Kπ scattering in
Kπ scattering in moving frame

Kπ scattering in moving frames

C. B. Lang
Univ. Graz, Austria

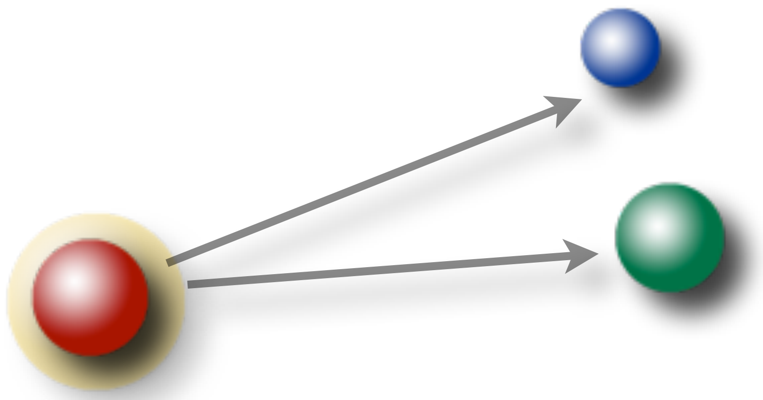
in collaboration with

Sasa Prelovsek, Luka Leskovec
Univ. Ljubljana, Slovenia

Daniel Mohler
Fermilab, USA

see also: arXiv: 1307.0736

Most hadrons are resonances



- The true ground state is a combination of (hadronically stable) hadrons like N, p, K, D..
- One needs to determine the energy spectrum in the quantum channel
- The observed energy levels are not located at the resonance "position"
- They depend on the volume and the coupled states
- In the limit V➛∞ and a➛0 the levels approach the spectral density of the continuum

Briefly: How to?

"Variational method"
Michael NPB259 (1985) 58
Lüscher/Wolff, NPB339 (1990) 222
Blossier et al., JHEP0904 (2009) 094

(Excited) energy levels:

- Determine correlation matrix for many interpolators (lattice operators coupling to the given quantum channel)

$$C_{ij}(t) = \langle \mathcal{O}_i(t) \mathcal{O}_j(0) \rangle$$

- Solve the generalized eigenvalue problem, then

$$\lambda^{(n)} \sim \exp(-E_n t)$$

- The eigenstates approach the physical eigenstates when the operator basis is sufficiently complete

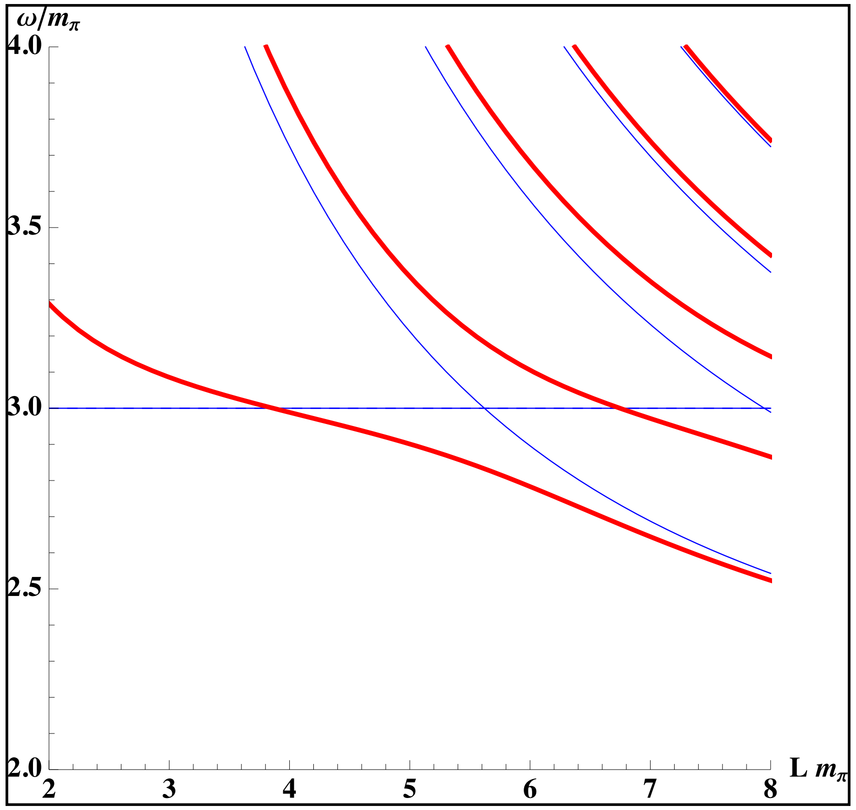
Briefly: How to?

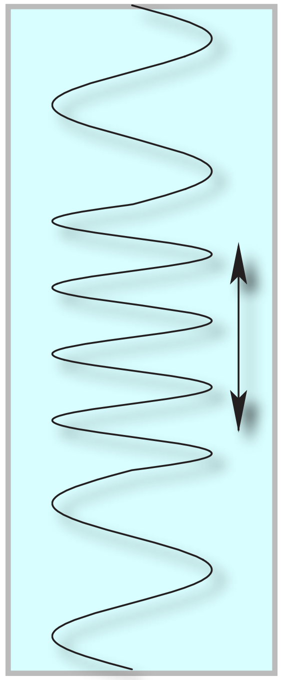
for Kπ plots see Döring et al., Eur.Phys.J. A48 (2012)114

(Excited) energy levels interpretation

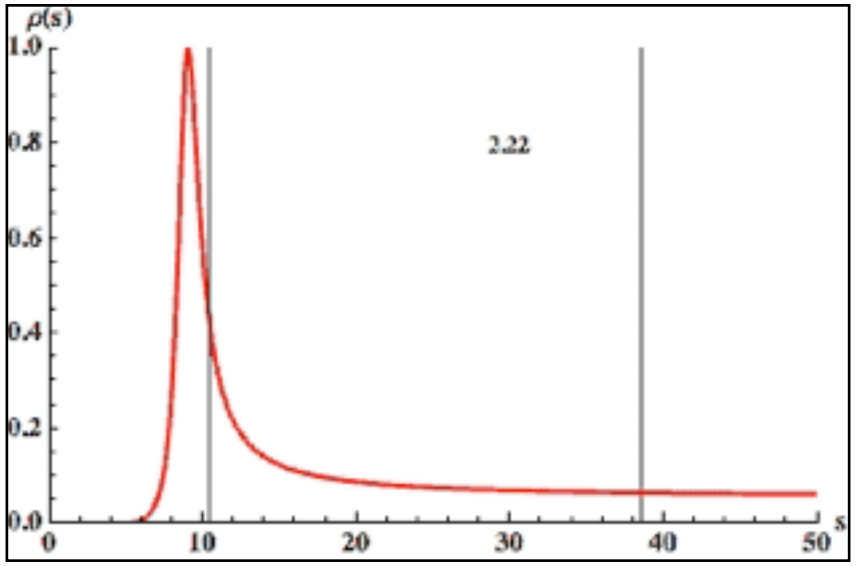
- Energy levels in finite volume ↔ phase shift in infinite volume (in the elastic region)

