Search for possible bound T_{cc} and T_{cs} on the lattice

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for HAL QCD Collaboration

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(Hadrons to Atomic nuclei from Lattice QCD)

31th INTERNATIONAL SYMPOSIUM ON LATTICE FIELD THEORY
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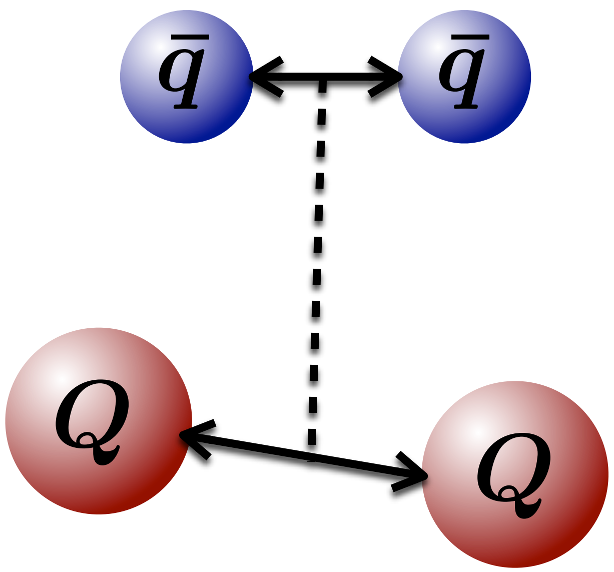
Bound tetraquarks $T_{QQ'}$?

Tetraquarks ($T_{QQ'} = QQ'\bar{q}\bar{q}$): $Q^{(\prime)}$ are strange, charm and bottom quarks

Possible candidates of exotic hadrons
--> Tetraquarks have not been experimentally discovered yet

Why can we expect possible bound T_{QQ}'s?

H. J. Lipkin, PLB172, 242 (1986).



Phenomenological quark models suggest bound states in
$T_{cc}(1^+)$, $T_{cs}(0^+, 1^+)$, $T_{bc}(0^+, 1^+)$, ...
because of strongly attractive color magnetic interactions

$$V_{\mathrm{CMI}} = -C \cdot \alpha_s \sum_{i<j} \frac{(\vec{\lambda}(i)\cdot\vec{\lambda}(j))(\vec{\sigma}(i)\cdot\vec{\sigma}(j))}{M_i M_j}\delta^3(\vec{r}_i - \vec{r}_j)$$

$$\langle v_{ij}\rangle = -\langle(\vec{\lambda}(i)\cdot\vec{\lambda}(j))(\vec{\sigma}(i)\cdot\vec{\sigma}(j))\rangle$$

$\langle v_{ij}\rangle$	C=1	C=8	C=$\bar{3}$	C=6
S=0	-16	2	-8	4
S=1	16/3	-2/3	8/3	-4/3

CMI in diquarks

- C=$\bar{3}$,S=0 (I=0) : -8
- C=6, S=1 (I=0) : -4/3
- C=$\bar{3}$, S=1 (I=1) : 8/3
- C=6, S=0 (I=1) : 4

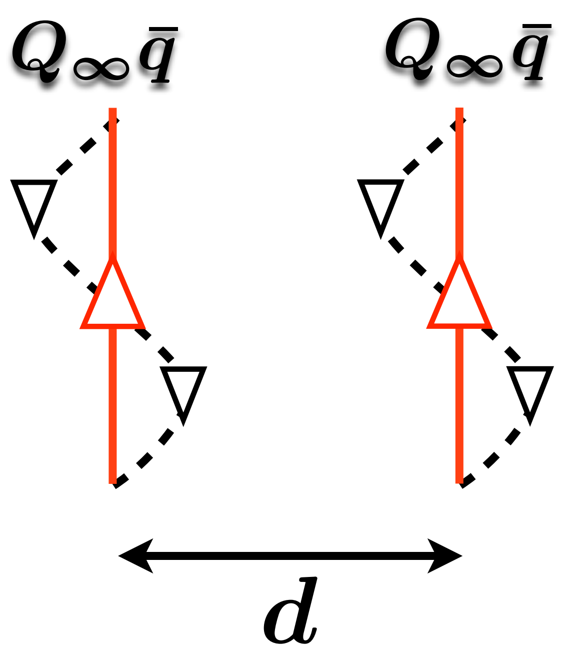
attractive ↕ repulsive

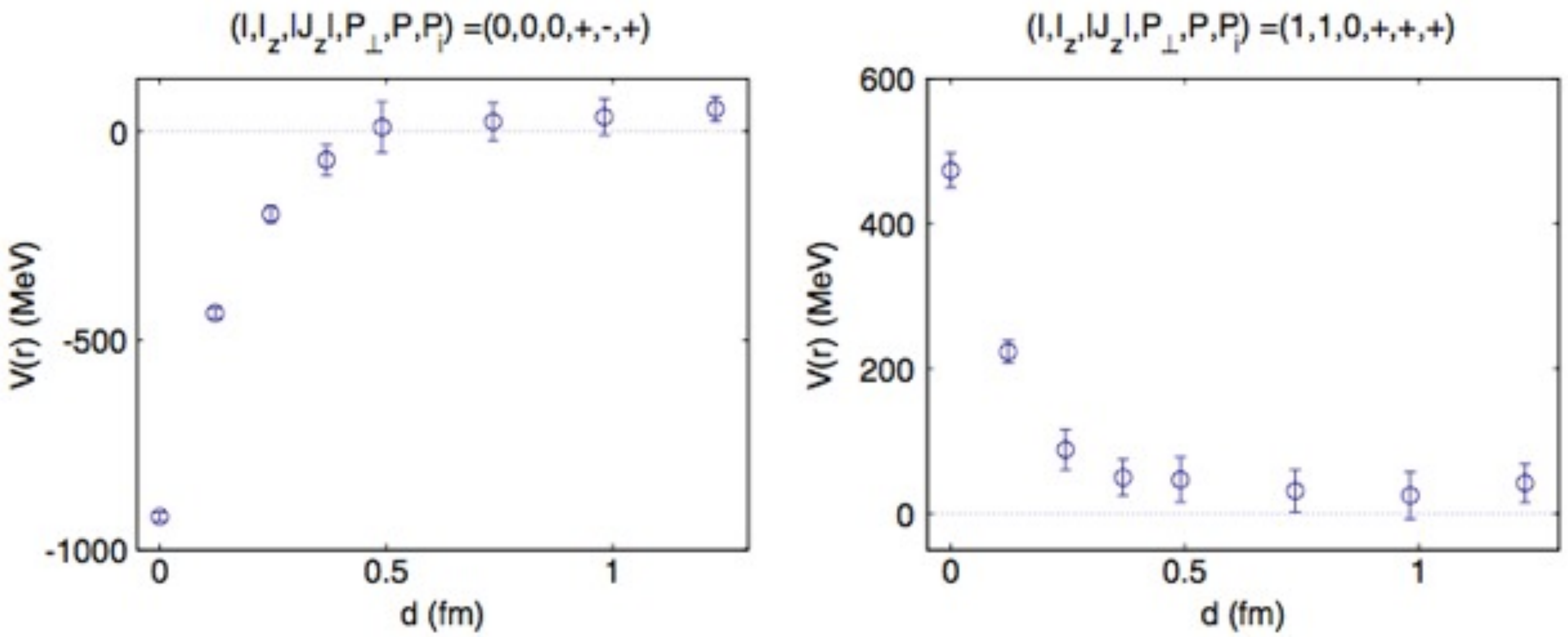
Scalar ud-diquark in I=0 is strongly attractive

Lattice QCD studies of T_{QQ}

Interaction energies from Wilson line approach for [Qq^{bar}-Qq^{bar}]

Z. Brown, K Orginos, PRD86, 114506 (2012); P. Bicudo, M. Wagner, PRD87, 114511 (2013).





➡ **used for T_{bb} search**

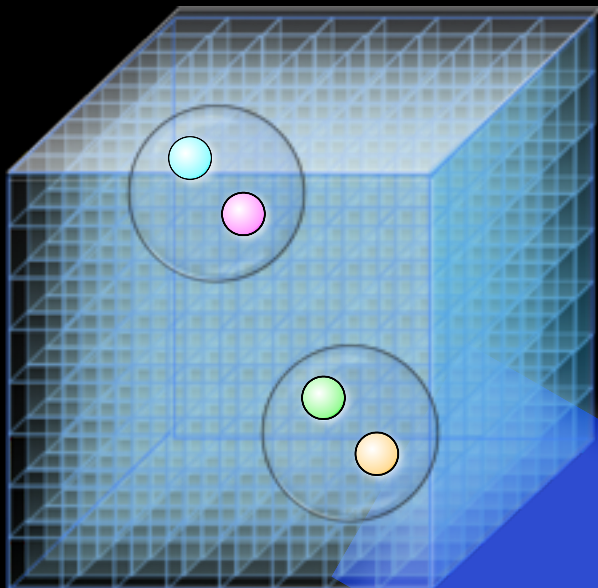
Our work: applying HAL QCD method to search for bound Tcc & Tcs

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007); Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

➡ **Tcc & Tcs channels are experimentally accessible --> Analysis by Belle Coll.**
➡ **Which channel is better to analyze?**
➡ **Dynamics of charm quarks should be appropriately taken into account, since charm quarks are relatively “light”**

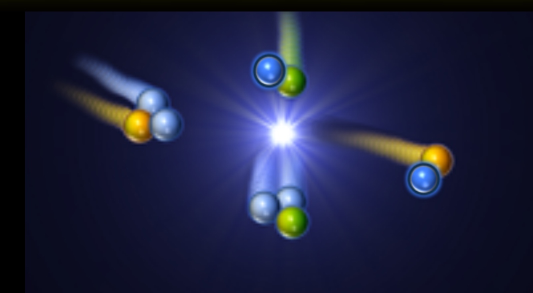
HAL QCD strategy

Hadrons to Atomic nuclei
HAL
from Lattice QCD



Nambu-Bethe-Saltpeter wave function

--> phase shift, T-matrix

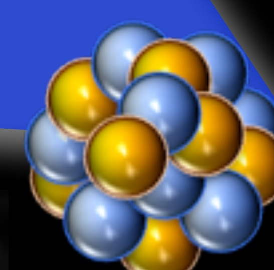


Potential defined on the lattice

Baryon-Baryon, Baryon-Baryon-Baryon,
Meson-Baryon, Meson-Meson, ...

Many applications:

nuclei, exotics,
astrophysics input...



T. Doi, T. Inoue (Mon.)
S. Aoki, B. Charron, H. Nemura (Tue.)
N. Ishii, K. Murano, K. Sasaki, M. Yamada (Fri.)

HAL QCD method

Ishii et al.(HAL QCD Coll.), PLB712, 437 (2012).

