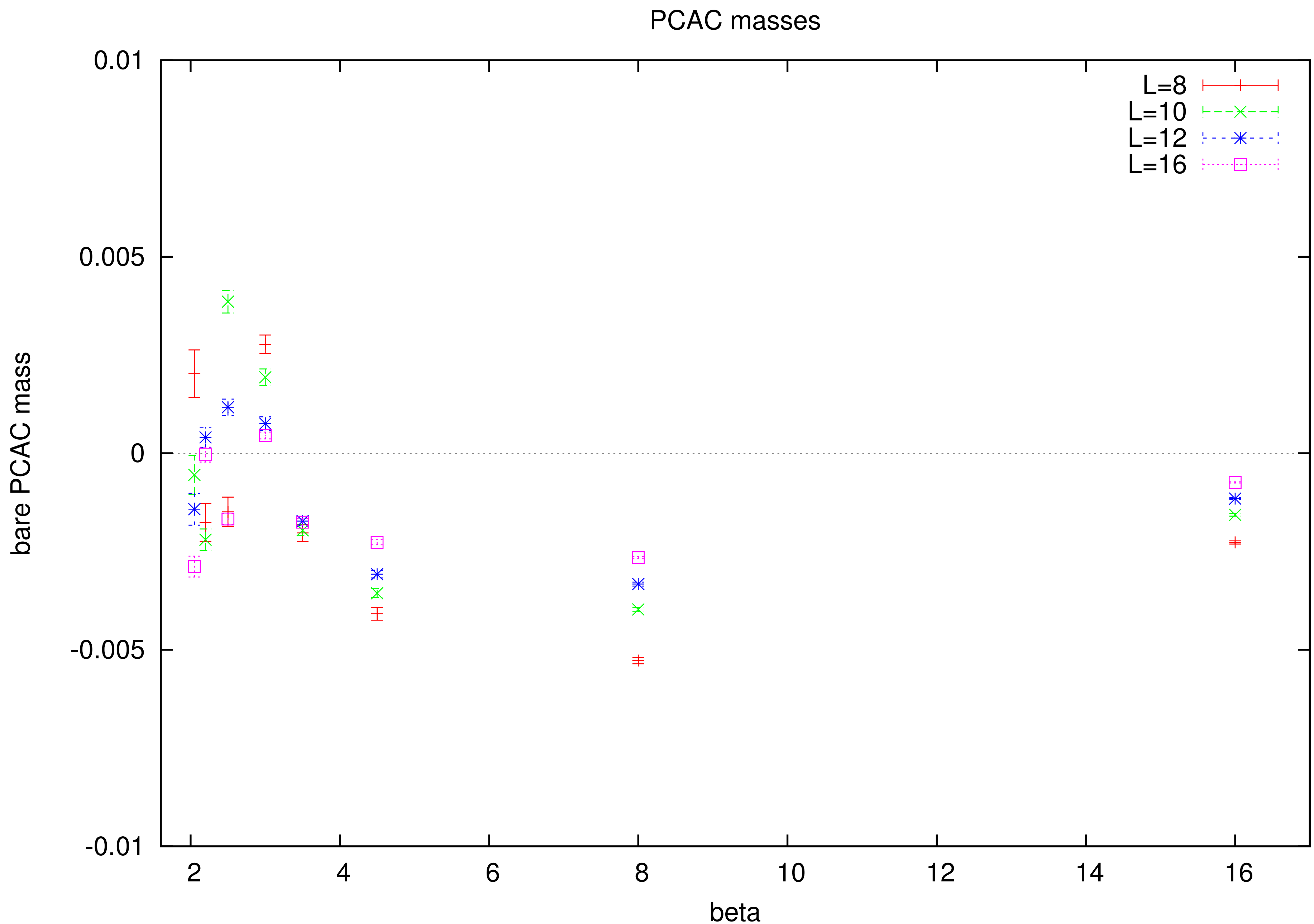


$$\log s = \int_{\bar{g}^2(L^{-1})}^{\sigma(s,\bar{g}^2(L^{-1}))} \mathrm{d}g^2 \, \frac{1}{\beta(g^2)}$$



$$\sigma_\Gamma(s, \bar{g}^2(L^{-1})) = \exp\left\{ \int_{\bar{g}^2(L^{-1})}^{\bar{g}^2((sL)^{-1})} \mathrm{d}g^2 \, \frac{\gamma(g^2)}{\beta(g^2)} \right\}$$

determination of chiral point for mass-independence



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