  Lüscher, CMP 105(86) 153,
  NP B354 (91) 531, NP B 364 (91) 237

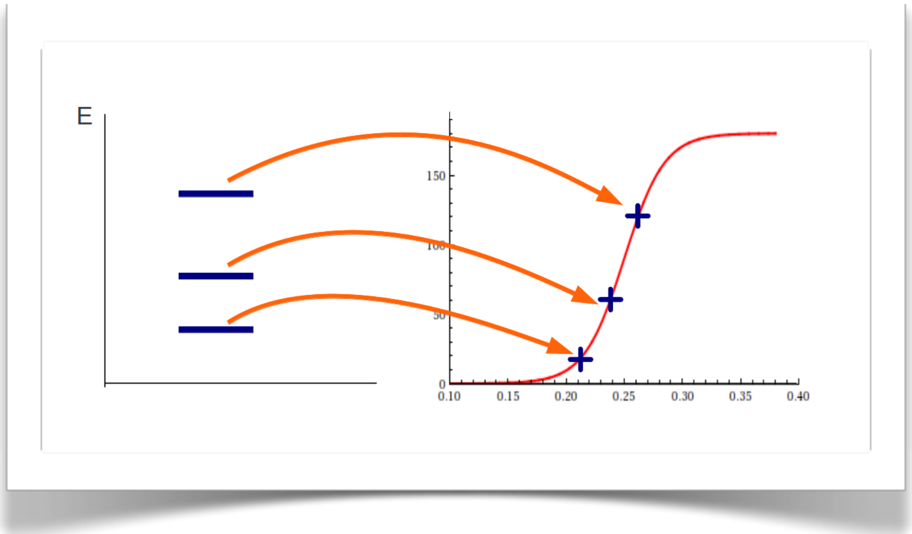




- m_1=m_2, P=0
- m_1=m_2, P≠0
- m_1≠m_2, P≠0



Briefly: How to?



(Excited) energy levels interpretation

- $m_1=m_2$, P=0

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237

$$\tan\delta(q) = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

- $m_1=m_2$, $P\neq 0$

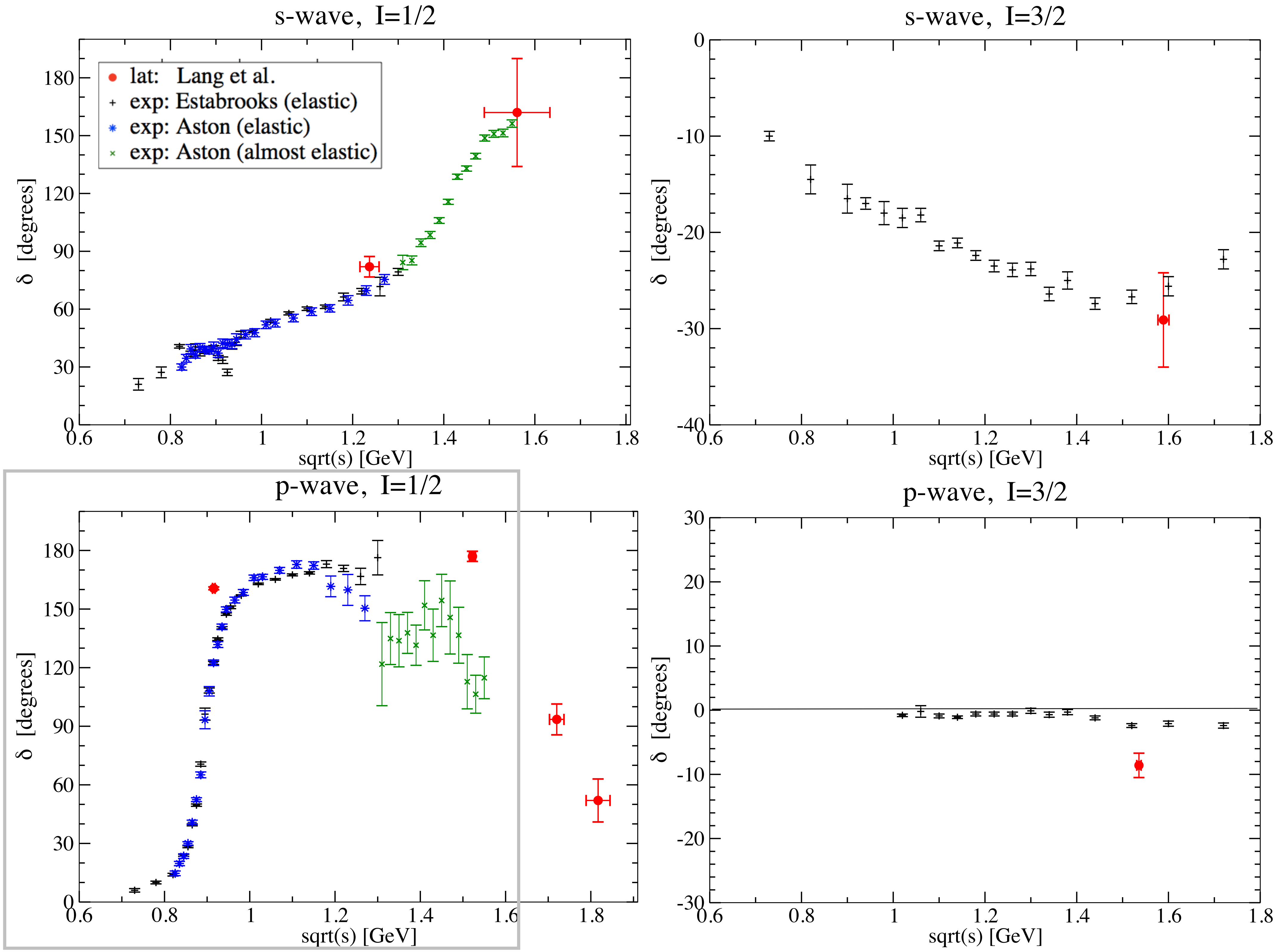
Rummukainen, Gottlieb: NP B 450(1995) 397
Kim, Sachrajda,Sharpe: NP B 727 (2005) 218
Feng, Jansen, Renner: PoS LAT10 (2010) 104
Fu, PR D85 (2012) 014506

- $m_1\neq m_2$, $P\neq 0$

Leskovec, Prelovsek, PRD85 (2012) 114507
Göckeler et a., PRD86 (2012) 094513
Döring et al., Eur.Phys.J. A48 (2012)114

Kπ for P=0

CBL, Leskovec, Mohler, Prelovsek; PRD86 (2012) 054508



Setup

Configurations

$N_L^3 \times N_T$	β	a[fm]	L[fm]	#cfgs	m_π[MeV]	m_K[MeV]
$16^3 \times 32$	7.1	0.1239(13)	1.98	276	266(4)	552(6)

(Thanks! See *Hasenfratz et al., PRD78(08)014515,054511*)

Operators

$$\mathcal{O}^{\bar{q}q}_{I=1/2} = \sum_x e^{iPx}\,\bar{s}(x)\hat{\Gamma}u(x)\ ,$$

5 Operators
(n_v=96)

$$\mathcal{O}^{K\pi}_{I=1/2} = \sum_j f_j\Big[\sqrt{\tfrac{1}{3}}K^+(p_{Kj})\pi^0(p_{\pi j})\ ,$$
$$+\sqrt{\tfrac{2}{3}}K^0(p_{Kj})\pi^+(p_{\pi j})\Big]\ ,$$