1) Start with normalized correlation functions (R-correlators)

$$R(\vec{r},t) = e^{(m_1+m_2)t}\sum_{\vec{x}}\langle 0|\phi_1(\vec{x}+\vec{r},t)\phi_2(\vec{x},t)\overline{\mathcal{J}}_{\rm src}(t=0)|0\rangle$$

$$= \sum_{\vec{k}} A_{\vec{k}} \exp\left[-\Delta W(\vec{k})t\right]\psi_{\vec{k}}(\vec{r})$$

NBS wave function : phase shift

2) define energy-independent non-local potentials

$$(-\frac{\partial}{\partial t} - H_0 + \cdots)R(\vec{r},t) = \int d\vec{r}' U(\vec{r},\vec{r}')R(\vec{r}',t) \qquad H_0 = -\frac{\nabla_r^2}{2\mu}$$

Relativistic correction: $\delta W(\vec{k})_{\rm rel} = \Delta W(\vec{k}) - \vec{k}^2/2\mu$

3) leading order potential of velocity expansion:

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$

4) Calculate observable: phase shift, binding energy, mean-square radius, ...

Lattice QCD Setup : light quarks

N_f=2+1 full QCD configurations generated by PACS-CS Coll.

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & Wilson clover
- Gauge coupling : β=1.90
- Lattice spacing : a=0.0907(13) (fm) ($\Lambda_{lat.}$=2176(MeV))
- Box size : 32^3x64 -> L~2.9 (fm)
- Hopping parameters :
  - set1 : (κ_{ud}, κ_s)=(0.13700, 0.13640)
  - set2 : (κ_{ud}, κ_s)=(0.13727, 0.13640)
  - set3 : (κ_{ud}, κ_s)=(0.13754, 0.13640)
- Conf. # : [set1]:399, [set2]:400, [set 3]:450
- Wall source

Light meson mass [set1, set2, set3] (MeV)
M_π=699(1), 572(2), 411(2) [PDG:135 (π^0)]
M_K=787(1), 714(1), 635(2) [PDG:498 (K^0)]

Lattice QCD Setup : charm quarks

✤ Tsukuba-type Relativistic Heavy Quark (RHQ) action

Aoki et al., PTP109, 383 (2003)

Cutoff errors, $O((ma)^n)$ and $O(a\Lambda_{QCD})$, are removed by adjusting RHQ parameters, $\{m_0, \nu, r_s, C_E, C_B\}$.

$$S^{\mathrm{RHQ}} = \sum_{x,y} \bar{q} D_{x,y} q(y)$$

$$D_{x,y} = m_0 + \gamma_0 D_0 + \nu \gamma_i D_i - a r_t D_0^2 - a r_s D_i^2 - a C_E \sigma_{0i} F_{0i} - a C_B \sigma_{ij} F_{ij}$$

- We are allowed to choose r_t=1
- We are left with $O((a\Lambda_{QCD})^2)$ error (~ a few %)

We use RHQ parameters tuned by Namekawa et al.

Y. Namekawa et al., PRD84, 074505 (2011)

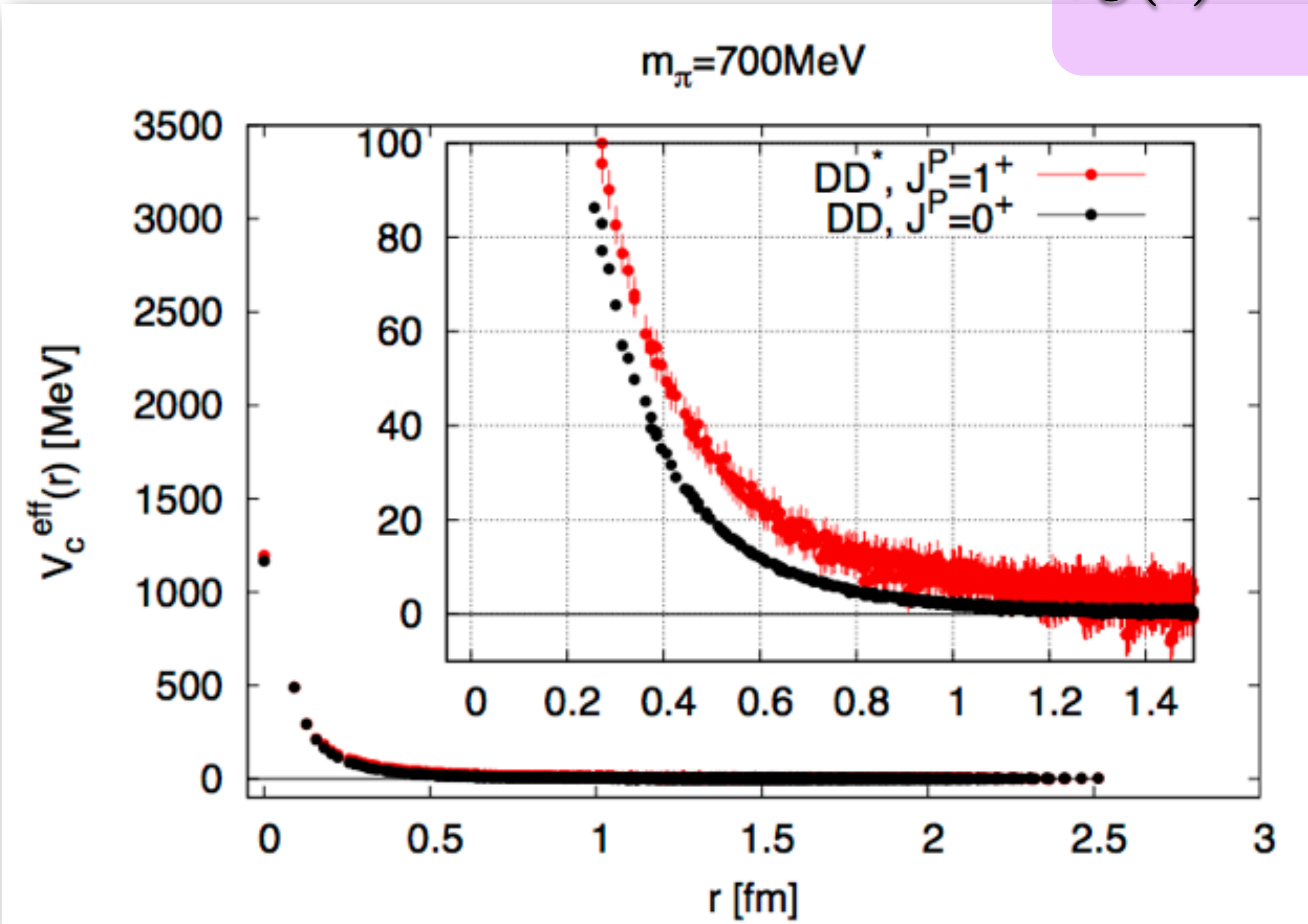
Charmed meson mass [set1, set2, set3] (MeV)

$M_{\eta c}$=3024(1), 3005(1), 2988(2) [PDG:2981]
$M_{J/\psi}$=3142(1), 3118(1), 3097(2) [PDG:3097]
M_D=1999(1), 1946(1), 1912(1) [PDG:1865 (D^0)]
M_{D^*}=2159(4), 2099(6), 2059(8) [PDG:2007 (D^{*0})]

Results : isospin 1 channels

S-wave $DD^{(*)}$ potentials: $T_{cc}(0^+, 1^+(1))$

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$



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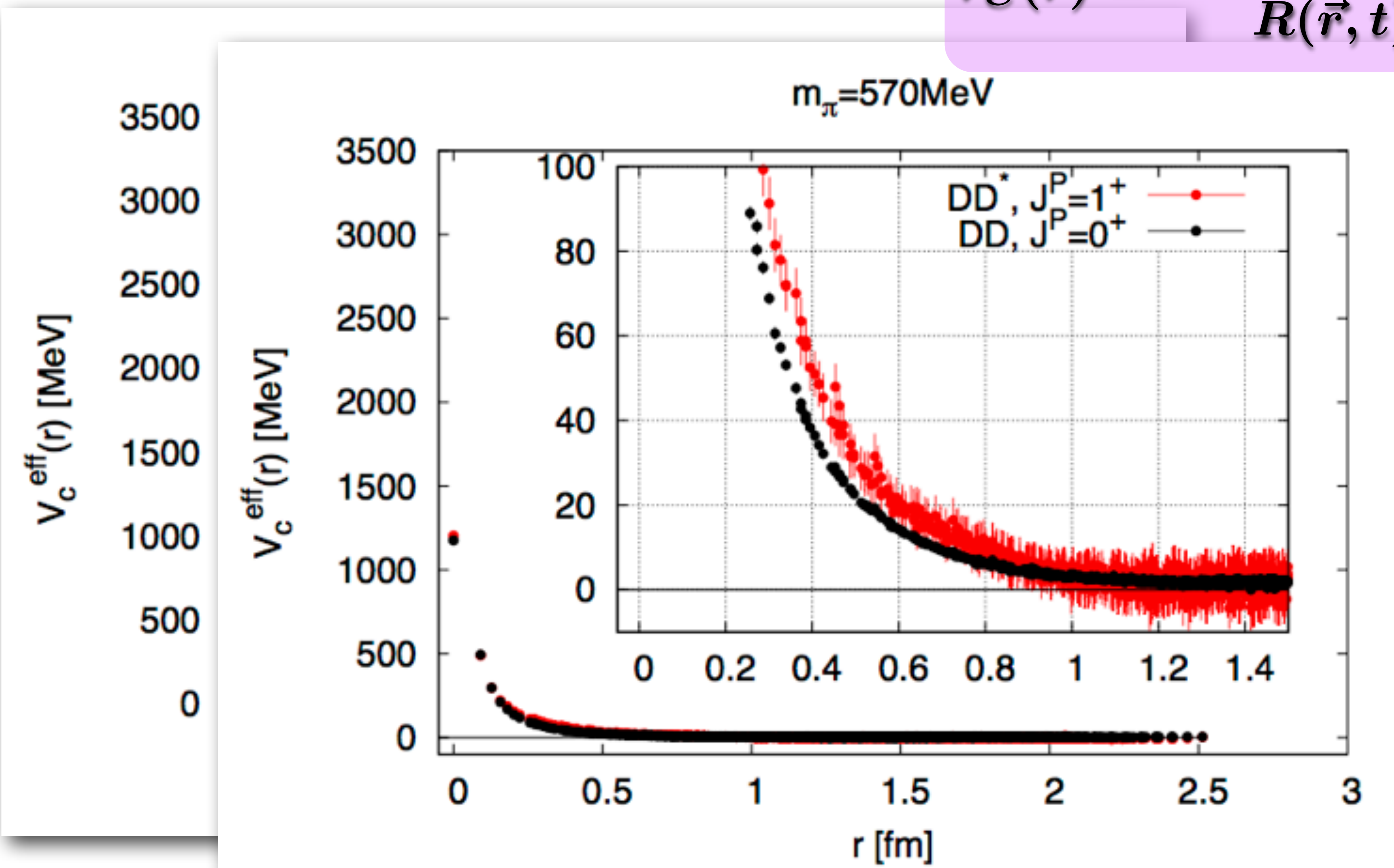