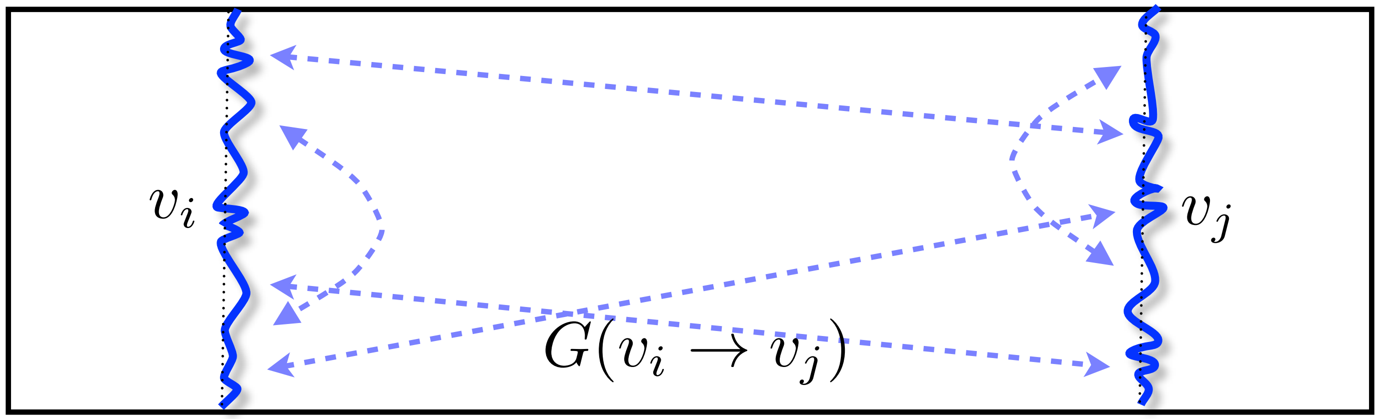
3 Operators,
various momenta ≤3
(units 2π/L), (n_v=64)

$$\mathcal{O}^{K\pi}_{I=3/2} = \sum_j f_j\ K^+(p_{Kj})\pi^+(p_{\pi j}),\quad p_{Kj}+p_{\pi j}=P$$

3 Operators,
various momenta ≤3
(units 2π/L), (n_v=64)

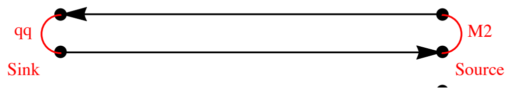
Operators

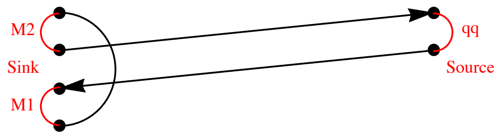
- Gauge links HYP smeared
- Quarks: 96 distillation (heaviside) sources
- all time slices to all time slices
- flexible source/sink operators, backtrackers

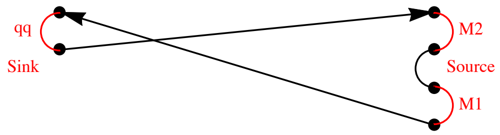


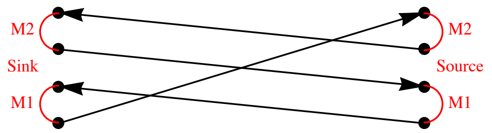
eigenmode ➛ eigenmode

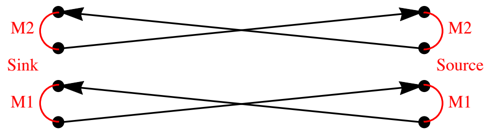
M. Peardon et al. (HSC), PRD 80, 054506 (2009).

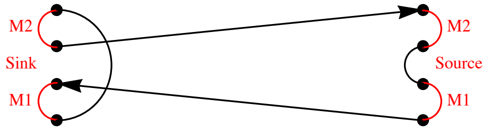












- P+A method (valence quarks): effective periodicity = 64

Moving frames

$P=0:$ $O_h\,,$ irrep $T_1^-\,,$ $l=1$

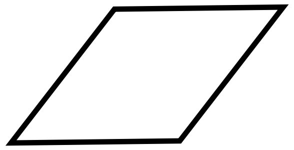
$$\tan\delta(q)=\frac{\pi^{3/2}q}{\mathcal{Z}_{00}(1;q^2)}$$

$P=\frac{2\pi}{L}e_z:$ $C_{4v}\,,$ irreps $E(e_{x,y}),\ E(e_x\pm e_y),$ $l=1,2$

(irrep A_1 mixes $l=0,1\ldots$)

Fu & Fu, PRD86 (2012) 094507

$P=\frac{2\pi}{L}(e_x+e_y):$ $C_{2v}\,,$ irreps $B_2,\ B_3,$ $l=1,2$



Relativistic distortion

$$\mathbf{d} = e_z, \quad C_{4v}: \quad E$$

Leskovec, Prelovsek, PRD85 (2012) 114507

$$\tan\delta_1(p^*) = \frac{\pi^{3/2}\gamma q}{Z_{00}^{\mathbf{d}}(1;q^2) - \frac{1}{\sqrt{5}} q^{-2} Z_{20}^{\mathbf{d}}(1;q^2)}$$

$$(\mathcal{O}_E^{\bar{q}q})_k = V_k(e_z), \qquad k = x, y$$

$$(\mathcal{O}_E^{P_1P_2})_k^I = P_1(e_z + e_k)P_2(-e_k) - P_1(e_z - e_k)P_2(e_k),$$

$$k = x, y$$

$$(\mathcal{O}_E^{P_1P_2})_k^{II} = P_1(e_z + u_k)P_2(-u_k) - P_1(e_z - u_k)P_2(u_k),$$

$$u_k = e_x + e_y, e_x - e_y$$

$\mathbf{d} = e_x + e_y, \quad C_{2v}:$

Leskovec, Prelovsek,
PRD85 (2012) 114507

B_2

$$\tan\delta_1(p^*) = \frac{\pi^{3/2}\gamma q}{Z^{\mathbf{d}}_{00}(1;q^2) - \frac{1}{\sqrt{5}} q^{-2} Z^{\mathbf{d}}_{20}(1;q^2) - \frac{\sqrt{6}}{\sqrt{5}} q^{-2} Im[Z^{\mathbf{d}}_{22}(1;q^2)]}.$$