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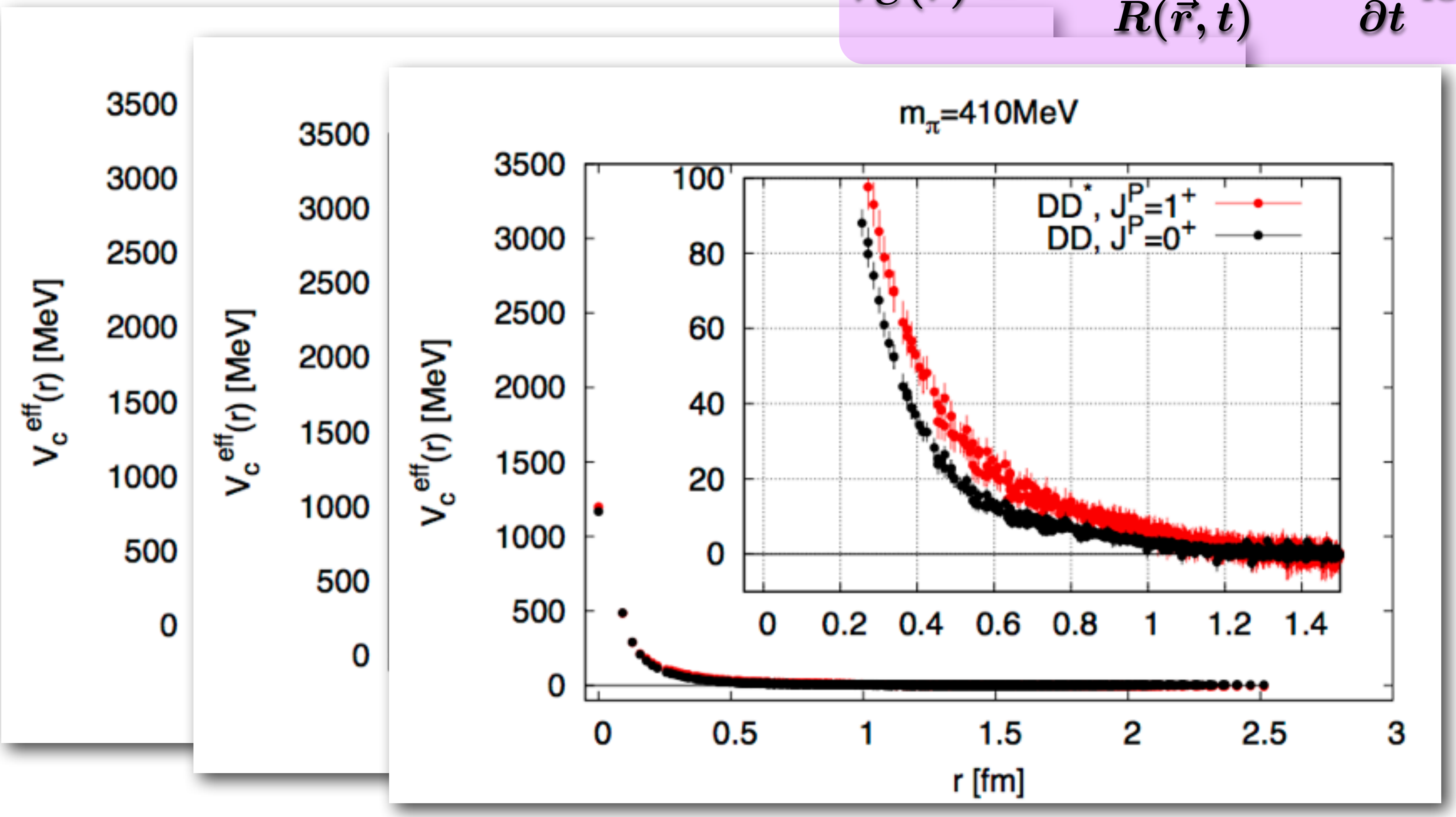
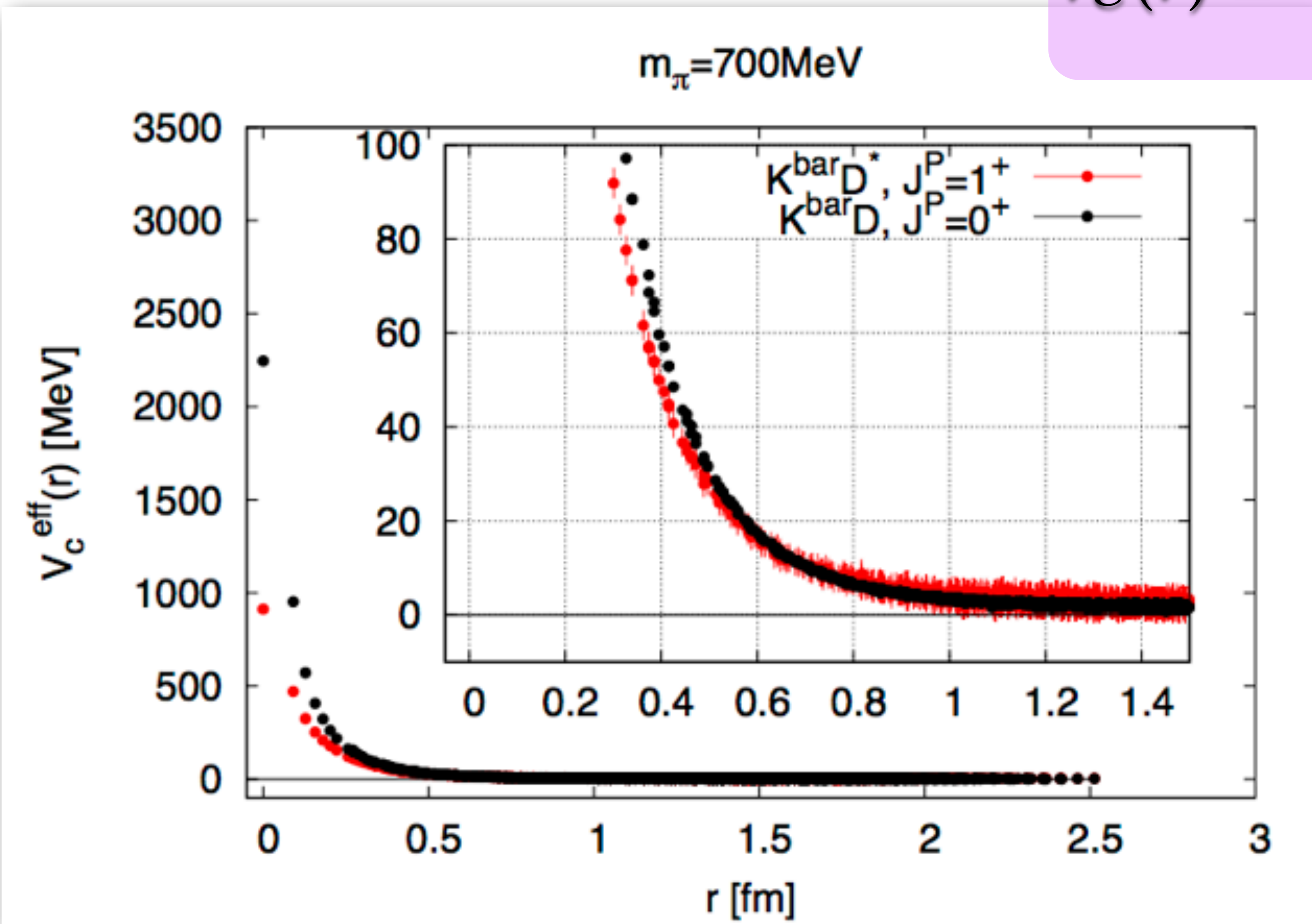


- Repulsive DD and DD* potentials
- Weak quark mass dependence
- It is unlikely to form bound state even at physical point

S-wave $D^{(*)}K^{bar}$ potential : $T_{cs}(0^+, 1^+(1))$

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$



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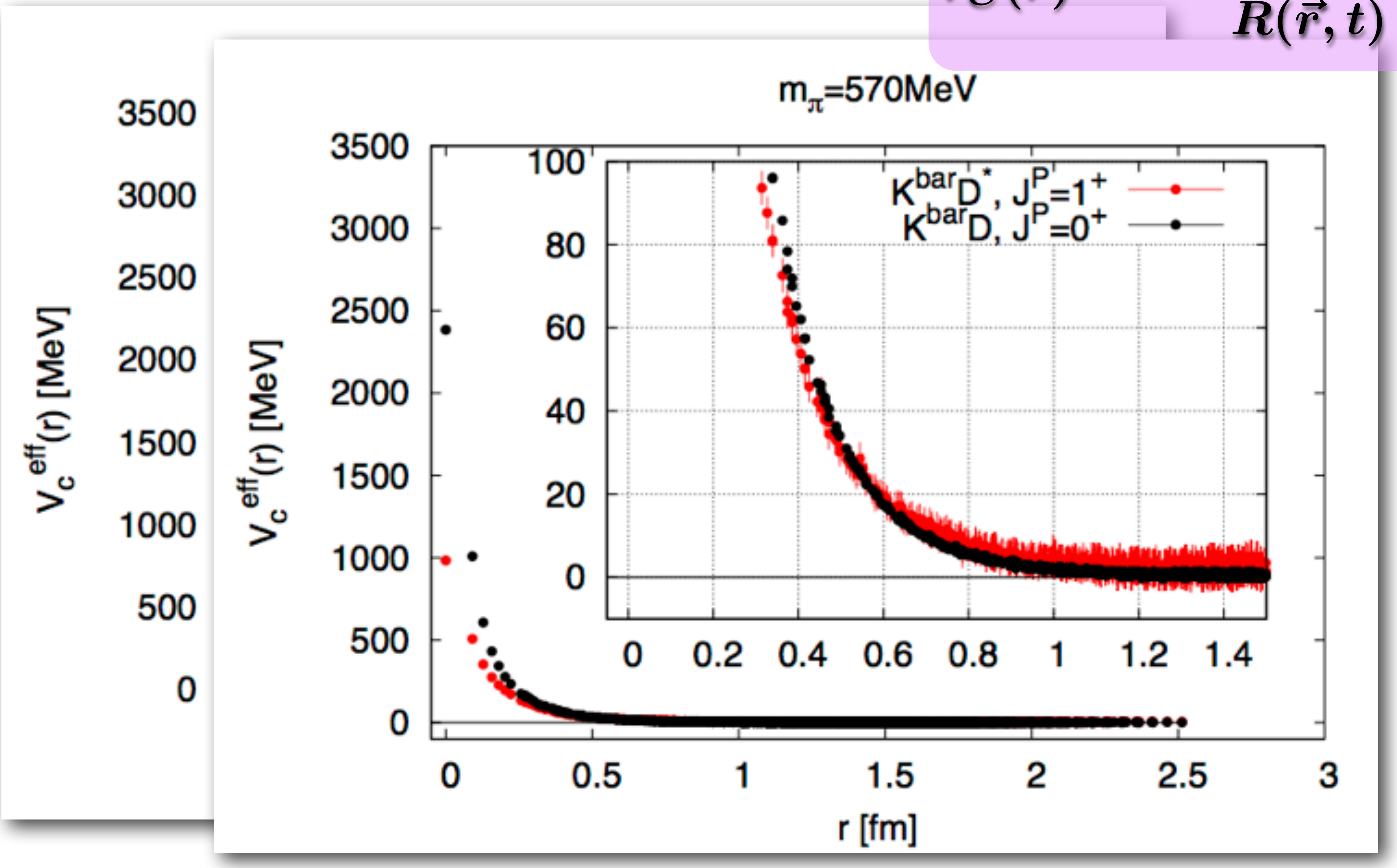