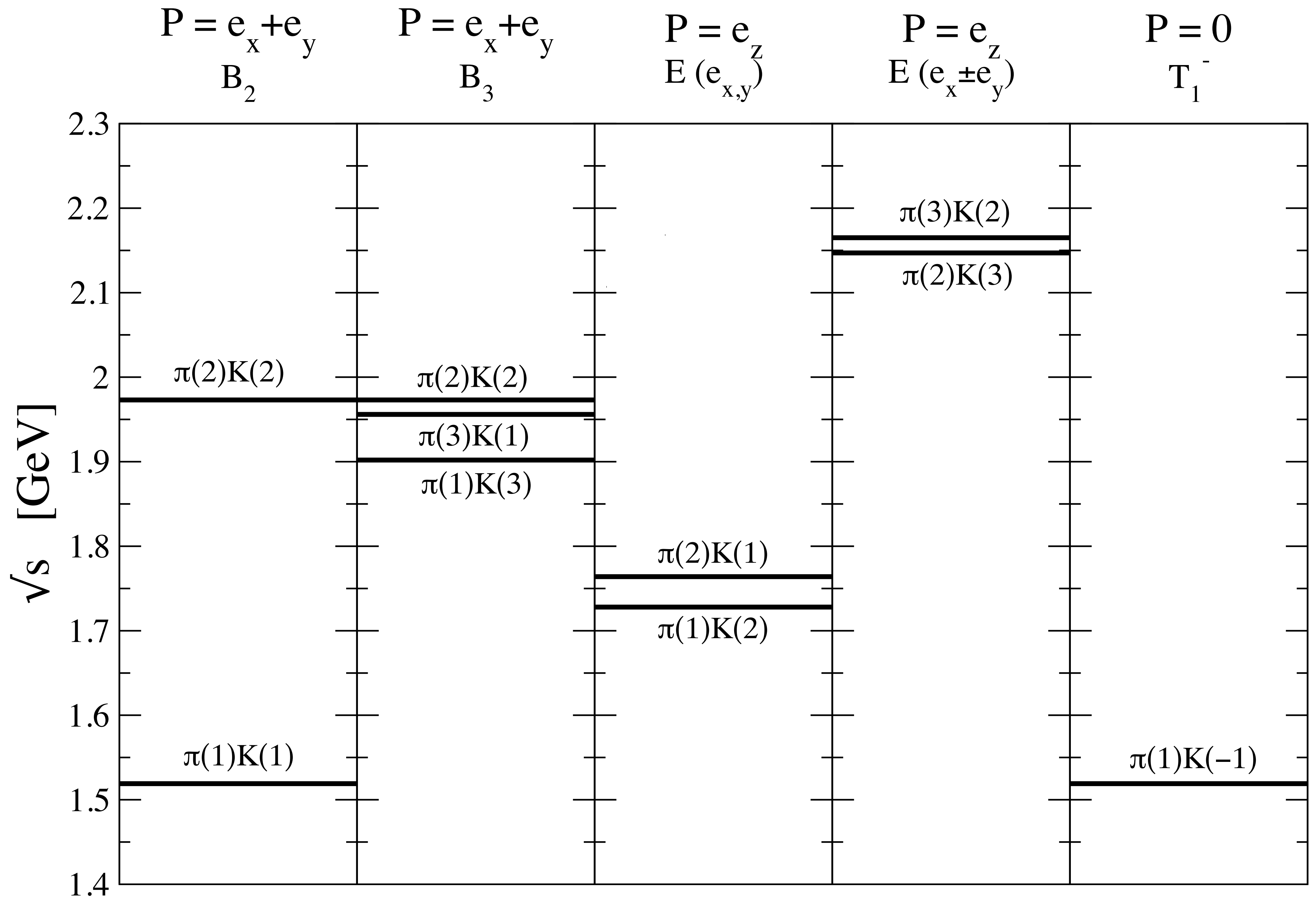
B_3

$$\tan\delta_1(p^*) = \frac{\pi^{3/2}\gamma q}{Z^{\mathbf{d}}_{00}(1;q^2) + \frac{2}{\sqrt{5}} q^{-2} Z^{\mathbf{d}}_{20}(1;q^2)}$$

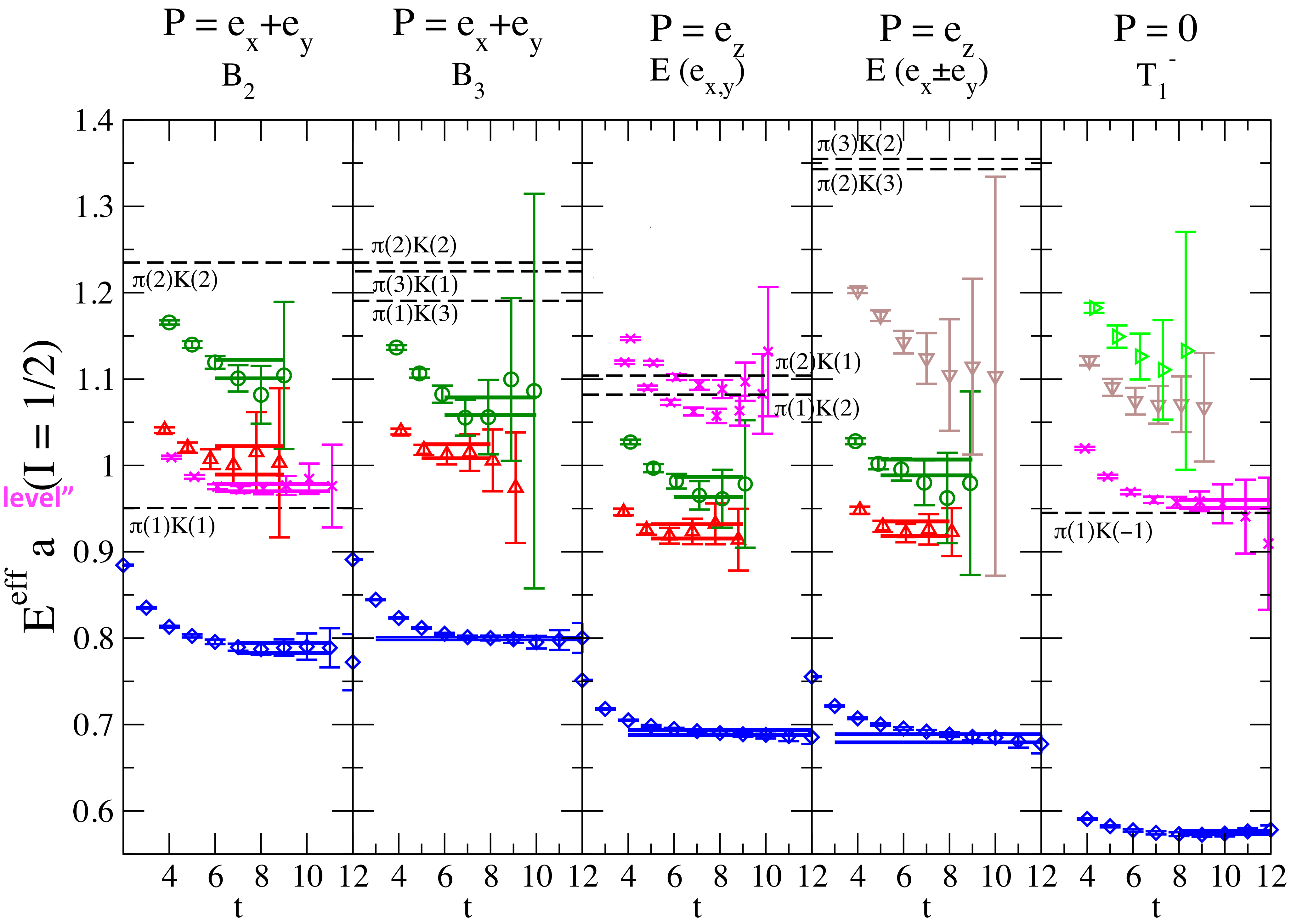
$$\mathcal{O}^{\bar{q}q}_{B_2} = V_x(e_x + e_y) - V_y(e_x + e_y),$$
$$\mathcal{O}^{\bar{q}q}_{B_3} = V_z(e_x + e_y),$$

$$\begin{aligned}
(\mathcal{O}^{P_1P_2}_{B_2})^{I} &= P_1(e_x)P_2(e_y) - P_1(e_y)P_2(e_x),\\
(\mathcal{O}^{P_1P_2}_{B_2})^{II} &= P_1(e_x + e_z)P_2(e_y - e_z)\\
&\quad - P_1(e_y + e_z)P_2(e_x - e_z) + \{e_z \leftrightarrow -e_z\},\\
(\mathcal{O}^{P_1P_2}_{B_3})^{I} &= P_1(e_x + e_y + e_z)P_2(-e_z)\\
&\quad - P_1(e_x + e_y - e_z)P_2(e_z),\\
(\mathcal{O}^{P_1P_2}_{B_3})^{II} &= P_1(e_x + e_z)P_2(e_y - e_z)\\
&\quad + P_1(e_y + e_z)P_2(e_x - e_z) - \{e_z \leftrightarrow -e_z\},
\end{aligned}$$

Effective energies: non-interacting



Effective energies (I=1/2)