S-wave $D^{(*)}K^{bar}$ potential : $T_{cs}(0^+, 1^+(1))$

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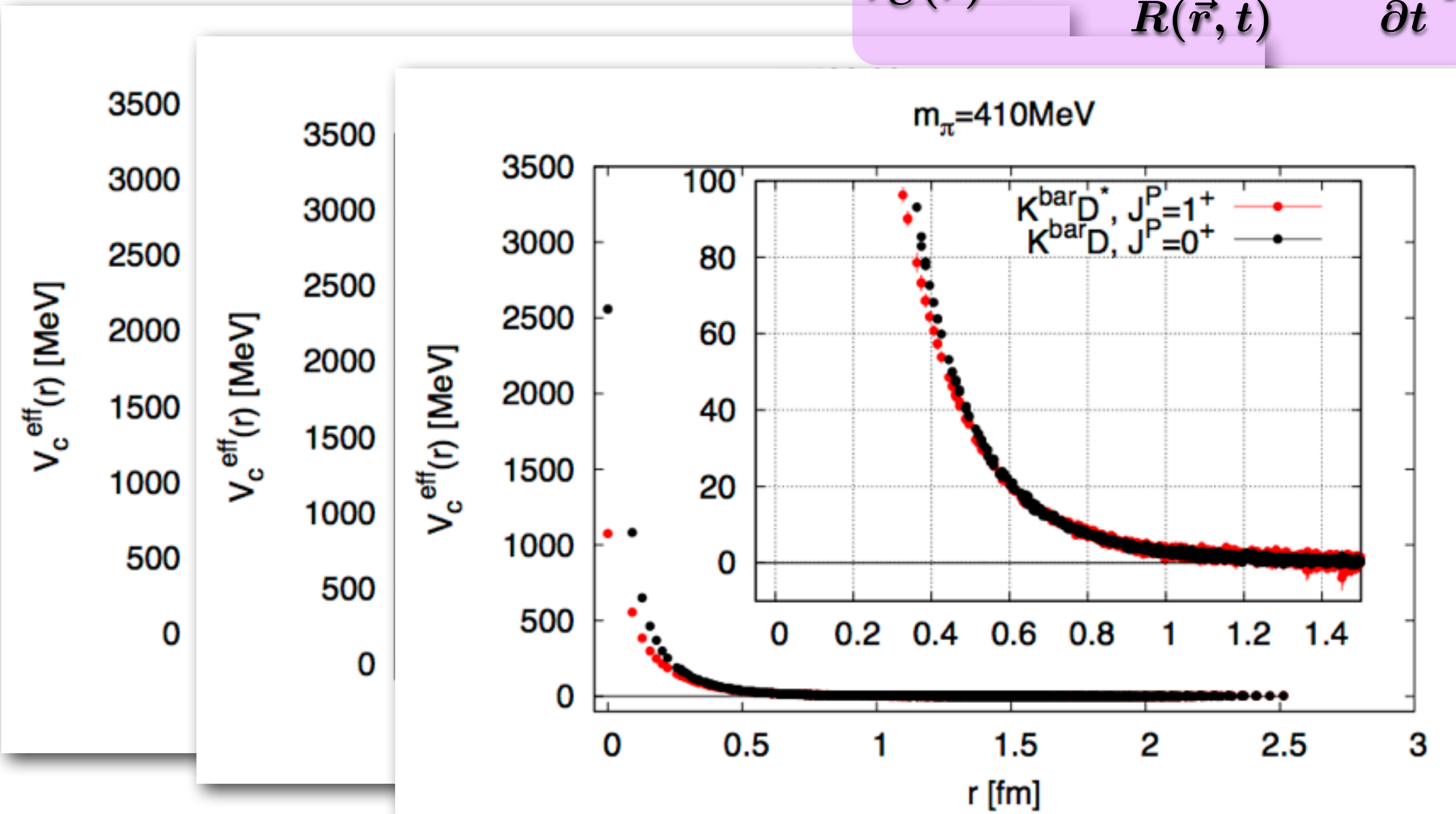
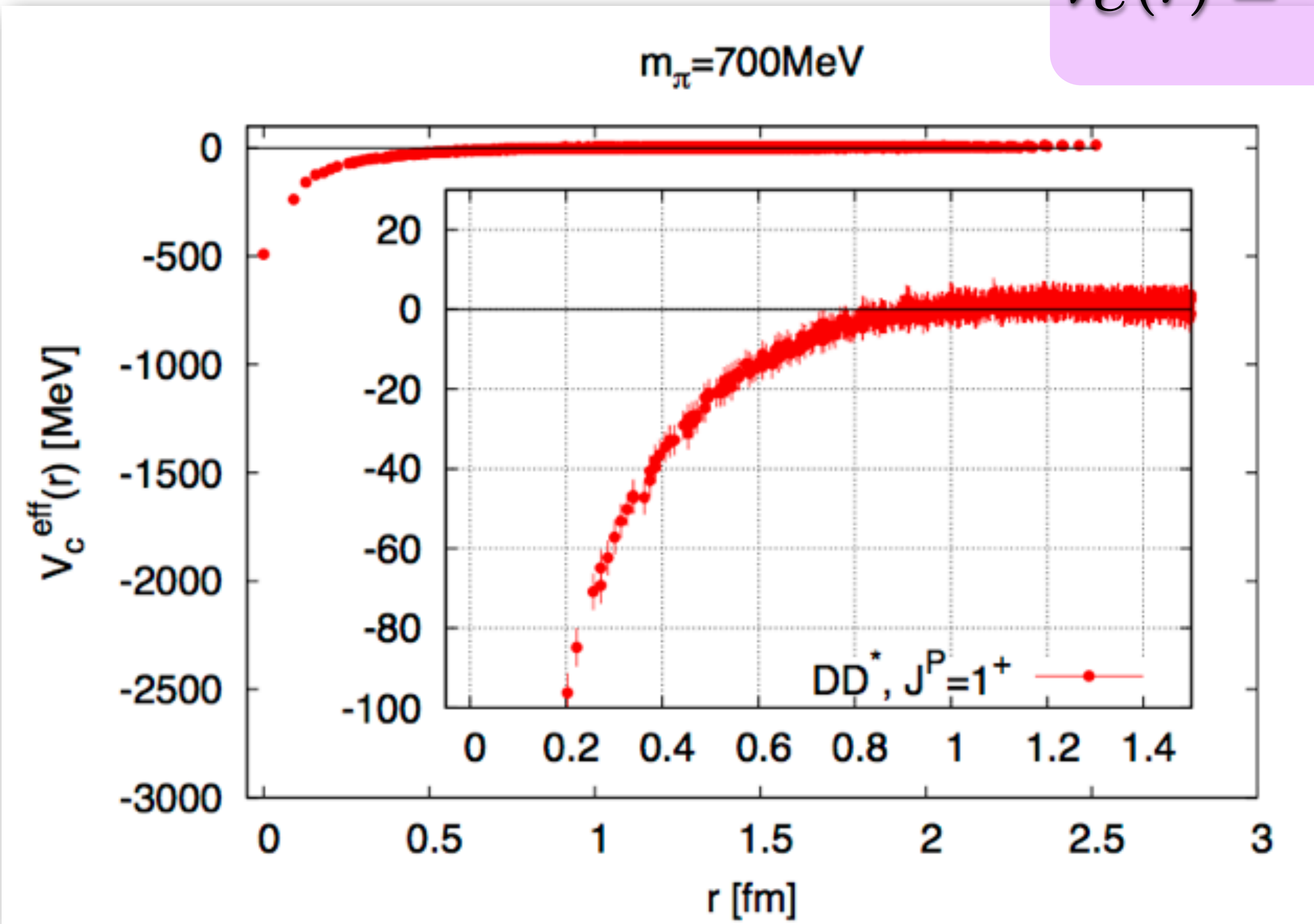


- Repulsive $K^{bar}D$ and $K^{bar}D^*$ potentials
- Weak quark mass dependence
- It is unlikely to form bound state even at physical point

Results : isospin 0 channel

S-wave DD* potential : $T_{cc}(1^+(0))$

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$



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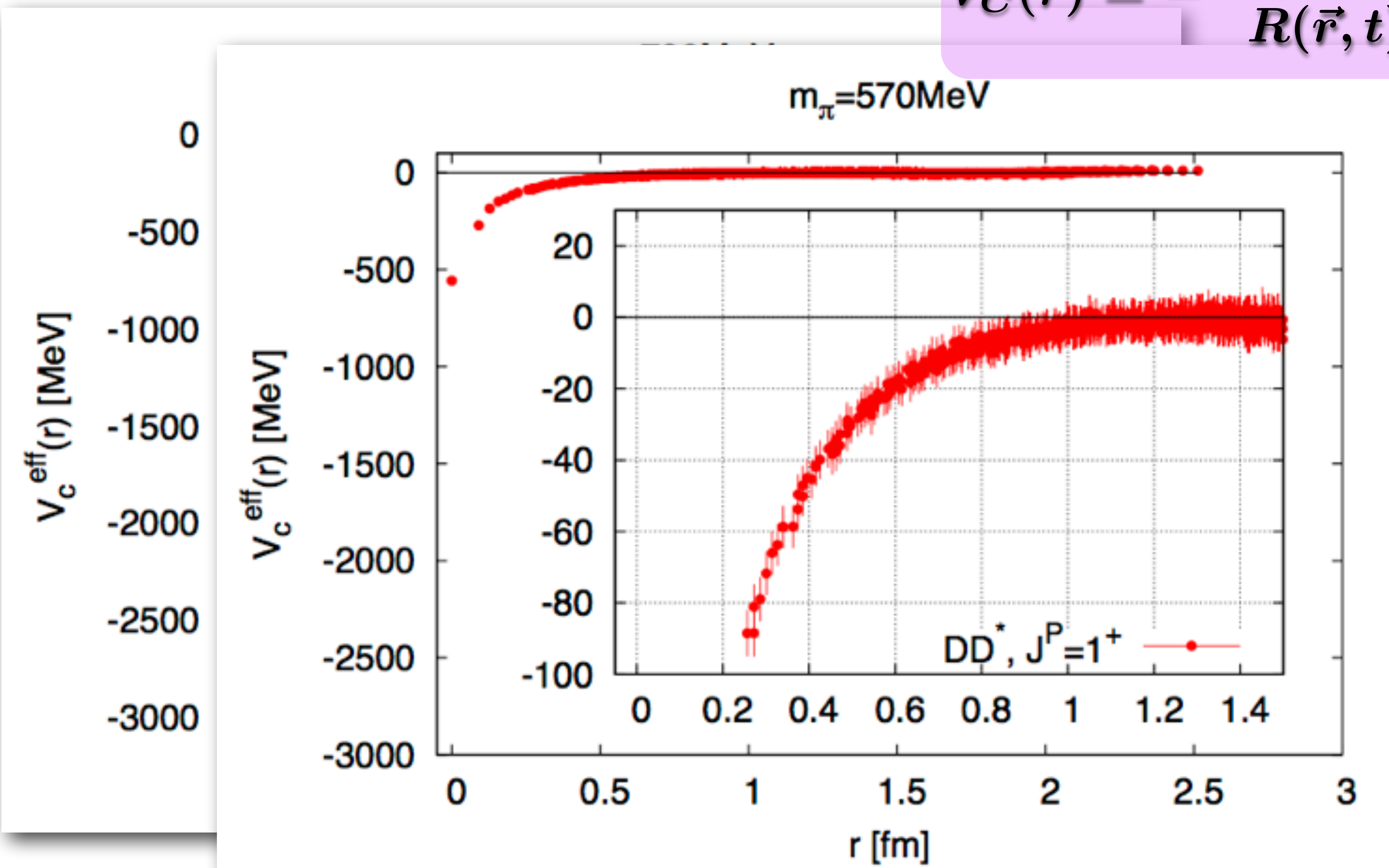