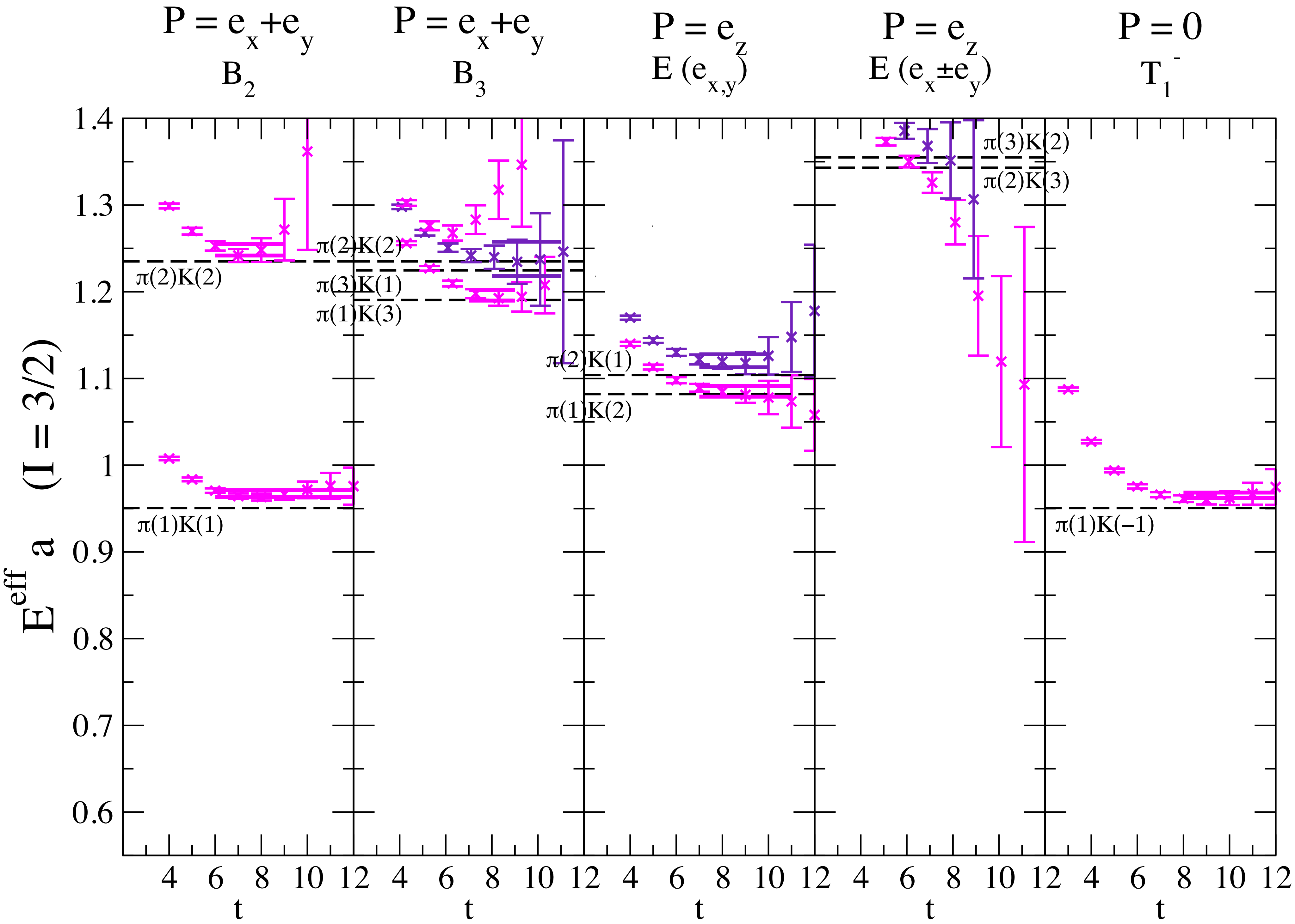
K_2*(1430)

K*(1410)

"scattering level"

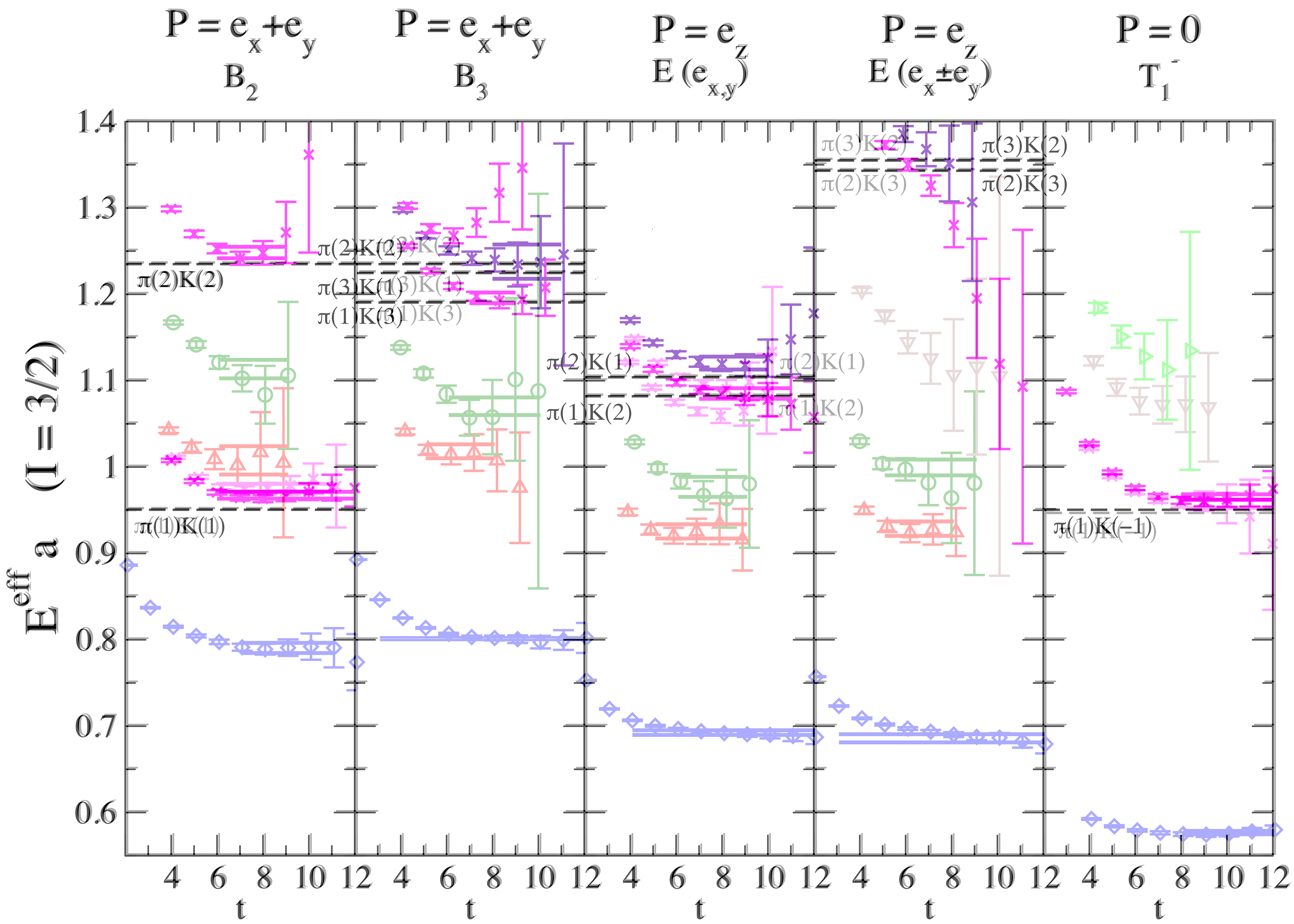
K*(892)

Effective energies (I=3/2)



all levels are close to non-interacting values;
repulsive interaction: upwards shift