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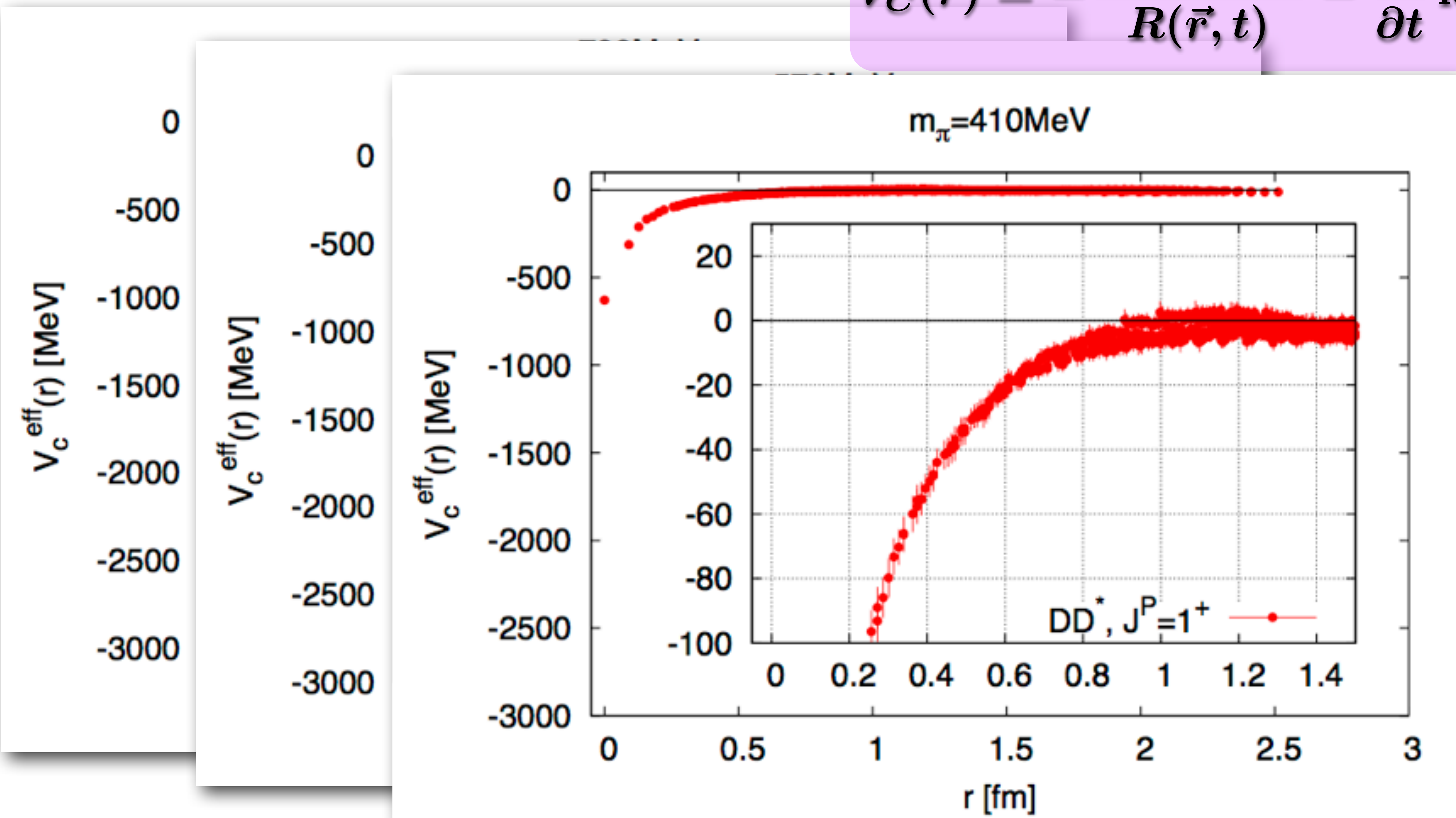
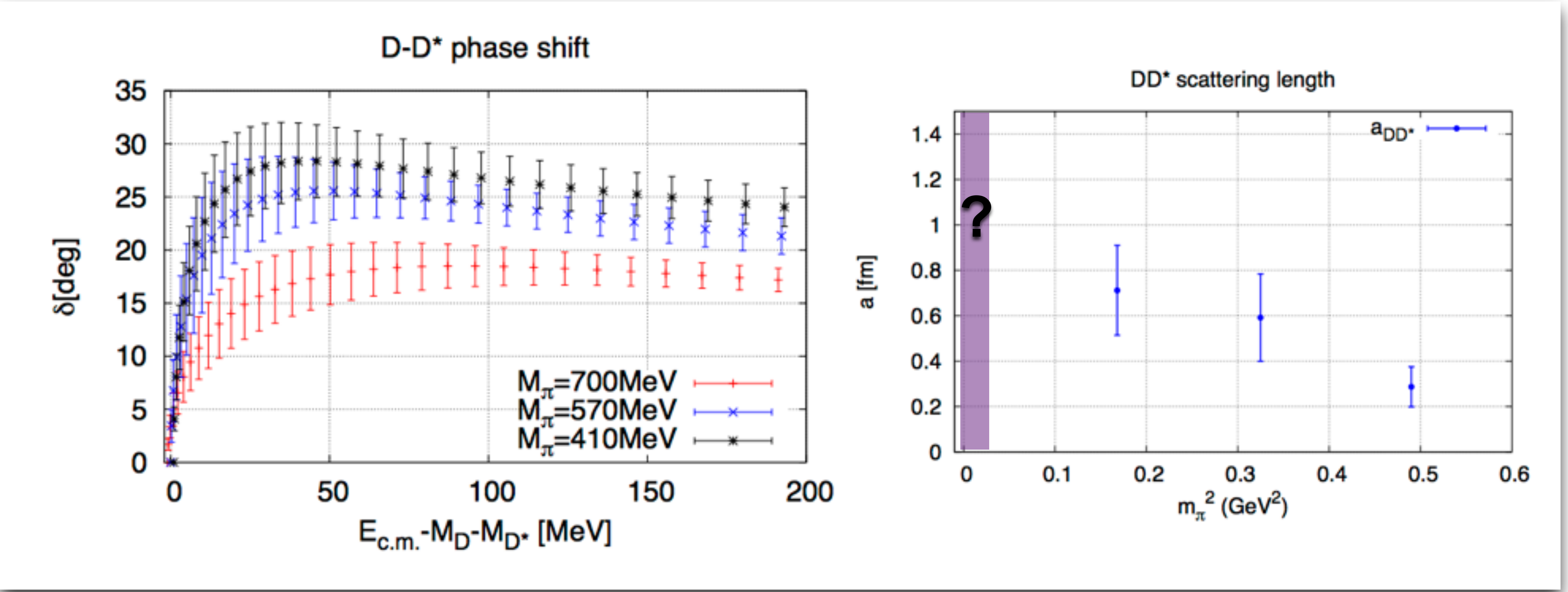


- Attractive DD* potential
- Check whether bound Tcc exist or not --> phase shift analysis

S-wave phase shift : $T_{cc}(1^+(0))$

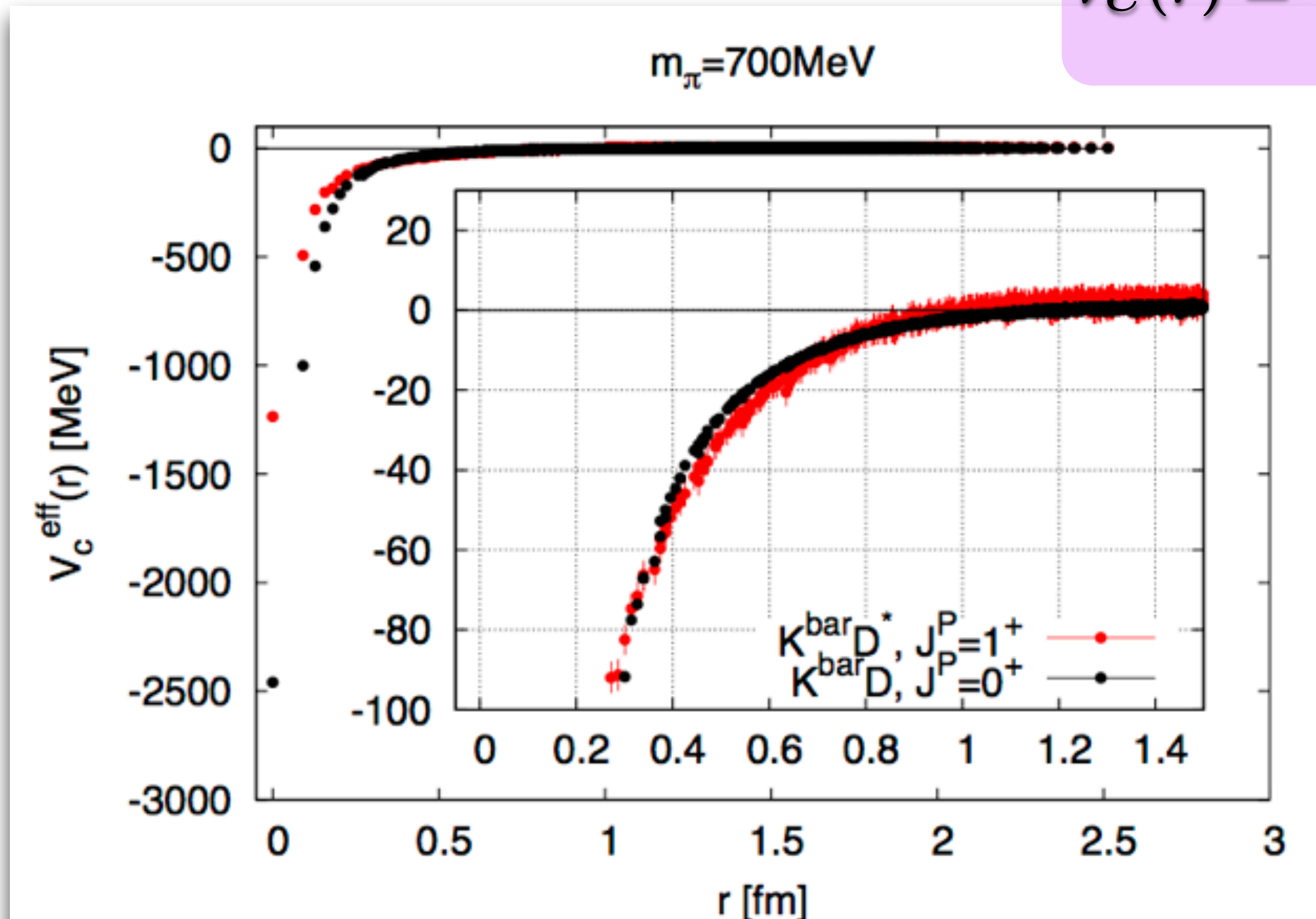
- fit multi-range gaussian: $f(\mathrm{r}) = \sum_i a_i e^{-\nu_i r^2}$
- solve Schrodinger equation in an infinite volume