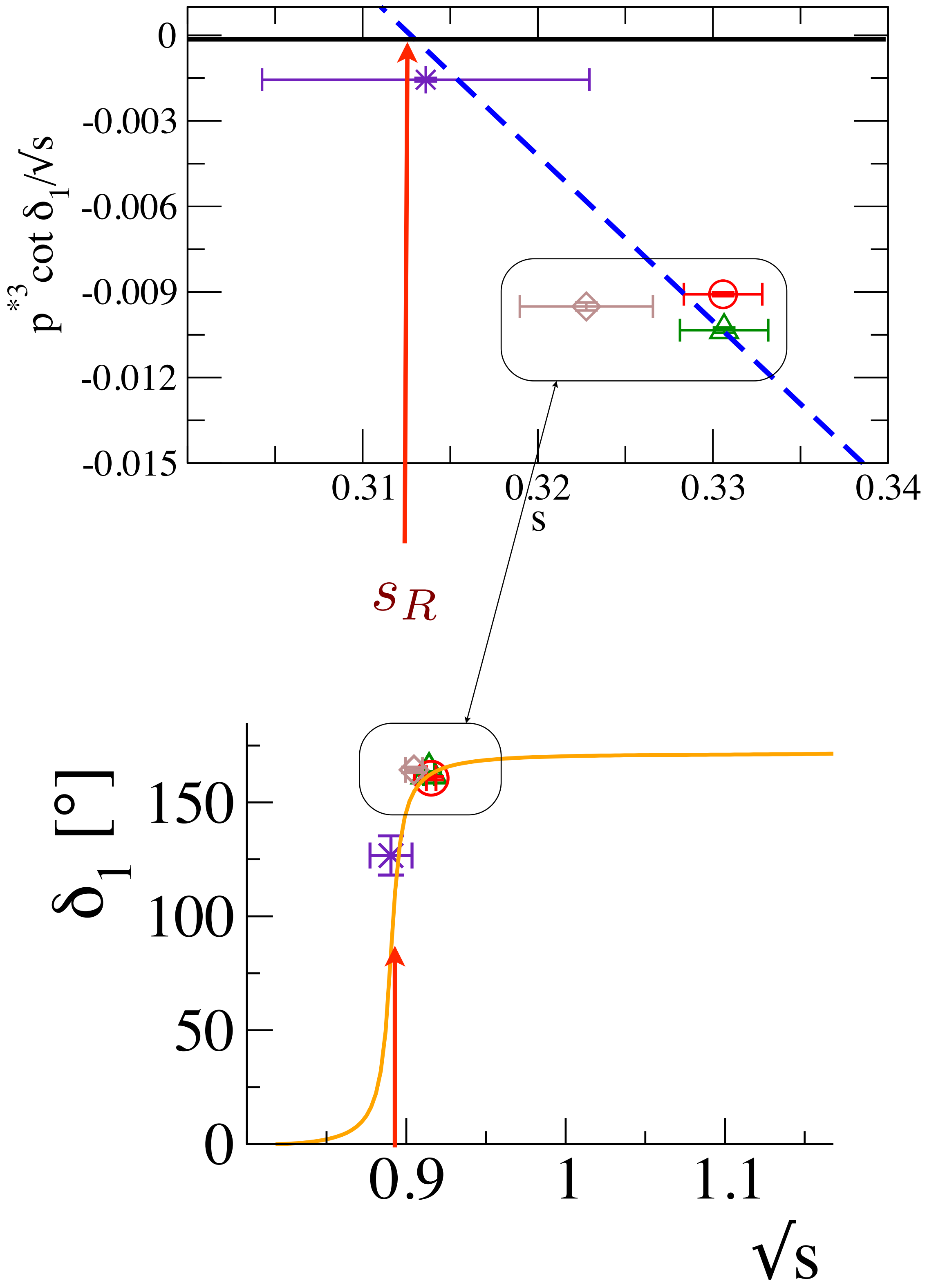
Effective energies



Elastic region (<1300 MeV): K*(892)

Neglect d-wave

$$\frac{p^{*3}}{\sqrt{s}}\cot\delta_1(s) = \frac{6\pi}{g^2}(m_{K^*}^2 - s)$$



	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6
exp	891.66 ± 0.26	5.72 ± 0.06

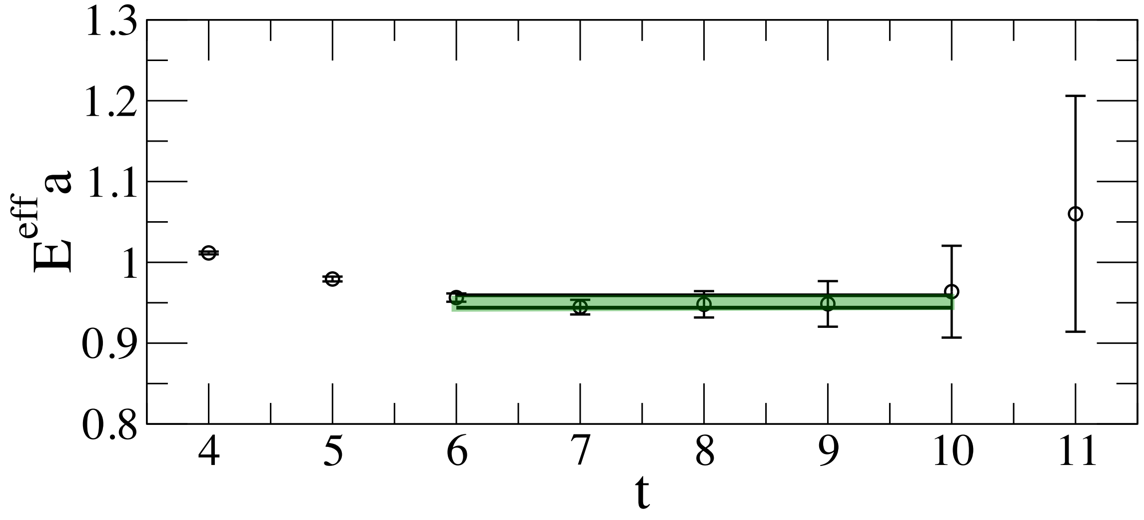
cf. 6.25, from UKQCD,
McNeile et al,PLB556 (2003) 177

Above the elastic region (>1300 MeV): K*(1410)

l=1, 2 mixing: $K_2^*(1430)$ couples as additional level

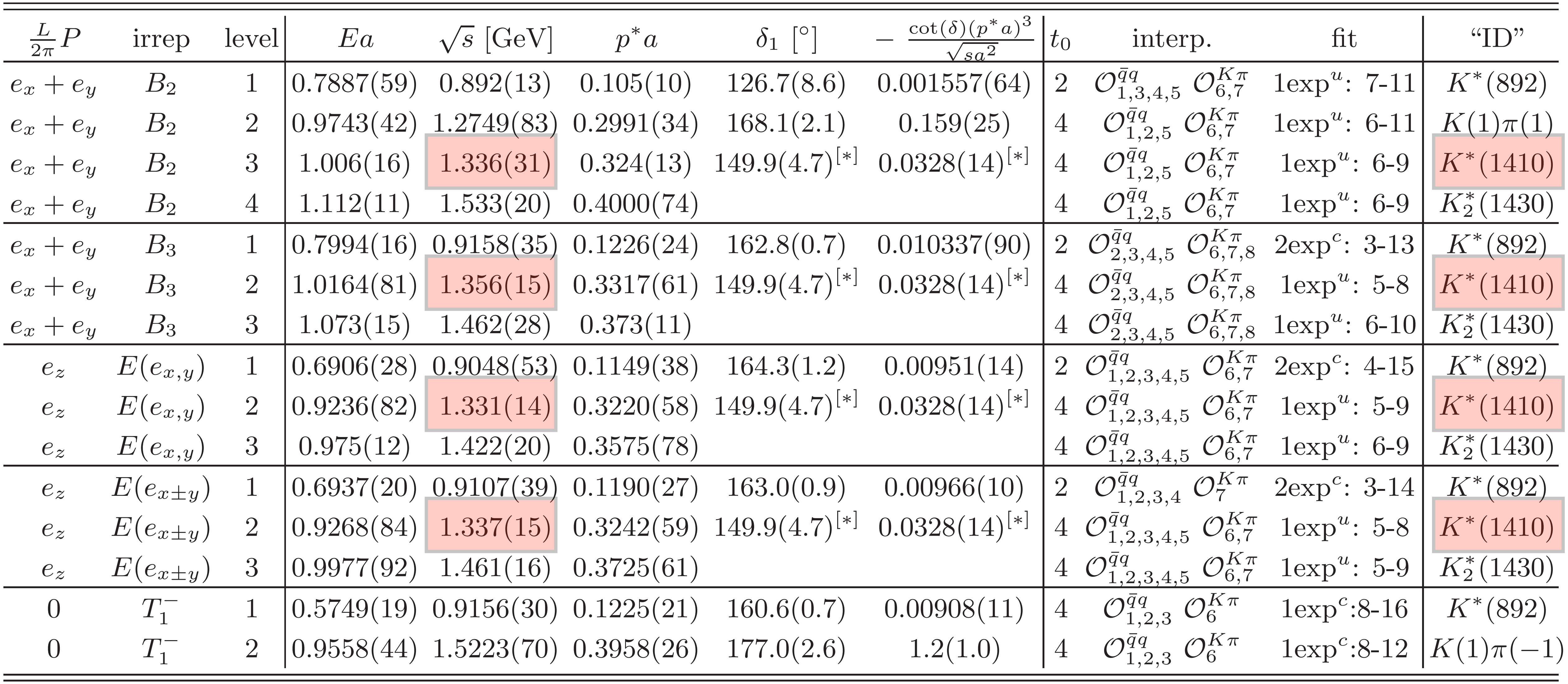
$\frac{L}{2\pi}P$	irrep	level	Ea	$\sqrt{s}$ [GeV]	p^*a	δ_1 [°]	$-\frac{\cot(\delta)(p^*a)^3}{\sqrt{s}a^2}$	t_0	interp.	fit	"ID"
e_x+e_y	B_2	1	0.7887(59)	0.892(13)	0.105(10)	126.7(8.6)	0.001557(64)	2	$\mathcal{O}^{\bar{q}q}_{1,3,4,5}$ $\mathcal{O}^{K\pi}_{6,7}$	1exp^u: 7-11	$K^*(892)$
e_x+e_y	B_2	2	0.9743(42)	1.2749(83)	0.2991(34)	168.1(2.1)	0.159(25)	4	$\mathcal{O}^{\bar{q}q}_{1,2,5}$ $\mathcal{O}^{K\pi}_{6,7}$	1exp^u: 6-11	$K(1)\pi(1)$
e_x+e_y	B_2	3	1.006(16)	1.336(31)	0.324(13)	149.9(4.7)$^{[*]}$	0.0328(14)$^{[*]}$	4	$\mathcal{O}^{\bar{q}q}_{1,2,5}$ $\mathcal{O}^{K\pi}_{6,7}$	1exp^u: 6-9	$K^*(1410)$
e_x+e_y	B_2	4	1.112(11)	1.533(20)	0.4000(74)			4	$\mathcal{O}^{\bar{q}q}_{1,2,5}$ $\mathcal{O}^{K\pi}_{6,7}$	1exp^u: 6-9	$K_2^*(1430)$
e_x+e_y	B_3	1	0.7994(16)	0.9158(35)	0.1226(24)	162.8(0.7)	0.010337(90)	2	$\mathcal{O}^{\bar{q}q}_{2,3,4,5}$ $\mathcal{O}^{K\pi}_{6,7,8}$	2exp^c: 3-13	$K^*(892)$
e_x+e_y	B_3	2	1.0164(81)	1.356(15)	0.3317(61)	149.9(4.7)$^{[*]}$	0.0328(14)$^{[*]}$	4	$\mathcal{O}^{\bar{q}q}_{2,3,4,5}$ $\mathcal{O}^{K\pi}_{6,7,8}$	1exp^u: 5-8	$K^*(1410)$
e_x+e_y	B_3	3	1.073(15)	1.462(28)	0.373(11)			4	$\mathcal{O}^{\bar{q}q}_{2,3,4,5}$ $\mathcal{O}^{K\pi}_{6,7,8}$	1exp^u: 6-10	$K_2^*(1430)$
e_z	$E(e_{x,y})$	1	0.6906(28)	0.9048(53)	0.1149(38)	164.3(1.2)	0.00951(14)	2	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}$ $\mathcal{O}^{K\pi}_{6,7}$	2exp^c: 4-15	$K^*(892)$
e_z	$E(e_{x,y})$	2	0.9236(82)	1.331(14)	0.3220(58)	149.9(4.7)$^{[*]}$	0.0328(14)$^{[*]}$	4	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}$ $\mathcal{O}^{K\pi}_{6,7}$	1exp^u: 5-9	$K^*(1410)$
e_z	$E(e_{x,y})$	3	0.975(12)	1.422(20)	0.3575(78)			4	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}$ $\mathcal{O}^{K\pi}_{6,7}$	1exp^u: 6-9	$K_2^*(1430)$
e_z	$E(e_{x\pm y})$	1	0.6937(20)	0.9107(39)	0.1190(27)	163.0(0.9)	0.00966(10)	2	$\mathcal{O}^{\bar{q}q}_{1,2,3,4}$ $\mathcal{O}^{K\pi}_{7}$	2exp^c: 3-14	$K^*(892)$
e_z	$E(e_{x\pm y})$	2	0.9268(84)	1.337(15)	0.3242(59)	149.9(4.7)$^{[*]}$	0.0328(14)$^{[*]}$	4	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}$ $\mathcal{O}^{K\pi}_{6,7}$	1exp^u: 5-8	$K^*(1410)$
e_z	$E(e_{x\pm y})$	3	0.9977(92)	1.461(16)	0.3725(61)			4	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}$ $\mathcal{O}^{K\pi}_{6,7}$	1exp^u: 5-9	$K_2^*(1430)$
0	T_1^-	1	0.5749(19)	0.9156(30)	0.1225(21)	160.6(0.7)	0.00908(11)	4	$\mathcal{O}^{\bar{q}q}_{1,2,3}$ $\mathcal{O}^{K\pi}_{6}$	1exp^c:8-16	$K^*(892)$
0	T_1^-	2	0.9558(44)	1.5223(70)	0.3958(26)	177.0(2.6)	1.2(1.0)	4	$\mathcal{O}^{\bar{q}q}_{1,2,3}$ $\mathcal{O}^{K\pi}_{6}$	1exp^c:8-12	$K(1)\pi(-1)$

$\mathbf{P} = 0, \;\; O_h : T_2^+, \; l = 2$ $\qquad K_2^*(1430)$



Coupled channel methods:
Bernard et al., JHEP 01(2011) 019;
Hansen & Sharpe, PRD86 (2012) 016007;
Liu et al., I.J.Mod.Phys. A21(2006) 847;
Döring et al., EJPA47 (2011) 139;
Briceno & Davoudi, arXiv 1204.110.