- Attraction is not enough strong to generate bound state
- Attraction gets stronger as decreasing quark mass
- For definite conclusion, physical point simulations are necessary

S-wave $D^{(*)}K^{bar}$ potential : $T_{cs}(0^+, 1^+(0))$

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$



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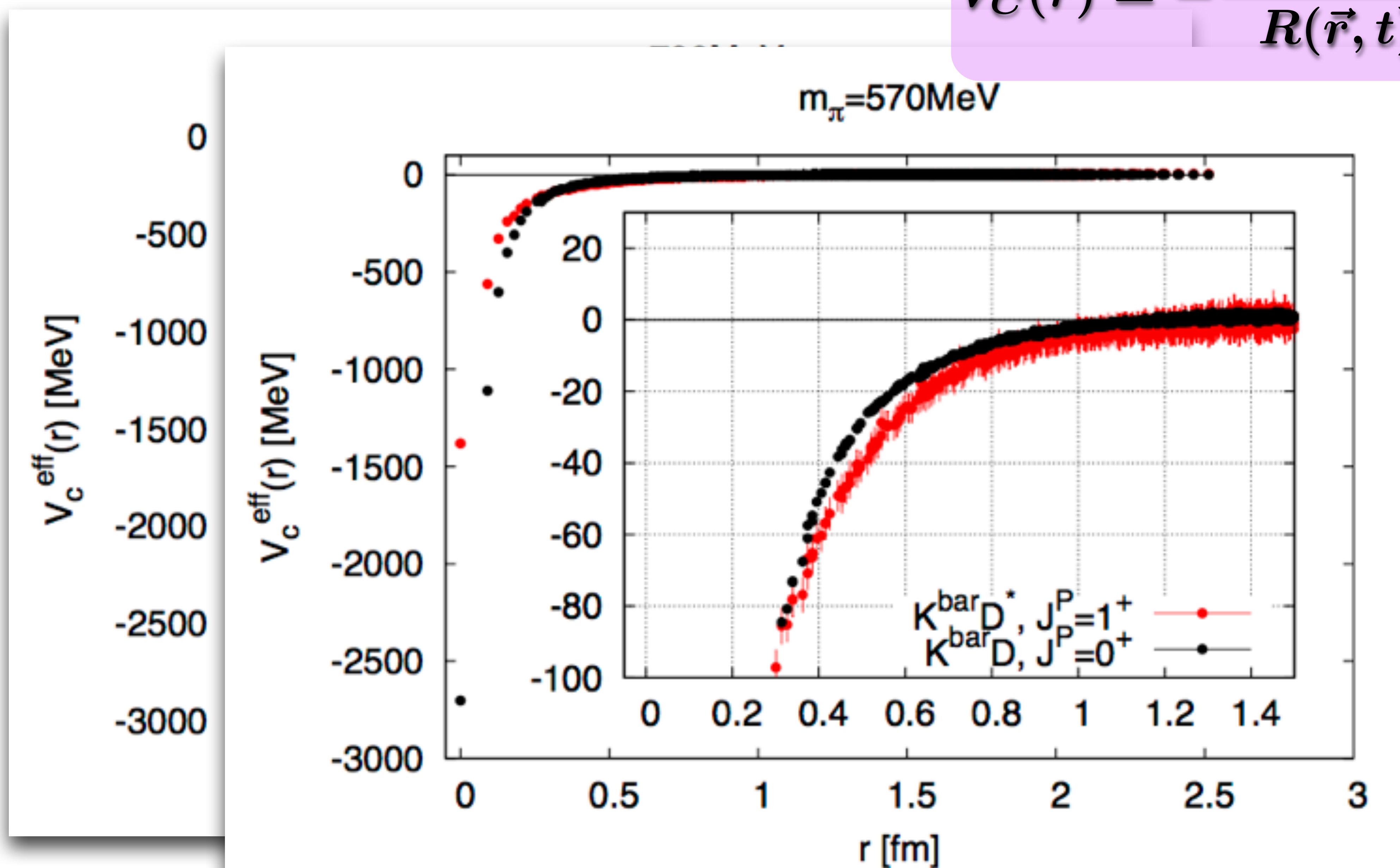

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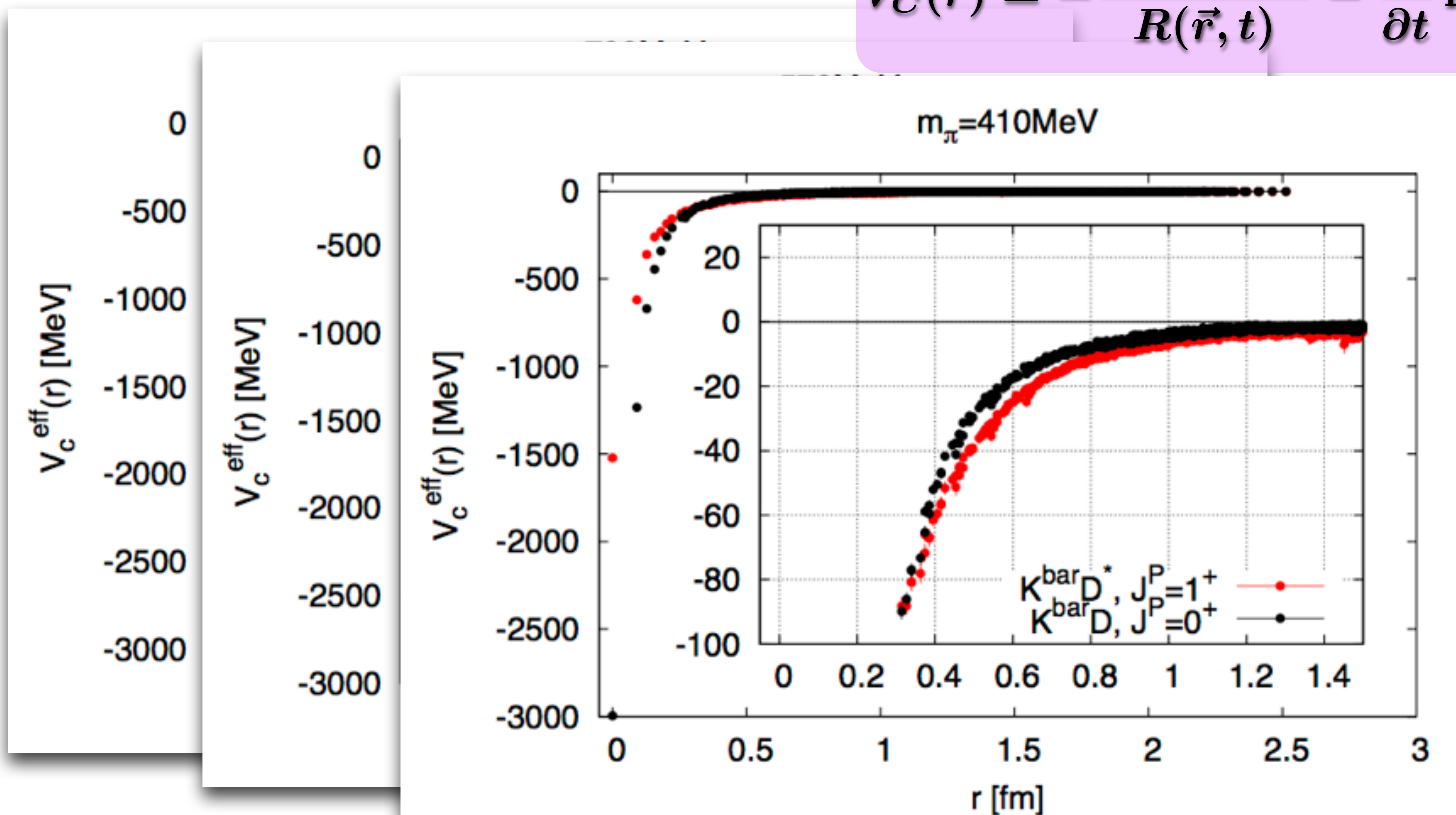
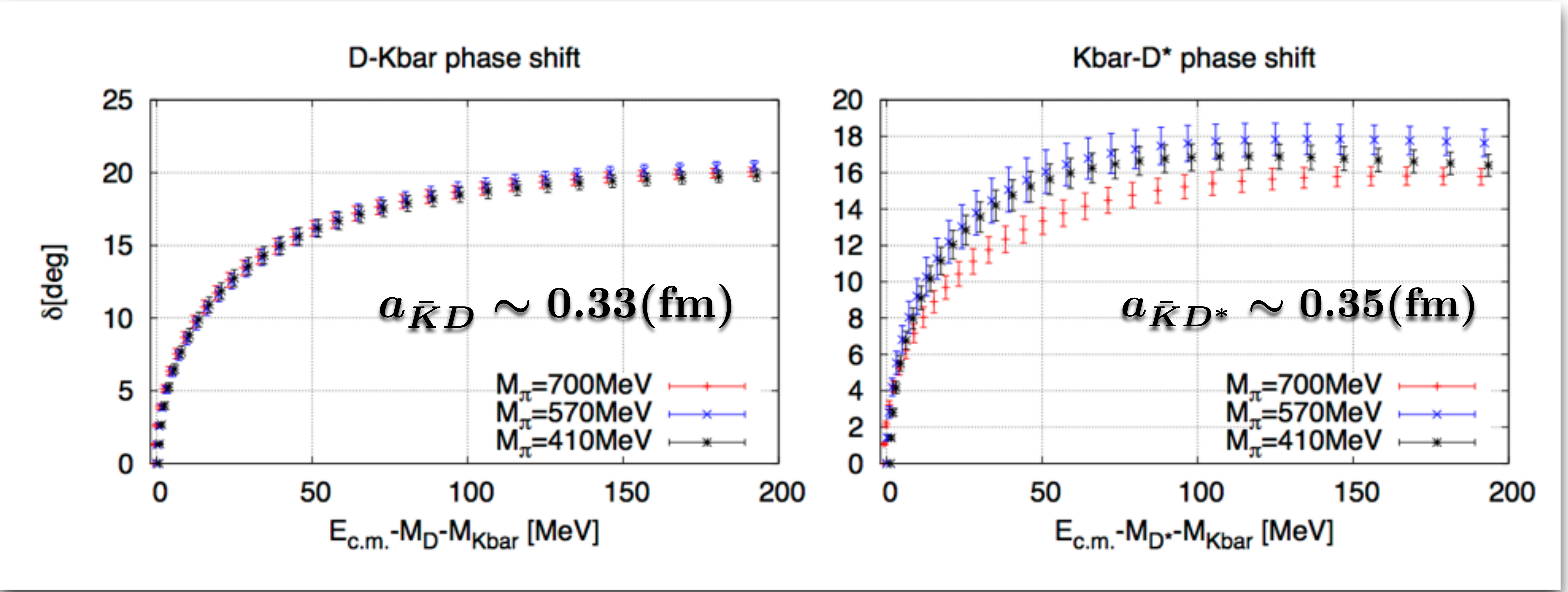


- Attractive $K^{bar}D$ and $K^{bar}D^*$ potentials
- Weak quark mass dependence
- Check whether bound Tcs exist or not --> phase shift analysis

S-wave phase shift : $T_{cs}(0^+, 1^+(0))$

- fit multi-range gaussian: $f(\mathrm{r}) = \sum_i a_i e^{-\nu_i r^2}$

- solve Schrodinger equation in an infinite volume



- **Attractions are not enough strong to generate bound states**
- **Weak quark mass dependence of phase shifts**

Summary

- Search for Tcc, Tcs on the lattice@m_π=410, 570, 700MeV

➡ N_f=2+1 full QCD simulation (PACS-CS configuration)
➡ Charm quarks: Relativistic Heavy Quark action

➡ Tcc, Tcs($J^P=0^+, 1^+$, I=1) : s-wave MM channels are repulsive
Bound states are unlikely...

➡ Tcc, Tcs($J^P=0^+, 1^+$, I=0) : s-wave MM channels are attractive,
but not enough strong to form bound states@m_π=410, 570, 700MeV
➡ $a_{DD^*} > a_{KbarD} \sim a_{KbarD^*}$ (attraction: Tcc(1^+) channel > Tcs($0^+, 1^+$) channel)
Large kinetic energy due to kaon in Tcs channels

- Future plan

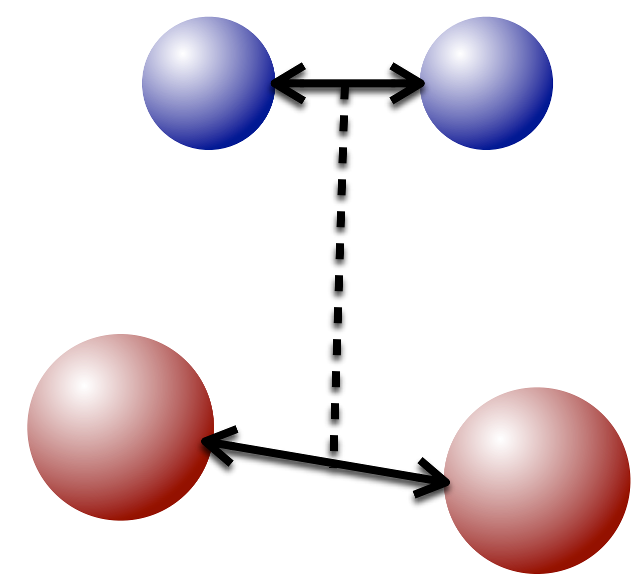
➡ Physical point simulation
➡ Coupled-channel analysis (DD*-D*D*,...)

Backup

$T_{QQ'}$ classification

Good di-quark : attractive $\bar{q}\bar{q}$ (C=3, S=0) pair

	$\bar{q}\bar{q}$ (light: u, d)		QQ' (heavy: s, c, b)	
	Anti-sym.	Sym.	Anti-sym.	Sym.
Color	3	$\bar{6}$	$\bar{3}$	6
Spin	S=0	S=1	S=0	S=1
Flavor	I=0	I=1	Anti-sym.	Sym.
Radial	L : odd	L : even	L : odd	L : even
Total	must be anti-symmetric			



<-- s-wave QQ pair

Possible QQ'-pair : C=$\bar{3}$

- C=$\bar{3}$, S=0, Anti-sym. : -8 (attractive)
- C=$\bar{3}$, S=1, Sym. : 8/3 (repulsive)

$\langle v_{ij} \rangle$	C=1	C=8	C=$\bar{3}$	C=6
S=0	-16	2	-8	4
S=1	16/3	-2/3	8/3	-4/3

Relativistic Heavy Quark Action

Aoki et al., PTP109, 383 (2003)

$$S^{\mathrm{RHQ}} = \sum_{x,y} \overline{q}(x) D_{x,y} q(y),$$

$$D_{x,y} = \delta_{xy} - \kappa \sum_{k=1,3} \left\{ (r_s - \nu\gamma_k) U_{x,k} \delta_{x+\hat{k},y} + (r_s + \nu\gamma_k) U^{\dagger}_{x,k} \delta_{x,y+\hat{k}} \right\}$$
$$- \kappa \left\{ (r_t - \gamma_4) U_{x,4} \delta_{x+\hat{4},y} + (r_t + \gamma_4) U^{\dagger}_{x,4} \delta_{x,y+\hat{4}} \right\}$$
$$- \delta_{xy} c_B \kappa \sum_{i<j} \sigma_{ij} F_{ij}(x) - \delta_{xy} c_E \kappa \sum_{i} \sigma_{4i} F_{4i}(x),$$

Namekawa et al., PRD84, 074505 (2011)

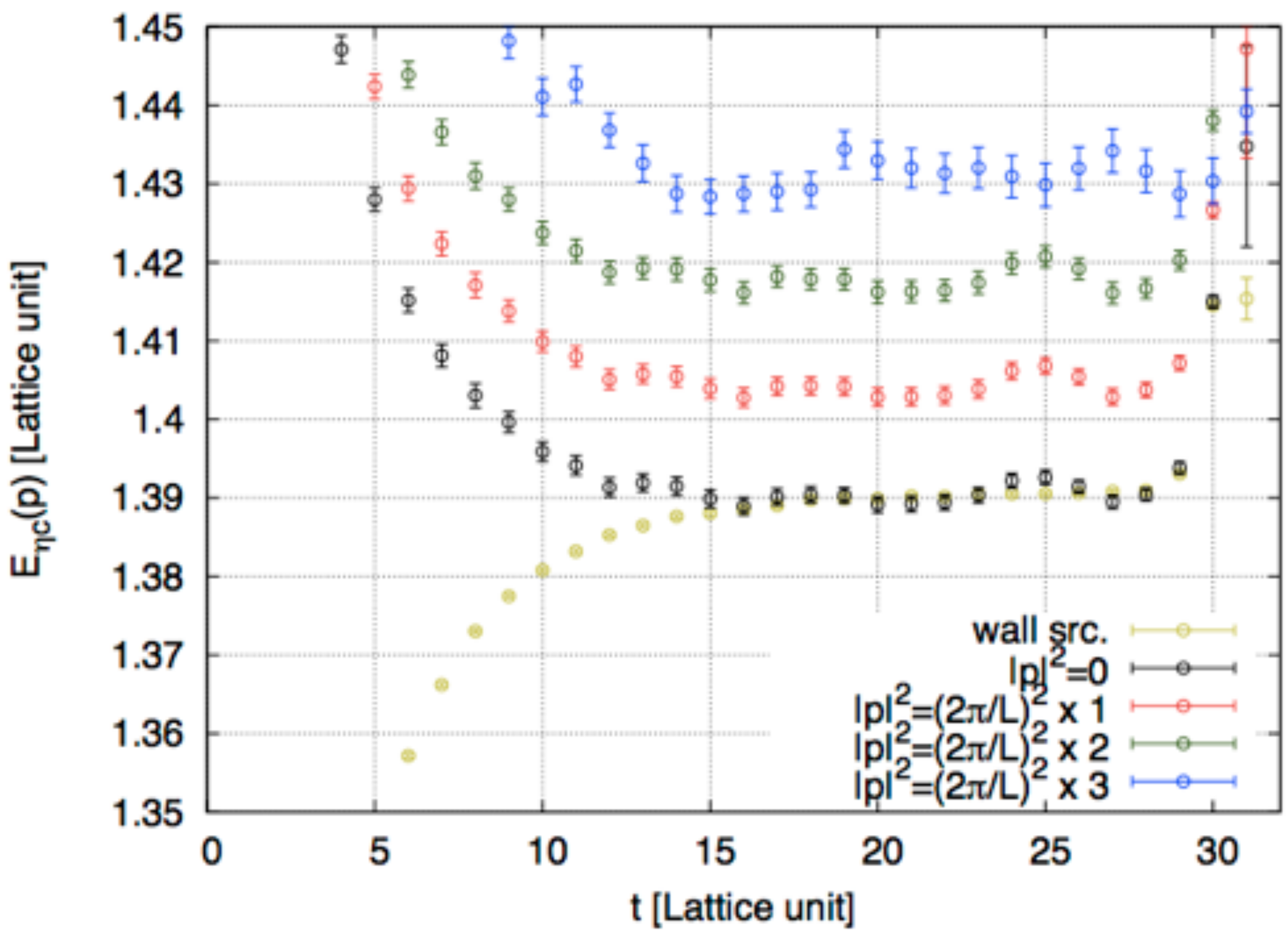
- **κ : hopping parameter for charm quark (1S averaged mass)**
- **r_t=1 choice (redundant parameter)**
- **r_s : one-loop perturbative value**
- **clover coefficients c_B, c_E :** $c_{B,E} = (c_{B,E}(m_Q a) - c_{B,E}(0))^{\mathrm{PT}} + c_{\mathrm{SW}}^{\mathrm{NP}}$.
- **ν : dispersion relations of 1S states**