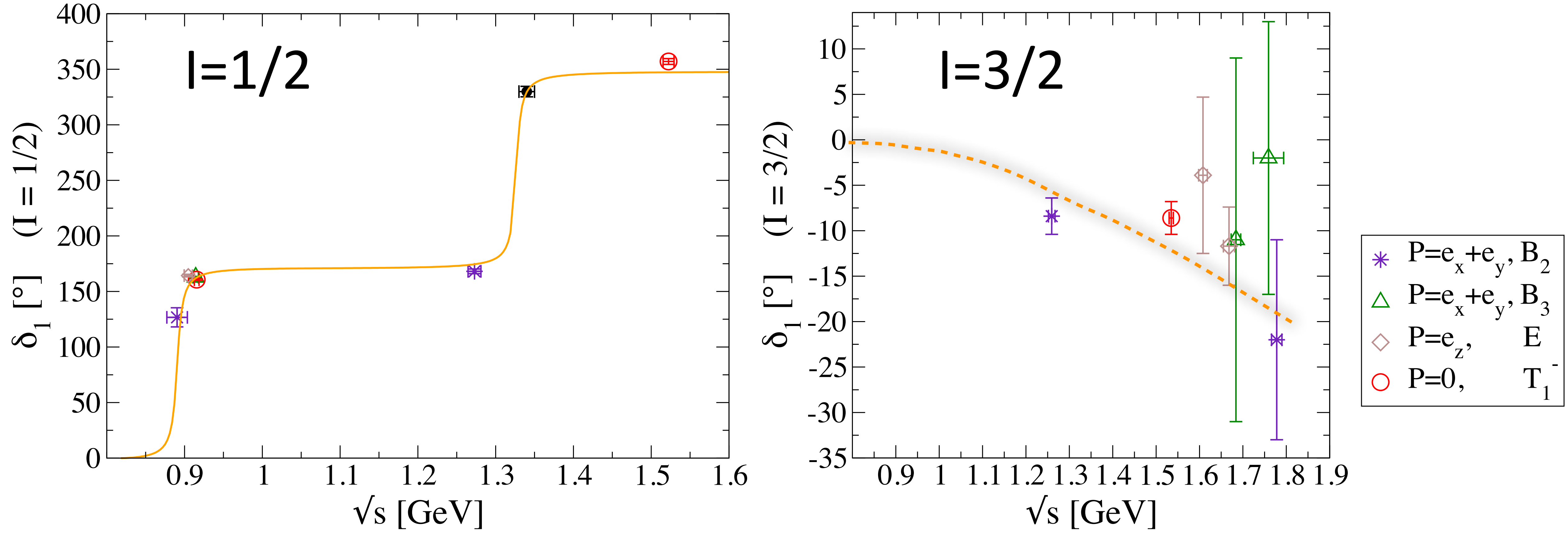
Lüscher relations couple δ_1 and δ_2 -
would need two measurements at the same energy -
let us assume that the red marked energies agree (Table)
and that the inelastic channels (K*π ,Kρ)can be neglected



$\frac{L}{2\pi}P$	irrep	level	Ea	$\sqrt{s}$ [GeV]	p^*a	δ_1 [°]	$-\frac{\cot(\delta)(p^*a)^3}{\sqrt{s}a^2}$	t_0	interp.	fit	"ID"
e_x+e_y	B_2	1	0.7887(59)	0.892(13)	0.105(10)	126.7(8.6)	0.001557(64)	2	$\mathcal{O}^{\bar{q}q}_{1,3,4,5}\ \mathcal{O}^{K\pi}_{6,7}$	1exp^u: 7-11	$K^*(892)$
e_x+e_y	B_2	2	0.9743(42)	1.2749(83)	0.2991(34)	168.1(2.1)	0.159(25)	4	$\mathcal{O}^{\bar{q}q}_{1,2,5}\ \mathcal{O}^{K\pi}_{6,7}$	1exp^u: 6-11	$K(1)\pi(1)$
e_x+e_y	B_2	3	1.006(16)	1.336(31)	0.324(13)	149.9(4.7)$^{[*]}$	0.0328(14)$^{[*]}$	4	$\mathcal{O}^{\bar{q}q}_{1,2,5}\ \mathcal{O}^{K\pi}_{6,7}$	1exp^u: 6-9	$K^*(1410)$
e_x+e_y	B_2	4	1.112(11)	1.533(20)	0.4000(74)			4	$\mathcal{O}^{\bar{q}q}_{1,2,5}\ \mathcal{O}^{K\pi}_{6,7}$	1exp^u: 6-9	$K_2^*(1430)$
e_x+e_y	B_3	1	0.7994(16)	0.9158(35)	0.1226(24)	162.8(0.7)	0.010337(90)	2	$\mathcal{O}^{\bar{q}q}_{2,3,4,5}\ \mathcal{O}^{K\pi}_{6,7,8}$	2exp^c: 3-13	$K^*(892)$
e_x+e_y	B_3	2	1.0164(81)	1.356(15)	0.3317(61)	149.9(4.7)$^{[*]}$	0.0328(14)$^{[*]}$	4	$\mathcal{O}^{\bar{q}q}_{2,3,4,5}\ \mathcal{O}^{K\pi}_{6,7,8}$	1exp^u: 5-8	$K^*(1410)$
e_x+e_y	B_3	3	1.073(15)	1.462(28)	0.373(11)			4	$\mathcal{O}^{\bar{q}q}_{2,3,4,5}\ \mathcal{O}^{K\pi}_{6,7,8}$	1exp^u: 6-10	$K_2^*(1430)$
e_z	$E(e_{x,y})$	1	0.6906(28)	0.9048(53)	0.1149(38)	164.3(1.2)	0.00951(14)	2	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}\ \mathcal{O}^{K\pi}_{6,7}$	2exp^c: 4-15	$K^*(892)$
e_z	$E(e_{x,y})$	2	0.9236(82)	1.331(14)	0.3220(58)	149.9(4.7)$^{[*]}$	0.0328(14)$^{[*]}$	4	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}\ \mathcal{O}^{K\pi}_{6,7}$	1exp^u: 5-9	$K^*(1410)$
e_z	$E(e_{x,y})$	3	0.975(12)	1.422(20)	0.3575(78)			4	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}\ \mathcal{O}^{K\pi}_{6,7}$	1exp^u: 6-9	$K_2^*(1430)$
e_z	$E(e_{x\pm y})$	1	0.6937(20)	0.9107(39)	0.1190(27)	163.0(0.9)	0.00966(10)	2	$\mathcal{O}^{\bar{q}q}_{1,2,3,4}\ \mathcal{O}^{K\pi}_{7}$	2exp^c: 3-14	$K^*(892)$
e_z	$E(e_{x\pm y})$	2	0.9268(84)	1.337(15)	0.3242(59)	149.9(4.7)$^{[*]}$	0.0328(14)$^{[*]}$	4	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}\ \mathcal{O}^{K\pi}_{6,7}$	1exp^u: 5-8	$K^*(1410)$
e_z	$E(e_{x\pm y})$	3	0.9977(92)	1.461(16)	0.3725(61)			4	$\mathcal{O}^{\bar{q}q}_{1,2,3,4,5}\ \mathcal{O}^{K\pi}_{6,7}$	1exp^u: 5-9	$K_2^*(1430)$
0	T_1^-	1	0.5749(19)	0.9156(30)	0.1225(21)	160.6(0.7)	0.00908(11)	4	$\mathcal{O}^{\bar{q}q}_{1,2,3}\ \mathcal{O}^{K\pi}_{6}$	1exp^c:8-16	$K^*(892)$
0	T_1^-	2	0.9558(44)	1.5223(70)	0.3958(26)	177.0(2.6)	1.2(1.0)	4	$\mathcal{O}^{\bar{q}q}_{1,2,3}\ \mathcal{O}^{K\pi}_{6}$	1exp^c:8-12	$K(1)\pi(-1)$

Lüscher-type equation for B_2 & E, coupling δ_1 and δ_2 assuming same values of E (see above)

Leskovec, Prelovsek,
PRD85 (2012) 114507



$$\frac{p^{*3}}{\sqrt{s}} \cot \delta_1(s) = \left[\sum_{K_i^*} \frac{g^2_{K_i^*}}{6\pi} \frac{1}{m^2_{K_i^*} - s} \right]^{-1}$$

$$K_i^* = K^*(892),\ K^*(1410)\ . \qquad \Gamma[K^* \to K\pi] = \frac{g^2}{6\pi} \frac{p^{*3}}{s}$$

	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]	$m_{K^*(1410)}$ [GeV]	$g_{K^*(1410)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6	1.33 ± 0.02	input
exp	891.66 ± 0.26	5.72 ± 0.06	1.414 ± 0.0015	1.59 ± 0.03

Batavia-Graz-Ljubljana-Vancouver collaboration:

Kπ ⟷ K*	*this talk* and arXiv:1307.0736
DK ⟷ D_s	*D. Mohler, par. session 5G*
DD* ⟷ X(3872)	*S. Prelovsek, par. session 8G* and arXiv:1307.5172
Nπ ⟷ N*	*V. Verduci, par. session 10 C* and PRD87(2013) 054502