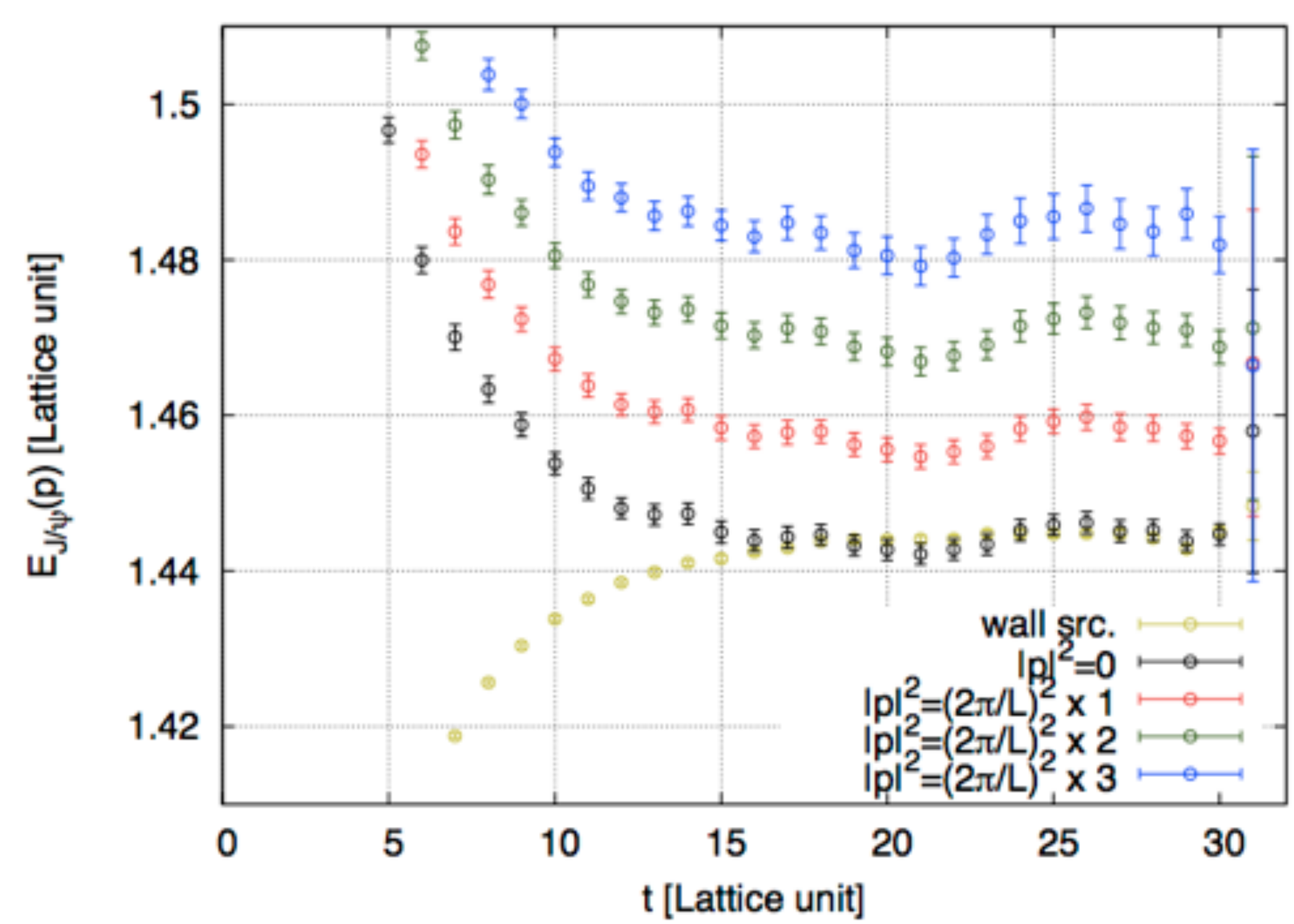
TABLE III. Parameters for the relativistic heavy quark action.

κ_{charm}	ν	r_s	c_B	c_E
0.10959947	1.145 051 1	1.188 160 7	1.984 913 9	1.781 951 2

1S charmonium energies@m_π=700MeV

$$Q(t_{\rm src}) = \sum_{\vec{X}} q(\vec{X}, t_{\rm src}) e^{-\alpha \vec{X}^2}, \quad \alpha = 1/5 \text{ [Lat. unit]}$$





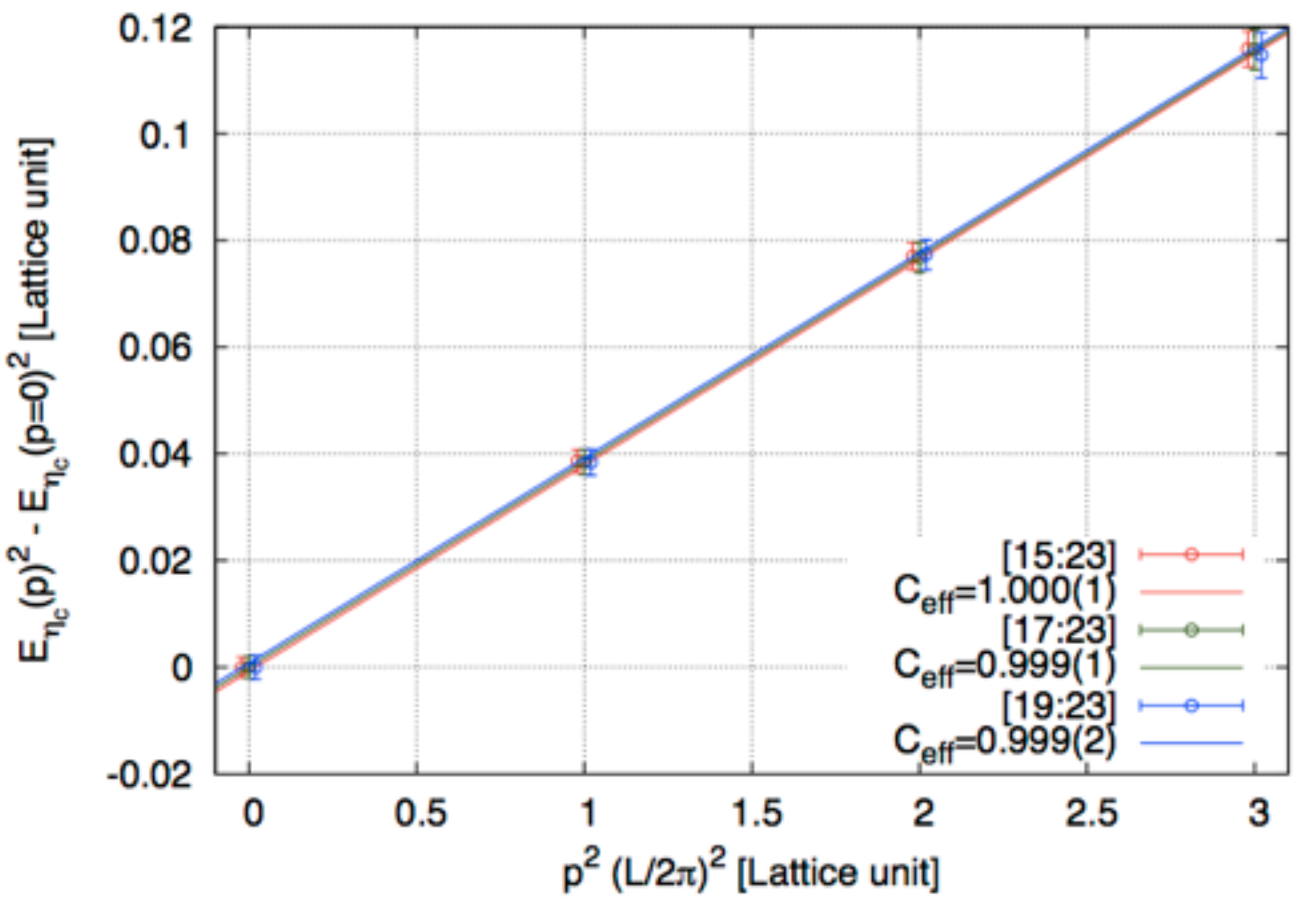
➡ **We choose three different fit ranges to calculate $E(p)^2$**

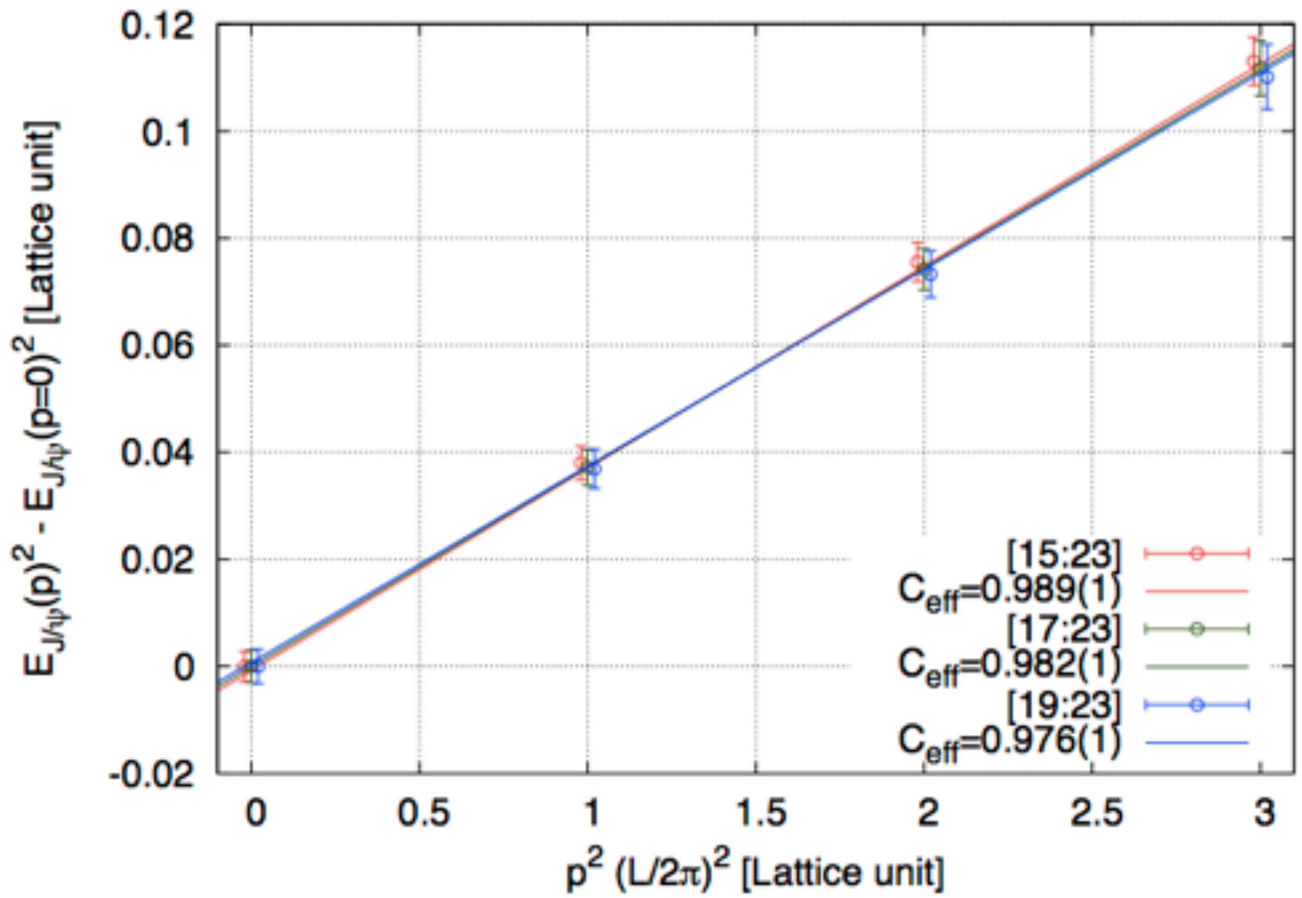
➡ **check dispersion relation with different fit ranges**

Dispersion relation@m_π=700MeV

- **Speed of light (c_{eff}) to be unity**

$$E(\vec{p}^2)^2 = E(0)^2 + c_{\text{eff}}^2 |\vec{p}|^2$$





➡ **c_{eff} only deviates maximally 2.5% from unity depending on fit range**